Published for SISSA by Springer

Received: April 23, 2014
Revised: June 28, 2014
Accepted: July 9, 2014
Published: August 5, 2014

Fermion EDMs with minimal flavor violation

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Abstract: We study the electric dipole moments (EDMs) of fermions in the standard model supplemented with right-handed neutrinos and its extension including the neutrino seesaw mechanism under the framework of minimal flavor violation (MFV). In the quark sector, we find that the current experimental bound on the neutron EDM does not yield a significant restriction on the scale of MFV. In addition, we consider how MFV may affect the contribution of the strong theta-term to the neutron EDM. For the leptons, the existing EDM data also do not lead to strict limits if neutrinos are Dirac particles. On the other hand, if neutrinos are Majorana in nature, we find that the constraints become substantially stronger. Moreover, the results of the latest search for the electron EDM by the ACME Collaboration are sensitive to the MFV scale of order a few hundred GeV or higher. We also look at constraints from CP-violating electron-nucleon interactions that have been probed in atomic and molecular EDM searches.

Keywords: Beyond Standard Model, Neutrino Physics, CP violation, Electromagnetic Processes and Properties

ArXiv ePrint: 1404.4436

doi:10.1007/JHEP08(2014)019

JHEP08(2014)019

Contents

1 Introduction

Searches for electric dipole moments (EDMs) are a powerful means of probing new sources of the violation of charge parity (CP) and time reversal (T) symmetries beyond the standard model (SM) of particle physics [1–6]. Recently the ACME experiment [7], which utilized the polar molecule thorium monoxide to look for the EDM of the electron, d_e, has produced a new result of $d_e = (-2.1 \pm 3.7_{\rm stat} \pm 2.5_{\rm syst}) \times 10^{-29}\, e\,{\rm cm}$, which corresponds to an upper limit of $|d_e| < 8.7 \times 10^{-29}\, e\,{\rm cm}$ at 90% confidence level (CL). This is more stringent than the previous best bound by about an order of magnitude, but still way above the SM expectation for d_e, which is at the level of $10^{-44}\, e\,{\rm cm}$ [8]. Hence there is abundant room between the current limit and SM value of d_e where potential new physics may be observed in future measurements. In the quark sector, the EDM of the neutron, d_n, plays an analogous role in the quest of new physics. At present its experimental limit is $|d_n| < 2.9 \times 10^{-26}\, e\,{\rm cm}$ at 90% CL [9], while the SM predicts it to be in the range of 10^{-32}-$10^{-31}\, e\,{\rm cm}$ [10–13].

Extra ingredients beyond the SM can increase the electron and neutron EDMs tremendously with respect to their SM predictions, even up to their existing measured bounds. Such substantial enlargement may have various causes which could greatly differ from model to model. It is, therefore, of interest to analyze fermion EDMs arising from possible nonstandard origins under a framework that allows one to deal with some general features of the physics without getting into model specifics. This turns out to be feasible under the context of the so-called minimal flavor violation (MFV) which presupposes that the sources of all flavor-changing neutral currents (FCNC) and CP violation reside in renormalizable Yukawa couplings defined at tree level [14–22]. Thus the MFV framework offers a systematic way to explore SM-related new interactions which do not conserve flavor and CP symmetries.

In an earlier paper [23], motivated by the recent ACME data, we have adopted the MFV hypothesis in order to examine d_e in the SM slightly expanded with the inclusion of three right-handed neutrinos and in its extension incorporating the seesaw mechanism for light neutrino mass generation. In the present work, we would like to provide a more extensive treatment of our previous study, covering the EDMs of the other charged leptons as well. For d_e particularly, we demonstrate in greater detail how various factors may affect it within the MFV context, taking into account extra empirical information on neutrino masses. Moreover, we address the possibility that d_e is correlated with the effective Majorana mass that is testable in ongoing and upcoming searches for neutrinoless double-beta decay. We will also perform an MFV analysis on the quark EDMs and estimate the resulting neutron EDM. In addition, we consider the MFV effect on the contribution of the theta term in QCD to the neutron EDM.

The structure of the paper is as follows. In section 2, we describe the MFV framework and its aspects which are relevant to our evaluation of fermion EDMs as probes for the scale of MFV. In section 3 we derive the expressions for quark and lepton EDMs from several effective operators satisfying the MFV principle. Section 4 contains our numerical analysis. After determining the neutron EDM from the quark contributions and inferring the constraint on the MFV scale from the neutron data, we examine how the contribution of the QCD theta-term is altered in the presence of MFV. In the lepton sector, we devote much of our attention to the electron EDM in light of the ACME data and briefly address its muon and tau counterparts. The acquired constraints on the MFV scale depend considerably on whether the light neutrinos are Dirac or Majorana in nature, the EDMs in the former case being much smaller than the latter. Subsequently, we look at CP-violating electron-nucleon interactions, which were also investigated by ACME and other experiments looking for atomic or molecular EDMs. Lastly, we discuss potential constraints from flavor-changing and other flavor-conserving processes. We make our conclusions in section 5. An appendix collects some useful lengthy formulas.

2 Minimal flavor violation framework

In the SM supplemented with three right-handed neutrinos, the renormalizable Lagrangian for the quark and lepton masses can be expressed as

$$\begin{aligned}\mathcal{L}_{\rm m} \;=\; & -\bar{Q}_{k,L}\,(Y_u)_{kl}\,U_{l,R}\,\tilde{H} - \bar{Q}_{k,L}\,(Y_d)_{kl}\,D_{l,R}\,H - \bar{L}_{k,L}\,(Y_\nu)_{kl}\,\nu_{l,R}\,\tilde{H} - \bar{L}_{k,L}\,(Y_e)_{kl}\,E_{l,R}\,H \\ & - \tfrac{1}{2}\,\overline{\nu^{\rm c}_{k,R}}\,(M_\nu)_{kl}\,\nu_{l,R} \;+\; {\rm H.c.}\,, \end{aligned} \tag{2.1}$$

where summation over $k,l=1,2,3$ is implicit, $Q_{k,L}$ $(L_{k,L})$ represents left-handed quark (lepton) doublets, $U_{k,R}$ and $D_{k,R}$ $\big(\nu_{k,R}$ and $E_{k,R}\big)$ denote right-handed up- and down-type quarks (neutrinos and charged leptons), respectively, $Y_{u,d,\nu,e}$ are matrices containing the Yukawa couplings, M_ν is the Majorana mass matrix of the right-handed neutrinos, H is the Higgs doublet, and $\tilde{H} = i\tau_2 H^*$ involving the second Pauli matrix τ_2. The Higgs' vacuum expectation value $v \simeq 246\,\mathrm{GeV}$ breaks the electroweak symmetry as usual, which makes the weak gauge bosons and charged leptons massive and also induces Dirac mass terms for the neutrinos. The M_ν part in $\mathcal{L}_\mathrm{m}$ plays an essential role in the type-I seesaw mechanism [24–32].[1] If neutrinos are Dirac particles, however, the M_ν terms are absent.

For the quark sector, the MFV hypothesis [21] implies that the Lagrangian in eq. (2.1) is formally invariant under the global group $\mathrm{U}(3)_Q \times \mathrm{U}(3)_U \times \mathrm{U}(3)_D = G_q \times \mathrm{U}(1)_Q \times \mathrm{U}(1)_U \times \mathrm{U}(1)_D$, where $G_q = \mathrm{SU}(3)_Q \times \mathrm{SU}(3)_U \times \mathrm{SU}(3)_D$. This entails that the three generations of $Q_{k,L}$, $U_{k,R}$, and $D_{k,R}$ transform as fundamental representations of the $\mathrm{SU}(3)_{Q,U,D}$, respectively, namely

$$Q_L \to V_Q Q_L\,, \qquad U_R \to V_U U_R\,, \qquad D_R \to V_D D_R\,, \qquad V_{Q,U,D} \in \mathrm{SU}(3)\,. \tag{2.2}$$

Moreover, the Yukawa couplings are taken to be spurions which transform according to

$$Y_u \to V_Q Y_u V_U^\dagger\,, \qquad Y_d \to V_Q Y_d V_D^\dagger\,. \tag{2.3}$$

Consequently, to arrange nontrivial FCNC and CP-violating interactions satisfying the MFV principle and involving no more than two quarks, one puts together an arbitrary number of the Yukawa coupling matrices $Y_u \sim (3,\bar{3},1)$ and $Y_d \sim (3,1,\bar{3})$ as well as their Hermitian conjugates to set up the G_q representations $\Delta_q \sim (1\oplus 8,1,1)$, $\Delta_{u8} \sim (1,1\oplus 8,1)$, $\Delta_{d8} \sim (1,1,1\oplus 8)$, $\Delta_u \sim (\bar{3},3,1)$, and $\Delta_d \sim (\bar{3},1,3)$, combines them with two quark fields to build the G_q-invariant objects $\bar{Q}_L\gamma_\alpha\Delta_q Q_L$, $\bar{U}_R\gamma_\alpha\Delta_{u8}U_R$, $\bar{D}_R\gamma_\alpha\Delta_{d8}D_R$, $\bar{U}_R(1,\sigma_{\alpha\beta})\Delta_u Q_L$, and $\bar{D}_R(1,\sigma_{\alpha\beta})\Delta_d Q_L$, includes appropriate numbers of the Higgs and gauge fields to arrive at singlets under the SM gauge group, and contracts all the Lorentz indices. Since $\bar{Q}_L\gamma_\alpha\Delta_q Q_L$, $\bar{U}_R\gamma_\alpha\Delta_{u8}U_R$, and $\bar{D}_R\gamma_\alpha\Delta_{d8}D_R$ in this case must be Hermitian, $\Delta_{q,u8,d8}$ must be Hermitian as well.

The Lagrangian describing the EDM d_f of a fermion f is $\mathcal{L}_{f\,\mathrm{edm}} = -\frac{i}{2}d_f\bar{f}\sigma^{\kappa\omega}\gamma_5 f F_{\kappa\omega}$, where $F_{\kappa\omega}$ is the photon field strength tensor. Accordingly, among the combinations listed in the preceding paragraph, only $\bar{U}_R\sigma_{\alpha\beta}\Delta_u Q_L$ and $\bar{D}_R\sigma_{\alpha\beta}\Delta_d Q_L$ pertain to our examination of quark EDMs. For $\Delta_{u,d}$, one can take $\Delta_u = Y_u^\dagger\Delta$ and $\Delta_d = Y_d^\dagger\Delta$, where Δ is built up of terms in powers of $\mathsf{A} = Y_u Y_u^\dagger$ and $\mathsf{B} = Y_d Y_d^\dagger$, which transform as $(1\oplus 8,1,1)$ under G_q.

Formally Δ comprises an infinite number of terms, namely $\Delta = \sum \xi_{jkl\cdots}\mathsf{A}^j\mathsf{B}^k\mathsf{A}^l\cdots$ with coefficients $\xi_{jkl\cdots}$ expected to be at most of $\mathcal{O}(1)$. The MFV hypothesis requires that $\xi_{jkl\cdots}$ be real because complex $\xi_{jkl\cdots}$ would introduce new CP-violation sources beyond that in the Yukawa couplings. Using the Cayley-Hamilton identity

$$X^3 \;=\; X^2\,\mathrm{Tr}X \;+\; \tfrac{1}{2}\,X\big[\mathrm{Tr}X^2-(\mathrm{Tr}X)^2\big] \;+\; \mathbb{1}\mathrm{Det}X \tag{2.4}$$

[1]An analogous situation occurs in the type-III seesaw model [33].

for an invertible 3×3 matrix X, one can resum the infinite series into a finite number of terms [34, 35]

$$\begin{aligned}\Delta = {} & \xi_1 \mathbb{1} + \xi_2\,\mathsf{A} + \xi_3\,\mathsf{B} + \xi_4\,\mathsf{A}^2 + \xi_5\,\mathsf{B}^2 + \xi_6\,\mathsf{AB} + \xi_7\,\mathsf{BA} + \xi_8\,\mathsf{ABA} + \xi_9\,\mathsf{BA}^2 \\ & + \xi_{10}\,\mathsf{BAB} + \xi_{11}\,\mathsf{AB}^2 + \xi_{12}\,\mathsf{ABA}^2 + \xi_{13}\,\mathsf{A}^2\mathsf{B}^2 + \xi_{14}\,\mathsf{B}^2\mathsf{A}^2 + \xi_{15}\,\mathsf{B}^2\mathsf{AB} \\ & + \xi_{16}\,\mathsf{AB}^2\mathsf{A}^2 + \xi_{17}\,\mathsf{B}^2\mathsf{A}^2\mathsf{B}\,, \end{aligned} \tag{2.5}$$

where $\mathbb{1}$ denotes a 3×3 unit matrix. One can then also utilize this to devise Hermitian combinations such as $\Delta_q = \Delta + \Delta^\dagger$.

Even though one starts with all $\xi_{jkl\cdots}$ being real, the resummation process will render the coefficients ξ_r in eq. (2.5) generally complex due to imaginary parts generated among the traces of the matrix products $\mathsf{A}^j\mathsf{B}^k\mathsf{A}^l\cdots$ with $j{+}k{+}l{+}\cdots \geq 6$ upon the application of the Cayley-Hamilton identity. In appendix A we show the detailed reduction of one of the lowest-order products which give rise to the imaginary components of ξ_r. We find that the imaginary contributions are always reducible to factors proportional to $\mathrm{Im}\,\mathrm{Tr}\big(\mathsf{A}^2\mathsf{BAB}^2\big) = (i/2)\,\mathrm{Det}[\mathsf{A},\mathsf{B}]$ which is a Jarlskog invariant and much smaller than one [34].

Taking advantage of the invariance under G_q, we will work in the basis where Y_d is diagonal,

$$Y_d \;=\; \frac{\sqrt{2}}{v}\,\mathrm{diag}\big(m_d, m_s, m_b\big)\,, \tag{2.6}$$

and the fields $U_{k,L}$, $U_{k,R}$, $D_{k,L}$, and $D_{k,R}$ belong to the mass eigenstates. Hence we can write $Q_{k,L}$ and Y_u in terms of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix $V_{\rm CKM}$ as

$$Q_{k,L} \;=\; \begin{pmatrix} (V_{\rm CKM}^\dagger)_{kl}\,U_{l,L} \\ D_{k,L} \end{pmatrix}, \qquad Y_u \;=\; \frac{\sqrt{2}}{v}\,V_{\rm CKM}^\dagger\,\mathrm{diag}\big(m_u, m_c, m_t\big)\,, \tag{2.7}$$

where in the standard parametrization [9]

$$V_{\rm CKM} \;=\; \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta} & c_{23}\,c_{13} \end{pmatrix}, \tag{2.8}$$

with δ being the CP violation phase, $c_{kl} = \cos\theta_{kl}$, and $s_{kl} = \sin\theta_{kl}$. We note that, as a consequence, Δ_{u8} and Δ_{d8}, whose basic building blocks are $Y_u^\dagger Y_u$ and $Y_d^\dagger Y_d$, respectively, are all diagonal and thus will not bring about new flavor- and CP-violating interactions.

