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Pre-Darcy flow revisited under experimental investigation

Fahd Siddiqui*, Mohamed Y. Soliman, Waylon House and Akif Ibragimov

Abstract

Background: Sufficient literature has been published about Pre-Darcy flow in non-petroleum disciplines. Investigators dissent about the significance of deviation of Darcy's law at very low fluid velocities. Most of their investigations are based on coarse, unconsolidated porous media with an aqueous fluid. However little has been published regarding the same for consolidated oil and gas reservoirs. If a significant departure from Darcy's law is observed, then this could have multiple implications on: reservoir limit tests, under prediction of reserves, unrecognized prospecting opportunities etc.

Methods: This study performs a comprehensive review of the literature. Experiments were conducted to confirm the presence and the significance of Pre-Darcy flow effect in petroleum rocks.

Results: The review of literature and experiments indicate the presence of Pre-Darcy effect. Contributing factors to Pre-Darcy effect are discussed and some reasons causing this effect are postulated. The experiments also show that this effect is significant.

Conclusion: Pre-Darcy effect is significant because it is the dominant flow regime in typical petroleum reservoirs.

Keywords: Pre-Darcy flow, Non-Darcy flow, Fluid flow through porous media

Background

Modern petroleum engineers have used many equations to describe the physics behind the fluid flow through porous media. Under ideal situations, these equations, which form the basis of modern software, yield accurate results. However, ever so often, engineers are faced with challenging problems that seemingly defy physics: be it a well-test problem, a history-matched simulation model, or even a tool as simple as the material balance. Upon further investigation, engineers have to concede to the simple explanation that the assumptions behind those equations were violated. Even further discomforting is the admission that engineers have not yet properly characterized the physics behind the fluid flow through porous media.

Darcy's pioneering work is at the heart of all equations related to porous media. Often engineers use it without question. Forchheimer (Forchheimer 1901) demonstrated the departure from linearity for high-velocity flows. However, little has been said about the validity of Darcy's law at low velocities. Considerable amount of work (Fishel 1935; Dudgeon 1966; Soni et al. 1978) has already been published in this area outside of petroleum, but it has not seeped through the petroleum engineering literature.

This paper aims to give a comprehensive non-petroleum literature review of low velocity or pre-Darcy flow. The second part deals with showing experimentally if this effect is significant.

Darcy's law

Darcy's law is based on the experimental observation that the apparent fluid velocity is proportional to the applied pressure gradient on a porous medium. This observation is analogous to the flow of fluid through pipes and capillaries (Poisseuille's law) and also to that of the flow of current through a resistive conductor. It is also similar to Fourier's law of heat conduction, and this similarity causes much confusion in understanding the resulting solutions (discussed at the end of this paper). Wyckoff et al. (Wyckoff, et al. 1933) separated Darcy's original constant of proportionality into permeability

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* Correspondence: fahdsiddiqui@gmail.com
Texas Tech University, 2500 Broadway, Lubbock, TX 79409, USA

(a property of the porous medium) and viscosity (a property of the fluid). The final form of equation is (Eq. 1)

$$v = \frac{q}{A} = -\frac{k}{\mu}\nabla p \tag{1}$$

The following sections discuss some of the well-known departures from Darcy's law.

Post-Darcy flow effect

Forchheimer (Forchheimer 1901) made observations that Darcy's law deviated from linearity for high velocities. He attributed this to the inertial losses. He proposed a velocity squared term to account for this non-linearity. Even in 1901, Forchheimer noted that some experimental data does not fit his newly proposed quadratic flow equation. He then proposed the addition of a cubic term to describe those data. Due to the less than proportional increase in flow velocity with respect to applied pressure gradient, this effect has shown a significant influence on well performance (Firoozabadi and Katz 1979; Evans and Civan 1994). This effect is generally termed as non-Darcy flow; however, in this study, we will refer to it as post-Darcy flow. Later studies (Holditch and Morse 1976; Guppy et al. 1982; Martins et al. 1990) have published the impact of post-Darcy flow on fractured gas wells. The literature already has effectively dealt with post-Darcy flow, and the reader is suggested to consult elsewhere for a more comprehensive treatment of the subject.

