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Mathematical modeling and analysis of a dark money network

Christopher Couch, William P Fox, and Sean F Everton

Abstract

In this article, the authors present background and analysis on a dark money network. An AHP/TOPSIS (analytical hierarchy process/technique of order preference by similarity to ideal solution) hybrid model is used to find the key nodes of the network. The analysis of the key nodes leads to improved targeting strategies against the network. Game theory applications using kinetic versus non-kinetic strategies in dealing with the network are developed after using AHP to obtain cardinal utility from the ordinal ranking originally provided. These methods provide an additional metric that can be employed when dealing with and analyzing any dark network.

Keywords

networks, dark network, dark money network, analytical hierarchy process (AHP), technique of order preference by similarity to ideal solution (TOPSIS), game theory, cardinal utility, Nash equilibrium, Nash arbitration, security levels

1. Introduction

Social scientists have long considered the nature of dark networks, which are typically defined as covert and illegal networks, that is, groups that seek to conceal themselves from authorities, such as terrorist networks, drug cartels, and criminal organizations.[1] Georg Simmel, for instance, was one of the first to explore their structure in his essay on secret societies,[2] a study that Bonnie Erickson later expanded and modified.[3] A decade later, Malcolm Sparrow considered the usefulness of social network analysis (SNA) for tracking criminal networks,[4] and Wayne Baker and Richard Faulkner used SNA to examine three price-fixing conspiracy networks in the heavy electrical equipment industry.[5] Since 9/11, analysts have become increasingly drawn to the use of SNA as a tool for understanding dark networks,[6–10] largely because of Valdis Krebs's analysis of the 9/11-hijacker network.[11] Not all dark networks are malignant. Some are benign. Take, for example, Żegota, the predominantly Roman Catholic underground organization that addressed the social welfare needs of Jews in German-occupied Poland from 1942 to 1945.[12] Most would consider it a dark network because it was covert and, at least from the perspective of the Nazis, illegal.

Not all dark networks are illegal, however. Some simply seek to keep their activities hidden from the wider public. Examples of such are 'dark money' networks, which are networks of politically active non-profit organizations that can receive unlimited donations from corporations, individuals, and unions but are not required to disclose who those donors are.[13] Although non-profit organizations 'may not attempt to influence legislation as a substantial part of its activities and it may not participate in any campaign activity for or against political candidates',[14] organizations are able to sidestep this law through a loose interpretation of the phrase 'substantial part of its activities', as well as by donating to other social welfare non-profits (which then can contribute to political causes), and by purchasing advertising for educational or single issues that do not explicitly favor certain political candidates, but clearly do. These companies do not have to disclose their spending until the following year's tax returns, and the individual donors to the non-profits remain anonymous.[15] To confuse the sources of money, the

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organizations funnel money through single-member limited liability corporations (SMLLCs, also called disregarded entities), which, for income tax purposes, exist under a parent organization or individual. For this reason, when an SMLLC appears on a donor list, it is nearly impossible to know who controls it without knowing who or what created its parent LLC. To make matters even more confusing, LLCs can also be created and dissolved relatively quickly and easily.[16]

In this paper, we examine a prominent dark money network (DMN), and explore strategies for disrupting it on the one hand and building it up on the other. It provides a methodology for a social network analysis of this network, the results of which may be illuminating for the disruption of other similar dark money networks, such as money-laundering and reverse money-laundering schemes. In the military's special operations world of unconventional warfare, there are potential missions where it may be necessary to strengthen such networks, or even build them. The insights gained from looking at these kinds of network from both sides are potentially useful for military operations and law enforcement.