For the lepton sector, since it is still unknown whether light neutrinos are Dirac or Majorana particles, we address the two possibilities separately. In the Dirac case, the M_ν part is absent from $\mathcal{L}_{\rm m}$ in eq. (2.1), which is therefore, in the MFV language, formally invariant under the global group $\mathrm{U}(3)_L \times \mathrm{U}(3)_\nu \times \mathrm{U}(3)_E = G_\ell \times \mathrm{U}(1)_L \times \mathrm{U}(1)_\nu \times \mathrm{U}(1)_E$ with $G_\ell = \mathrm{SU}(3)_L \times \mathrm{SU}(3)_\nu \times \mathrm{SU}(3)_E$. This means that the three generations of $L_{k,L}$, $\nu_{k,R}$, and $E_{k,R}$ transform as fundamental representations of $\mathrm{SU}(3)_{L,\nu,E}$ in G_ℓ, respectively,

$$L_L \,\to\, V_L L_L\,, \qquad \nu_R \,\to\, V_\nu \nu_R\,, \qquad E_R \,\to\, V_E E_R\,, \tag{2.9}$$

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where $V_{L,\nu,E} \in \mathrm{SU}(3)$, whereas the Yukawa couplings are spurions transforming according to

$$Y_\nu \to V_L Y_\nu V_\nu^\dagger\,, \qquad Y_e \to V_L Y_e V_E^\dagger\,. \tag{2.10}$$

We will work in the basis where Y_e is already diagonal,

$$Y_e = \frac{\sqrt{2}}{v}\,\mathrm{diag}\big(m_e, m_\mu, m_\tau\big)\,, \tag{2.11}$$

and the fields $\nu_{k,L}$, $\nu_{k,R}$, $E_{k,L}$, and $E_{k,R}$ refer to the mass eigenstates. We can then express $L_{k,L}$ and Y_ν in terms of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix U_{PMNS} as

$$L_{k,L} = \begin{pmatrix} (U_{\mathrm{PMNS}})_{kl}\,\nu_{l,L} \\ E_{k,L} \end{pmatrix}, \quad Y_\nu = \frac{\sqrt{2}}{v}\,U_{\mathrm{PMNS}}\,\hat{m}_\nu\,, \quad \hat{m}_\nu = \mathrm{diag}\big(m_1, m_2, m_3\big)\,, \tag{2.12}$$

where $m_{1,2,3}$ are the light neutrino eigenmasses and U_{PMNS} has the same standard parametrization as in eq. (2.8). Thus the discussion for the down-type quarks can be easily applied to the charged leptons by replacing V_{CKM} with $U_{\mathrm{PMNS}}^\dagger$ and employing the building blocks $\mathsf{A} = Y_\nu Y_\nu^\dagger$ and $\mathsf{B} = Y_e Y_e^\dagger$ to construct Δ_ν and Δ_e, which are the lepton counterparts of Δ_u and Δ_d, respectively.

If neutrinos are of Majorana nature, the M_ν part in eq. (2.1) is allowed. As a consequence, for $M_\nu \gg M_{\mathrm{D}} = vY_\nu/\sqrt{2}$ the seesaw mechanism [24–32] becomes operational involving the $6{\times}6$ neutrino mass matrix

$$\mathsf{M} = \begin{pmatrix} 0 & M_{\mathrm{D}} \\ M_{\mathrm{D}}^{\mathrm{T}} & M_\nu \end{pmatrix} \tag{2.13}$$

in the $\big(U_{\mathrm{PMNS}}^*\nu_L^{\mathrm{c}}, \nu_R\big)^{\mathrm{T}}$ basis. The resulting matrix of light neutrino masses is

$$m_\nu = -\frac{v^2}{2}\,Y_\nu M_\nu^{-1} Y_\nu^{\mathrm{T}} = U_{\mathrm{PMNS}}\,\hat{m}_\nu\,U_{\mathrm{PMNS}}^{\mathrm{T}}\,, \tag{2.14}$$

where now U_{PMNS} contains the diagonal matrix $P = \mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$ multiplied from the right, $\alpha_{1,2}$ being the Majorana phases. It follows that Y_ν in eq. (2.12) is no longer valid, and one can instead take Y_ν to be [36]

$$Y_\nu = \frac{i\sqrt{2}}{v}\,U_{\mathrm{PMNS}}\,\hat{m}_\nu^{1/2} O M_\nu^{1/2}\,, \tag{2.15}$$

where O is a matrix satisfying $OO^{\mathrm{T}} = \mathbb{1}$ and $M_\nu = \mathrm{diag}(M_1, M_2, M_3)$. As we will see later, O can provide a potentially important new source of CP violation besides U_{PMNS}. We comment that the presence of M_ν breaks the global $\mathrm{U}(3)_\nu$ completely if $M_{1,2,3}$ are unequal and partially into $\mathrm{O}(3)_\nu$ if $M_{1,2,3}$ are equal [22].

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3 Fermion EDMs in MFV framework

To explore the MFV contribution to the EDMs of quarks and charged leptons, one needs to construct the relevant operators using $\Delta_{u,d,e}$ in combination with the quark, lepton, Higgs, and gauge fields. At leading order, the operators can be written as [21, 22]

$$\begin{aligned} O_{RL}^{(u1)} &= g'\bar{U}_R Y_u^\dagger \Delta_{qu1}\sigma_{\kappa\omega}\tilde{H}^\dagger Q_L B^{\kappa\omega}\,, & O_{RL}^{(u2)} &= g\bar{U}_R Y_u^\dagger \Delta_{qu2}\,\sigma_{\kappa\omega}\tilde{H}^\dagger \tau_a Q_L W_a^{\kappa\omega}\,, \\ O_{RL}^{(d1)} &= g'\bar{D}_R Y_d^\dagger \Delta_{qd1}\sigma_{\kappa\omega}H^\dagger Q_L B^{\kappa\omega}\,, & O_{RL}^{(d2)} &= g\,\bar{D}_R Y_d^\dagger \Delta_{qd2}\,\sigma_{\kappa\omega}H^\dagger \tau_a Q_L W_a^{\kappa\omega}\,, \end{aligned} \tag{3.1}$$

$$O_{RL}^{(e1)} = g'\bar{E}_R Y_e^\dagger \Delta_{\ell 1}\sigma_{\kappa\omega}H^\dagger L_L B^{\kappa\omega}\,, \qquad O_{RL}^{(e2)} = g\,\bar{E}_R Y_e^\dagger \Delta_{\ell 2}\,\sigma_{\kappa\omega}H^\dagger \tau_a L_L W_a^{\kappa\omega}\,, \tag{3.2}$$

where W and B denote the usual $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ gauge fields with coupling constants g and g', respectively, τ_a are Pauli matrices, $a=1,2,3$ is summed over, and $\Delta_{qu\varsigma,qd\varsigma,\ell\varsigma}$ with $\varsigma=1,2$ have the same form as Δ in eq. (2.5), but generally different ξ_r. One can express the effective Lagrangian containing these operators as

$$\mathcal{L}_{\rm eff} \;=\; \frac{1}{\Lambda^2}\Big(O_{RL}^{(u1)}+O_{RL}^{(u2)}+O_{RL}^{(d1)}+O_{RL}^{(d2)}+O_{RL}^{(e1)}+O_{RL}^{(e2)}\Big) \;+\; \text{H.c.}\,, \tag{3.3}$$

where Λ is the MFV scale. In general the operators in $\mathcal{L}_{\rm eff}$ have their own coefficients which have been absorbed by ξ_r in their respective Δ's. These coefficients also take into account the possibility that the MFV scale in the quark sector may differ from that in the lepton sector.

Expanding eq. (3.3), one can identify the terms relevant to fermion EDMs. In the quark sector the resulting EDMs of up- and down-type quarks are, respectively, proportional to $\mathrm{Im}\big(Y_u^\dagger\Delta_{qu\varsigma}V_{\rm CKM}^\dagger\big)_{kk}$ and $\mathrm{Im}\big(Y_d^\dagger\Delta_{qd\varsigma}\big)_{kk}$. The contributions of $\Delta_{qu\varsigma,qd\varsigma}$ to the EDMs come not only from some of the products of the A and B matrices therein, but also from the imaginary parts of ξ_r. As mentioned earlier, $\mathrm{Im}\,\xi_r$ are always proportional to $J_\xi \equiv \mathrm{Im\,Tr}\big(\mathsf{A}^2\mathsf{B}\mathsf{A}\mathsf{B}^2\big) = (i/2)\,\mathrm{Det}[\mathsf{A},\mathsf{B}]$, or explicitly

$$J_\xi \;=\; \frac{-64\big(m_u^2-m_c^2\big)\big(m_c^2-m_t^2\big)\big(m_t^2-m_u^2\big)\big(m_d^2-m_s^2\big)\big(m_s^2-m_b^2\big)\big(m_b^2-m_d^2\big)}{v^{12}}\,J_q\,, \tag{3.4}$$

where $J_q = \mathrm{Im}\big(V_{us}V_{cb}V_{ub}^*V_{cs}^*\big) = c_{12}\,s_{12}\,c_{23}\,s_{23}\,c_{13}^2\,s_{13}\,\sin\delta$ is a Jarlskog parameter for $V_{\rm CKM}$.

Not all of the products of $\mathsf{A}=Y_uY_u^\dagger$ and $\mathsf{B}=Y_dY_d^\dagger$ in $\Delta_{qu\varsigma,qd\varsigma}$ will contribute to quark EDMs. Since Y_u has the form in eq. (2.7) and Y_d is diagonal, the Hermiticity of A and B implies that only certain combinations of them are relevant. For example, $\big(Y_d^\dagger\mathsf{A}\big)_{kk} = \sqrt{2}\,m_{D_k}\mathsf{A}_{kk}/v$ is purely real and hence does not affect d_{D_k}. In this case, one needs to have terms in $\Delta_{qd\varsigma}$ which are not Hermitian in order to have imaginary components in $\big(Y_d^\dagger\Delta_{qd\varsigma}\big)_{kk}$. We find that only two terms, proportional to $\mathsf{B}^2\mathsf{A}\mathsf{B}$ and $\mathsf{B}^2\mathsf{A}^2\mathsf{B}$, are pertinent to the up-type quarks' EDMs and only the $\mathsf{A}\mathsf{B}\mathsf{A}^2$ and $\mathsf{A}\mathsf{B}^2\mathsf{A}^2$ terms are pertinent to the EDM's of down-type quarks.

The preceding discussions show that the contributions of $\mathrm{Im}\,\xi_r$ to the EDM of, say, the u (d) quark are suppressed by a factor of m_c^2/v^2 $\big(m_s^2m_b^2/v^4\big)$ compared to the contributions from $\mathsf{B}^2\mathsf{A}\mathsf{B}$ $\big(\mathsf{A}\mathsf{B}\mathsf{A}^2\big)$, which has the least number of suppressive factor from Y_u (Y_d) among

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the products in eq. (2.5) that can potentially contribute. Hence we can neglect the impact of $\mathrm{Im}\,\xi_r$ on the quark EDMs. One, however, needs to take $\mathrm{Im}\,\xi_r$ into account when considering how MFV affects the contribution of the strong theta-term to the neutron EDM, as we demonstrate later.

Simplifying things, we arrive at the leading-order contributions to the u- and d-quarks' EDMs

$$
\begin{aligned}
d_u &= \frac{\sqrt{2}\,e\,v}{\Lambda^2}\,\mathrm{Im}\big[Y_u^\dagger(\Delta_{qu1}+\Delta_{qu2})V_{\rm CKM}^\dagger\big]_{11} \\
&= \frac{32\,e\,m_u}{\Lambda^2}\left[\xi_{15}^u + \frac{2\big(m_c^2+m_t^2\big)}{v^2}\,\xi_{17}^u\right]\frac{(m_c^2-m_t^2)(m_d^2-m_s^2)(m_s^2-m_b^2)(m_d^2-m_b^2)}{v^8}\,J_q\,, \qquad (3.5)\\
d_d &= \frac{\sqrt{2}\,e\,v}{\Lambda^2}\,\mathrm{Im}\big[Y_d^\dagger(\Delta_{qd1}-\Delta_{qd2})\big]_{11} \\
&= \frac{32\,e\,m_d}{\Lambda^2}\left[\xi_{12}^d + \frac{2\big(m_s^2+m_b^2\big)}{v^2}\,\xi_{16}^d\right]\frac{\big(m_s^2-m_b^2\big)\big(m_u^2-m_c^2\big)\big(m_c^2-m_t^2\big)\big(m_u^2-m_t^2\big)}{v^8}\,J_q\,, \qquad (3.6)
\end{aligned}
$$

where $\xi_r^u=\xi_r^{u1}+\xi_r^{u2}$ and $\xi_r^d=\xi_r^{d1}-\xi_r^{d2}$. The expressions for $d_{c,t}$ and $d_{s,b}$ can be simply derived from eqs. (3.5) and (3.6), respectively, by cyclically changing the quark labels.[2]

In the lepton sector, we get from eq. (3.3) the electron EDM

$$
\begin{aligned}
d_e &= \frac{\sqrt{2}\,e\,v}{\Lambda^2}\,\mathrm{Im}\big(Y_e^\dagger\Delta_{\ell1} - Y_e^\dagger\Delta_{\ell2}\big)_{11} \\
&= \frac{\sqrt{2}\,e\,v}{\Lambda^2}\Big[\xi_{12}^\ell\,\mathrm{Im}\big(Y_e^\dagger\mathsf{ABA}^2\big)_{11} + \xi_{16}^\ell\,\mathrm{Im}\big(Y_e^\dagger\mathsf{AB}^2\mathsf{A}^2\big)_{11}\Big]\,, \qquad (3.7)
\end{aligned}
$$

where $\xi_r^\ell=\xi_r^{\ell1}-\xi_r^{\ell2}$, we have ignored $\mathrm{Im}\,\xi_r^\ell$, and here $\mathsf{A}=Y_\nu Y_\nu^\dagger$ and $\mathsf{B}=Y_e Y_e^\dagger$. If neutrinos are Dirac particles, analogously to d_d, we obtain

$$
d_e^{\rm D} = \frac{32e\,m_e}{\Lambda^2}\left[\xi_{12}^\ell + \frac{2\big(m_\mu^2+m_\tau^2\big)}{v^2}\,\xi_{16}^\ell\right]\frac{\big(m_\mu^2-m_\tau^2\big)\big(m_1^2-m_2^2\big)\big(m_2^2-m_3^2\big)\big(m_3^2-m_1^2\big)}{v^8}\,J_\ell\,, \qquad (3.8)
$$

where $J_\ell=\mathrm{Im}\big(U_{e2}U_{\mu3}U_{e3}^*U_{\mu2}^*\big)$ is a Jarlskog invariant for $U_{\rm PMNS}$.

In the case of Majorana neutrinos, if $\nu_{k,R}$ are degenerate, $M_\nu=\mathcal{M}\mathbb{1}$, and O is a real orthogonal matrix,[3] from eq. (2.15) we have

$$
\mathsf{A} = \frac{2}{v^2}\,\mathcal{M}\,U_{\rm PMNS}\,\hat{m}_\nu U_{\rm PMNS}^\dagger \qquad (3.9)
$$

and consequently

$$
d_e^{\rm M} = \frac{32e\,m_e\mathcal{M}^3}{\Lambda^2 v^8}\big(m_\mu^2-m_\tau^2\big)\big(m_1-m_2\big)\big(m_2-m_3\big)\big(m_3-m_1\big)\,\xi_{12}^\ell\,J_\ell\,, \qquad (3.10)
$$

[2] It is worth commenting that since $\mathrm{Im}\,\xi_r\propto \mathrm{Det}[\mathsf{A},\mathsf{B}]$, due to the reality of the coefficients $\xi_{jkl\cdots}$ in the infinite series expansion of Δ, and since A and B are Hermitian, d_q would be identically zero if there were only one generation of fermions. The same applies to the lepton sector.