Low-pressure Klinkenberg's effect

This effect is also well known and ascribed to the Knudsen effect (slippage effect). Also known as the Klinkenberg effect (Klinkenberg 1941), who demonstrated that the permeability of a porous medium is a function of gas pressure. Well-known published procedures exist to deal with this effect.

Non-Newtonian fluid effect

Darcy's law does not apply to non-Newtonian fluid flow. For non-Newtonian fluids, viscosity is a function of applied shear rate. Bird et al. (1960) and Savins (1969) gave an analogous expression of Darcy's law for power law fluids (Eq. 2).

$$v^n = -\frac{k}{\mu_{\mathrm{eff}}}\nabla p \tag{2}$$

Siddiqui et al. (Siddiqui et al. 2014) applied the above equation and solved the radial diffusivity equation for analyzing pressure transient tests. They validated the above equation with real field injection data.

Pre-Darcy flow

Oil and gas flowing at very low velocity will be referred to as pre-Darcy flow in this text. Many authors (Longmuir 2004) have already pointed out the necessity of considering the pre-Darcy flow. The actual fluid flow velocity in a real reservoir is very slow especially for radial flow. The Darcy velocity is superficial velocity and is related to the continuity equation by (Eq. 3)

$$v = \frac{q}{A} \tag{3}$$

In radial flow, the cross-sectional area to flow increases, which causes a decrease in fluid velocity for any, given constant flow rate.

Figure 1 shows a plot of typical oil/gas wells under steady state radial flow regime for 160 acre spacing with a net height of 100 ft. Figure 1 shows that at smaller radial distances, the velocity is large, but it rapidly drops to small values for intermediate to large radial distances. Another important conclusion from Fig. 1 is that only for the initial 5 to 10 % of the radial distance, the fluid is flowing with a high velocity, whereas more than 90 % of the fluid in the porous medium is actually moving with a very low velocity of 0.1 ft/day or less (shaded on the figure). For the low rate (low permeability) case, the 90 % of the fluid is moving with a velocity of 0.005 ft/day or less. Therefore, it is important to experimentally investigate the validity of Darcy's law in this region, because this flow velocity occupies a significant portion of the flow regime in the reservoir. In later sections, we demonstrate, experimentally, that below a certain velocity (or pressure gradient) the pre-Darcy effect becomes apparent and a deviation from Darcy straight line is observed depending on the particular rock/fluid system.

Researchers (Hubbert 1956; Tek 1957; Das 1997; Prada and Civan 1999) over the years have also realized that not only is Pre-Darcy a deviation from linearity, but it also indicates the presence of a "threshold pressure gradient" and that Darcy's law should be corrected for that. Figure 2 summarizes some of the proposed models for Pre-Darcy flow in the literature (with and without the threshold gradient). Kutilek (Kutilek 1972) classified these regimes into seven types, four of which are shown in Fig. 2.

Various non-petroleum engineering literature (Fishel 1935; Dudgeon 1966; Soni et al. 1978) have already demonstrated deviations from Darcy linearity under very small velocity fluid flow. However, most of those studies were conducted on unconsolidated samples. The following sections examine their experiments and conclusions.