The two broad sets of disruption strategies analyzed here are termed 'direct' and 'non-direct'. Direct strategies (sometimes called kinetic or targeting strategies) usual refer to the removal of a node by killing, capturing or defeating it.[17] However, for the purposes of this paper, direct means prosecuting or fining any organization or individual who is found to have violated the Internal Revenue Service (IRS) restrictions on political campaign spending or non-profit activities. It also includes efforts to legislate limits as to what non-profits can and cannot do politically, as well as creating transparency in donations to such organizations, regulating the ability of the liability companies to remain anonymous, and creating laws that expand what is considered to be political spending to include single-issue advocacy. The prosecution, or attempted prosecution, of non-profits has serious pitfalls, however. One only needs to look at the political backlash that came when conservatives discovered that the IRS was investigating conservative non-profit eligibility.[18]

Traditionally, non-direct strategies are a less aggressive approach; they are more patient, subtle, and will often use partners. Frequently, the goal is to secure the population's support and marginalize the dark network.[17] In the context of this paper, this means generating social outcry against the influence of such a small group. The main point is to ally with the press to illuminate the individuals and companies that donate to or constitute the network. A primary reason why the DMN analyzed here has been successful is because of the anonymity it offers. Donors like the idea of being able to support causes without experiencing the backlash that often happens when they do. An example of why anonymity is important to donors is that when the news became public that *Target* had supported political candidates who held anti-homosexual agendas, *Target* had to deal with calls for boycotts.[19] Thus, if opponents invest in a concerted effort to illuminate the names of donors, the network is likely lose donors for the simple fact that there are easier ways to donate money and, if anonymity is lost, those avenues are likely to be taken.

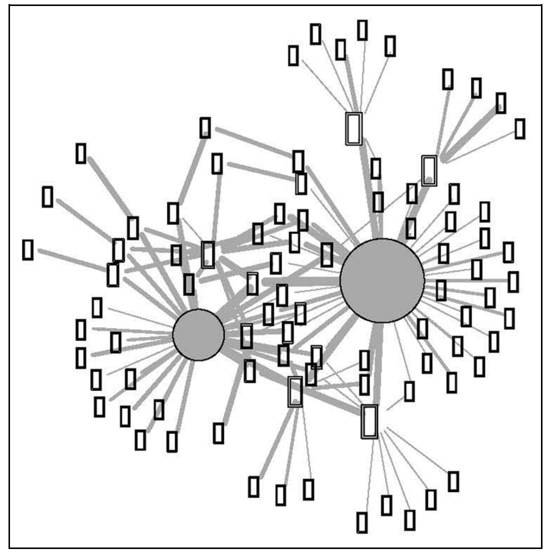

Figure 1. DMN money flow network.

What about from the perspective of the DMN? If we assume that the reason it has been successful is because it is operating close to an optimal level, the obvious thing it can do is maintain its current organizational structure. It has a small number of highly central organizations that control the flow of funds through the network, and representatives from most of these attended a key network-strategizing event. This is captured in Figure 1, which presents a network map of the DMN (generated in R with igraph library[20]) where nodes are DMN organizations and the ties between them indicate that they exchanged funds. The names have been left off the network as presented, the size of the nodes reflects *betweenness centrality* (an SNA metric that is often used to capture the ability of network members to control the flow of resources), and the width of the ties reflects the dollar amount exchanged between the organizations. Finally, the gray colored nodes are those organizations that attended the strategizing event. This network structure has the added bonus of allowing the network leadership to control the focus of the network's spending. However, it leaves the network vulnerable to the action by opponents.

If anonymity and survival is the network's goal, eventually the network's opponents will discover the names of the donors to the organization. Thus, the DMN may want to diversify and decentralize the source of the money and guidance. This will probably make it less efficient, reduce its overall effectiveness, and limit the command and control ability of the network's leaders; but it will make it harder for the DMN's opponents to disrupt it.

If anonymity and survival is the goal of the network, eventually opponents of the network will discover the names of the donors to the organizations. The other option for the DMN is to diversify and decentralize the source of the money and guidance. This will make the network less efficient, may reduce the overall effectiveness, and will definitely reduce the control the network may have; but it will make it much harder for opponents to disrupt it.

2. AHP and TOPSIS application

In previous research work,[21–27] the authors have shown the practicality and usefulness of AHP (analytical hierarchy process) and TOPSIS (technique of order preference by similarity to ideal solution) for solving real problems and improving analysis in social networks.

In a previous study of this situation, there was no systematic prioritization of the key measures used in the SNA of the DMN. This meant that the identification of which nodes were important relied on a purely subjective choice of which measures indicated importance in the network. Here we will re-analyze the results of the previous work by using an AHP and a TOPSIS to improve the analysis and selection of the strategies for both the DMN and the State.