[3] Since the lepton Lagrangian with $\nu_{k,R}$ being degenerate is $\mathrm{O}(3)_\nu$ symmetric, one could transform this real O into a unit matrix [37].

the ξ_{16}^{ℓ} term having been neglected. Since $m_k \ll \mathcal{M}$, we can see that $d_e^{\rm D}$ is highly suppressed relative to $d_e^{\rm M}$. The formulas for $d_{\mu,\tau}^{\rm D}$ and $d_{\mu,\tau}^{\rm M}$ can be readily found from eqs. (3.8) and (3.10), respectively, by cyclically changing the mass subscripts.

In the discussion above, d_e arises from the CP-violating Dirac phase δ in $U_{\rm PMNS}$, and the Majorana phases $\alpha_{1,2}$ therein do not participate. However, if $\nu_{k,R}$ are not degenerate, nonzero $\alpha_{1,2}$ can bring about an additional effect on d_e, even with a real $O \neq \mathbb{1}$. With a complex O, the phases in it may give rise to an extra contribution to d_e, whether or not $\nu_{k,R}$ are degenerate. The formulas for d_e in these scenarios are more complicated than eq. (3.10) and are not shown here, but we will explore some of them numerically in the next section.

The various contributions to the fermion EDMs that we have considered have high powers in Yukawa couplings. Since the MFV hypothesis presupposes that all CP-violation effects originate from the Yukawa couplings, the high orders in them reflect the fact that nonvanishing EDMs in the SM begin to appear at the three-loop level for quarks and in higher loops for the electron. One may wonder whether these are the only contributions to fermion EDMs under the MFV framework. The answer is no because one can realize fermion EDMs by combining some lower-order Yukawa terms from the MFV operators with SM loop diagrams, such as those contributing to quark EDMs in the SM. Nevertheless, hereafter we will not include such type of possible contributions. The contributions that we have already covered should provide a good idea about how fermion EDMs are generated in the presence of MFV. For definiteness, we will apply numerically the results we have acquired and discuss some of their implications.

4 Numerical analysis

We will first treat the neutron EDM, d_n, evaluated from the quark contributions and infer from its data a bound on the scale of quark MFV. We will also look at how MFV affects the contribution of the strong θ-term to d_n. Proceeding to the lepton sector, we will devote much of the section to the electron EDM, and briefly deal with the muon and tau EDMs, in order to explore limitations on the scale of leptonic MFV. Afterwards, we will examine constraints from CP-violating electron-nucleon interactions which were probed by recent searches for atomic and molecular EDMs. Finally, we will address potential restrictions from some CP-conserving processes.

4.1 Neutron EDM

In calculating quark EDMs, as in eqs. (3.5) and (3.6), one needs to take into account the running of the quark masses due to QCD evolution. We adopt the mass ranges $m_u = 0.00139^{+0.00042}_{-0.00041}$, $m_d = 0.00285^{+0.00049}_{-0.00048}$, $m_s = 0.058^{+0.018}_{-0.012}$, $m_c = 0.645^{+0.043}_{-0.085}$, $m_b = 2.90^{+0.16}_{-0.06}$, and $m_t = 174.2 \pm 1.2$, all in GeV, at a renormalization scale $\mu = m_W$ from ref. [38]. With the central values of these masses and the quark Jarlskog parameter

$J_q = \left(3.02^{+0.16}_{-0.19}\right) \times 10^{-5}$ from the latest fit by CKMfitter [39], we arrive at

$$
\begin{aligned}
d_u &= \frac{1.4\times 10^{-35}\, e\,\mathrm{cm}}{\Lambda^2/\mathrm{GeV}^2}\left(\xi^u_{15} + \xi^u_{17}\right), \qquad d_d = \frac{1.3\times 10^{-29}\, e\,\mathrm{cm}}{\Lambda^2/\mathrm{GeV}^2}\left(\xi^d_{12} + 0.00028\, \xi^d_{16}\right), \\
d_s &= \frac{-2.6\times 10^{-28}\, e\,\mathrm{cm}}{\Lambda^2/\mathrm{GeV}^2}\left(\xi^d_{12} + 0.00028\, \xi^d_{16}\right),
\end{aligned} \tag{4.1}
$$

where $\xi^u_r = \xi^{u1}_r + \xi^{u2}_r$ and $\xi^d_r = \xi^{d1}_r - \xi^{d2}_r$. Evidently, the s-quark effect may be dominant.

To determine the neutron EDM, one needs to connect it to the quark-level quantities. The relation between d_n and $d_{u,d,s}$ can be parameterized as

$$
d_n = \eta_n\left(\rho^u_n\, d_u + \rho^d_n\, d_d + \rho^s_n\, d_s\right), \tag{4.2}
$$

where $\eta_n = 0.4$ accounts for corrections due to the QCD evolution from $\mu = m_W$ down to the hadronic scale [40] and the values of the parameters $\rho^{u,d,s}_n$ depend on the model for the matrix elements $\langle n|\bar{q}\sigma^{\kappa\omega}q|n\rangle = \rho^q_n\, \bar{u}_n\sigma^{\kappa\omega}u_n$. For instance, in the constituent quark model $\rho^d_n = \frac{4}{3} = -4\rho^u_n$ and $\rho^s_n = 0$ [1], whereas in the parton quark model $\rho^u_n = -0.508$, $\rho^d_n = 0.746$, and $\rho^s_n = -0.226$ [5]. From the various models proposed in the literature [1, 5, 41], we may conclude that

$$
-0.78 \le \rho^u_n \le -0.17\,, \qquad 0.7 \le \rho^d_n \le 2.1\,, \qquad -0.35 \le \rho^s_n \le 0\,. \tag{4.3}
$$

In view of these numbers and eq. (4.1), we can ignore the d_u and ξ^d_{16} terms. Hence, taking the extreme values $\rho^d_n = 2.1$ and $\rho^s_n = -0.35$, as well as scanning over the quark mass and J_q ranges quoted above to maximize d_n, we get

$$
d_n = \frac{8.4\times 10^{-29}\, e\,\mathrm{cm}}{\Lambda^2/\mathrm{GeV}^2}\,\xi^d_{12}\,. \tag{4.4}
$$

It is then interesting to note that $\Lambda/\left|\xi^d_{12}\right|^{1/2} = 100\,\mathrm{GeV}$ translates into $d_n = 8.4 \times 10^{-33}\, e\,\mathrm{cm}$, which is roughly similar to the SM expectation $d^{\mathrm{SM}}_n \sim 10^{-32}$-$10^{-31}\, e\,\mathrm{cm}$ [10–13]. Comparing eq. (4.4) with the current data $|d_n|_{\mathrm{exp}} < 2.9 \times 10^{-26}\, e\,\mathrm{cm}$ at 90% CL [9], we extract

$$
\frac{\Lambda}{\left|\xi^d_{12}\right|^{1/2}} > 0.054\ \mathrm{GeV}\,, \tag{4.5}
$$

which is not strict at all. Less extreme choices of $\rho^{d,s}_n$ would lead to even weaker bounds. We conclude that the present neutron-EDM limit cannot yield a useful restriction on Λ.

One can also look at the contributions of quark chromo-EDMs to the neutron EDM [1]. The relevant operators are obtainable from the MFV quark-EDM operators by replacing $W^{\mu\nu}_a$ and τ_a with the gluon field strength tensor $G^{\mu\nu}_c$ and the color SU(3) generators λ_c, respectively. The extracted constraints on Λ are similar.

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4.2 MFV contribution to strong theta term

Besides the quark (chromo-)EDMs, another contributor to the neutron EDM is the theta term of QCD [42–44], which in the SM is given by [5]

$$\mathcal{L}_{\bar\theta} \;=\; \frac{-\bar\theta\, g_{\rm s}^2}{32\pi^2}\, \epsilon_{\kappa\upsilon\phi\omega} G_c^{\kappa\upsilon} G_c^{\phi\omega}\,, \tag{4.6}$$

where $\bar\theta = \theta + \arg{\rm Det}(Y_u Y_d)$ involves the bare θ-parameter, $g_{\rm s}$ is the strong coupling constant, and $\epsilon_{0123} = +1$. The inclusion of MFV causes $\bar\theta$ to be modified to

$$\bar\theta_{\rm MFV} \;=\; \theta + \arg{\rm Det}\big(\Delta_{qu}^\dagger Y_u\, \Delta_{qd}^\dagger Y_d\big) \;=\; \bar\theta + \arg{\rm Det}\,\Delta_{qu}^\dagger + \arg{\rm Det}\,\Delta_{qd}^\dagger\,, \tag{4.7}$$

where $\Delta_{qu,qd}$ have the same expression as Δ in eq. (2.5), but generally different coefficients ξ_r. Although the addition of these new factors to the Yukawa Lagrangian amounts only to a redefinition of $Y_{u,d}$ and hence has no direct experimental implications after the quark mass matrices are diagonalized, we can expect that $\Delta_{qu,qd}$ are close to the unit matrix. Our interest is in investigating the size of $\arg{\rm Det}\,\Delta_{qu,qd}$ in eq. (4.7) and thus whether or not their presence makes the fine tuning between the two terms in $\bar\theta$ worse.

To compute ${\rm Det}\,\Delta_{qu}$, we first write the real and imaginary parts of ξ_r in terms of real constants ϱ_r and $\imath_r$ as

$${\rm Re}\,\xi_r \;=\; \varrho_r\,, \qquad {\rm Im}\,\xi_r \;=\; \imath_r\, J_\xi \tag{4.8}$$

with J_ξ given in eq. (3.4). Upon applying the Cayley-Hamilton identity, we then get

$${\rm Det}\,\Delta_{qu} \;=\; \tfrac{1}{6}\big({\rm Tr}\Delta_{qu}\big)^3 - \tfrac{1}{2}\,{\rm Tr}\Delta_{qu}\,{\rm Tr}\big(\Delta_{qu}^2\big) + \tfrac{1}{3}\,{\rm Tr}\big(\Delta_{qu}^3\big)\,, \tag{4.9}$$

which leads us to

$${\rm Re}\big({\rm Det}\,\Delta_{qu}\big) \;\simeq\; \varrho_1^3 + \varrho_1^2\big(\varrho_2\, y_t^2 + \varrho_4\, y_t^4\big)\,, \tag{4.10}$$

$$\begin{aligned} J_\xi^{-1}\,{\rm Im}\big({\rm Det}\,\Delta_{qu}\big) \;\simeq\;& -\varrho_2\big[\varrho_2\,\varrho_{15} + \varrho_3\big(\varrho_{13}-\varrho_{14}\big) + \varrho_5\,\varrho_9 + \varrho_7\,\varrho_{11}\big] - \varrho_3\big(\varrho_3\,\varrho_{12} - \varrho_4\,\varrho_{11} - \varrho_6\,\varrho_9\big) \\ & - \big(\varrho_6-\varrho_7\big)\big(\varrho_2\,\varrho_{10} + \varrho_3\,\varrho_8 + \varrho_4\,\varrho_5 - \varrho_6\,\varrho_7\big) - \varrho_2\big(\varrho_4\,\varrho_{17} + \varrho_9\,\varrho_{13}\big) y_t^4 \\ & + \big[-\varrho_2\big(\varrho_2\,\varrho_{17} + \varrho_4\,\varrho_{15} - \varrho_6\,\varrho_{14} + \varrho_7\,\varrho_{13} + \varrho_9\,\varrho_{11}\big) - \varrho_3\big(\varrho_6\,\varrho_{12} - \varrho_8\,\varrho_9\big) \\ & \quad - \big(\varrho_6-\varrho_7\big)\big(\varrho_4\,\varrho_{10} - \varrho_6\,\varrho_9\big)\big] y_t^2 \\ & + \varrho_1\big\{-\varrho_2\,\varrho_{17} - \varrho_3\,\varrho_{16} + \varrho_4\,\varrho_{15} + \varrho_5\,\varrho_{12} + \varrho_6\,\varrho_{13} - \varrho_7\,\varrho_{14} + \varrho_8\,\varrho_{11} - \varrho_9\,\varrho_{10} \\ & \qquad + \big[2\,\varrho_2\,\imath_1 - \varrho_6\,\varrho_{16} + \varrho_8\big(\varrho_{13}-\varrho_{14}\big) + \varrho_{11}\,\varrho_{12}\big] y_t^2 \\ & \qquad + \big(2\,\varrho_4\,\imath_1 + \varrho_{12}\,\varrho_{13}\big) y_t^4\big\} \\ & + \varrho_1^2\big(3\,\imath_1 + \imath_2\, y_t^2 + \imath_4\, y_t^4\big)\,, \end{aligned} \tag{4.11}$$

where $y_q = \sqrt{2}\,m_q/v$ and on the right-hand sides we have ignored terms suppressed by powers of $y_{u,c,d,s,b}$. The formulas for ${\rm Det}\,\Delta_{qd}$ are similar.

Since $y_t^2 \sim 1 \gg y_{u,c,d,s,b}^2$, the requirement that $\Delta_{qu,qd} \simeq \mathbb{1}$ implies that

$$\varrho_1 \;\simeq\; 1\,, \qquad \big|\varrho_{2,4}\big| \;\ll\; 1\,, \qquad \big|\varrho_{3,5,6,\dots,17}\big| \;\le\; \mathcal{O}(1)\,, \qquad \big|\imath_{1,2,\dots,17}\big| \;\le\; \mathcal{O}(1)\,, \tag{4.12}$$

Using these conditions and the quark parameter values employed earlier, we have checked numerically that eqs. (4.10) and (4.11) approximate well the exact (but much lengthier) expressions, especially if $|\varrho_{2,4}| \le \mathcal{O}(0.001)$. Moreover, we get $|\arg{\rm Det}\,\Delta_{qu,qd}| < 10^{-21}$. Obviously, the MFV effect is negligible compared to the present bound $\bar\theta_{\rm exp} < 10^{-10}$ [5].

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Observable	NH	IH
$\sin^2\theta_{12}$	0.308 ± 0.017	0.308 ± 0.017
$\sin^2\theta_{23}$	$0.425^{+0.029}_{-0.027}$	$0.437^{+0.059}_{-0.029}$
$\sin^2\theta_{13}$	$0.0234^{+0.0022}_{-0.0018}$	0.0239 ± 0.0021
δ/π	$1.39^{+0.33}_{-0.27}$	$1.35^{+0.24}_{-0.39}$
$\Delta m^2_{21} = m_2^2 - m_1^2$	$\big(7.54^{+0.26}_{-0.22}\big) \times 10^{-5}\ \mathrm{eV}^2$	$\big(7.54^{+0.26}_{-0.22}\big) \times 10^{-5}\ \mathrm{eV}^2$
$\Delta m^2 = \lvert m_3^2 - (m_1^2 + m_2^2)/2\rvert$	$\big(2.44^{+0.08}_{-0.06}\big) \times 10^{-3}\ \mathrm{eV}^2$	$(2.40 \pm 0.07) \times 10^{-3}\ \mathrm{eV}^2$

Table 1. Results of a recent fit to the global data on neutrino oscillations [45]. The neutrino mass hierarchy may be normal $(m_1 < m_2 < m_3)$ or inverted $(m_3 < m_1 < m_2)$.