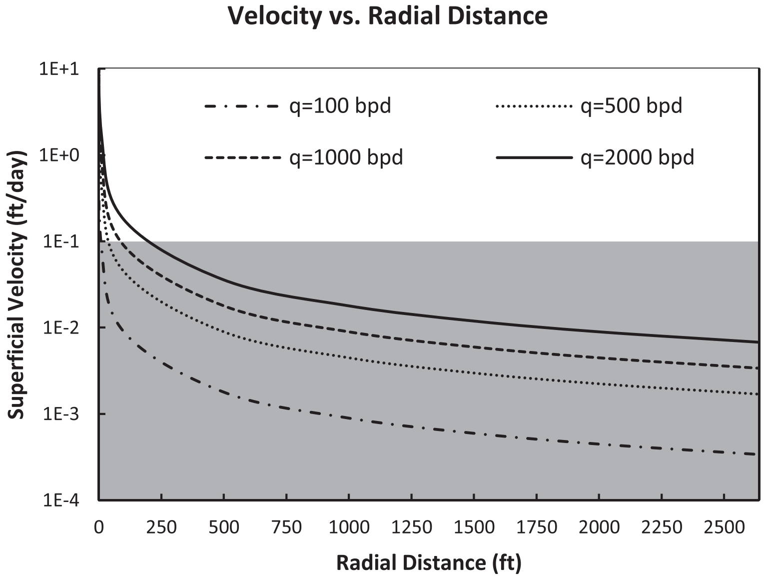

Fig. 1 Superficial velocity vs. radial distance for various flow rates and net-pay thicknesses

Published datasets

Figure 3 summarizes some of the datasets collected for pre-Darcy flow. It also yields a comparison of real reservoirs velocities vs. the lab experiments.

Fishel (Fishel 1935) observed that laboratory tests for permeability are made with much higher pressure gradients than those encountered in water-bearing formations. He conducted experiments with sand samples and water as the working fluid in a U-tube apparatus. His conclusions were that Darcy's law is valid for very low velocities (10^{-4} ft/day and above).

Dudgeon (Dudgeon 1966) conducted permeability tests on coarse-grained material including river gravels, crushed rock particles, and glass marbles with water as the working fluid. He was able to discern three different regimes from his experiments: pre-linear (pre-Darcy), linear (Darcy), and post-linear (post-Darcy). These flow regimes are distinguishable with abrupt changes in linearity. Therefore, Dudgeon proposed an empirical fit based on Eq. 4 (Escande 1953; Slepicka 1961; Anandakrishan and Varadarajulu 1963). Where the coefficients a and m are different for each flow regime and that $m = 1$ for Darcy flow and $m < 1$ for pre-Darcy flow. Dudgeon's explanation for the pre-linear flow was ascribed to non-Newtonian characteristics caused by interfacial tension.

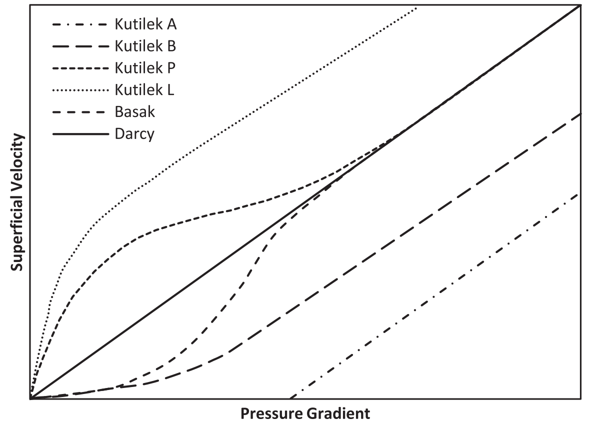

Fig. 2 Pre-Darcy regime models and classification (Kutilek 1972; Basak 1977)

$$\nabla p = a v^m \tag{4}$$

Soni et al. (Soni et al. 1978) conducted various experiments on different particle sized porous media. The objective of their study was to better correlate the values of a and m for particle size and porosities. Their experiments also suggested abrupt changes in flow regimes and categorized them into pre-linear, linear, and post-linear flow regimes, and they too concluded that $m < 1$ for pre-Darcy flow. Their experiments were conducted with particle sizes in the range of 0.074 to 1.19 mm and with porosities as high as 48.75 %. They were able to identify pre-Darcy regime for velocity as high as 100 ft/day. This kind of information is relevant to unconsolidated reservoirs and shows qualitatively that even the near wellbore region might be experiencing pre-Darcy flow phenomenon.

