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Research Article

A Variational Approach to the Modeling of MIMO Systems

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Motivated by the study of the optimization of the quality of service for multiple input multiple output (MIMO) systems in 3G (third generation), we develop a method for modeling MIMO channel $\mathcal{H}$. This method, which uses a statistical approach, is based on a variational form of the usual channel equation. The proposed equation is given by $\delta^2 = \langle \delta\mathbf{R} | \mathcal{H} | \delta\mathbf{E}\rangle + \langle \delta\mathbf{R} | (\delta\mathcal{H}) | \mathbf{E}\rangle$ with scalar variable $\delta = \|\delta\mathbf{R}\|$. Minimum distance $\delta_{\min}$ of received vectors $|\mathbf{R}\rangle$ is used as the random variable to model MIMO channel. This variable is of crucial importance for the performance of the transmission system as it captures the degree of interference between neighbors vectors. Then, we use this approach to compute numerically the total probability of errors with respect to signal-to-noise ratio (SNR) and then predict the numbers of antennas. By fixing SNR variable to a specific value, we extract informations on the optimal numbers of MIMO antennas.

1. INTRODUCTION

Digital communication of the third generation (3G) using multi-input multi-output (MIMO) is one of the important techniques used to exploit the spatial diversity in a rich scattering environment [1]. This revival interest in MIMO is primarily dictated by the objective of improving the network's quality of service and the operator's revenues significantly [2]. Due to the great spectral efficiency gain, MIMO systems have known a great interest nowadays and have been defined by IEEE 802.16 [3], for fixed broadband wireless access and 3G partnership project (3GPP) for mobile applications. Using MIMO, it has been shown in [4] that spectral efficiency can be improved significantly in wireless communications in fading environment.

Recall that the main objective of the optimization process of the MIMO network is to improve the quality of services for network and to be sure of optimal exploitation of the resources of network efficiency. For instance, an essential shutter in MIMO and which will be discussed in this work is the theoretical determination of the optimal numbers N_T and N_R of transmitter and receiver antennas respectively. To our knowledge, few studies in literature have been devoted to theoretical approach of MIMO systems. It would be then interesting to deeper this issue.

To study MIMO system, we will use Rayleigh model as it is the most widely used method for indoor and urban channels [5]. When the bandwidth is narrow (flat fading) [6], our system can be modeled by $N_R * N_T$ random matrix $\mathcal{H}$. The received N_R-vector $|\mathbf{R}\rangle$, describing received signals at reception, is related to the transmitted one $|\mathbf{E}\rangle$ as $|\mathbf{R}\rangle = \mathcal{H}|\mathbf{E}\rangle + |\mathbf{N}\rangle$, where $|\mathbf{N}\rangle$ is the noise vector with covariance matrix $\sigma^2 I_{N_R}$ with I_{N_R} being the $N_R \times N_R$ unit matrix. When the bandwidth is large as in WCDMA, OFDM (orthogonal frequency division multiplexing) [7] can be used to divide the large bandwidth into a narrow ones and for each subband, the previous model is used. The gains $(\mathcal{H}_{a\alpha})$ $(a = 1, \ldots, N_R,\ \alpha = 1, \ldots, N_T)$ of channel matrix are supposed to be independent identically distributed (iid) and are governed by a circular complex Gaussian random variables with zero mean and unit variance.

In this work, we use differential analysis and borrow ideas from quantum scattering theory [8–10] to develop a new way to deal with MIMO channel. We first derive a scalar equation for modeling MIMO channel; then we study the performance of MIMO systems for indoor and urban channels by varying SNR and the antennas numbers N_T and N_R. Associating the MIMO channel $\mathcal{H}$ with the random minimum distance $\delta_{\min}$ between two generic received vectors $|\mathbf{R}_a >$ and $|\mathbf{R}_b >$, we first study the theoretical expression of total

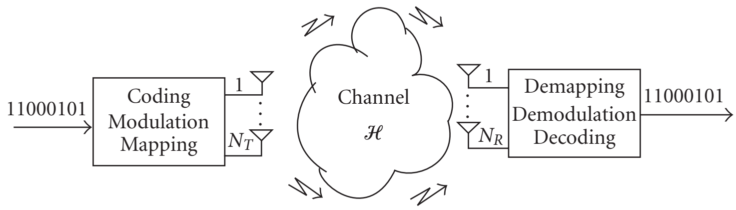

FIGURE 1: Principle of MIMO techniques. Remark that $\mathscr{H}$ plays a quite similar role of the quantum scattering theory S-matrix of particle physics.

probability of errors $\mathscr{P}_e = \mathscr{P}_e(N_T, N_R, d_0, \sigma)$, where d_0 stands for the minimal distance between transmitted vectors and $\mathrm{SNR} = -20\log_{10}(\sigma)$. We show in Section 4 that $\mathscr{P}_e$ reads in general as follows:

$$\mathscr{P}_e = \frac{1}{\sigma\sqrt{\pi}}\int_0^{\infty} e^{-t^2/4\sigma^2}\left[1-\left(1-\Gamma_{t^2/d_0^2}(N_R)\right)^{N_T}\right]dt. \tag{1}$$

Then we compute numerically $\mathscr{P}_e$ with respect to SNR for fixed N_T and d_0 and varying N_R. By fixing bounds on $\mathscr{P}_e$ for a given SNR, we study the determination of the theoretical value of the optimal number of antennas. This theoretical analysis, which uses a statistical approach, allows to predict the number of antennas without using long simulations. It permits as well to optimize the conception of MIMO systems and the reduction of the cost of its implementation. Notice by the way that great changes are envisaged to evaluate and migrate to third generation systems (3G). In the fixed network, an evolutionary path is envisaged, whereas in the radio interface a revolutionary approach is needed to support high data services [11]. The price to pay for the evolution towards 3G is exorbitant. Research for methods aiming the reduction of the cost of optimization is then essential.

The presentation of this paper is as follows: in Section 2, we give preliminaries on MIMO systems and a brief overview on the third generation (3G) of telecommunications. In Section 3, we study the performances of MIMO systems dealing with 3G. We first develop the modeling of MIMO channel by using the random minimum distance variable $\delta_{\min}$. Then we compute numerically the probability of errors $\mathscr{P}_e$ in terms of the signal-to-noise ratio variable (SNR). In Section 4 we give our conclusion.

2. PRELIMINARIES

We begin by describing briefly the principle of MIMO; then we give an overview on the 3G mobile systems that support circuit and packet oriented.