4.3 Electron EDM

To evaluate the EDMs of charged leptons, we need the values of the various pertinent quantities, such as the elements of the neutrino mixing matrix $U_{\rm PMNS}$ as well as the masses of neutrinos and charged leptons. If neutrinos are Dirac in nature, the parametrization of $U_{\rm PMNS}$ is the same as $V_{\rm CKM}$ in eq. (2.8). In table 1, we have listed $\sin^2\theta_{kl}$ and δ from a recent fit to global neutrino data [45]. Most of these numbers depend on whether neutrino masses fall into a normal hierarchy (NH), where $m_1 < m_2 < m_3$, or an inverted one (IH), where $m_3 < m_1 < m_2$. If neutrinos are Majorana particles, $U_{\rm PMNS}$ contains an additional matrix $P = \mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$ multiplied from the right, where $\alpha_{1,2}$ are the Majorana phases which remain unknown.

Also listed in table 1 are the differences in neutrinos' squared masses, which are well determined. In contrast, our knowledge about the absolute scale of the masses is still poor. Some information on the latter is available from tritium β-decay experiments [46, 47]. In particular, their latest results imply an upper limit on the (electron based) antineutrino mass of $m_{\bar\nu_e} < 2\,\mathrm{eV}$ [9]. Planned measurements will be more sensitive by an order of magnitude [46, 47]. Indirectly, stronger bounds on the total mass $\Sigma_k m_k = m_1 + m_2 + m_3$ can be inferred from cosmological observations. Specifically, the Planck Collaboration extracted $\Sigma_k m_k < 0.23\,\mathrm{eV}$ at 95% CL from cosmic microwave background (CMB) and baryon acoustic oscillation (BAO) measurements [48]. Including additional observations can improve this limit to $\Sigma_k m_k < 0.18\,\mathrm{eV}$ [49]. On the other hand, there are also recent analyses that have turned up tentative indications of bigger masses and hence quasidegeneracy (QD) among the neutrinos. The South Pole Telescope Collaboration reported $\Sigma_k m_k = (0.32 \pm 0.11)\,\mathrm{eV}$ from the combined CMB, BAO, Hubble constant, and Sunyaev-Zeldovich selected galaxy cluster abundances dataset [50]. This is compatible with the later finding $\Sigma_k m_k = (0.36 \pm 0.10)\,\mathrm{eV}$ favored by the Baryon Oscillation Spectroscopic Survey CMASS Data Release 11 [51]. In the following numerical work, we take this QD possibility into consideration.

If neutrinos are of Dirac nature, we first note that the mass difference definitions in table 1 imply that $\big(m_1^2 - m_2^2\big)\big(m_2^2 - m_3^2\big)\big(m_3^2 - m_1^2\big) = \Delta m^2_{21}\big(\Delta m^2\big)^2 - \frac{1}{4}\big(\Delta m^2_{21}\big)^3$, which is independent of m_k individually. Then, scanning the parameter ranges in table 1 to

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maximize $d_e^{\rm D}$ in eq. (3.8), we obtain for the NH (IH) of neutrino masses [23]

$$d_e^{\rm D} \;=\; \frac{1.3\;(1.3)\times 10^{-99}\, e\,{\rm cm}}{\Lambda^2/{\rm GeV}^2}\,\xi_{12}^\ell\,, \tag{4.13}$$

after dropping the ξ_{16}^ℓ part. This is negligible compared to the latest data $|d_e|_{\rm exp} < 8.7\times 10^{-29}\, e\,{\rm cm}$ reported by ACME [7], and the smallness is due to the light neutrino masses being tiny.

In contrast, if neutrinos are Majorana particles, d_e can be sizable. To see this, we begin with the simplest possibility that $\nu_{k,R}$ are degenerate, $M_\nu = \mathcal{M}\mathbb{1}$, and the O matrix in eq. (2.15) is real. For this scenario, d_e is already given in eq. (3.10), which depends on the choice for one of $m_{1,2,3}$ after the mass data are included. Scanning again the empirical parameter ranges in table 1 to maximize $d_e^{\rm M}$, we obtain for $m_1 = 0$ $\big(m_3 = 0\big)$ in the NH (IH) case

$$\frac{d_e^{\rm M}}{e\,{\rm cm}} \;=\; 4.7\;(0.52)\times 10^{-23}\bigg(\frac{\mathcal{M}}{10^{15}\,{\rm GeV}}\bigg)^3\bigg(\frac{\rm GeV}{\hat\Lambda}\bigg)^2, \tag{4.14}$$

where $\hat\Lambda = \Lambda/\big|\xi_{12}^\ell\big|^{1/2}$. Then $|d_e^{\rm exp}| < 8.7\times 10^{-29}\; e\,{\rm cm}$ [7] implies

$$\hat\Lambda \;>\; 0.74\;(0.24)\;{\rm TeV}\,\bigg(\frac{\mathcal{M}}{10^{15}\,{\rm GeV}}\bigg)^{3/2}. \tag{4.15}$$

Although this might suggest that $\hat\Lambda$ could be extremely high with an excessively large $\mathcal{M}$, there are limitations on $\mathcal{M}$. Since the series in eq. (2.5), which implicitly incorporates arbitrarily high powers of A and B, has to converge, their eigenvalues need to be capped [23, 35]. Otherwise, the coefficients ξ_r might not converge to finite numbers after the reduction of Δ from its infinite series expansion to eq. (2.5). In the lepton sector, we only need to be concerned with $\mathsf{A} = Y_\nu Y_\nu^\dagger$, as $\mathsf{B} = Y_e Y_e^\dagger$ already has diminished eigenvalues. Thus one may demand that the eigenvalues of A are at most 1. However, since MFV may emerge from calculations of SM loops, the expansion quantities may be more naturally be $\mathsf{A}/(16\pi^2)$ and $\mathsf{B}/(16\pi^2)$, in which case the maximum eigenvalue of A cannot be more than $16\pi^2$. As another alternative, one may impose the perturbativity condition on the Yukawa couplings, namely $(Y_\nu)_{jk} < \sqrt{4\pi}$ [52], implying a cap of 4π instead.

In this paper we require the eigenvalues of $\mathsf{A} = Y_\nu Y_\nu^\dagger$ not to exceed unity. Furthermore, in our illustrations we will choose the largest eigenmasses of the right-handed neutrinos subject to this condition. For the example resulting in eq. (4.15), this translates into the maximal value $\mathcal{M} = 6.16\;(6.22)\times 10^{14}\,{\rm GeV}$ in the NH (IH) case and consequently

$$\hat\Lambda \;>\; 0.36\;(0.12)\;{\rm TeV}\;. \tag{4.16}$$

This constraint would weaken if $m_{1(3)} > 0$. For comparison with later illustrations, the $\mathcal{M}$ numbers above translate into $d_e^{\rm M}\hat\Lambda^2 = 1.1\;(0.13)\times 10^{-23}\, e\,{\rm cm}$.

Now, with $\nu_{k,R}$ still degenerate, $M_\nu = \mathcal{M}\mathbb{1}$, but O complex, A has a less simple expression,

$$\mathsf{A} \;=\; \frac{2}{v^2}\,\mathcal{M}\,U_{\rm PMNS}\,\hat m_\nu^{1/2} O O^\dagger \hat m_\nu^{1/2} U_{\rm PMNS}^\dagger\,, \tag{4.17}$$

which is to be applied to $d_e^{\rm M}$ in eq. (3.7). From now on, we ignore the ξ_{16}^{ℓ} parts. We can always write $OO^\dagger = e^{2i\mathsf{R}}$ with a real antisymmetric matrix

$$\mathsf{R} = \begin{pmatrix} 0 & r_1 & r_2 \\ -r_1 & 0 & r_3 \\ -r_2 & -r_3 & 0 \end{pmatrix} . \tag{4.18}$$

Since $OO^\dagger$ is not diagonal, A will in general have dependence on the Majorana phases in $U_{\rm PMNS}$ if they are not zero. To concentrate first on demonstrating how O can give rise to CP violation beyond that induced by the Dirac phase δ in $U_{\rm PMNS}$, we switch off the Majorana phases, $\alpha_{1,2} = 0$. Subsequently, for illustrations, we pick two possible sets of $r_{1,2,3}$, namely, (i) $r_1 = -r_2 = r_3 = -\rho$ and (ii) $r_1 = 2r_2 = 3r_3 = \rho$, and employ the central values of the data in table 1, particularly

$$\delta = 1.39\pi \text{ (NH) or } 1.35\pi \text{ (IH)} . \tag{4.19}$$

We present in figure 1(a)-(d) the resulting $d_e^{\rm M}\hat{\Lambda}^2$ versus ρ for the NH (IH) of light neutrino masses with $m_{1(3)} = 0$. Since δ is not yet well-determined, we also depict the variations of $d_e^{\rm M}$ over the one-sigma ranges of δ quoted in table 1 with the lighter blue and red bands. We remark that the boundaries of the bands do not necessarily correspond to the upper or lower ends of the δ ranges. Within these bands, the blue and red solid curves belong, respectively, to the NH and IH central values in eq. (4.19). We also graph the (dashed) curves for $\delta = 0$ to reveal the CP-violating role of O alone. The solid and dashed curves in figure 1(a,b) are roughly the mirror images about $\rho = 0$ of the corresponding curves given in ref. [23] for $r_{1,2,3} = \rho$.

In figure 1(a)-(d), as well as in ref. [23], we have only examples where the lightest neutrinos are massless and, consequently, the neutrino masses sum up to $\Sigma_k m_k = 0.059$ eV and 0.099 eV in the NH and IH cases, respectively. These numbers satisfy the aforementioned bound from cosmological data, $\Sigma_k m_k < 0.18\,\text{eV}$ [49]. In light of the hints of quasidegenerate neutrinos with $\Sigma_k m_k \sim 0.3\,\text{eV}$ from other cosmological observations [50, 51], which still need confirmation by future measurements, here we also provide a couple of instances in figure 1(e,f) after making the NH choice $m_1 = 0.1\,\text{eV} < m_2 < m_3$, which translates into $\Sigma_k m_k = 0.31\,\text{eV}$.

All these examples in figure 1 clearly indicate that O can generate potentially significant new effects of CP violation which can exceed those of δ. The latter point is most noticeable in figure 1(b,d) from comparing the IH $\delta \neq 0$ regions at $\rho \sim 0$ with the extreme values of the corresponding IH $\delta = 0$ curves.

With $\alpha_{1,2} = 0$, the CP-violating impact of O can still materialize even if it is real provided that $\nu_{k,R}$ are not degenerate. In that case

$$\mathsf{A} = \frac{2}{v^2} U_{\rm PMNS} \hat{m}_\nu^{1/2} O M_\nu O^\dagger \hat{m}_\nu^{1/2} U_{\rm PMNS}^\dagger \tag{4.20}$$

based on eq. (2.15). For instance, assuming that O is real, $O = e^{\mathsf{R}}$ with $r_1 = -r_2 = r_3 = -\rho$, and that $M_\nu = \mathcal{M}\,\text{diag}(1, 0.8, 1.2)$, we show the resulting $d_e^{\rm M}\hat{\Lambda}^2$ versus ρ in figure 2(a),

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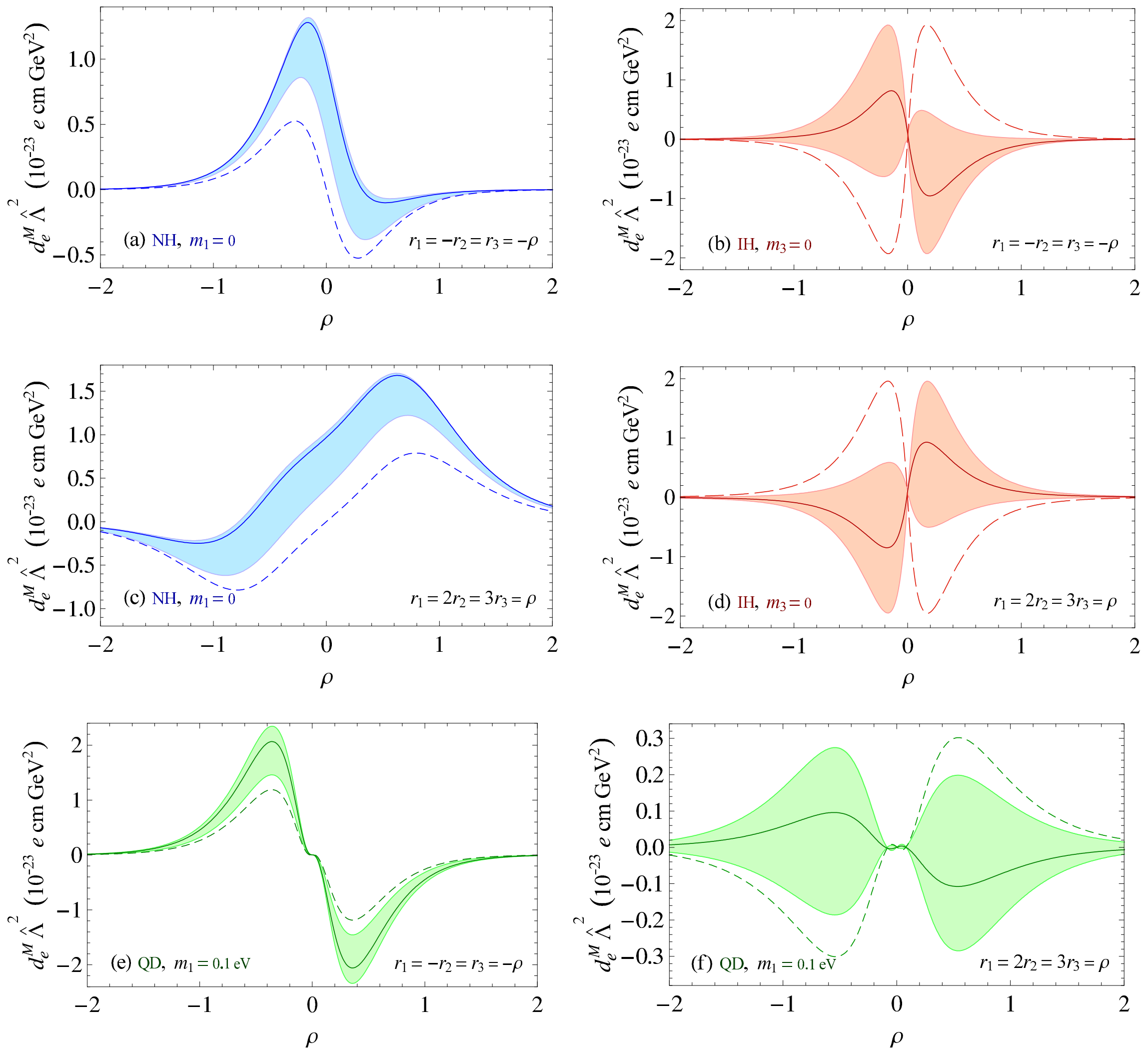

Figure 1. Dependence of $d_e^{\rm M}$ times $\hat{\Lambda}^2 = \Lambda^2/\left|\xi_{12}^{\ell}\right|$ on the O-matrix parameter ρ in the absence of the Majorana phases, $\alpha_{1,2} = 0$, for degenerate $\nu_{k,R}$ and complex O with (a,b,e) $r_1 = -r_2 = r_3 = -\rho$ and (c,d,f) $r_1 = 2r_2 = 3r_3 = \rho$, as explained in the text. The lighter blue, red, and green bands reflect the one-sigma ranges of δ, while the solid and dashed curves correspond, respectively, to its central values in eq. (4.19) and to $\delta = 0$. In (e,f) and other QD plots below, only the NH scenario is assumed, unless stated otherwise.

where only the $\delta \neq 0$ curves are nonvanishing and the sinusoidal behavior of d_e is visible. As in the previous figure, we also display the variations of $d_e^{\rm M}$ over the one-sigma ranges of δ from table 1. The solid curves in figure 2(a) are similar to their $r_{1,2,3} = \rho$ counterparts in ref. [23]. As another example, we select again $r_1 = 2r_2 = 3r_3 = \rho$, keeping the other input parameters unchanged, and plot figure 2(b) which differs somewhat qualitatively from figure 2(a). In figure 2(c,d) we graph the QD cases with $m_1 = 0.1\,{\rm eV} < m_2 < m_3$, which turn out to have much smaller $d_e^{\rm M}$ ranges. All of these results further demonstrate the importance of O as an extra source of CP violation.