Neuzil (Neuzil 1986) attributed the departures from Darcy's law in the pre-Darcy range to subtle experimental errors: changes in water viscosity, measurement errors, small leaks, bacterial activity, incorrect assumption of steady state flow, gas generation and dissolution, and

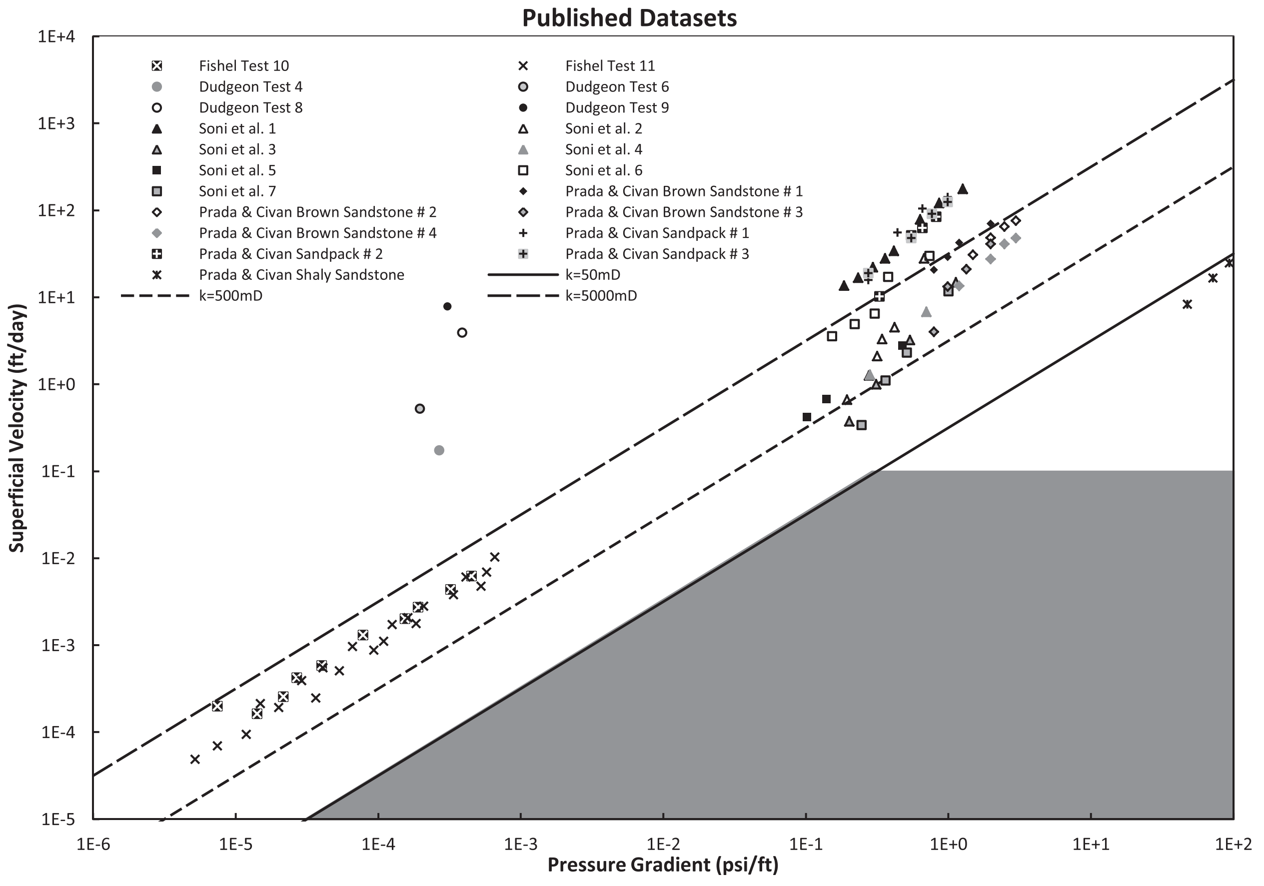

Fig. 3 Published datasets compared with real reservoirs

changes in medium matrix. However, he also conceded that an observational gap exists and that flow measurements have only been made at gradients several orders of magnitudes higher than in actual nature. Therefore, applicability of Darcy's law can only be *inferred* at small gradients.