2.1. Principle of MIMO

From a diagram of a MIMO wireless transmission system (see Figure 1), a compressed digital source in the form of a binary data stream is fedin to a simplified transmitting block encompassing the functions of error control coding and mapping to complex modulation symbols (BPSQ, QPSK, M-QAM, etc.). Each separate symbol stream is mapped onto one of the multiple N_T antennas. After filtering and amplification, the signals are launched into the wireless channel. At the receiver, the signals are captured by N_R antennas and inverse functions are performed to recover the message. For a SISO (single input single output, $N_T = N_R = 1$) channel, the capacity $C = C(\rho)$ reads, in terms of SNR ($\mathrm{SNR} = \rho$), as $C = \log_2(1+\rho)$ bits/sec/Hz [12]. For a SIMO (single input multiple output, $N_T = 1$, $N_R > 1$) system, information theory can be used to demonstrate that the capacity is given by $C_{N_R}(\rho) = \log_2(1+\rho N_R^2)$ bits/sec/Hz [13]. Figure A.1 shows the variation of capacity in terms of SNR for a SISO and SIMO systems.

For a MIMO channel, the capacity C of the system is given by the following general relation [13]:

$$C_{N_T,N_R}(\rho) = \log_2\left[\det\left(I_{N_R} + \frac{\rho}{N_T}\mathscr{H}\mathscr{H}^+\right)\right], \tag{2}$$

where N_T is the number of transmitters and N_R the number of receivers. The variable ρ is the signal-to-noise ratio (SNR), $\mathscr{H}$ is the $N_R * N_T$ channel matrix with adjoint conjugate $\mathscr{H}^+$ and the capacity C is expressed with unit bits/sec/Hz. Note that this equation is based on N_T equal power uncorrelated sources. Foschini and Gans [4] demonstrated that capacity C grows linearly in $\min(N_R, N_T)$. In the particular case where N_T and N_R are large, the average capacity is given by $E(C) \simeq N_R\log_2(1+\rho)$. Figure A.2 shows clearly the improvement of the profit in capacity of a system MIMO for $N_T = 4$ and N_R varying in the interval $[5, 6, \ldots, 10]$. MIMO systems advantages are numerous; in particular their ability to turn multipath propagation, traditionally qualified as a problem of wireless communications, into a benefit for the user. MIMO may be also used to increase operator's revenues. We also recall several techniques, seen as complementary to MIMO in improving throughput, performance and spectrum efficiency subject to a growing interest [14], especially the enhancement of 3G mobile systems; for example, high speed digital packet access (HSDPA). In Table 1, we recall some simulated MIMO results in 3GPP based on a link level simulation of a combination of V-Blast and spreading reuse [4]. The table gives the peak data rates achieved by the downlink shared channel using MIMO techniques in the 2 GHz bandwith with a 5 MHz carrier spacing under conditions of flat fading.

Table 1

(N_R, N_T)	Code rate	Modulation	Data rate
$(1, 1)$	$\frac{3}{4}$	64 QAM	10.8 Mbs
$(2, 2)$	$\frac{3}{4}$	64 QAM	14.4 Mbs
$(2, 2)$	$\frac{3}{4}$	QPSK	14.4 Mbs
$(4, 4)$	$\frac{1}{2}$	8 PSK	21.6 Mbs

Notice moreover that there is a price to pay for improving quality and revenues since additional antenna increases the complexity of the system. This is because of the additional circuits for processing (equalization or interference cancelation) needed due to dispersing channel conditions resulting from delay spread of the environment surrounding the MIMO receiver [4].

2.2. Third generation (3G)

A great demand for a wide range of services (voice, high rate data services, mobile multimedia) is expressed by many users. This leads to a new generation (3G) of mobile systems, IMT-2000, that support circuit and packet-oriented. One of the air-interfaces developed within the frame work of the international Mobile Telecommunications (IMT-2000) is WCDMA (wide code division multiple access) using a direct spread technology that spread encoded user data over wider bandwidth (5 MHz), a sequence of pseudo-random units called chips at higher rate (3.84 Mcps) is used.

The basic idea of the 3G system is to integrate all the networks of 2G whole world in only one network and to associate it multimedia capacities (high flow for the data). Recall also that CDMA is a modulation and multiple-access using a spread spectrum communication which is used in civilian and military communication. It has the ability to combat multipath interference and increase performance systems. Within 3G (third generation partnership project) WCDMA is known as UTRA (universal terrestrial radio access). UTRA is designed to operate in either TDD mode (Time Division Duplex) or FDD mode (frequency division duplex). The FDD mode uplink (from user equipment (UE) to the base station (node B)) and downlink transmission (from node B to UE) deploys separated frequency bands. TDD is used when uplink and downlink transmissions are performed within the same frequency band in different time slots. In terms of capacity and receiver complexity, the downlink is more critical than the uplink.

WCDMA systems suffer from multiple access interference (MAI), because the same frequency band is shared by different users. The desired signal is extracted from its code, while other signals from system users in the home cell and other cells covering the service area appear as additive interference. This received interference is a factor which limits the radio capacity of the system.

3. PERFORMANCE OF MIMO

One of the basic tasks in dealing with MIMO systems is the modeling of the channel generally represented by the random $N_R * N_T$ matrix $\mathcal{H}$. Guided by the analysis of [15] and borrowing ideas from particles scattering theory of quantum mechanics [8–10], we develop, in the first part of this section, a way to approach $\mathcal{H}$ using the random variable $\delta_{\min}$ introduced earlier. In the second part, we use the results of this method to study the performance of MIMO by varying the N_R and N_T numbers and the SNR variable σ. To that purpose, we first consider the simple case $N_R = N_T = 1$, $h \equiv \mathcal{H}$ as a matter to fix the ideas and to make some useful comparisons with scattering theory and give our equation to approach the channel. Then we focus on special aspects of the channel matrices $\mathcal{H}$ with $N_R, N_T \geq 2$. We give, amongst others, the general form of the differential scattering equation for MIMO.

3.1. MIMO channels

We start by illustrating ideas on a simple example. This allows us to show how results on scattering theory of quantum mechanics (QM) can be used to approach MIMO channels. Thus consider a single-input single-output (SISO) system and focus on the channel of the system with matrix h.

Having seen the link between MIMO systems and QM scattering theory, it is interesting to start by recalling some useful QM tools. An incoming wave e is generally represented by a Hilbert space vector denoted as $|e\rangle$ and called ket. The outcoming vector r, belonging to the dual Hilbert space H, is represented by $\langle r|$ and is called bra. The latter is just the adjoint vector of the ket $|r\rangle$, ($\langle r| = (|r\rangle)^\dagger$). The Hilbert space H is an Euclidean space endowed with the inner product $H \times H \rightarrow C$ which associates to the two vectors $f \in H^*$ and $g \in H$ the scalar $\langle f \mid g\rangle$. The ket and bra notations satisfy the usual properties of the Hilbert space including linearity and normalization; they are very useful in the study of scattering theory and their power comes from the fact that they are representation independent; one may work either in the real space or in the Fourier dual and can move from one representation together without difficulty; for details see Appendix A.2.