Turning our attention now to the contribution of the Majorana phases, we first illustrate it for $M_\nu = \mathcal{M}\mathbb{1}$ and $O = e^{i\mathrm{R}}$ with the two sets of $r_{1,2,3}$ chosen in the previous

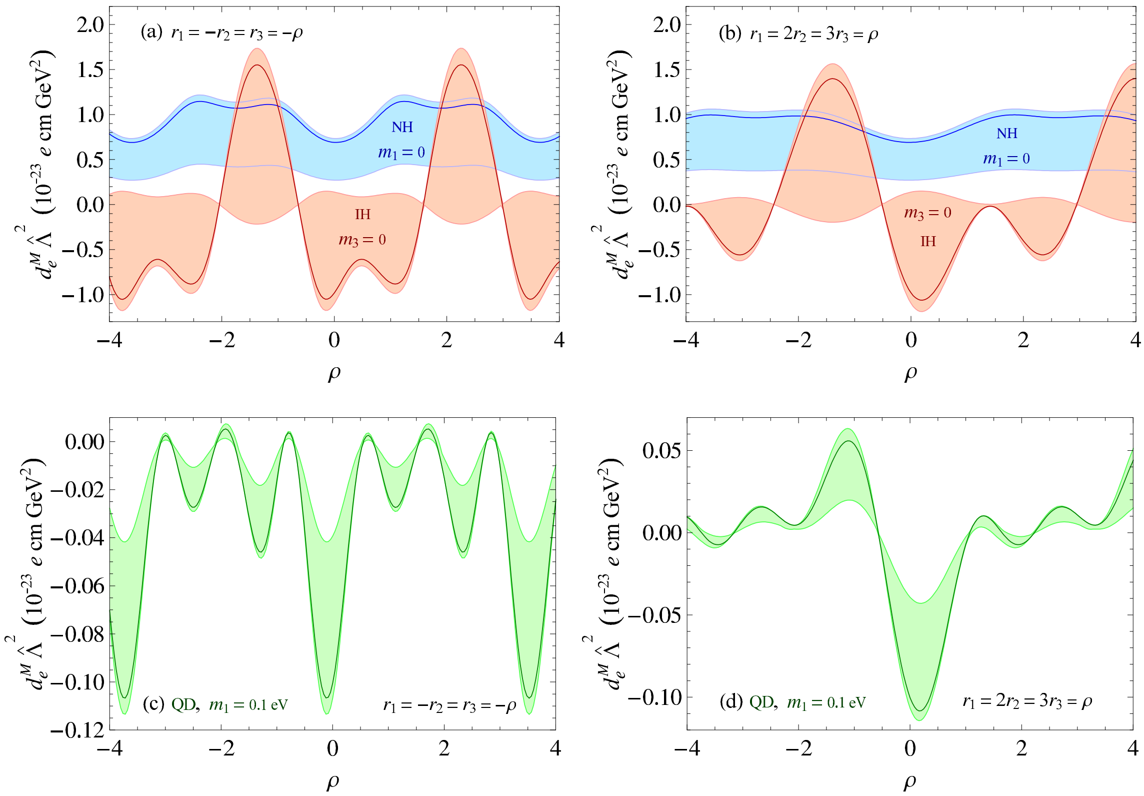

Figure 2. Dependence of $d_e^{\rm M}\hat{\Lambda}^2$ on O-matrix parameter ρ in the absence of Majorana phases, $\alpha_{1,2} = 0$, for nondegenerate $\nu_{k,R}$ with $M_\nu = \mathcal{M}\,{\rm diag}(1, 0.8, 1.2)$ and real $O = e^{\rm R}$ with (a,c) $r_1 = -r_2 = r_3 = -\rho$ and (b,d) $r_1 = 2r_2 = 3r_3 = \rho$, as explained in the text. The lighter blue, red, and green bands reflect the one-sigma ranges of δ, while the solid curves correspond to its central values in eq. (4.19).

paragraph. Thus, fixing $\alpha_1 = 0$ and $\rho = \frac{1}{2}$, we depict the resulting dependence of $d_e^{\rm M}$ on α_2 in figure 3 for nonzero δ within its one-sigma ranges from table 1 and also for $\delta = 0$. For further illustrations, we do the same with $M_\nu = \mathcal{M}\,{\rm diag}(1, 0.8, 1.2)$ and $O = e^{\rm R}$, displaying the results in figure 4. It is noticeable that each of the solid or dashed curves in figures 3 and 4 repeats itself after α_2 changes by 4π, which is attributable to the $e^{i\alpha_2/2}$ dependence of $d_e^{\rm M}$ in these cases. Also, one can verify visually that the solid curves in figures 1 and 3 (2 and 4) are consistent with each other at $\rho = \frac{1}{2}$ and $\alpha_{1,2} = 0$. It is evident from the instances in figures 3 and 4, as well as their counterparts in ref. [23], that the Majorana phases yield additional important CP-violating effects on d_e beyond δ.

It is interesting that some of the CP-violating variables which enter $d_e^{\rm M}$ also affect neutrinoless double-β decay due to the Majorana nature of the electron neutrino. This process is of fundamental importance because it does not conserve lepton number and thus will be evidence for new physics if detected [46, 47]. If there are no other contributions, the rate of neutrinoless double-β decay increases with the square of the effective Majorana mass

$$\begin{aligned} \langle m_{\beta\beta} \rangle &= \left| \sum_k U_{ek}^2\, \hat{m}_k \right| = \left| \left(U_{\rm PMNS}\, \hat{m}_\nu U_{\rm PMNS}^{\rm T} \right)_{11} \right| \\ &= \left| c_{12}^2\, c_{13}^2\, m_1\, e^{i\alpha_1} + s_{12}^2\, c_{13}^2\, m_2\, e^{i\alpha_2} + s_{13}^2\, m_3\, e^{-2i\delta} \right| . \end{aligned} \tag{4.21}$$

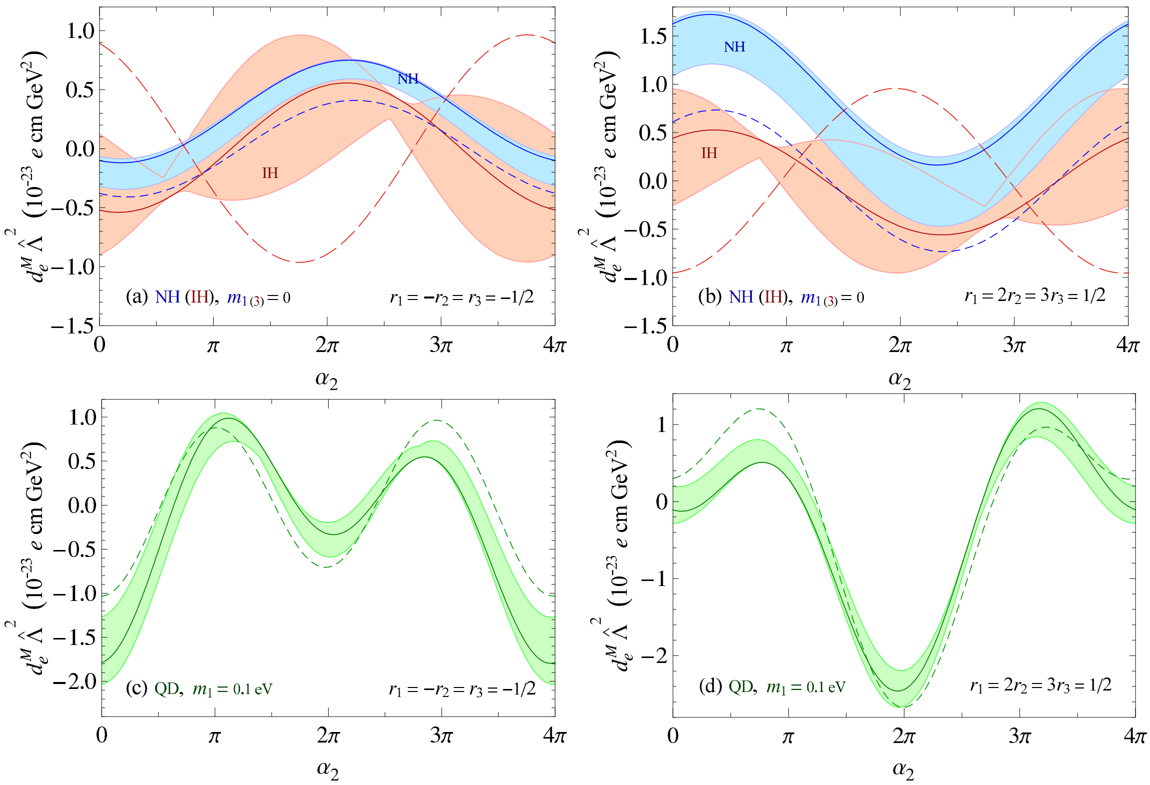

Figure 3. Dependence of $d_e^{\mathrm{M}} \hat{\Lambda}^2$ on α_2 for $\alpha_1 = 0$, degenerate $\nu_{k,R}$, and $O = e^{i\mathsf{R}}$ with (a,c) $r_{1,3} = -r_2 = -\frac{1}{2}$ and (b,d) $r_1 = 2r_2 = 3r_3 = \frac{1}{2}$, as explained in the text. The bands and curves have the same meanings as in preceding figures.

In figure 5 we display several examples of $\langle m_{\beta\beta} \rangle$ versus α_2 for $\alpha_1 = 0$, but not those for $\delta = 0$ to avoid crowding the plots. It is obvious that each of the curves repeats itself after α_2 changes by 2π, which is due to the presence of $e^{i\alpha_2}$ in $\langle m_{\beta\beta} \rangle$, unlike the d_e^{M} curves in figures 3 and 4. The peak values in the third plot of figure 5 are already close to the existing experimental upper limits on $\langle m_{\beta\beta} \rangle$, the best one being 0.12 eV [53–56]. Thus the QD possibility will be tested by forthcoming searches within the next decade, which are expected to have sensitivities reaching 0.04 eV to 0.01 eV [57].

From figures 3–5, one can conclude that d_e^{M} and $\langle m_{\beta\beta} \rangle$ may be correlated. For the MFV scenario under consideration and the parameter choices we made with the central values from table 1, we show in figure 6 some sample relations between the two observables. One can see in particular that the plots in figure 6(a,c) [(d,f)] are related to the solid curves in the first and third (green) graphs of figure 5, respectively, and the corresponding solid curves in figure 3 [figure 4] for $r_1 = 2r_2 = 3r_3 = \frac{1}{2}$. In figure 6 we have also indicated a projected sensitivity of 0.04 eV in future hunts for neutrinoless double-β decay which may be achieved after several years.

The illustrations in figure 6 suggest that, if searches in coming years still yield null results, the acquired limits on d_e and $\langle m_{\beta\beta} \rangle$ will impose significant restrictions on various scenarios based on lepton MFV. On the other hand, unambiguous observations of d_e^{M} and/or neutrinoless double-beta decay will help pin down the favored underlying model and parameter space, under the assumption that the latter process is mediated by a light

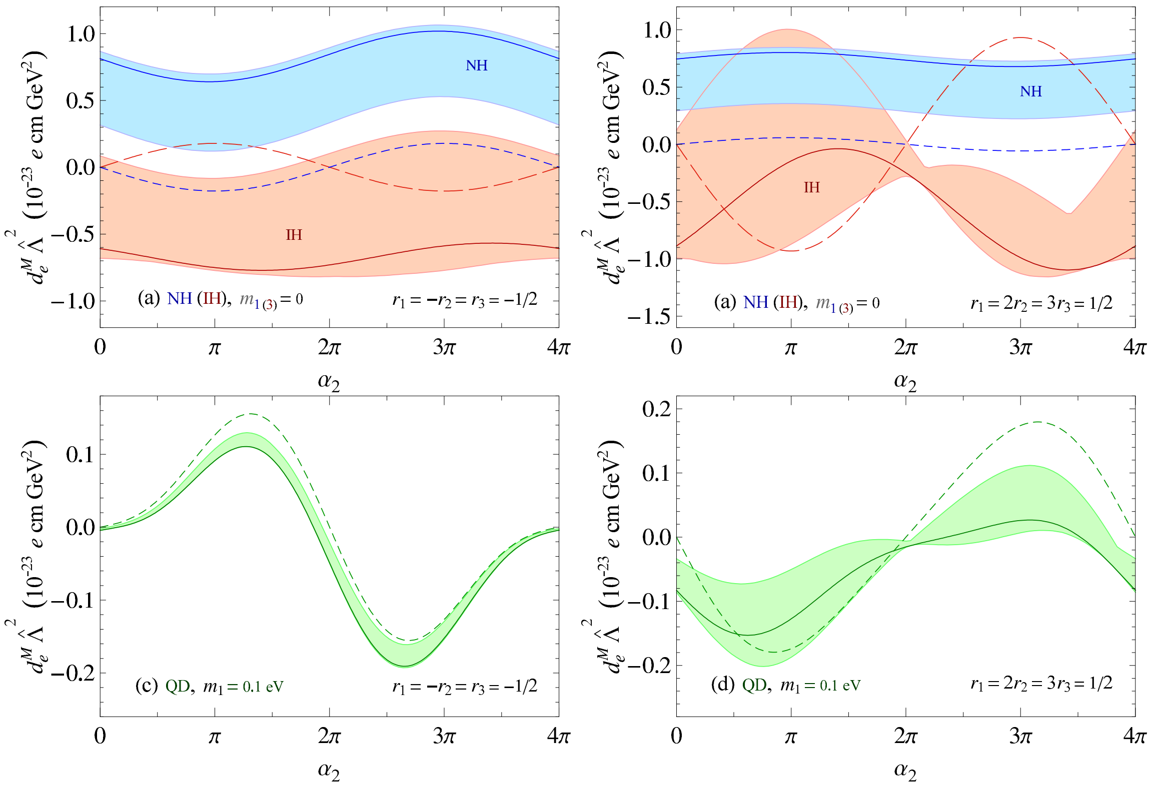

Figure 4. The same as figure 3, except $\nu_{k,R}$ are nondegenerate with $M_\nu = \mathcal{M}\,\mathrm{diag}(1, 0.8, 1.2)$ and $O = e^{\mathsf{R}}$.

Majorana neutrino [46, 47]. The information to be gained from the direct neutrino-mass determination in planned tritium β-decay experiments, with expected sensitivities as low as $0.2\,\mathrm{eV}$ [46, 47], and the total neutrino mass to be inferred from upcoming cosmological data with improved precision will supply complementary constraints and cross checks.