Fand et al. (Fand, et al. 1987) conducted experiments in the high velocity range, but also hinted the existence of pre-Darcy. Bear (Bear 1972) attributed the pre-Darcy phenomena to non-Newtonian behavior of fluids at low velocities. Bear also postulated that small countercurrents are generated along the pre-walls in the direction opposite to the main flow giving rise the pre-Darcy effect.

Liu and Masliyah (Liu and Masliyah 1996) divided the flow in four regions: pre-Darcy, Darcy, Forchheimer flow, and turbulent flow. They suggested that the transitions between the flow regimes are smooth. They attributed the pre-Darcy effect to "surface-interactive flows" and that this effect is strongly dependent on the type of porous media and the flowing fluid.

Prada and Civan (Prada and Civan 1999) experimentally demonstrated the existence of a threshold pressure gradient for liquids. They attributed this threshold to frictional effects. Their experiments were conducted on consolidated sandstones, sand-packs, and shaly sandstone, with brine as the working fluid. They demonstrated that the threshold pressure gradient is an inverse power law of mobility.

Figure 3 also shows the velocity vs. gradient lines for different permeabilities encountered in petroleum reservoirs. These lines show that almost all of the experiments conducted were on high permeability (k > 500 mD) media and also confirm that most of the experimental data is not parallel to these lines (hence under pre-Darcy flow as discussed above). The shaded region shows the reservoirs with a permeability of 50 mD or less. Inspection of Fig. 3 alongside Fig. 1 reveals that only the experiments conducted by Fishel (1935) were in the low velocity range (v < 0.1 ft/day). As described earlier, at least 80 % of the porous media is flowing fluid with the velocity of 0.1 ft/day or lower.

Noting that most petroleum reservoirs have a permeability of 50 mD or less and that 80 % of the fluid in a typical reservoir is flowing with a velocity of 0.1 ft/day or less; a "region of interest" can be constructed on Fig. 3 (shaded) based on these constraints. This region describes a real petroleum reservoir having a permeability of 50 mD (or less) dominated with low velocity flow (0.1 ft/day or less). It becomes apparent that none of the experiments were conducted in this region of interest.

Methods

Most of the published work was concerned with coarse unconsolidated material with water as the working fluid. Petroleum reservoirs consist of consolidated rocks. In this study, authors carried out experiments on consolidated porous media with an organic (Soltrol-130) fluid to match real field reservoir rock/fluid system. Use of consolidated samples also avoids the errors due to solids movement associated with unconsolidated samples. Organic fluid was chosen to observe the pre-Darcy effect without the polar interaction effects associated with aqua based fluids.

Fand et al. (Fand, et al. 1987) used a steel tube to contain the porous media (glass beads) and the fluid was allowed to flow through the media by a constant head gravity tank for low velocity flows. They used orifice plates to measure the flow rates. Meyer and Krause (Meyer and Krause 1998) used the traditional Hassler-sleeve type permeameter. However, their experiments consisted of finding the low velocity gas flow effect. Fishel (Fishel 1935) described a U-tube type apparatus (similar to the one used in this study) to apply low pressure gradient on a porous medium.

Experimental setup

The U-tube type apparatus (Fig. 4) holds the porous media (core) in the Hassler-sleeve coreholder, and the pressure gradient is applied on the saturated core sample through the difference of fluid levels in the columns. This difference in head is then converted to pressure difference through the density of the working fluid. The pressure difference is converted to pressure gradient by dividing by the length of the core sample. The flow rate is calculated by noting the change in head with respect to time. By using the diameter of the tube (cross-sectional area), this rate of head change is converted to rate of volume change (flow rate). Superficial velocity can be found by dividing the flow rate by the cross-sectional area of the core sample (Eq. 3).

Consolidated core samples from sandstone reservoirs of various permeabilities were used to study the pre-Darcy effect in these experiments. Samples with natural/induced fractures were not considered in this study.

The required data of head vs. time were recorded for core samples with different geometry and permeabilities.