Using the input vectors $\langle e|$ and output ones $|r\rangle$, the h matrix reads in particular as $h = |r\rangle\langle e|$; but in general like[1]

$$h = |r'\rangle\langle e|, \quad \langle e \mid e\rangle = 1, \tag{3}$$

where $|r'\rangle$ captures also the noise vector $|n\rangle$ which should be thought as an external source. We suppose that the channel gains are identical and independently distributed. Therefore given a transmitted vector $|e\rangle$, which reads explicitly in

[1] The ket and bra notations are conventions borrowed from quantum scattering theory.

M-ary modulation ($M = 2^n$) as $|b_1, \ldots, b_n\rangle$ where the bit is taken as $b_i = \pm 1$, then the received signal vector $|r\rangle$ is,

$$|r\rangle = h|e\rangle + |n\rangle. \tag{4}$$

In the above relations $|r'\rangle$ is equal to $|r\rangle - |n\rangle$ and like for $|e\rangle$, the vector $|r\rangle$ has the M-ary modulation $|j_1, \ldots, j_n\rangle$ with $j_i = \pm 1$. Before going ahead, note that as far as links with quantum are concerned, one can make a remarkable correspondence between MIMO channel h and quantum scattering theory of particles. We have, amongst others, the two following:

(1) Bits ± 1 are in one to one with the quantum states $\psi_{s,m}$ of particles of spin $s = \hbar/2$ and spin projection $m = \pm\hbar/2$, where $\hbar$ is the Planck constant. This opens an issue to borrow methods used to describe spin particles to approach the channel. Note that the state $\psi_{s,m}$ is often denoted as $|m = \pm 1\rangle$. This vector may be also used to describe the bits vector of BPSK modulation.

(2) Equation (3) can be interpreted as just the usual T_{if} transition amplitude

$$T_{if} = \langle r_f | h | e_i \rangle \tag{5}$$

of quantum scattering theory of spin particles $s = \hbar/2$ and $m = \pm\hbar/2$ moving in some potential. Within this view one may use QM methods (Green functions) to compute T_{if} to approach SISO channel. We will not develop this issue here; for a review of the QM methods; see, for instance, Appendix A.2 and [8–10].

3.1.1. *Variational channel equation*

Instead of modeling the channel by the typical scattering equation (4), we propose to rather use the following variational one,

$$|\delta r\rangle = h|\delta e\rangle + \delta h|e\rangle, \tag{6}$$

where the variation vectors are as $|\delta v\rangle = |v'\rangle - |v\rangle$ with $|v\rangle$ standing for $|r\rangle$ and $|e\rangle$ and where δh describes a fluctuation of the channel. In the above relation, we have also supposed a constant static noise ($|\delta n\rangle = 0$). Moreover, since $|\delta r\rangle$ is an arbitrary vector, one can put the vector equation (6) into the following scalar form:

$$\delta^2 = \langle \delta r | h | \delta e \rangle + \langle \delta r | \delta h | e \rangle, \tag{7}$$

where now $\delta = \sqrt{\langle \delta r \mid \delta r \rangle}$ is a random variable that captures information on the channel and where $\langle \delta r| = \langle \delta e | h^+ + \langle e | (\delta h^+)$ with h^+ *being the* adjoint conjugate of h. Instead of (6), the channel is now modeled by the above scalar relation. We will turn later on the way this relation can be used in practice; for the moment let us say few words about the extension to MIMO.

3.1.2. *MIMO case*

The channel matrix $\mathcal{H}$ of MIMO systems is described by the following random $N_R * N_T$ complex matrix which reads in the bra-ket notations as follows:

$$\mathcal{H} = |\mathbf{R}'\rangle\langle \mathbf{E}|, \quad |\mathbf{R}'\rangle = |\mathbf{R}\rangle - |\mathbf{N}\rangle, \tag{8}$$

where $|\mathbf{R}'\rangle$ may be thought as an $N_R \times 1$ column vector and $\langle \mathbf{E}|$ a row $1 \times N_T$ one (see (9) below). The matrix $\mathcal{H}$ involves N_T transmitters, N_R receivers, and obeys quite similar relations to SISO; except that in MIMO the previous vectors $|r\rangle$ and $|e\rangle$ get now promoted to larger vectors, namely,

$$|\mathbf{R}\rangle = \begin{pmatrix} |r_1\rangle \\ \vdots \\ |r_{N_R}\rangle \end{pmatrix}, \qquad |\mathbf{E}\rangle = \begin{pmatrix} |e_1\rangle \\ \vdots \\ |e_{N_T}\rangle \end{pmatrix}, \tag{9}$$
$$\langle \mathbf{R}| = (\langle r_1|, \ldots, \langle r_{N_R}|), \qquad \langle \mathbf{E}| = (\langle e_1|, \ldots, \langle e_{N_T}|).$$

As such, MIMO channel obeys the following generalized equation $|\mathbf{R}\rangle = \mathcal{H}|\mathbf{E}\rangle + |\mathbf{N}\rangle$, forming N_R equations of type MISO. The differential version of this equation extending (6) reads then as follows:

$$|\delta \mathbf{R}\rangle = \mathcal{H}|\delta \mathbf{E}\rangle + (\delta \mathcal{H})|\mathbf{E}\rangle. \tag{10}$$

By taking the norm, we can bring this relation into various forms; in particular,

$$\delta^2 = \langle \delta \mathbf{R} | \mathcal{H} | \delta \mathbf{E} \rangle + \langle \delta \mathbf{R} | (\delta \mathcal{H}) | \mathbf{E} \rangle, \tag{11}$$

where now $\delta^2 = \langle \delta \mathbf{R} \mid \delta \mathbf{R} \rangle$. Using these differential equations, we will develop, in what follows, the method to model MIMO channel and its optimization. A closed approach has been also considered in [15]. The method involves the two following ingredients: (a) the minimum distance $\delta_{\min}$ between signal vectors $|\mathbf{R}\rangle$ and $|\mathbf{R}'\rangle = |\mathbf{R}\rangle + |\delta \mathbf{R}\rangle$ at reception and (b) the optimization of the total probability of errors $\mathcal{P}_e = \mathcal{P}_e(\sigma)$ to predict the optimal number of MIMO antennas.

3.2. *Minimum distance as a channel variable*

As noted so far, the key point in dealing with MIMO and its performance is how to model the random channel matrix $\mathcal{H}$. The latter is in general a non Hermitian rectangular matrix with the unique data is that it satisfies the scattering equation $\mathcal{H}|\mathbf{E}\rangle = |\mathbf{R}'\rangle$. This lack of information makes the study of MIMO and in particular its $\mathcal{H}$ matrix not an easy task. However, there is a clever way to extract information from this matrix without going into involved mathematical analysis. The idea is to optimize the above scattering equation using a variation approach (11) together with special properties of the space of the complex vectors $|\mathbf{E}\rangle$ and $|\mathbf{R}'\rangle$. The idea of the method involves three steps; two of them are summarized just below and the third one will be exposed in next section 3.3.