Before moving on, we would like to make some remarks on the situation in which only two right-handed neutrinos are added into the theory. In that case, Y_ν and M_ν as defined in eq. (2.1) are $3{\times}2$ and $2{\times}2$ matrices, respectively. As a natural consequence [58], it is straightforward to realize from eq. (2.14) that $|\mathrm{Det}\,m_\nu| = m_1 m_2 m_3 = 0$, indicating that one of $m_{1,2,3}$ has to vanish. Another difference is that the O matrix in eq. (2.12) is now $3{\times}2$. Accordingly, with $m_1 = 0$ or $m_3 = 0$ we can write respectively [59]

$$O = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} O_2 \qquad \text{or} \qquad O = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} O_2 \tag{4.22}$$

where O_2 is a complex $2{\times}2$ matrix satisfying $O_2 O_2^{\mathrm{T}} = \mathbb{1}_2$, where $\mathbb{1}_2$ is $2{\times}2$ unit matrix. Thus O_2 has 2 free real parameters, whereas O in the presence of 3 right-handed neutrinos has six. All this implies that the specific examples we have provided so far with m_1 or m_3 set to zero are applicable to the situation with only 2 right-handed neutrinos, as the 2 parameters of O_2 are functions of the 6 parameters of O in the case of 3 right-handed

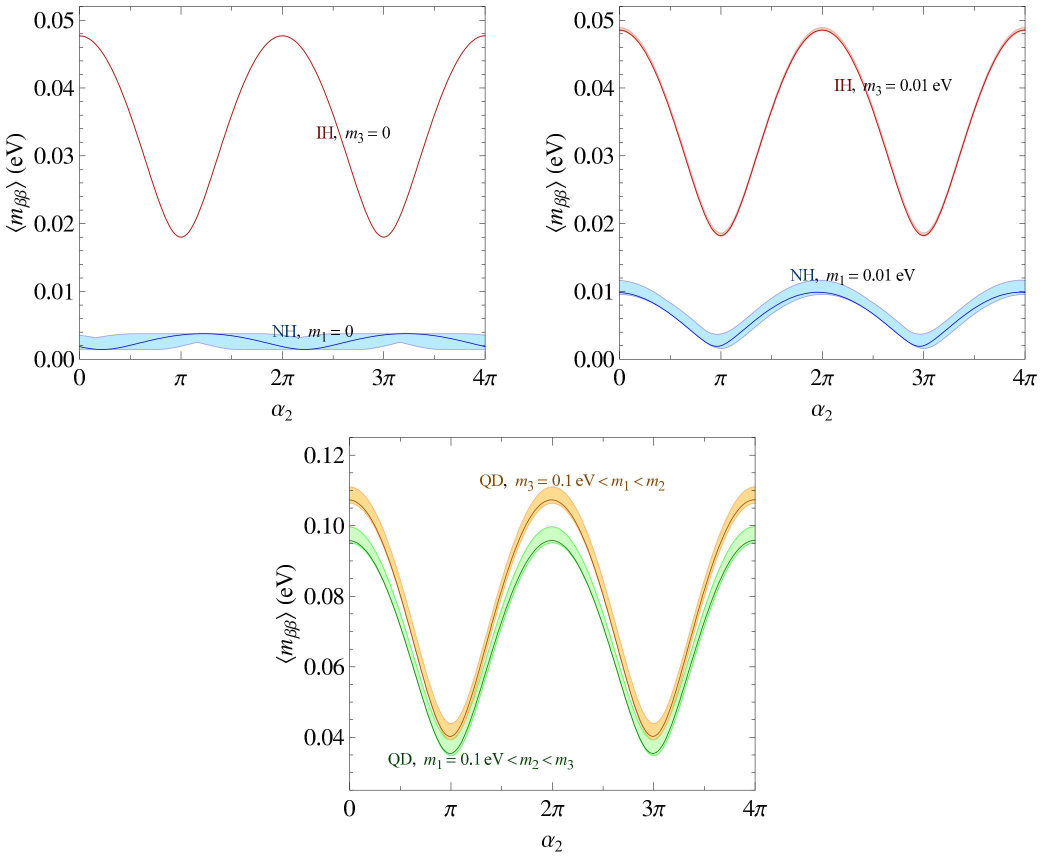

Figure 5. Dependence of effective Majorana mass $\langle m_{\beta\beta}\rangle$ on α_2 for $\alpha_1 = 0$, nonzero δ, and some selections of $m_{1\text{ or }3}$. The bands and solid curves have the same meanings as in previous figures.

neutrinos with $m_{1\,\text{or}\,3} = 0$. We conclude that for d_e the situations with 2 and 3 right-handed neutrinos are similar.

4.4 Muon and tau EDMs

If neutrinos are of Dirac nature, the muon and tau EDMs will be tiny, like $d_e^{\rm D}$. Therefore here, and in the rest of the section, we suppose that neutrinos are Majorana fermions. Furthermore, for definiteness and simplicity, we consider only the scenario in which the right-handed neutrinos are degenerate, $M_\nu = \mathcal{M}\mathbb{1}$, and the orthogonal matrix O is real. For the neutrino parameters, we will adopt the specific values which yielded eq. (4.14) and $\mathcal{M} = 6.16\ (6.22) \times 10^{14}\,\text{GeV}$ in the NH (IH) case with $m_{1(3)} = 0$.

Accordingly, from eq. (3.10) we easily infer the muon and tau EDMs, respectively, to be

$$d_\mu^{\rm M} = d_e^{\rm M}\,\frac{m_\mu\big(m_\tau^2 - m_e^2\big)}{m_e\big(m_\mu^2 - m_\tau^2\big)}\,, \qquad d_\tau^{\rm M} = d_e^{\rm M}\,\frac{m_\tau\big(m_e^2 - m_\mu^2\big)}{m_e\big(m_\mu^2 - m_\tau^2\big)}\,, \tag{4.23}$$

with $d_e^{\rm M}$ in eq. (4.14). Since $d_\tau^{\rm M} \sim -0.06\,d_\mu^{\rm M}$ and the experimental information on d_τ is still imprecise [9], we will not deal with it further.

Hence we get $d_\mu^{\rm M} = -2.3\ (-0.26) \times 10^{-21}\ \text{GeV}^2/\hat\Lambda^2$. Currently the best measured limit on the muon EDM is $|d_\mu|_{\rm exp} < 1.8 \times 10^{-19}\,e\,\text{cm}$ at 95% CL, set by the Muon $(g-2)$

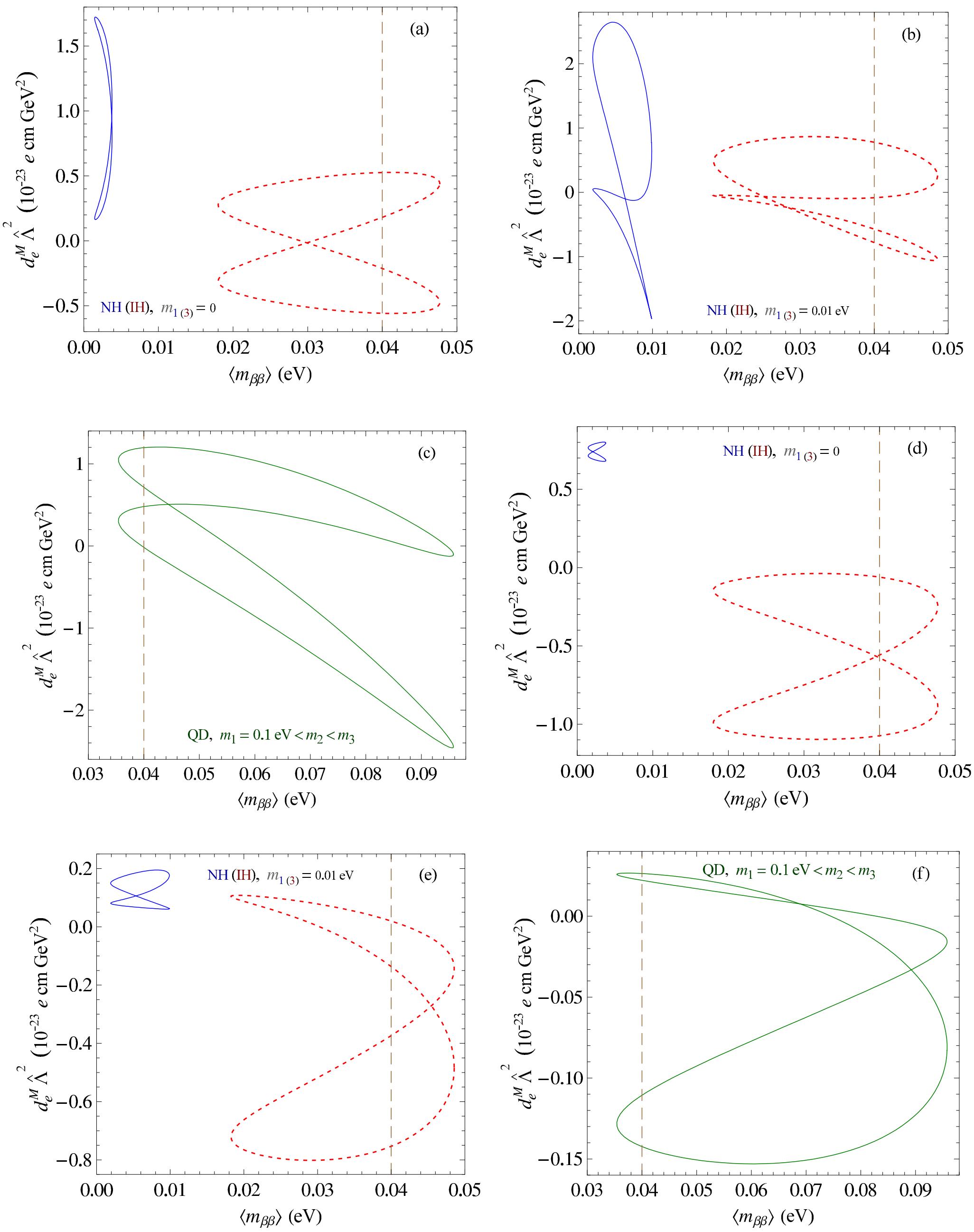

Figure 6. Sample correlations between $d_e^{\mathrm{M}} \hat{\Lambda}^2$ and $\langle m_{\beta\beta} \rangle$ over $0 \leq \alpha_2 \leq 4\pi$ for $\alpha_1 = 0$ and the central values of δ in the cases of (a,b,c) degenerate $\nu_{k,R}$ and $O = e^{i\mathrm{R}}$ and (d,e,f) nondegenerate $\nu_{k,R}$ and $O = e^{\mathrm{R}}$, all with $r_1 = 2r_2 = 3r_3 = \frac{1}{2}$, as described in the text. The vertical dashed lines mark a possible sensitivity in future searches for neutrinoless double-β within decay the next decade.

Collaboration [60]. This implies

$$\hat{\Lambda} \; > \; 0.11\ (0.038)\ \mathrm{GeV}\,, \tag{4.24}$$

which are not competitive to the bounds in eq. (4.16) from $|d_e|_{\mathrm{exp}}$.

4.5 CP-violating electron-neutron interactions

Searches for atomic and molecular EDMs may be sensitive to other mechanisms possibly responsible for them besides the electron EDM, such as the EDMs of nuclei and CP-violating electron-nucleon interactions. In this section we are interested in the third possibility, particularly that described by [5, 6]

$$\mathcal{L}_{eN} = \frac{-iC_S G_{\rm F}}{\sqrt{2}}\,\bar{e}\gamma_5 e\,\bar{N}N - \frac{iC_P G_{\rm F}}{\sqrt{2}}\,\bar{e}e\,\bar{N}\gamma_5 N - \frac{iC_T G_{\rm F}}{\sqrt{2}}\,\bar{e}\sigma^{\kappa\omega}\gamma_5 e\,\bar{N}\sigma_{\kappa\omega}N\,. \quad (4.25)$$

The recent ACME experiment has set the best limit on the first coupling, $|C_S|_{\rm exp} < 5.9\times10^{-9}$ at 90% CL [7]. The strictest limits on the other two, $|C_P|_{\rm exp} < 5.1\times10^{-7}$ and $|C_T|_{\rm exp} < 1.5\times10^{-9}$ at 95% CL, were based on the latest search for the EDM of the ^{199}Hg atom [61].

These interactions may originate from MFV in the lepton sector as well as the quark sector, which has to be included for a consistent analysis. The Lagrangian for the relevant lowest-order operators is

$$\begin{aligned}\mathcal{L}_{\ell q} = \frac{1}{\Lambda^2}\Big(&\bar{U}_R Y_u^\dagger \bar{\Delta}_{qu1} i\tau_2 Q_L\,\bar{E}_R Y_e^\dagger \bar{\Delta}_{\ell1} L_L + \bar{Q}_L \bar{\Delta}_{qd1}^\dagger Y_d D_R\,\bar{E}_R Y_e^\dagger \bar{\Delta}_{\ell2} L_L \\ &+ \bar{U}_R \sigma^{\kappa\omega} Y_u^\dagger \bar{\Delta}_{qu2} i\tau_2 Q_L\,\bar{E}_R \sigma_{\kappa\omega} Y_e^\dagger \bar{\Delta}_{\ell3} L_L \\ &+ \bar{Q}_L \sigma^{\kappa\omega} \bar{\Delta}_{qd2}^\dagger Y_d D_R\,\bar{E}_R \sigma_{\kappa\omega} Y_e^\dagger \bar{\Delta}_{\ell4} L_L\Big) + \text{H.c.}\,, \end{aligned} \quad (4.26)$$

where $\bar{\Delta}_{qu\varsigma,qd\varsigma}$ $(\bar{\Delta}_{\ell1,\ell2,\ell3,\ell4})$ are the same in form as Δ in eq. (2.5) and contain the quark (lepton) Yukawa couplings. The leptonic contributions to $C_{S,P,T}$ turn out to be dominant.

To determine C_S, we need the matrix elements $\langle N|m_q\bar{q}q|N\rangle = g_q^N \bar{u}_N u_N v$. Thus, we derive

$$\begin{aligned}C_S = &\frac{16\sqrt{2}\,m_e\mathcal{M}^3}{\Lambda^2 G_{\rm F} v^9}\big(m_\tau^2 - m_\mu^2\big)\big(m_1 - m_2\big)\big(m_2 - m_3\big)\big(m_3 - m_1\big) \\ &\times\Big[\big(g_u^N + g_c^N + \kappa_{u1}\,g_t^N\big)\,\bar{\xi}_{12}^{\ell1} - \big(g_d^N + g_s^N + \kappa_{d1}\,g_b^N\big)\,\bar{\xi}_{12}^{\ell2}\Big]J_\ell\,, \end{aligned} \quad (4.27)$$

where $\bar{\xi}_{12}^{\ell1,\ell2}$ belong to $\bar{\Delta}_{\ell1,\ell2}$ and have absorbed the first coefficients $\bar{\xi}_1^{u1,d1}$ of $\bar{\Delta}_{qu1,qd1}$, respectively, and $\kappa_x \simeq 1 + \big(\bar{\xi}_2^x + \bar{\xi}_4^x\big)/\bar{\xi}_1^x$ are numbers expected to be at most of $\mathcal{O}(1)$. Numerically, we adopt the chiral Lagrangian estimate [62, 63]

$$g_u^N = 0.04\ (0.12)\times10^{-3}\,, \qquad g_d^N = 0.08\ (0.21)\times10^{-3}\,, \quad (4.28)$$

$$g_s^N = 0.25\ (2.88)\times10^{-3}\,, \qquad g_{c,b,t}^N = 0.26\ (0.05)\times10^{-3}\,, \quad (4.29)$$

corresponding to the so-called pion-nucleon sigma term $\sigma_{\pi N} = 30\ (80)$ MeV, which is not yet well-determined [64–67].[4] Then, using the maxima of g_q^N and assuming $\kappa_x = 1$, we can neglect the $\bar{\xi}_{12}^{\ell1}$ part in eq. (4.27) to obtain from $|C_S|_{\rm exp} < 5.9\times10^{-9}$

$$\frac{\Lambda}{\big|\bar{\xi}_{12}^{\ell2}\big|^{1/2}} > 0.27\ (0.091)\ \text{GeV} \quad (4.30)$$

[4]Lattice QCD computations [64, 65] tend to produce results smaller than those of chiral Lagrangian calculations and some other methods [66, 67]. As a consequence, employing the lattice values of g_q^N in eq. (4.27) would yield even looser limits than in eq. (4.30).

in the NH (IH) neutrino parameter values specified in the preceding subsection. These restraints are far weaker than those from $|d_e|_{\rm exp}$.