In order to improve the social network analysis in 'A Dark Money Network Study' (unpublished), the initial project that later became this article, the AHP and TOPSIS hybrid approach is used to determine the key nodes the DMN network. The dark money network strategy group (DMNSG) only has two measures, in- and out-degree centrality, so there is no need to analyze it further. As mentioned in the study, the key members of the DMN were also the organizations most represented at the DMNSG, so a better understanding of the DMN helps the overall analysis. Previously, the most important organizations in the DMN were assumed to be the ones with the greatest out-degree centrality. After analysis with the AHP and TOPSIS, this assumption turned out to be true for only the two most important organizations. However, it was worthwhile to analyze all of the other organizations across the top eight degree centrality. This showed that all eight should be utilized in the analysis.

When looking again at the DMN, we will analyze the eight organizations with the highest overall degree centrality as calculated by the SNA tool ORA. Those organizations are listed in Table 1.

The eight criteria that will be used for the AHP and TOPSIS hybrid analysis will be the *total degree* (number of connections), *in-degree* (number of connections directed towards), *out-degree* (number of connections directed away), *eigenvector* (how connected to other highly connected nodes), *closeness* (average of how close the node is to all other nodes), *betweenness* (how many shortest paths between hubs does the node fall on), *hub* (out-links are connected to nodes with many in-links), and *authority centrality* (in-links are connected to nodes with many out-links). Because all of these criteria have specific values for each organization, we will input the unscaled measure for each criterion as calculated by ORA. In order to rank the different criteria, Saaty's standard nine-point preference table is used.[28]

Table 1. Alternatives for SNA in the dark network under overall degree centrality.

Center to Protect Patients' Rights
Freedom Partners Chamber of Commerce
TC4 Trust
American Future Fund
Americans for Prosperity
Americans for Responsible Leadership
Corner Table LLC
Americans for Job Security

Table 2. Social network measures used.

	Criterion
1	Out-degree
2	Degree
3	Eigenvector
4	Betweenness
5	Hub
6	In-degree
7	Closeness
8	Authority

Table 2 represents our order of priority for the different centrality measurements. We prioritized these measures in accordance with protocols we felt were essential to the analysis of this dark network. As mentioned above, for the DMN, out-degree centrality logically should be the most important. We then put total degree as the second most important because the total amount of money moving in or out of one of the DMN organizations logically should also be important. Eigenvector centrality was placed next because it shows how many other nodes with high centrality a node is connected to. Logically, betweenness is important in a network with a sole purpose of moving money: however, there are important organizations, namely, the originators of much of the money, who would have low betweenness, which is why it is not a top three. The same rationale goes for hub centrality. The remaining three are not very important measures for this network because of its directional nature.

	Element			More Important	Intensity (1-9)
	A		B		
1	Out Degree	compared with	Degree	A	5
2			Eigenvector	A	6
3			Betweenness	A	7
4			Hub	A	8
5			In Degree	A	9
6			Closness	A	9
7			Authority	A	9
1	Degree	compared with	Eigenvector	A	2
2			Betweenness	A	3
3			Hub	A	5
4			In Degree	A	8
5			Closness	A	9
6			Authority	A	9
1	Eigenvector	comp. with	Betweenness	A	2
2			Hub	A	5
3			In Degree	A	8
4			Closness	A	9
5			Authority	A	9
1	Betweenness	comp. with	Hub	A	2
2			In Degree	A	3
3			Closness	A	8
4			Authority	A	8
1	Hub	vs	In Degree	A	2
2			Closness	A	6
3			Authority	A	6
1	In Degree	vs	Closness	A	5
2			Authority	A	5
1	Closness	vs	Authority	A	2

Figure 2. AHP pairwise comparison template for inputs.