(1) First use a variational approach which deals with the channel not through the usual scattering equation $\mathcal{H}|\mathbf{E}\rangle = |\mathbf{R}'\rangle$, but rather in terms of its variation as shown below:

$$\mathcal{H}|\delta \mathbf{E}\rangle = |\delta \mathbf{R}'\rangle, \tag{12}$$

where we have supposed

$$\mathcal{H}_{a\alpha} \gg \delta\mathcal{H}_{a\alpha}, \quad a = 1,\ldots,N_R, \ \alpha = 1,\ldots,N_T. \tag{13}$$

So the above variational scattering equations reduce to $\mathcal{H}|\delta\mathbf{E}\rangle = |\delta\mathbf{R}\rangle$.

(2) Take the norm of the simplified vector equation (12) reducing it into a scalar relation $\langle\delta\mathbf{E}|\mathcal{H}^\dagger\mathcal{H}|\delta\mathbf{E}\rangle = \langle\delta\mathbf{R}'|\delta\mathbf{R}'\rangle$ where $\mathcal{H}^\dagger\mathcal{H}$ is an $N_T * N_T$ square Hermitian matrix. Then minimize both sides of the resulting scalar equation leading to the typical relation

$$\delta_{\min} \simeq d_0\sqrt{\langle \mathrm{h}_{m_0} \mid \mathrm{h}_{m_0}\rangle}, \tag{14}$$

for some integer m_0 belonging to the set $[1,\ldots,N_T]$ and where we have set $\delta_{\min} = \min(\langle\delta\mathbf{R}' \mid \delta\mathbf{R}'\rangle)$, $d_0 = \langle\delta\mathbf{E} \mid \delta\mathbf{E}\rangle$ and $|\mathrm{h}_{m_0}\rangle = \min(\mathcal{H}|\delta\mathbf{E}\rangle/d_0)$ with $\langle\mathrm{h}_{m_0} \mid \mathrm{h}_{m_0}\rangle = \||\mathrm{h}_{m_0}|\|^2$.

To see how this works in practice, consider two generic vectors $|r_a\rangle$ and $|r_b\rangle$ and their difference $|r_{ab}\rangle \equiv |r_a\rangle - |r_b\rangle$ with $a \neq b$. To make contact with the variational analysis given above, this difference can also be read as $|r_{ab}\rangle = |r_a\rangle + |\delta r_a\rangle$. Then compute the minimum of the distance $\||r_{ab}\rangle\|_{\min}$ in terms of the transmitted symbols $|e_{ab}\rangle$ and the channel matrix $\mathcal{H}$. We have

$$\min(\||r_{ab}\rangle\|) \simeq \min\|\mathcal{H}|e_{ab}\rangle\|, \quad a \neq b, \tag{15}$$

where $|e_{ab}\rangle = |e_a\rangle - |e_b\rangle$. Setting $\delta_{\min} = \min\||r_{ab}\rangle\|$ and $d_0 = \||e_{ab}\rangle\|$ which is solved as

$$|e_{ab}\rangle = d_0 e^{i\theta_m}|u_m\rangle, \quad (|u_m\rangle)_n = \delta_{n,m}, \quad n,m = 1,\ldots,N_T, \tag{16}$$

where $\||u_m\rangle\| = 1$ and where the phase $e^{i\theta_m}$ depends on the M-ary modulation ($\theta_m = 2p\pi/M$, $0 \le p \le M-1$). Substituting this change back into $\||r_{ab}\rangle\| \simeq \|\mathcal{H}|e_{ab}\rangle\|$ gives at a first stage $\||r_{ab}\rangle\| \simeq d_0\|\mathcal{H}|u_m\rangle\|$; then using the identity[2] $\mathcal{H}|u_m\rangle = |\mathbf{R}'\rangle\langle\mathbf{E}|u_m\rangle \equiv |\mathrm{h}_m\rangle$, we get $\||r_{ab}\rangle\| \simeq d_0\||\mathrm{h}_m\rangle\|$. Therefore, the minimum distance $\delta_{\min}$ equation (15) is given by

$$\delta_{\min} \simeq d_0 \min_{m=1,\ldots,N_T} \||\mathrm{h}_m\rangle\|. \tag{17}$$

Notice that $|\mathrm{h}_m\rangle$ is a vector with components $(|\mathrm{h}_m\rangle)_a \equiv \mathrm{h}_{am}$, $a = 1,\ldots,N_R$. Its Hermitian norm vector is $\|\mathrm{h}_m\|^2 = \sum_{a=1}^{N_R}|\mathrm{h}_{am}|^2$. Notice also that the distribution law for each channel gain is given by $\rho_{X_a}(x_a) = 2\sqrt{1/\pi}e^{-x_a^2}$, with $X_a = |\mathrm{h}_{am}|$. Therefore, the probability density of $\|\mathrm{h}_m\|$ is given by a chi-square distribution

$$\rho_{Y_m}(y) = \frac{1}{\Gamma(N_R)} y^{N_R-1}e^{-y}, \quad Y_m = \|\mathrm{h}_m\|^2, \tag{18}$$

[2] Notice that $\{|u_m\rangle\}$ is the canonical vector basis and $|\mathrm{h}_m\rangle = \mathcal{H}|u_m\rangle$ is just the mth column of the $\mathcal{H}$ matrix.

where $\Gamma(N_R) = (N_R - 1)!$. Moreover, the cumulative distribution function $F_{Y_m}(u)$ (cdf) associated with Y_m is

$$F_{Y_m}(u) = P(Y_m < u) = \Gamma_u(N_R), \tag{19}$$

where $\Gamma_u(p)$ is the incomplete gamma function defined as $\Gamma_u(p) = (1/\Gamma(p))\int_0^u x^{p-1}e^{-x}dx$. Then the quantity $\min_{m=1,\ldots,N_T} F_{Y_m}(u) = P(\min_{m=1,\ldots,N_T} Y_m < u)$ can be also written, using independence property of Y_m's, as $(\min_{m=1,\ldots,N_T} F_{Y_m}(u)) = 1 - \Pi_{m=1,\ldots,N_T}P(Y_m > u)$, which, upon using channel gains identity and $\min_{m=1,\ldots,N_T}(Y_m) = \delta_{\min}/d_0$, reads as well as $1 - [P([\delta_{\min}/d_0] > u)]^{N_T}$. Thus, we have the result

$$\min_{m=1,\ldots,N_T}[F_{Y_m}] = 1 - \left[1 - P\left(\left[\frac{\delta_{\min}}{d_0}\right]^2 < u^2\right)\right]^{N_T}. \tag{20}$$

By using (19), we finally get

$$\min_{m=1,\ldots,N_T}[F_{Y_m}(u)] = 1 - \left[1 - \Gamma_{(\delta_{\min}/d_0)^2}(u)\right]^{N_T}, \tag{21}$$

where $\Gamma_u(p)$ stands for the incomplete gamma function defined above. Thus the cdf for a generic $\delta_{\min}$ reads as follows:

$$F(\delta_{\min}) = 1 - \left[1 - \Gamma_{(\delta_{\min}/d_0)^2}(N_R)\right]^{N_T}. \tag{22}$$

Notice that strictly speaking, the cdf is a function depends on the variables N_T, N_R, and the ratio $\delta_{\min}/d_0$. But later on we will fix N_T and look for the optimal values of N_R by studying the variation of the probability of errors P_e with respect to SNR.