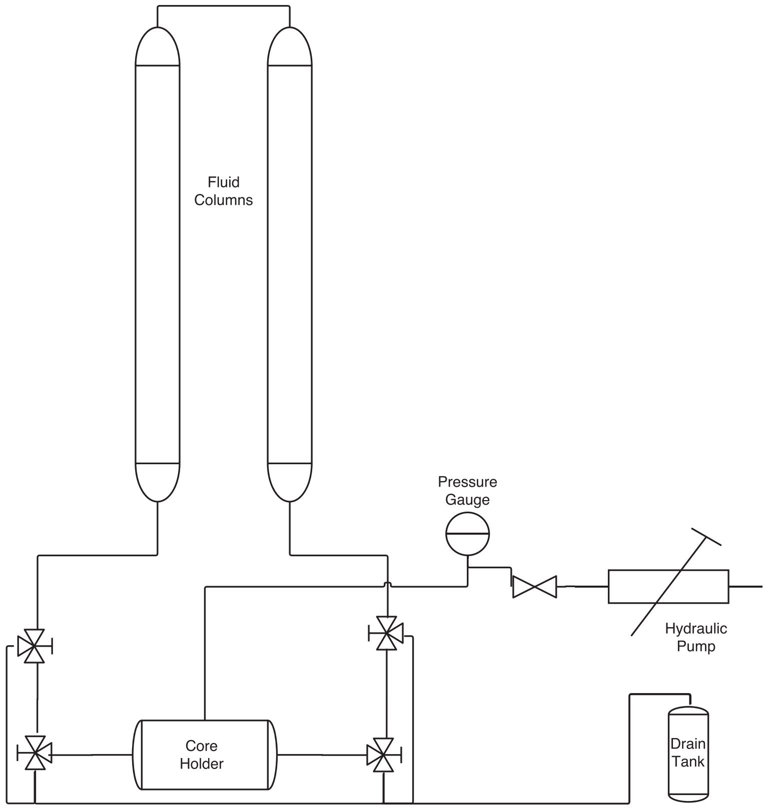

Fig. 4 Schematic of the U-tube type apparatus

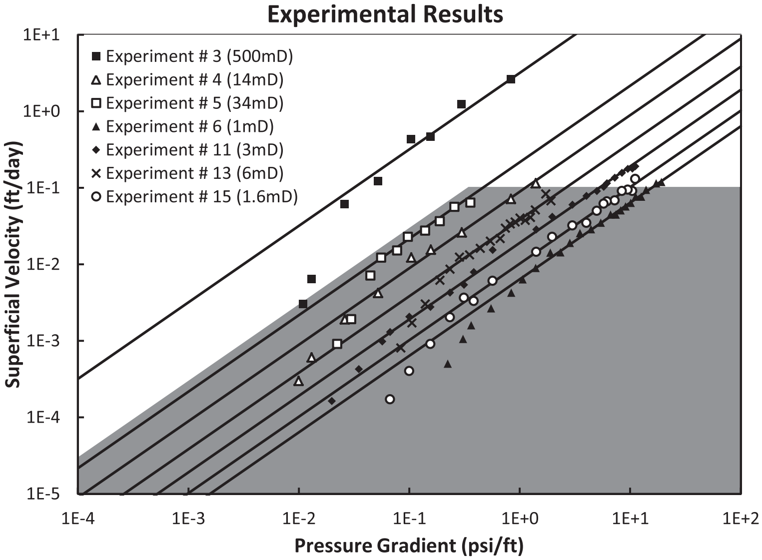

Fig. 5 Experimental results targeting the "Region of Interest"

Velocity vs. gradient data (after calculations) is plotted on Fig. 5.

Laboratory precision and uncertainty

The main sources of error in the experiment arise from the challenge of measurement of low flow rates and low pressure gradients. The setup described above can read pressures down to 0.5 mm head (4.9 Pa or 7×10^{-4} psi, assuming water head, Soltrol-130 would result in even smaller least count). Low flow rates can also be directly read off down to zero with this setup.