Table 3. Saaty's nine-point scale.[28]

Intensity of importance in pairwise comparisons	Definition
1	Equal importance
3	Moderate importance
5	Strong importance
7	Very strong importance
9	Extreme importance
2,4,6,8	For comparing between the above
Reciprocals of above	In comparison of elements *i* and *j* if *i* is 3 compared to *j*, then *j* is 1/3 compared to *i*
Rationale	Force consistency; measure values available

Table 3 represents the process to obtain the criteria weights using an analytic hierarchy process to determine how to weight each criterion for the TOPSIS analysis. Again, using Saaty's nine-point reference scale,[28] we applied subjective judgment to weight each criterion against all other criteria lower in importance. Figure 2 displays the template and inputs used.

This pairwise comparison analysis resulted in the initial criterion weights shown in Table 4. The consistency ratio, *CR*, for these pairwise comparisons is 0.062, which is less than the required 0.10. These criterion weights were then applied to the centrality values calculated by ORA in the SNA of the DMN. Those values are shown in Figure 3.

Finally, the results of the TOPSIS analysis reveal the order of the eight most important nodes in the network. Table 5 shows the order of importance of the organizations, their associated weights, and their out-degree centrality.

As we see, the two most important organizations in the DMN are also the organizations with the highest out-degree centrality. However, other important organizations are clearly important but do not have high out-degree centrality. Looking at Corner Table LLC, it is clear that even though it does not have high out-degree centrality, it is the third most important organization. On the other hand, TC4 Trust had the third highest out-degree centrality, yet it is the seventh most important node.

These findings improve the analysis in 'Social Network Analysis of the Dark Money Network' (unpublished), another project that was the forerunner of this article. In our targeting strategies, we conducted an analysis of what would happen if all LLCs were removed from the network, and what would happen if the Center to Protect Patients' Rights was removed from the network. Originally we

		Out Degree	Degree	Eigenvector	Betweenness	Hub	In Degree	Closness	Authority
Alternatives	Criterion	C1	C2	C3	C4	C5	C6	C7	C8
Center to Protect Patient's Rights		78	117	0.442	785.5	0.516	39	0.024	0
Freedom Parners Chamber of Commerce		90	95	0.484	240	1.225	0	0.023	0
TC4 Trust		38	38	0.219	0	0.467	0	0.111	0
American Future Fund		22	33	0.246	296.5	0.099	10	0.023	0.429
Americans for Prosperity		13	27.5	0.14	134	0.039	10	0.022	0
American for Responsible Leadership		16	36.2	0	44	0.034	0	0.022	0
Corner Table LLC		7	19	0.328	0	0.022	12	0	0.538
American for Job Security		13	18	0	55	0.02	0	0.022	0

Figure 3. Social network values for TOPSIS.

Table 4. Criterion weights from pairwise comparisons.

Criterion number	Criterion name	Criterion weight
1	Out-degree	0.50199
2	Degree	0.14517
3	Eigenvector	0.11045
4	Betweenness	0.07489
5	Hub	0.05459
6	In-degree	0.04378
7	Closeness	0.03525
8	Authority	0.03418

Table 5. TOPSIS output.

Name	Larger better	Out-degree
Freedom Partners Chamber of Commerce	0.753817996	90
Center to Protect Patients' Rights	0.619213658	78
Corner Table LLC	0.490923764	7
American Future Fund	0.488711591	23
Americans for Prosperity	0.486941592	13
Americans for Responsible Leadership	0.48367178	16
TC4 Trust	0.483520063	38
Americans for Job Security	0.481719794	13

thought the Center to Protect Patients' Rights was the most important of the organizations because ORA predicted it was the most important throughout all of its measures. After AHP and TOPSIS analysis, our recommendations for kinetic targeting would be different. We would run analysis of the removal of the Freedom Partners Chamber of Commerce, and Corner Table LLC as well as the removal of all LLCs.

We argue that, in pursuing a non-kinetic strategy, the investigative reporters should not waste their efforts trying to uncover the identities of all donors to all of the organizations. The investigators should focus on the top five organizations in terms of out-degree centrality. If they could illuminate the donor list of just the Center to Protect Patients' Rights it is likely to reveal the largest number of donors.

After our TOPSIS analysis, we would now recommend that investigators focus on the top five organizations as determined by TOPSIS.