3.3. *Probability of errors for the minimum distance*

We begin this section by noting that given a transmitted vector $|\mathbf{E}_i\rangle$ of a package $\mathcal{E}$ including the closed neighbors $|\mathbf{E}_i + \boldsymbol{\delta}\mathbf{E}_i\rangle$, one also has the three following vectors at reception.

(a) The basic received noise free vector $|\mathbf{R}_i\rangle = \mathcal{H}|\mathbf{E}_i\rangle$.

(b) Its closed neighbors; that is, received noise free vectors $|\mathbf{R}_i + \boldsymbol{\delta}\mathbf{R}_i\rangle = \mathcal{H}|\mathbf{E} + \boldsymbol{\delta}\mathbf{E}\rangle$ with $\delta\mathcal{H}$ ignored.

(c) The basic received noisy vector

$$|\mathbf{R}'\rangle = |\mathbf{R}\rangle + |\mathbf{N}\rangle \tag{23}$$

with noise vector $|\mathbf{N}\rangle$.

With these received vectors we are in position to complete the third stage of the three steps mentioned in Section 3.2. We require the following condition:

$$\langle\mathbf{N} \mid \mathbf{N}\rangle \text{ less than } \langle\mathbf{N} - \boldsymbol{\delta}\mathbf{R} \mid \mathbf{N} - \boldsymbol{\delta}\mathbf{R}\rangle. \tag{24}$$

This constraint relation is the condition for disregarding interference between the received noisy vector $|\mathbf{R}'\rangle$ and the received vector $|\mathbf{R} + \boldsymbol{\delta}\mathbf{R}\rangle$ associated with the transmitted $|\mathbf{E} + \boldsymbol{\delta}\mathbf{E}\rangle$ neighbor to $|\mathbf{E}\rangle$. To better see this condition, let us consider a noise-free received vector $|\mathbf{R}_q\rangle$ and its neighbors

$|\mathbf{R}_p\rangle$ at $\delta_{\min}$. Then error appears whenever we have confusion between two neighbors vectors

$$||(|\mathbf{R}_q\rangle + |N\rangle - |\mathbf{R}_p\rangle)||^2 < ||(|\mathbf{R}_q\rangle + |N\rangle - |\mathbf{R}_q\rangle)||^2 \quad (25)$$

which leads to the inequality $\|(|\mathbf{R}_q\rangle - |\mathbf{R}_p\rangle)\|^2 < 2\,\mathrm{Re}\langle N \mid (\mathbf{R}_q - \mathbf{R}_p)\rangle$. Substituting $|R_q\rangle = \mathcal{H}|E_q\rangle$ and $|R_p\rangle = \mathcal{H}|\mathbf{E}_p\rangle$ and using the identity

$$|\mathbf{E}_p\rangle - |\mathbf{E}_q\rangle = d_0 \exp(i\theta_{m_0}) \,|\, u_{m_0}\rangle \quad (26)$$

we get the condition

$$||\mathcal{H}(|\delta\mathbf{E}\rangle)||^2 < 2d_0 \exp(i\theta_{m_0}) \,\mathrm{Re}\,\langle \mathbf{N} \mid \mathrm{h}_{m_0}\rangle, \quad (27)$$

where we have set $|\delta\mathbf{E}\rangle = |\mathbf{E}_q\rangle - |\mathbf{E}_p\rangle$. At the minimum distance $\delta_{\min} = d_0 \| |\mathrm{h}_{m_0}\rangle \|$, the variation of the transmitted vector $|\delta\mathbf{E}\rangle$ obeys then the following constraint equation:

$$|\delta\mathbf{E}\rangle = d_0 \exp(i\theta_{m_0}) \,|\, u_{m_0}\rangle. \quad (28)$$

Usually, the reason of error comes from the similarity between the received noisy vector $\mathcal{H}|\mathbf{E}\rangle + |\mathbf{N}\rangle$ and its nearest neighbors $\mathcal{H}|\mathbf{E} + \delta\mathbf{E}\rangle$. The error appears whenever the norm of the noisy vector is greater than the distance between noise received vector and neighbor noise-free received vectors. Thus error occurs when we have $\|\mathbf{N}\| \geq \|(\mathbf{N} - \delta\mathbf{R})\|$, that is,

$$[\langle \mathbf{N} \mid \delta\mathbf{R}\rangle + \langle \delta\mathbf{R} \mid \mathbf{N}\rangle] \geq \langle \delta\mathbf{R} \mid \delta\mathbf{R}\rangle, \quad (29)$$

where $\langle \mathbf{V}|$ is the adjoint conjugate of $|\mathbf{V}\rangle$ and where $|\delta\mathbf{R}\rangle = \mathcal{H}|\delta\mathbf{E}\rangle$ and $\langle\delta\mathbf{R}| = \langle\delta\mathbf{E}|\mathcal{H}^\dagger$. Using (16), we have $\mathcal{H}|\delta\mathbf{E}\rangle = d_0 \exp(i\theta)\mathcal{H}|u_{m_0}\rangle$ which we can rewrite as well, by help of (15), as $|\delta\mathbf{R}\rangle = (\delta_{\min}/\sqrt{\langle h_{m_0} \mid h_{m_0}\rangle})e^{i\theta}|h_{m_0}\rangle$, where we have used (17). Putting back into (29), we obtain

$$\frac{1}{\sqrt{\langle h_{m_0} \mid h_{m_0}\rangle}} \mathrm{Re}\,[e^{i\theta}\langle \mathbf{N} \mid h_{m_0}\rangle] > \frac{\delta_{\min}}{2}. \quad (30)$$

We can rewrite this relation by remembering that the complex random variable $v = \langle \mathbf{N} \mid h_{m_0}\rangle/\sqrt{\langle h_{m_0} \mid h_{m_0}\rangle}$ is a Gaussian complex circular variable with zero mean and variance σ^2. Thus, we have $\mathrm{Re}(e^{i\theta}v) > \delta_{\min}/2$, with probability of errors $\mathrm{P}_{e,\inf}(\mathrm{Re}(e^{i\theta}v) > \delta_{\min}/2) = \int_{\delta_{\min}/2}^{\infty} \rho_v(x)dx$ where $\rho_v(x) = (1/\sigma\sqrt{\pi})\exp(-x^2/\sigma^2)$. Note that for BPSK constellation, θ takes only one value for a given $|\mathbf{E}\rangle$, while for constellation other than BPSK θ takes more than one value. Using the expression of $\rho_v(x)$, the probability of errors above the lower bound reads then as follows,

$$\mathrm{P}_{e,\inf}\left(\mathrm{Re}\,(e^{i\theta}v) > \frac{\delta_{\min}}{2}\right) = \frac{1}{2}\,\mathrm{erf}\,c\left(\frac{\delta_{\min}}{2\sigma}\right). \quad (31)$$