For C_P, the expression is the same as that for C_S in eq. (4.27), except g_q^N is replaced by $\varsigma_q\, h_q^N m_N/v$ with $\varsigma_q = +1\,(-1)$ if $q = u,c,t\,(d,s,b)$ and h_q^N defined by $\langle N|m_q\bar{q}\gamma_5 q|N\rangle = h_q^N m_N\,\bar{u}_N\gamma_5 u_N$. Since for mercury C_P is estimated to be mostly from the neutron contribution [6], we focus on it. Ignoring the effects of $h^n_{c,b,t}$, we can relate $h^n_{u,d,s}$ to the axial-vector charges $g_A^{(0,3,8)}$ by $6h_u^n = 2g_A^{(0)} - 3g_A^{(3)} + g_A^{(8)}$, $6h_d^n = 2g_A^{(0)} + 3g_A^{(3)} + g_A^{(8)}$, and $3h_s^n = g_A^{(0)} - g_A^{(8)}$, where $g_A^{(0)} = 0.33 \pm 0.06$, $g_A^{(3)} = 1.270 \pm 0.003$, and $g_A^{(8)} = 0.58 \pm 0.03$ were measured in baryon β-decay and deep inelastic scattering experiments [68]. Taking $\bar{\xi}_{12}^{\ell 1} = \bar{\xi}_{12}^{\ell 2}$ and maximizing C_P, we obtain from $|C_P|_{\rm exp} < 5.1\times 10^{-7}$

$$\frac{\Lambda}{|\bar{\xi}_{12}^{\ell 2}|^{1/2}} > 0.020\ (0.0068)\ {\rm GeV}\,, \tag{4.31}$$

less restrictive than eq. (4.30) by more than an order of magnitude.

To evaluate C_T, we need the matrix elements $\langle N|\bar{q}\sigma^{\kappa\omega}q|N\rangle = \rho_N^q\,\bar{u}_N\sigma^{\kappa\omega}u_N$, where ρ_N^q have the values in eq. (4.3) for light quarks, assuming isospin symmetry, and vanish for heavier quarks. This leads us to

$$C_T = \frac{32\sqrt{2}\,m_e\,\mathcal{M}^3}{\Lambda^2 G_{\rm F}\,v^{10}}\big(m_\tau^2 - m_\mu^2\big)\big(m_1 - m_2\big)\big(m_2 - m_3\big)\big(m_3 - m_1\big)\,\rho_u^n\, m_u\,\bar{\xi}_{12}^{\ell 3}\,J_\ell\,, \tag{4.32}$$

where $\bar{\xi}_{12}^{\ell 3}$ belongs to $\bar{\Delta}_{\ell 3}$ and has absorbed $\bar{\xi}_1^{u2}$ from $\bar{\Delta}_{qu2}$. The contributions of the down-type quarks cancel due to the relation $\bar{q}\sigma^{\kappa\omega}\gamma_5 q\,\bar{e}\sigma_{\kappa\omega}e = \bar{q}\sigma^{\kappa\omega}q\,\bar{e}\sigma_{\kappa\omega}\gamma_5 e$. Hence, with the largest m_u from section 4.1 and $\rho_n^u = -0.78$, we get from $|C_T|_{\rm exp} < 1.5\times 10^{-9}$

$$\frac{\Lambda}{|\bar{\xi}_{12}^{\ell 4}|^{1/2}} > 0.033\ (0.011)\ {\rm GeV}\,, \tag{4.33}$$

comparable to eq. (4.31).

4.6 Muon $g-2$, $\mu \to e\gamma$, nuclear $\mu \to e$ conversion, $\bar{B} \to X_s\gamma$

The MFV coefficient ξ_{12}^ℓ that determines the electron EDM also enters the anomalous magnetic moment of the muon $(g_\mu - 2)$ and the rates of the radiative decay $\mu \to e\gamma$ and nuclear $\mu \to e$ conversion, the latter two being still unobserved. Since $g_\mu - 2$ has been very precisely measured and the experimental limits of the flavor-changing transitions are stringent, it is important to check if these processes can yield stronger bounds on $\hat{\Lambda} = \Lambda/|\xi_{12}^\ell|^{1/2}$ than those evaluated in the preceding subsections. Although the other $\xi^\ell_{r\neq 12,16}$ terms may contribute to these processes as well and therefore may reduce the impact of the ξ_{12}^ℓ term, one also cannot rule out the possibility of a scenario in which the latter dominates the other contributions.

The anomalous magnetic moment a_l of lepton l is described by $\mathcal{L}_{a_l} = \big[e\,a_l/(4m_l)\big]\,\bar{l}\sigma^{\kappa\omega}lF_{\kappa\omega}$. From eq. (3.3) we have

$$\mathcal{L}_{E_i\to E_k\gamma} = \frac{e}{2\Lambda^2}\,\bar{E}_k\,\sigma_{\kappa\omega}\Big\{m_{E_k}(\Delta_\ell)_{ki} + m_{E_i}(\Delta_\ell)^*_{ik} - \Big[m_{E_k}(\Delta_\ell)_{ki} - m_{E_i}(\Delta_\ell)^*_{ik}\Big]\gamma_5\Big\}E_i\,F^{\kappa\omega}\,, \tag{4.34}$$

where $(E_1, E_2, E_3) = (e, \mu, \tau)$ and $\Delta_\ell = \Delta_{\ell 1} - \Delta_{\ell 2}$. It follows that

$$a_{E_k} \;=\; \frac{4m_{E_k}^2}{\Lambda^2}\,\mathrm{Re}(\Delta_\ell)_{kk}\;. \tag{4.35}$$

Thus, with the NH neutrino parameter values specified in section 4.4, we have

$$a_\mu \;=\; \frac{4m_\mu^2}{\Lambda^2}\,\mathrm{Re}(\Delta_\ell)_{22} \;=\; \Big(45\,\xi_1^\ell + 23\,\xi_2^\ell + 20\,\xi_4^\ell + 0.00085\,\xi_8^\ell + 0.00094\,\xi_{12}^\ell\Big)\frac{\mathrm{GeV}^2}{10^3\Lambda^2} \tag{4.36}$$

where terms with numerical factors much smaller than that of ξ_{12}^ℓ have been dropped. The corresponding numbers in the IH case are roughly similar. Currently the experimental and SM values differ by $a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = (249 \pm 87) \times 10^{-11}$ [69], which suggests that we can require $|a_\mu| < 3.4 \times 10^{-9}$. For the ξ_{12}^ℓ term alone, this translates into the rather loose limit $\hat{\Lambda} > 17$ GeV, which may be weakened in the presence of the other terms in eq. (4.36).

From eq. (4.34), one can also calculate the branching ratio $\mathcal{B}(\mu \to e\gamma)$ of $\mu \to e\gamma$. In the $m_e = 0$ limit

$$\mathcal{B}(\mu \to e\gamma) \;=\; \frac{\tau_\mu\, e^2 m_\mu^5}{4\pi\,\Lambda^4}\big|(\Delta_\ell)_{21}\big|^2\,, \tag{4.37}$$

where τ_μ is the muon lifetime. In the NH case

$$(\Delta_\ell)_{21} = (0.061 - 0.1i)\xi_2^\ell + (0.011 - 0.11i)\xi_4^\ell - \big[(26 + 48i)\xi_8^\ell + (6 + 51i)\xi_{12}^\ell\big] \times 10^{-7}\,, \tag{4.38}$$

where again terms with numerical factors less than that of ξ_{12}^ℓ have been ignored. The $(\Delta_\ell)_{21}$ numbers in the IH case are comparable in size. If only ξ_{12}^ℓ is nonvanishing in $(\Delta_\ell)_{21}$, then the experimental bound $\mathcal{B}(\mu \to e\gamma)_{\mathrm{exp}} < 5.7 \times 10^{-13}$ [70] implies

$$\hat{\Lambda} \;>\; 2.0\ \mathrm{TeV}\;. \tag{4.39}$$

This is stronger by up to $\sim$20 times than those in eq. (4.16) from the electron EDM data. However, the other terms in $(\Delta_\ell)_{21}$, some of which are potentially much bigger than the ξ_{12}^ℓ contribution, can in principle decrease the impact of the latter, thereby lessening the restriction on $\hat{\Lambda}$. Consequently, d_e provides a less ambiguous probe for $\hat{\Lambda}$.

Measurements on $\mu \to e$ conversion in nuclei can provide constraints on new physics competitive to those from $\mu \to e\gamma$ searches [71]. The relation between the rates of $\mu \to e$ conversion and $\mu \to e\gamma$ produced by possible new physics is available from ref. [72]. Assuming that the MFV dipole interactions described by eq. (3.2) saturate $\mu \to e$ conversion in nucleus $\mathcal{N}$, we can express its rate divided by the rate $\omega_{\mathrm{capt}}^{\mathcal{N}}$ of μ capture in $\mathcal{N}$ as

$$\mathcal{B}(\mu\mathcal{N} \to e\mathcal{N}) \;=\; \frac{e^2 m_\mu^5\,\big|(\Delta_\ell)_{21}\,D_{\mathcal{N}}\big|^2}{4\Lambda^4\,\omega_{\mathrm{capt}}^{\mathcal{N}}}\,, \tag{4.40}$$

where $D_{\mathcal{N}}$ represents the dimensionless overlap integral for $\mathcal{N}$ and for the NH parameter choices $(\Delta_\ell)_{21}$ is given in eq. (4.38). Based on the existing experimental limits on $\mu \to e$ transition in various nuclei [9] and the corresponding $D_{\mathcal{N}}$ and $\omega_{\mathrm{capt}}^{\mathcal{N}}$ values [72], significant restrictions can be expected from $\mathcal{B}(\mu\,\mathrm{Ti} \to e\,\mathrm{Ti})_{\mathrm{exp}} < 6.1 \times 10^{-13}$ [73] and $\mathcal{B}(\mu\,\mathrm{Au} \to$

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$e\,\mathrm{Au})_{\rm exp} < 7\times10^{-13}$ [9]. From these data, if only the ξ_{12}^ℓ term in $(\Delta_\ell)_{21}$ is nonvanishing, employing $D_{\rm Ti}=0.087$, $D_{\rm Au}=0.189$, $\omega_{\rm capt}^{\rm Ti}=2.59\times10^6/{\rm s}$, and $\omega_{\rm capt}^{\rm Au}=13.07\times10^6/{\rm s}$ [72], we extract

$$\hat\Lambda_{\rm Ti} \;>\; 0.49~{\rm TeV}\,, \qquad \hat\Lambda_{\rm Au} \;>\; 0.47~{\rm TeV}\,, \tag{4.41}$$

which are stricter than the results in eq. (4.30) by up to a few times, but weaker than eq. (4.39). Upcoming searches for $\mu\to e$ in the next several years will, if it still eludes detection, lower the limits to the 10^{-16} level or better [71], which will push $\hat\Lambda$ higher. Nevertheless, since again the other ξ_r^ℓ terms are generally present in $(\Delta_\ell)_{21}$, these bounds on $\hat\Lambda$ are not unambiguous. Thus d_e provides the best probe for $\hat\Lambda$ in connection with CP violation.

Since there is a possibility that the MFV scales in the lepton and quark sectors are equal or related to each other, it is of interest to check if there are any quark processes that can also offer bounds stronger than those on $\hat\Lambda$ from d_e. Since, as we saw in section 4.1, the neutron EDM could not provide a competitive constraint, we need to look at other processes. The most stringent restriction on the quark MFV scale turns out to be from the rare decay $\bar B\to X_s\gamma$ [21]. Its experimental and SM branching ratios are $\mathcal{B}(\bar B\to X_s\gamma)_{\rm exp}=(3.43\pm0.22)\times10^{-4}$ [74, 75] and $\mathcal{B}(\bar B\to X_s\gamma)_{\rm SM}=(3.15\pm0.23)\times10^{-4}$ [76] both for the photon energy $E_\gamma>1.6\,{\rm GeV}$. To isolate the MFV contribution, we adopt from ref. [21] the relation

$$\mathcal{B}(\bar B\to X_s\gamma)_{\rm exp} \;\simeq\; \big(1-2.4\,C_{7\gamma}^{\rm MFV}\big)\mathcal{B}(\bar B\to X_s\gamma)_{\rm SM}\,, \tag{4.42}$$

where $C_{7\gamma}^{\rm MFV}$ is evaluated at $\mu=m_W$ and enters the effective Lagrangian

$$\mathcal{L}_{b\to s\gamma} \;=\; \frac{eG_{\rm F}m_b}{8\sqrt2\,\pi^2}\,V_{ts}^*V_{tb}\left(C_{7\gamma}^{\rm SM}+C_{7\gamma}^{\rm MFV}\right)\bar s\sigma^{\kappa\omega}(1+\gamma_5)b\,F_{\kappa\omega}\,, \tag{4.43}$$

implying that

$$C_{7\gamma}^{\rm MFV} \;=\; \frac{4\sqrt2\,\pi^2}{\Lambda^2 G_{\rm F}}\,\frac{(\Delta_{qd})_{32}^*}{V_{ts}^*V_{tb}}\,. \tag{4.44}$$

For the central values of the quark masses quoted in section 4.1

$$\frac{(\Delta_{qd})_{32}^*}{V_{ts}^*V_{tb}} \;=\; \xi_2^d y_t^2+\xi_4^d y_t^4+y_b^2\left(\xi_7^d y_t^2+\xi_8^d y_t^4+\xi_9^d y_t^4+\xi_{12}^d y_t^6\right), \tag{4.45}$$

where the imaginary parts and other ξ_r^d terms are negligible, $y_t^2\simeq1$, and $y_b^2\simeq0.0003$. Combining the errors in quadrature for the ratio of branching ratios in eq. (4.42) and assuming that $\xi_{r\neq12}^d=0$, we obtain at 90% CL

$$\frac{\Lambda}{\left|\xi_{12}^d\right|^{1/2}} \;>\; 0.19~(0.11)~{\rm TeV} \tag{4.46}$$

if $C_{7\gamma}^{\rm MFV}$ has destructive (constructive) interference with the SM term. These numbers are somewhat lower than those in eq. (4.16) and, as in the lepton cases, may go down in the presence of the other ξ_r^d terms in eq. (4.45).