3. Sensitivity analysis

The decision weights are subjective and lend themselves to sensitivity analysis to determine how a change in the weights affects the final ranking. Sensitivity analysis is essential for good analysis. A model by Alinezhad and Amini[29] suggests sensitivity analysis for AHP and TOPSIS for changing a criterion weight and modifying all other weights proportionally. We use Equation (1) to perform our sensitivity analysis:

$$w_j' = \frac{1 - w_p'}{1 - w_p} w_j \tag{1}$$

where w_j' is the new weight, w_p is the original weight of the criterion to be adjusted, and w_p' is the value after the criterion was adjusted. We found this to be an easy method to adjust weights to re-enter into our model within the template.

We began with the most heavily weighted criterion, out-degree centrality. We modified it by 0.1 increments and ensured that it was no longer the most heavily weighted criterion. We provide a visualization of the results in Figure 4, which shows that Freedom Partners Chamber of Commerce remains the number one ranked alternative.

Figure 5 shows clearly that altering the lowest criterion to make it larger in 0.1 increments results in the Center to Protect Patients' Rights overtaking Freedom Partners Chamber of Commerce. This shows how modifications in decision weights affect ranking in this analysis.

4. Applying game theory

In order to improve the results, much more analysis using ORA is necessary. There were certain measurements

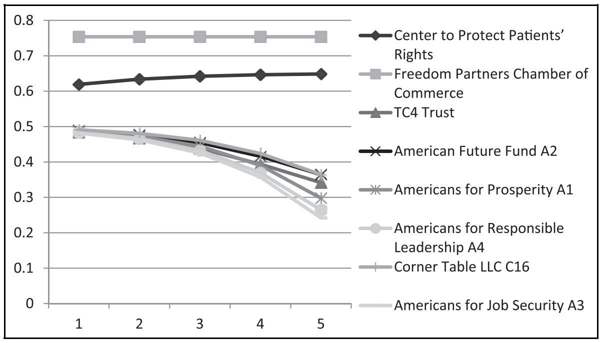

Figure 4. Sensitivity analysis on out-degree centrality.

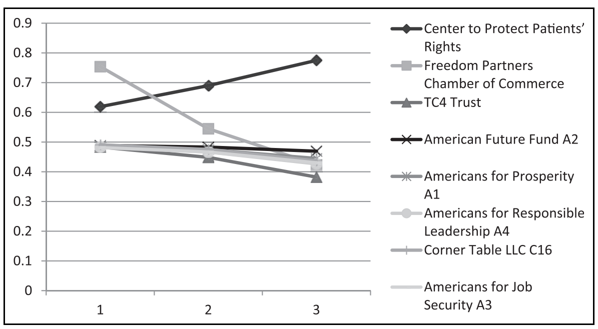

Figure 5. Sensitivity analysis on closeness.

where not all of the top organizations had listed data. This is because they were not in the top ten for that particular metric and ORA only provides the top ten.

Analysis of the strategies for the DMN and the State trying to defeat them leads to the application of game theory. When conducting game theory analysis, we were originally limited to using ordinal scaling, and to ranking each of the four options one through four. The game was set up as shown in Table 7. Strategy A is for the State to pursue a non-kinetic strategy, while B is a kinetic strategy. Strategy C is for the DMN to maintain its organization and D is for it to decentralize.

First, we define some meanings. Traditionally in network disruption, kinetic refers to the removal of a node by killing, capturing or defeating it. These are also called targeting strategies. Because this network is currently legal, and the Liberal leadership in the United States is unable to conduct these types of operation, for the purposes of this paper, kinetic means prosecuting or fining any organization or individual who is found to have violated the Internal Revenue Service's restrictions on political campaign spending or non-profit activities. Kinetic will also include creating legislation that clearly states what non-profits can and cannot do politically, as well as legislating for transparency in donations to such organization, regulating the ability of the liability companies to remain anonymous, and creating laws that expand what is considered political spending to include single-issue advocacy. The prosecution, or attempted prosecution, of non-profit organizations has serious pitfalls. One only needs to look at the political backlash that came about when conservatives found out that the IRS was investigating conservative non-profit eligibility.