For the upper bound, we should replace $\mathrm{Re}(e^{i\theta}v) > \delta_{\min}/2$ by $|v| > \delta_{\min}/2$. The cdf of $|v|^2$ is $F_{|v|^2}(u) = P(|v|^2 < u)$ or equivalently as $F_{|v|^2}(u) = \Gamma_{u/\sigma^2}(1) = 1 - \exp(-u/\sigma^2)$. For the upper bound $P_{e,\sup}(\delta_{\min}) = P(|v|^2 > (\delta_{\min}/2)^2)$, the probability of errors can be put into the form

$$\mathrm{P}_{e,\sup}(\delta_{\min}) = \left(1 - \Gamma_{|v|^2}\left[\left(\frac{\delta_{\min}}{2}\right)^2\right]\right) = e^{-(\delta_{\min}/2\sigma)^2}. \quad (32)$$

Ploting the curves of probability of errors $\mathrm{P}_{e,\sup}$ and $\mathrm{P}_{e,\sup}$ with respect to $\delta_{\min}$, we see that a good compromise providing quite good results for usual constellation (other than BPSK) is given by

$$\mathrm{P}_e(\delta_{\min}) = \mathrm{erf}\,c\left(\frac{\delta_{\min}}{2\sigma}\right) = \frac{2}{\sqrt{\pi}}\int_{\delta_{\min}/2\sigma}^{\infty} \exp(-x^2)dx. \quad (33)$$

So, the theoretical total probability of errors $\mathcal{P}_e$ can be expressed as $\mathcal{P}_e = \int_0^\infty [\rho(\delta_{\min})\mathrm{P}_e(\delta_{\min})]d(\delta_{\min})$, with $\rho(x) = (1/(N_R-1)!)x^{N_R-1}\exp(-x)$. By an integration by part, we have

$$\mathcal{P}_e = -\int_0^\infty F(t)\mathrm{P}_e'(t)dt, \quad t = \delta_{\min}, \quad (34)$$

where $F(\delta_{\min})$ is the cumulative distribution function of $\delta_{\min}$ and $\mathrm{P}_e'(\delta_{\min})$ is the derivative of $\mathrm{P}_e(\delta_{\min})$.

4. THEORETICAL RESULTS

We first describe the method for evaluating $\mathcal{P}_e$; then we give our numerical results.

4.1. *The method*

To compute the total probability of errors in terms of signal-to-noise ratio (SNR) variable, that is $\mathcal{P}_e = \mathcal{P}_e(\mathrm{SNR})$, we proceed in steps as follows: first we start from the integral expression of $\mathcal{P}_e$ (34), then substitute $F(\delta_{\min})$ as in (22) and $\mathrm{P}_e'(\delta_{\min})$ by

$$\mathrm{P}_e'(\delta_{\min}) = \frac{-1}{\sigma\sqrt{\pi}}\exp\left[-\left(\frac{\delta_{\min}}{2\sigma}\right)^2\right]; \quad (35)$$

we find

$$\mathcal{P}_e = \frac{1}{\sigma\sqrt{\pi}}\int_0^\infty e^{-t^2/4\sigma^2}\left[1 - \left(1 - \Gamma_{t^2/d_0^2}(N_R)\right)^{N_T}\right]dt. \quad (36)$$

Up on fixing d_0 for a given constellation, this is a real function depending on three parameters as shown below:

$$\mathcal{P}_e = \mathcal{P}_e(\sigma, N_T, N_R). \quad (37)$$

This expression is difficult to compute exactly although it can be simplified a little bit since a priori the numbers N_T and N_R are two inputs. But here we will deal with them as moduli fixed by physical considerations, statistics and desired services. Next, we adopt a numerical approach to evaluate this quantity. In our computation, we use the following method.

(1) We fix once for all the numbers of transmitter antennas as $N_T = 2$ reducing the previous $\mathcal{P}_e$ moduli dependence to $\mathcal{P}_e(\sigma, N_R)$. This is because of electromagnetic interaction of antenna elements on small platform and the expense of multiple down-conversion RF paths [16], the implementation of diversity at user mobile in 3G which cannot support more than two antennas is difficult. Note that N_R must be greater than N_T, otherwise some power is wasted. For instance, in case where the power is allocated uniformly over the transmitter, there will be an average power loss of $10\log_{10}(N_T/N_R)$.

(2) To deal with the two remaining moduli N_R and σ, we proceed as follows.

(a) We choose the number of receiver antennas N_R into an interval lying from 3 to 9 ($3 \leq N_R \leq 9$). For each choice of N_R, $\mathcal{P}_e(\sigma, N_R)$ becomes a one parameter function which we denote as $\mathcal{P}_{e,N_R}(\sigma)$.

(b) Comparing the values of $\mathcal{P}_{e,N_R}(\sigma)$ for each choice of N_R, one gets information on the optimal value of N_R for a given value of σ.

(3) To extract information on the optimal value of N_R, we draw the parametric curves $\mathcal{P}_{e,N_R} = \mathcal{P}_{e,N_R}(\text{SNR})$ with SNR(db) which is equal to $-20\log_{10}\sigma$. Recall by the way that $\text{SNR(db)} = 10\log_{10}(P_T/P_N)$ with P_T and P_N defining, respectively, the transmitted power and the power associated with noise at reception. By implementing the expression of the noise covariance matrix, namely $\sigma^2 I_{N_R}$, and normalizing the total transmitted power to 1, one gets the above relation between SNR and variance σ^2.

4.2. Numerical results

Below we give our numerical results. These are grouped in the form of figures illustrating the variation of the total probability of errors with respect to SNR.

Notice that as we are interested in 3G, we have adopted the structure of WCDMA physical layer that assumes QPSK modulated data streams assembled into 10 millisecond frames. We recall also that the minimum distance d_0 between the transmitted symbols in a QPSK modulation is $\sqrt{2}$.

From Figure 3, we learn the three following:

(i) For a given SNR and for a desired value of probability of errors, we can determine the number of antennas to install. Choosing a MIMO performance with total probability of errors as

$$\mathcal{P}_{e,N_R}(\text{SNR}) < 10^{-6} \qquad (38)$$

at SNR = 6 dB, we find that the required number of received antennas is at least $N_R = 9$. As we can see, this number is too high because of the required high performance. Relaxing this requirement by choosing for instance $P_{e,N_R}(\text{SNR}) < 10^{-3}$ we get $N_R = 4$. The number of antennas at reception strongly depends then on the precision of $\mathcal{P}_{e,N_R}(\text{SNR})$.