Constant temperature was achieved using the laboratory HVAC system to maintain the temperature at 19 °C. The temperature variations, near the apparatus, were monitored every half hour and found to be within 0.2 °C. The sample was cleaned, dried and vacuumed, before saturating with Soltrol-130 to remove impurities and air from the system. Constant confining pressure was applied with a hydraulic pump on the core holder to avoid annular flow and wall effects. The experiment was performed single phase to avoid multiphase effects. Viscosity of Soltrol-130 was measured every hour and found to be constant.

These precautions helped with taking precise readings to avoid the experimental errors suspected by Neuzil (Neuzil 1986). Moreover, since the authors used organic fluid instead of water, this study avoids the electrostatic effects associated with the polar nature of water and its interaction with the porous medium.

Material balance checks were performed to establish certainty and accuracy of data. Each experiment was repeated for redundancy and validation. The validity of experimental setup was also confirmed by comparing the obtained value of permeability, for each sample, against the permeability value obtained from a conventional core-flooding experiment. These checks helped mitigate much of the uncertainty in the measurements made at Darcy and pre-Darcy velocities.

Results and discussion

Experimental data and results are plotted on Fig. 5 (which is an extension of Fig. 3). The experiments were conducted on samples with a range of permeability targeting the "region of interest" (shaded). Pre-Darcy phenomenon is clearly observed, and it occurs abruptly (as pointed out by other researchers) below a certain velocity value that depends on the medium. At these velocities, the plot of superficial velocity vs. pressure gradient deviates from the linear relationship of the Darcy's law. Because all proper precautions were taken, this effect must be due to the nature of the fluid and rock interaction which manifests itself only at low velocities—pre-Darcy phenomena. The pre-Darcy deviation was observed in the range of 0.002 to 0.1 ft/day (for the tested samples with low to high permeabilities, respectively). And as observed earlier, at least 90 % of the fluid is moving with a velocity in that range (depending on rate/permeability). This observation confirms that the presence of pre-Darcy flow regime in a significant portion of the flow regime distributed in the reservoir.

Figure 6 shows the power law fit (described by Eq. 4) on the pre-Darcy regime of the experimental results. The value of m was found to vary between 0.3 and 0.6 for these particular experiments. Since these values are father from unity, it shows significant pre-Darcy effect at these velocities. However, m does not correlate well with the permeability, and therefore, m can only be

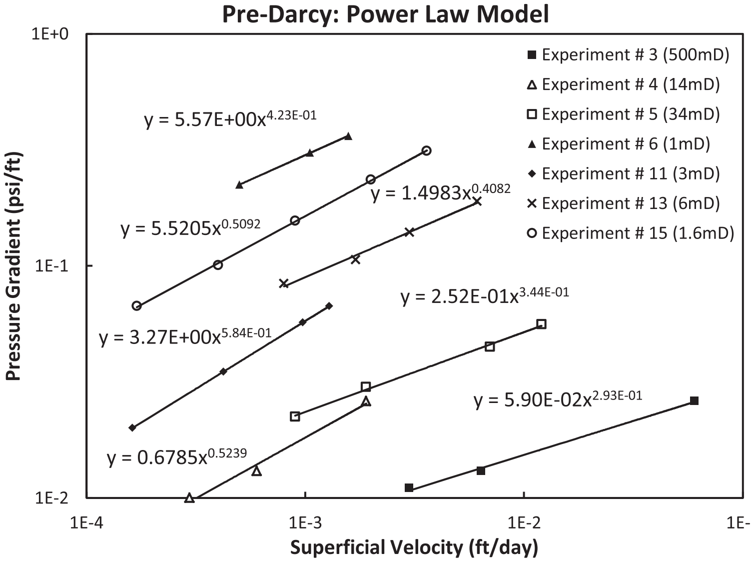

Fig. 6 Power law applied to pre-Darcy regime

determined experimentally. Clearly, since *a* is a function of permeability, our experiments also show that *a* varies inversely with permeability.

Deviation from unit slope also points towards the existence of a "threshold pressure gradient" as described by previous researchers. However, for this study, it was considered that Eq. 4 is sufficient to describe mathematically the physics behind the pre-Darcy phenomenon. However, our experimental data does corroborate the existence of a threshold pressure gradient.