Traditionally non-kinetic is a less aggressive approach: it is more patient, subtle, and often will use partners. Its goal is usually to secure the support of the population and to marginalize the dark network. Here, non-kinetic means creating a social outcry against the influence of such a small group. The main purpose, in this case, is to ally with the press to illuminate the individuals and companies that donate to the network, or make up the network. One of the main reasons that this network is successful is the anonymity it offers. Donors like the idea of funneling their money to conservative causes without the backlash that happens when they do.

Based on these definitions and the combination of strategies played by each player, these strategies are ranked ordinally from 1 (worst case) to 4 (best case) for each player.

First, we consider the State.

- AD: This is the most desirable course of action because it does not hurt its own ability to raise money by legislating away dark money and avoids the political mess of trying to prosecute. The DMN has decentralized, which reduces its efficiency, as well as reducing the control the network can exert.
- BD: This is the second most desirable situation because the DMN has decentralized, which is the most important thing. However, the State has degraded its ability to use dark money.
- AC: This is the second-worst-case scenario. The DMN did not decentralize and although they have had to make some difficult choices to maintain their organization, they are not significantly degraded. However, the good side is that the State can still raise money using dark money.
- BC: This is the worst-case scenario. They have degraded their own fund-raising ability, are likely to have dealt with significant political blowback from their 'attacks' on conservative groups, and the DMN is still operating efficiently.

For the DMN, the most important thing is maintaining anonymity. Without it, they will slowly lose donors, and are likely to face public backlash and business setbacks. Maintaining ultimate efficiency, as well as control by the network, is important, but not as important as maintaining the flow of money.

- BC: This is the best case for the DMN in this scenario. They simply do not have a complete-win scenario like the State does. They have proven that they can recover from the attack of key nodes. As for the legislation, unless all political activity is banned for non-profits, they will be able to maintain the flow of money. They also recognize the difficulty in legislation. Most importantly, they maintain their anonymity.
- BD: The kinetic strategies have worked well enough so that they have had to decentralize to make the targeting more difficult. The flow of money is degraded, but anonymity is maintained.
- AD: This is the second-worst-case scenario because the State has succeeded in lowering their anonymity enough so that they have had to decentralize. By decentralizing, they minimized the impact of the State's efforts to illuminate their donors. The more decentralized they are, the more difficult it will be for the State to reveal identities.
- AC: This is the worst-case scenario. They were unable or unwilling to decentralize and the State pursued a strategy of illuminating their donors. They have lost a large number of their donors, and their members and their businesses have received public backlash.

The ordinal values for the payoff matrix are depicted in Table 6. The equilibrium is found to be AD, which means the State does not use kinetic force and the DMN decentralizes to make it harder to track them.

This ordinal scaling worked well through all strategic moves. However, without a way of determining interval scaling, it was impossible to conduct proper analysis using prudential strategies, Nash arbitration, or Nash equalizing strategies because they all require cardinal scale values for the mathematics. This project will apply AHP, in lieu of the lottery method, in order to determine the interval-scaled payoffs of each strategy for both the DMN and the State. Again, we will the use the standard nine-point preference. The AHP templates were used in the analysis. For the State's the evaluation criteria we chose for the four possible outcomes were: how well it degraded the DMN, how well it maintained the states own ability to fundraise, how well the strategy would rally their base, and, finally, how well it removed nodes from the DMN. The evaluation criteria we chose for the DMN's four possible outcomes were: how anonymity was maintained, how much money the outcome would raise, and, finally, how well the DMN could maintain control of the network.

Table 6. Game theory payoffs using ordinal scale.