(ii) Knowing that the choice of the probability of errors depends on the type of service we want to send on the channel (voice, data, image), we can, by help of Figure 2, determine the optimal value of the received antennas as shown on Table 2.

(iii) The same approach may also be used for other techniques such as EDGE, HSDPA, and WIMAX using, respectively, the modulations 8-PSK, QAM, and QPSK. In these techniques, the same relation for the probability of errors (36) is valid except that now we have to vary the minimum inter-distance d_0 between transmitter signal vectors. For instance, for QPSK, $d_0 = \sqrt{2}$, and for 8-PSK we have $d_0 = \sqrt{2-\sqrt{2}}$.

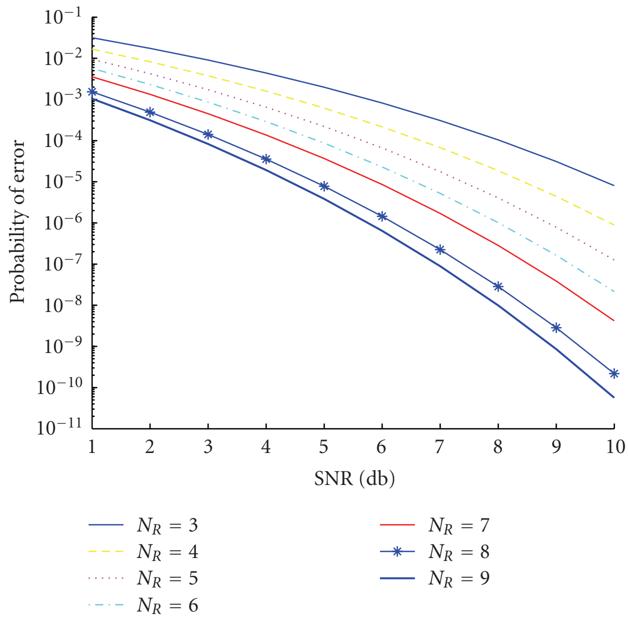

Figure 2: Theoretical probability of errors.

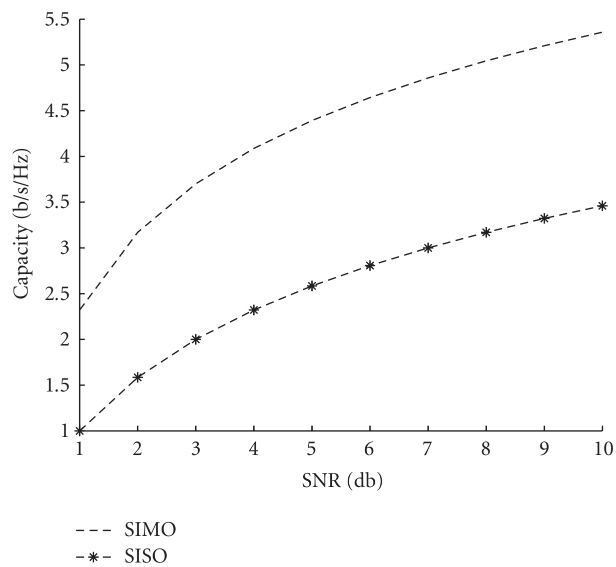

Figure 3: Variation of capacity C with respect to SNR. Upper curve describes SIMO and lower one is for SISO.

Table 2

SNR	5	5
$P_{e,N_R}(\text{SNR})$	10^{-2}	10^{-6}
Service	Voice	Multimedia
Required N_R	3	8

5. CONCLUSION

In this paper, we have developed a model proposal for studying MIMO channel and its performances. Instead of the usual channel vector equation $|\mathbf{R}\rangle = \mathcal{H}|\mathbf{E}\rangle + |\mathbf{N}\rangle$, we have proposed a variational relation for approaching MIMO channel; see (7)–(11). This is a scalar equation involving the minimum distance $\delta_{\min}$ as a random variable. Restricting our analysis to the case $\boldsymbol{\delta\mathcal{H}} \simeq 0$, we have shown that much on the MIMO channel is encoded in the minimum distance $\delta_{\min} = \min(\sqrt{\langle \delta\mathbf{R} \mid \delta\mathbf{R}\rangle})$ between received vectors $|\mathbf{R}\rangle$ and $|\mathbf{R}+\delta\mathbf{R}\rangle$.

Moreover, we have considered the theoretical determination of the number of antennas in MIMO systems combined to the third generation. This approach, which agrees with the study of [15], is important because it is easy to implement for predicting the theoretical optimal number of antennas.

Furthermore, if one succeeds to integrate some system parameters into the above theoretical result, this approach could also be used for other applications. Digital modulation such as QPSK (quadrature phase shift keying) and QAM (quadrature amplitude modulation) are used for many communication systems 3G, WIFI, HSDPA, and WIMAX. The probability of errors for constellations using HSDPA and WIMAX (QAM, QPSK) is given by the same equation (36). This means that the same analysis and quite similar results can be applied for other new technologies such as HSDPA and WIMAX technologies.

APPENDIX

We give two Appendices A.1 and A.2: in Appendix A.1 we give figures describing the variation of capacity C with respect to signal-to-noise ration (SNR). In Appendix A.2, we describe briefly the link between MIMO channel and wave scattering theory of quantum mechanics.

A.1. MIMO capacity

We give two figures; Figure A.1 illustrates the variation of MIMO (SISO and SIMO) capacity C with respect to SNR and Figure A.2 illustrates its average for various numbers N_R of received antennas.

A.2. General wave scattering theory

In this appendix Section, we show briefly how standard methods of scattering theory can be used to study MIMO channel. Here we exhibit rapidly the parallel between the channel equation of radio propagation and the so-called Born series of scattering theory of quantum physics; for references on applications of methods of scattering theory see for instance [17–19]. For other applications of methods of mathematical physics, such as large random matrices and maximum entropy principle, see Wigner proposal [20, 21]. Notice that as the subject of scattering theory is very huge, we will content ourselves here to expose the basic idea by giving the main lines of this correspondence. We hope to come back in a future occasion to give more details on how methods and results of scattering theory could be used in MIMO engineering.