5 Conclusions

We have explored CP violation beyond the SM via fermion EDMs under the framework of minimal flavor violation. The new physics scenarios covered are the standard model slightly expanded with the addition of three right-handed neutrinos and its extension including the seesaw mechanism for endowing neutrinos with light mass. Addressing the quark sector first, we find that the present empirical limit on the neutron EDM implies only a loose constraint on the scale of quark MFV. Moreover, we show that the impact of MFV on the contribution of the strong theta-term to the neutron EDM is insignificant. Turning to the lepton sector, we demonstrate that the current EDM data also yield unimportant restraints on the leptonic MFV scale if neutrinos are of Dirac nature. In contrast, if neutrinos are Majorana particles, the constraints become tremendously more stringent and, in light of the latest search for d_e by ACME, restrict the MFV scale to above a few hundred GeV or more. Furthermore, d_e can be connected in a complementary way to neutrinoless double-β decay if it is induced mainly or solely by the exchange of a light Majorana neutrino. We find in addition that constraints on the MFV scale inferred from the CP-violating electron-nucleon couplings probed by ACME and the most recent search for the EDM of mercury are relatively weak as well. Finally, we take into account potential restrictions from the measurements on the muon $g-2$, radiative decays $\mu \to e\gamma$ and $\bar{B} \to X_s\gamma$, and $\mu \to e$ conversion in nuclei, which are not sensitive to CP violation.

Acknowledgments

This research was supported in part by the MOE Academic Excellence Program (Grant No. 102R891505) and NSC of ROC and by NNSF (Grant No. 11175115) and Shanghai Science and Technology Commission (Grant No. 11DZ2260700) of PRC.

A Evaluation of some products of A and B matrices

From the Cayley-Hamilton identity in eq. (2.4) with $X = a\mathsf{A} + b\mathsf{B}$, where and a and b are free parameters, one can extract [34]

$$\begin{aligned} \mathsf{A}^2\mathsf{B} + \mathsf{ABA} + \mathsf{BA}^2 \;=\;& \mathsf{A}^2\langle\mathsf{B}\rangle + (\mathsf{AB}+\mathsf{BA})\langle\mathsf{A}\rangle + \mathsf{A}\big(\langle\mathsf{AB}\rangle - \langle\mathsf{A}\rangle\langle\mathsf{B}\rangle\big) + \tfrac{1}{2}\big(\langle\mathsf{A}^2\rangle - \langle\mathsf{A}\rangle^2\big)\mathsf{B} \\ &+ \mathbb{1}\Big[\tfrac{1}{2}\big(\langle\mathsf{A}\rangle^2 - \langle\mathsf{A}^2\rangle\big)\langle\mathsf{B}\rangle + \langle\mathsf{A}^2\mathsf{B}\rangle - \langle\mathsf{A}\rangle\langle\mathsf{AB}\rangle\Big] \end{aligned} \tag{A.1}$$

and an analogous expression for $\mathsf{ABB}+\mathsf{BAB}+\mathsf{BBA}$, where $\langle\cdots\rangle = \mathrm{Tr}(\cdots)$. These relations can be used to derive other combinations of A and B. For instance, by replacing B with B^2 ($[\mathsf{B},\mathsf{AB}]$) in eq. (A.1), we can write $\mathsf{A}^2\mathsf{B}^2 + \mathsf{AB}^2\mathsf{A} + \mathsf{B}^2\mathsf{A}^2$ $\big(\mathsf{A}^2\mathsf{BAB} + \mathsf{ABABA} + \mathsf{BABA}^2 - \mathsf{A}^2\mathsf{B}^2\mathsf{A} - \mathsf{AB}^2\mathsf{A}^2 - \mathsf{A}^3\mathsf{B}^2\big)$ in terms of lower-ordered products of these matrices. After further algebra, we arrive at

$$\begin{aligned} \mathsf{A}^2\mathsf{BAB}^2 \;=\;& \zeta_1\mathbb{1} + \zeta_2\,\mathsf{A} + \zeta_3\,\mathsf{B} + \zeta_4\,\mathsf{A}^2 + \zeta_5\,\mathsf{B}^2 + \zeta_6\,\mathsf{AB} + \zeta_7\,\mathsf{BA} + \zeta_8\,\mathsf{ABA} + \zeta_9\,\mathsf{BA}^2 \\ &+ \zeta_{10}\,\mathsf{BAB} + \zeta_{11}\,\mathsf{AB}^2 + \zeta_{12}\,\mathsf{ABA}^2 + \zeta_{13}\,\mathsf{A}^2\mathsf{B}^2 + \zeta_{14}\,\mathsf{B}^2\mathsf{A}^2 + \zeta_{15}\,\mathsf{B}^2\mathsf{AB} \\ &+ \zeta_{16}\,\mathsf{AB}^2\mathsf{A}^2 + \zeta_{17}\,\mathsf{B}^2\mathsf{A}^2\mathsf{B}\,, \end{aligned} \tag{A.2}$$

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where

$$
\begin{aligned}
\zeta_1 = {} & \frac{\langle \mathsf{A}^2\mathsf{B}\mathsf{A}\mathsf{B}^2\rangle + \langle \mathsf{A}\mathsf{B}\rangle\langle \mathsf{A}^2\mathsf{B}^2\rangle + \langle \mathsf{A}^2\mathsf{B}\rangle\langle \mathsf{A}\mathsf{B}^2\rangle}{3} + \langle \mathsf{A}\rangle\langle \mathsf{B}\rangle \frac{4\langle \mathsf{A}^2\rangle\langle \mathsf{B}^2\rangle - 6\langle \mathsf{A}^2\mathsf{B}^2\rangle - 3\langle \mathsf{A}\rangle^2\langle \mathsf{B}\rangle^2}{6} \\
& + \frac{\big(\langle \mathsf{B}\rangle^3 - \langle \mathsf{B}\rangle\langle \mathsf{B}^2\rangle\big)\mathrm{Det}\mathsf{A} + \big(\langle \mathsf{A}\rangle^3 - \langle \mathsf{A}\rangle\langle \mathsf{A}^2\rangle\big)\mathrm{Det}\mathsf{B}}{6} + \langle \mathsf{A}\mathsf{B}\rangle \frac{13\langle \mathsf{A}\rangle^2\langle \mathsf{B}\rangle^2 - 3\langle \mathsf{A}^2\rangle\langle \mathsf{B}^2\rangle}{12} \\
& + \langle \mathsf{A}\mathsf{B}\rangle \frac{5\langle \mathsf{A}\rangle^2\langle \mathsf{B}^2\rangle + \langle \mathsf{B}\rangle^2\langle \mathsf{A}^2\rangle - 8\langle \mathsf{A}\rangle\langle \mathsf{A}\mathsf{B}^2\rangle - 4\langle \mathsf{B}\rangle\langle \mathsf{A}^2\mathsf{B}\rangle}{12} + \langle \mathsf{A}\mathsf{B}^2\rangle\langle \mathsf{B}\rangle \frac{\langle \mathsf{A}\rangle^2 - \langle \mathsf{A}^2\rangle}{6} \\
& - \langle \mathsf{A}\rangle\langle \mathsf{B}^2\rangle \frac{\langle \mathsf{A}\rangle^2\langle \mathsf{B}\rangle + 2\langle \mathsf{A}^2\mathsf{B}\rangle}{6}\,,
\end{aligned} \tag{A.3}
$$

$$
\begin{aligned}
\zeta_2 = {} & \frac{-\langle \mathsf{A}^2\rangle\,\mathrm{Det}\mathsf{B}}{3} + \langle \mathsf{A}\mathsf{B}\rangle \frac{4\langle \mathsf{A}\mathsf{B}^2\rangle - 5\langle \mathsf{A}\rangle\langle \mathsf{B}\rangle^2 - 3\langle \mathsf{A}\rangle\langle \mathsf{B}^2\rangle}{6} + \langle \mathsf{B}\rangle\langle \mathsf{B}^2\rangle \frac{7\langle \mathsf{A}\rangle^2 + \langle \mathsf{A}^2\rangle}{12} \\
& - \langle \mathsf{B}\rangle \frac{2\langle \mathsf{A}\rangle\langle \mathsf{A}\mathsf{B}^2\rangle + \langle \mathsf{A}^2\mathsf{B}^2\rangle}{3} + \frac{\langle \mathsf{B}\rangle^2\langle \mathsf{A}^2\mathsf{B}\rangle}{3} + \langle \mathsf{B}\rangle^3\,\frac{9\langle \mathsf{A}\rangle^2 - \langle \mathsf{A}^2\rangle}{12}\,,
\end{aligned} \tag{A.4}
$$

$$
\begin{aligned}
\zeta_3 = {} & \frac{-\langle \mathsf{B}^2\rangle\,\mathrm{Det}\mathsf{A}}{3} + \langle \mathsf{A}\mathsf{B}\rangle \frac{2\langle \mathsf{A}^2\mathsf{B}\rangle - 5\langle \mathsf{A}\rangle^2\langle \mathsf{B}\rangle - \langle \mathsf{A}^2\rangle\langle \mathsf{B}\rangle}{6} + \langle \mathsf{A}\rangle\langle \mathsf{A}^2\rangle \frac{3\langle \mathsf{B}\rangle^2 - \langle \mathsf{B}^2\rangle}{12} \\
& - \langle \mathsf{A}\rangle \frac{\langle \mathsf{A}^2\mathsf{B}\rangle\langle \mathsf{B}\rangle + \langle \mathsf{A}^2\mathsf{B}^2\rangle}{3} + \langle \mathsf{A}\mathsf{B}^2\rangle \frac{\langle \mathsf{A}\rangle^2 + \langle \mathsf{A}^2\rangle}{6} + \langle \mathsf{A}\rangle^3\,\frac{9\langle \mathsf{B}\rangle^2 + \langle \mathsf{B}^2\rangle}{12}\,,
\end{aligned} \tag{A.5}
$$

$$
\zeta_4 = \frac{\langle \mathsf{A}\rangle\,\mathrm{Det}\mathsf{B}}{3} + \langle \mathsf{B}^2\rangle \frac{2\langle \mathsf{A}\mathsf{B}\rangle - 7\langle \mathsf{A}\rangle\langle \mathsf{B}\rangle}{6} - \frac{\langle \mathsf{A}\rangle\langle \mathsf{B}\rangle^3}{6} + \frac{\langle \mathsf{A}\mathsf{B}^2\rangle\langle \mathsf{B}\rangle}{3}\,, \tag{A.6}
$$

$$
\zeta_5 = \frac{\langle \mathsf{B}\rangle\,\mathrm{Det}\mathsf{A}}{3} + \langle \mathsf{A}^2\rangle \frac{2\langle \mathsf{A}\mathsf{B}\rangle - 5\langle \mathsf{A}\rangle\langle \mathsf{B}\rangle}{6} - \frac{\langle \mathsf{A}\rangle^3\langle \mathsf{B}\rangle}{6}\,, \tag{A.7}
$$

$$
\zeta_6 = \langle \mathsf{A}^2\rangle \frac{\langle \mathsf{B}^2\rangle + \langle \mathsf{B}\rangle^2}{6} - \langle \mathsf{A}\rangle^2\,\frac{\langle \mathsf{B}^2\rangle + 7\langle \mathsf{B}\rangle^2}{6} + \frac{2\langle \mathsf{A}\rangle\langle \mathsf{A}\mathsf{B}^2\rangle + \langle \mathsf{B}\rangle\langle \mathsf{A}^2\mathsf{B}\rangle - 2\langle \mathsf{A}^2\mathsf{B}^2\rangle}{3}\,, \tag{A.8}
$$

$$
\zeta_7 = \langle \mathsf{A}^2\rangle \frac{\langle \mathsf{B}^2\rangle - \langle \mathsf{B}\rangle^2}{12} - \langle \mathsf{A}\rangle^2\,\frac{5\langle \mathsf{B}^2\rangle + 11\langle \mathsf{B}\rangle^2}{12} + \frac{2\langle \mathsf{A}\rangle\langle \mathsf{A}\mathsf{B}^2\rangle + \langle \mathsf{B}\rangle\langle \mathsf{A}^2\mathsf{B}\rangle - \langle \mathsf{A}^2\mathsf{B}^2\rangle}{3}\,, \tag{A.9}
$$

$$
\zeta_8 = \langle \mathsf{A}\rangle \frac{5\langle \mathsf{B}\rangle^2 + 3\langle \mathsf{B}^2\rangle}{6} - \frac{2\langle \mathsf{A}\mathsf{B}^2\rangle}{3}\,, \qquad \zeta_9 = \frac{\langle \mathsf{A}\rangle\langle \mathsf{B}^2\rangle - \langle \mathsf{A}\mathsf{B}^2\rangle}{3}\,, \tag{A.10}
$$

$$
\zeta_{10} = \langle \mathsf{B}\rangle \frac{5\langle \mathsf{A}\rangle^2 + \langle \mathsf{A}^2\rangle}{6} - \frac{\langle \mathsf{A}^2\mathsf{B}\rangle}{3}\,, \qquad \zeta_{11} = \frac{\langle \mathsf{A}^2\mathsf{B}\rangle - \langle \mathsf{A}^2\rangle\langle \mathsf{B}\rangle}{3}\,, \tag{A.11}
$$

$$
\zeta_{12} = \frac{-\langle \mathsf{B}\rangle^2}{2} - \frac{\langle \mathsf{B}^2\rangle}{6}\,, \qquad \zeta_{13} = \frac{4\langle \mathsf{A}\rangle\langle \mathsf{B}\rangle + \langle \mathsf{A}\mathsf{B}\rangle}{3}\,, \tag{A.12}
$$

$$
\zeta_{14} = \langle \mathsf{A}\rangle\langle \mathsf{B}\rangle - \frac{\langle \mathsf{A}\mathsf{B}\rangle}{3}\,, \qquad \zeta_{15} = \frac{-\langle \mathsf{A}\rangle^2}{2} - \frac{\langle \mathsf{A}^2\rangle}{6}\,, \tag{A.13}
$$

$$
\zeta_{16} = \frac{2\langle \mathsf{B}\rangle}{3}\,, \qquad \zeta_{17} = \frac{2\langle \mathsf{A}\rangle}{3}\,. \tag{A.14}
$$

The Hermiticity of A and B implies that all the traces and determinants in $\zeta_{1,2,\cdots,17}$ are purely real, except $\langle \mathsf{A}^2\mathsf{B}\mathsf{A}\mathsf{B}^2\rangle$ in ζ_1 which has an imaginary component

$$
J_\xi = \mathrm{Im}\langle \mathsf{A}^2\mathsf{B}\mathsf{A}\mathsf{B}^2\rangle = \frac{i}{2}\,\mathrm{Det}[\mathsf{A},\mathsf{B}] \tag{A.15}
$$

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obtainable from the Cayley-Hamilton identity

$$[\mathsf{A},\mathsf{B}]^3 \;=\; \mathbb{1}\,\mathrm{Det}[\mathsf{A},\mathsf{B}] + \tfrac{1}{2}[\mathsf{A},\mathsf{B}]\big(\langle[\mathsf{A},\mathsf{B}]^2\rangle - \langle[\mathsf{A},\mathsf{B}]\rangle^2\big) + [\mathsf{A},\mathsf{B}]^2\,\langle[\mathsf{A},\mathsf{B}]\rangle\,. \qquad (A.16)$$

Clearly the reduction of $\mathsf{A}^2\mathsf{BAB}^2$ into a sum of matrix products with lower orders causes the coefficient ζ_1 to gain an imaginary component equal to J_ξ. It follows that higher-order matrix products containing $\mathsf{A}^2\mathsf{BAB}^2$ will lead to contributions to the coefficients ξ_r with imaginary parts which are always proportional to J_ξ

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