		Dark money network	
		Strategy C: centralize	Strategy D: decentralize
State	Strategy A: non-kinetic	(2,1)	(4,2)
	Strategy B: kinetic	(1,4)	(3,3)

For the State, we input our priority strategies and our AHP nine-point scales to obtain a matrix

	AD	BD	AC	BC
	1	2	3	4
AD	1	2	3	4
BD	1/2	1	3	4
AC	1/3	1/3	1	2
BC	1/4	1/4	1/2	1

We obtained the eigenvector as the weights and verified that the *CR* was less than 0.1. The results with a *CR* = 0.02244 were:

AD 0.480038
BD 0.28523
AC 0.139265
BC 0.095467

For the DMN we input our priority strategies and our nine-point scales to obtain a matrix

	BC	BD	AD	AC
	1	2	3	4
BC	1	2	3	4
BD	1/2	1	3	5
AD	1/3	1/3	1	2
AC	1/4	1/5	1/2	1

We obtained the eigenvector as the weights and verified that the *CR* was less than 0.1. The results with a *CR* = 0.03248 were:

BC 0.480645265
BD 0.290615129
AD 0.137848095
AC 0.09089151

After conducting the AHP analysis, we obtained a new payoff matrix with cardinal utility values replacing the ordinal values (Table 7).

With proper cardinal scaling for the players' utilities, it is now possible to conduct analysis such as to find prudential strategies, Nash equalizing strategies, and Nash

Table 7. Cardinal values for payoff matrix from AHP weights.

		Dark money network	
		Strategy C: centralize	Strategy D: decentralize
State	Strategy A: non-kinetic	(0.139, 0.091)	(0.48, 0.137)
	Strategy B: kinetic	(0.0955, 0.548)	(0.2853, 0.2906)

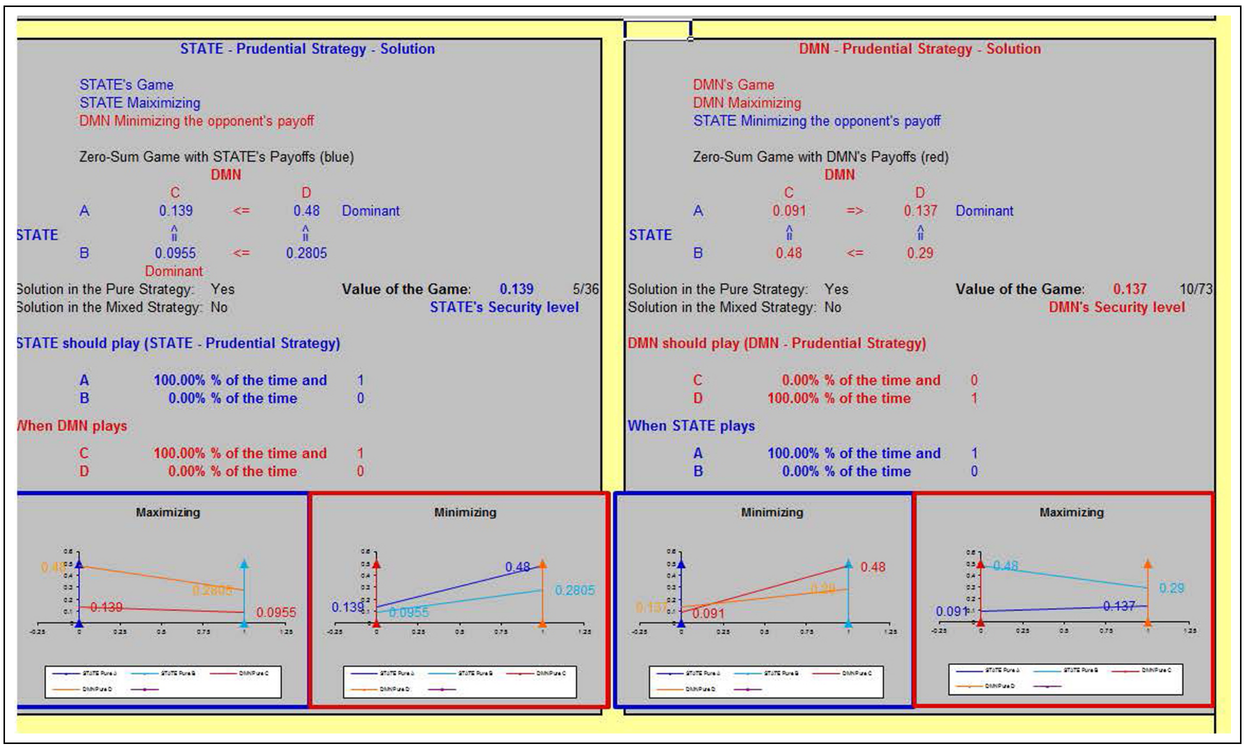

Figure 6. Prudential strategy solver by Feix.[30]

arbitration. Using a series of game theory solvers by Feix[30] and Fox,[31] we obtained the following results.