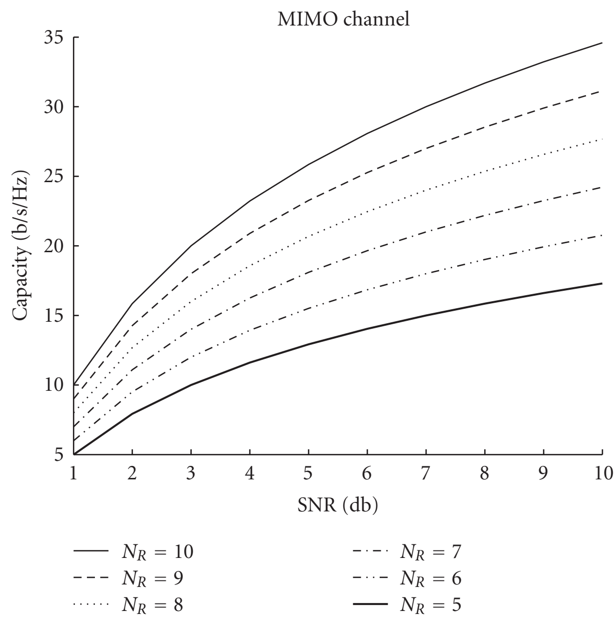

Figure A.1: Average capacity for MIMO systems. Curve in bottom is for $N_R = 5$ and top one for $N_R = 10$.

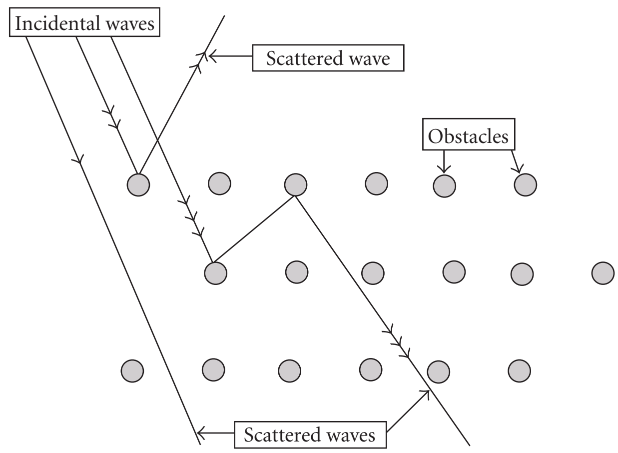

Figure A.2: Illustration of scattered phenomenon.

Link between channel equation and Born series

For readers who are not familiar with scattering theory and before going into technical details, we begin by noting that the usual MIMO channel equation (4),

$$|s\rangle = h|e\rangle + |n\rangle, \tag{A.1}$$

of radio propagation model looks like the following basic equation of wave scattering theory:

$$|\Psi_{\text{scat}}\rangle = (I_{\text{id}} + G_0 V + G_0 V G_0 V + G_0 V G_0 V G_0 V \ldots)\,|\Psi_{\text{inc}}\rangle. \tag{A.2}$$

In this relation, $|\Psi_{\text{inc}}\rangle$ is the incidental wave and $|\Psi_{\text{scat}}\rangle$ is the scattered wave resulting after multipath reflections on obstacles represented by a potential V. The function G_0 is the

Green distribution describing the line of sight (free space) propagation; see Figure A.2 for illustration. Notice that the objects $|\phi\rangle$ ($\langle\phi|$) with $\phi = \Psi_{\text{inc}}, \Psi_{\text{scat}}$, which in the context of radio propagation should be thought as $|\phi\rangle = |e\rangle, |s\rangle, |n\rangle$, are standard tools currently used in quantum mechanics. The objects $|\phi\rangle$ and $\langle\phi|$ are known as ket and bra vector waves (Dirac formalism); they constitute a clever way to study wave scattering and allow to avoid the usual complexity of integral computation.

To fix the ideas, let us give the link between the usual space wave function $\phi(x, y, z)$ and Dirac formalism. This is obtained by help of the resolution formula of the identity operator I_{id}. For one dimensional waves, for instance, $\phi(x)$ the resolution of the identity I_{id} reads as follows:

$$I_{\text{id}} = \int \rho_x dx, \quad \rho_x = |x\rangle\langle x|, \tag{A.3}$$

where ρ_x is the projector on the wave position x, we have

$$|\phi\rangle = I_{\text{id}}|\phi\rangle = \int \phi(x)|x\rangle dx, \quad \langle x \mid \phi\rangle. \tag{A.4}$$

With these conventions of notations, the usual Dirac-delta function

$$\delta(x_1 - x_2) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ik(x_1 - x_2)} dk \tag{A.5}$$

reads in real space as follows:

$$\delta(x_1 - x_2) = \langle x_2 \mid x_1\rangle. \tag{A.6}$$

Notice also that by using a normalized incidental wave $|e\rangle$, which reads in real 3-dimensional coordinate space as:

$$\int_{R^3} d^3\mathbf{x} \left| e(x, y, z) \right|^2 = 1, \quad \mathbf{x} = (x, y, z) \tag{A.7}$$

or equivalently in Dirac formalism just like

$$\langle e \mid e\rangle = 1. \tag{A.8}$$

Then substituting the noise vector $|n\rangle = |n\rangle \times 1$ by

$$|n\rangle\langle e \mid e\rangle = (|n\rangle\langle e|)|e\rangle, \tag{A.9}$$

we can rewrite (A.1) into the following equivalent form:

$$|s\rangle = \widetilde{h}|e\rangle \tag{A.10}$$

with

$$\widetilde{h} = (h + |n\rangle\langle e|). \tag{A.11}$$

By comparing (A.11) and (A.2), one has the two following results.

(1) The matrix $\widetilde{h}$ of the MIMO radio propagation channel is equal to the Born series of scattering theory

$$\widetilde{h} = (I_{\text{id}} + G_0 V + G_0 V G_0 V + G_0 V G_0 V G_0 V \ldots). \tag{A.12}$$

Under an assumption of the nature of the propagation environment associated with a hypothesis on the eigenvalues of the $G_0 V$, the matrix $\widetilde{h}$ can be read, for small V's, also as

$$\widetilde{h} = \frac{1}{1 - G_0 V}, \tag{A.13}$$

where G_0 is the Green function for line of sight and V potential barrier models the environment.

(2) From (A.2), one learns as well that the scattered wave $|\Psi_{\text{scat}}\rangle$ has the remarkable structure

$$|\Psi_{\text{scat}}\rangle = |\Psi_{\text{scat}}^{(0)}\rangle + |\Psi_{\text{scat}}^{(1)}\rangle + \cdots + |\Psi_{\text{scat}}^{(n)}\rangle + \cdots \tag{A.14}$$

with the identification

$$\begin{aligned} |\Psi_{\text{scat}}^{(0)}\rangle &= |\Psi_{\text{inc}}\rangle, && \text{line of sight (LOS),} \\ |\Psi_{\text{scat}}^{(1)}\rangle &= G_0 V |\Psi_{\text{inc}}\rangle, && \text{one diffusion,} \\ |\Psi_{\text{scat}}^{(2)}\rangle &= G_0 V G_0 V |\Psi_{\text{inc}}\rangle, && \text{two diffusions,} \end{aligned} \tag{A.15}$$

and so on.

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