Physical explanations

The pre-Darcy effect occurs due to fluid and rock interaction phenomena at very low velocities. At very low velocities, the fluid/rock system behaves more like the flow of non-Newtonian power law fluid through the porous media (Eq. 2) than a Newtonian fluid (Eq. 1). The evidence of this explanation is hinted by the mathematical similarity of Eq. 4 and Eq. 2. However, the power law index (*m* value) for the pre-Darcy effect is defined more by the fluid/rock system than just by fluid properties (which is the case for non-Newtonian fluid) and is only applicable at very low velocities. This is evident by the fact that the same (Newtonian) fluid yielded a different *m* value for different media; more importantly, this behavior is manifested only at low velocities.

The superficial/Darcy velocity is a function of permeability, viscosity (internal resistance), and also the friction between the surface area of the pores in the permeable media and the moving fluid. This friction begins to change at low pressure gradient because dynamic friction is smaller than static friction. At very low velocities, the static friction begins to dominate the fluid movement behavior (along with the permeability and viscosity), thus contributing to the pre-Darcy effect.

Fourier's law of heat conduction describes the flow of heat, under an applied temperature gradient, through a medium via the vibrations and diffusion of electrons. Mathematically, Darcy's law is analogous to Fourier's law of thermal conduction; however, they both describe a different physical phenomenon. Even though both laws yield the same partial differential equations, differences in the physics of the two processes become apparent at extreme ends. It is worth mentioning that at very low pressure gradients, fluid will not behave exactly same as electrons would under a very low temperature gradient. The vibration and diffusion of electrons follow the Fourier's law even at lowest temperature gradients (except below the quantum phenomenon) in contrast to the fluid flow through porous medium, which involves physical transport of mass. Fluid will require some "threshold gradient" to shear and to begin flowing. Not only is the threshold gradient hinted in our experiments (and others'), the transition to flow manifests itself as the pre-Darcy effect. Hence, to extrapolate Darcy's law down to lowest (zero), pressure gradients would be in gross error (as confirmed by the experiments in this study and others). The linearity will not hold for fluids the way it holds for electron diffusion (heat conduction). This idea must be factored in when understanding the solutions (e.g., well-test analysis) and the physics behind fluid flow as opposed to heat conduction.

Conclusions

1. Experiments from previous non-petroleum literature are inconclusive about the existence of significant pre-Darcy effect. Those experiments, that confirm the existence of a pre-Darcy effect, were conducted on coarse unconsolidated material with an aqueous fluid, which casts a shadow on applicability to petroleum reservoirs.
2. A "region of interest" (to petroleum engineers) was identified and none of the published experiments were conducted in that region. This study experimentally showed the existence of pre-Darcy effects on consolidated core samples with organic fluid, in that region of interest.
3. Because most of the fluid is moving with a very low velocity, pre-Darcy effect is present in the significant portion of the flow regime distributed in the reservoir.
4. Pre-Darcy effect is caused by fluid/rock interaction, and some of the contributing factors were postulated. However, further studies are needed to list, investigate, and quantify them.

Unit conversions

1	ft/day	3.528E-06	m/s
1	stb/day	1.840E-06	m^3/s
1	psi	6895	Pa
1	ft	0.3048	m
1	mD	9.872E-16	m^2
1	psi/ft	22621	Pa/m

Nomenclature

A cross-sectional area, m^2
a empirical parameter
h formation thickness, m
k permeability, m^2
K_v modified Bessel function of the second kind of order v
m empirical index
n flow behavior index (power law parameter)
p pressure, Pa
q flow rate, m^3/s
v superficial velocity, m/s
μ viscosity, $Pa.s$
μ_{eff} effective viscosity, $Pa.s^n.m^{1-n}$

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

FS: Carried out experiments and drafted the article. MYS: Provided supervision and direction to the article. WH: Design of experiments and physical insights. AI: Provided mathematical insights and direction. All authors read and approved the final manuscript.

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