- Nash equilibrium: AD (0.48, 0.137) using strategies of non-kinetic and decentralize.
- Mixed Nash equalizing strategies might be more applicable as is for the State to play non-kinetic 84.8385% of the time and kinetic 15.1615% of the time, and the DMN to play maintain organization in centralized mode.
- Prudential strategies (security levels) (0.139, 0.137). The prudential strategies are for the State to play A and the DMN to play C always, as shown in Figure 6.

Since there is no equalizing (mixed) strategy for the DMN, should the State attempt to equalize the DMN they should use non-kinetic strategies 84.8385% and kinetic strategies 15.1615% of the time (see Figure 7).

This is a significant departure from our original analysis prior to including the AHP pairwise comparisons in our analysis. The recommendations for the State were to use a kinetic strategy 50% of the time and a non-kinetic strategy 50% of the time. However, it is obvious that, with proper scaling, the recommendation should have been to execute a non-kinetic strategy the vast majority (85%) of the time, and only occasionally (15%) conduct kinetic targeting of network nodes. This greatly reinforces the recommendation to execute a non-kinetic strategy to defeat the DMN.

Finally, if the State and the DMN could enter into arbitration, the result would be close to BD but projected to the Pareto optimal line segment with values (0.31, 0.3192), as shown in Figure 8.

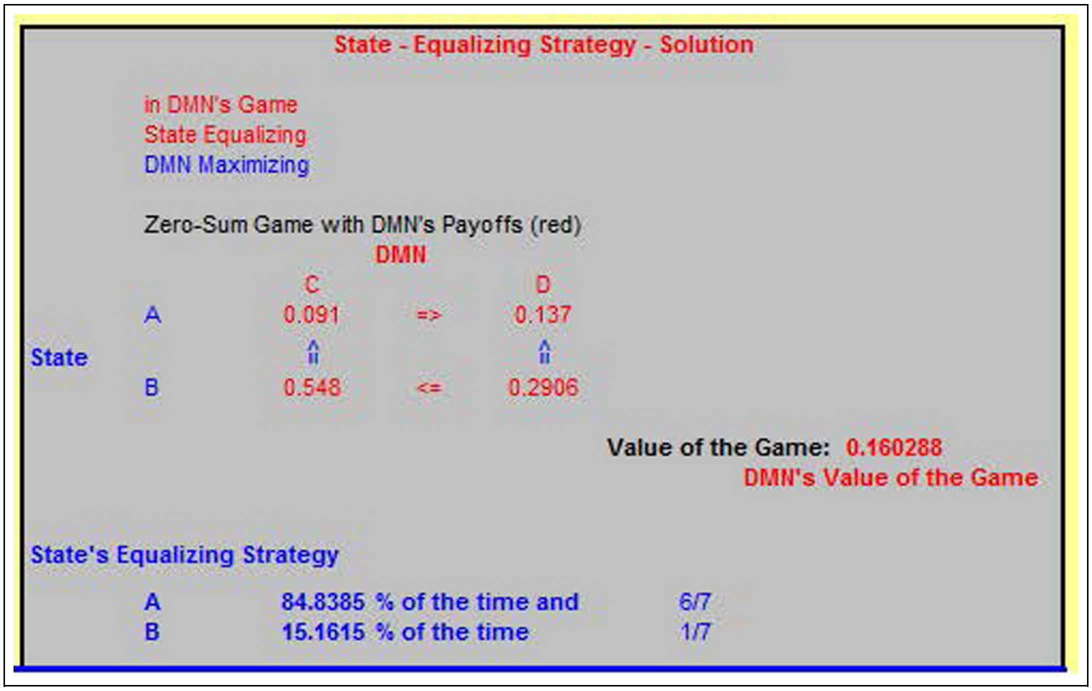

Figure 7. State's equalizing strategy.

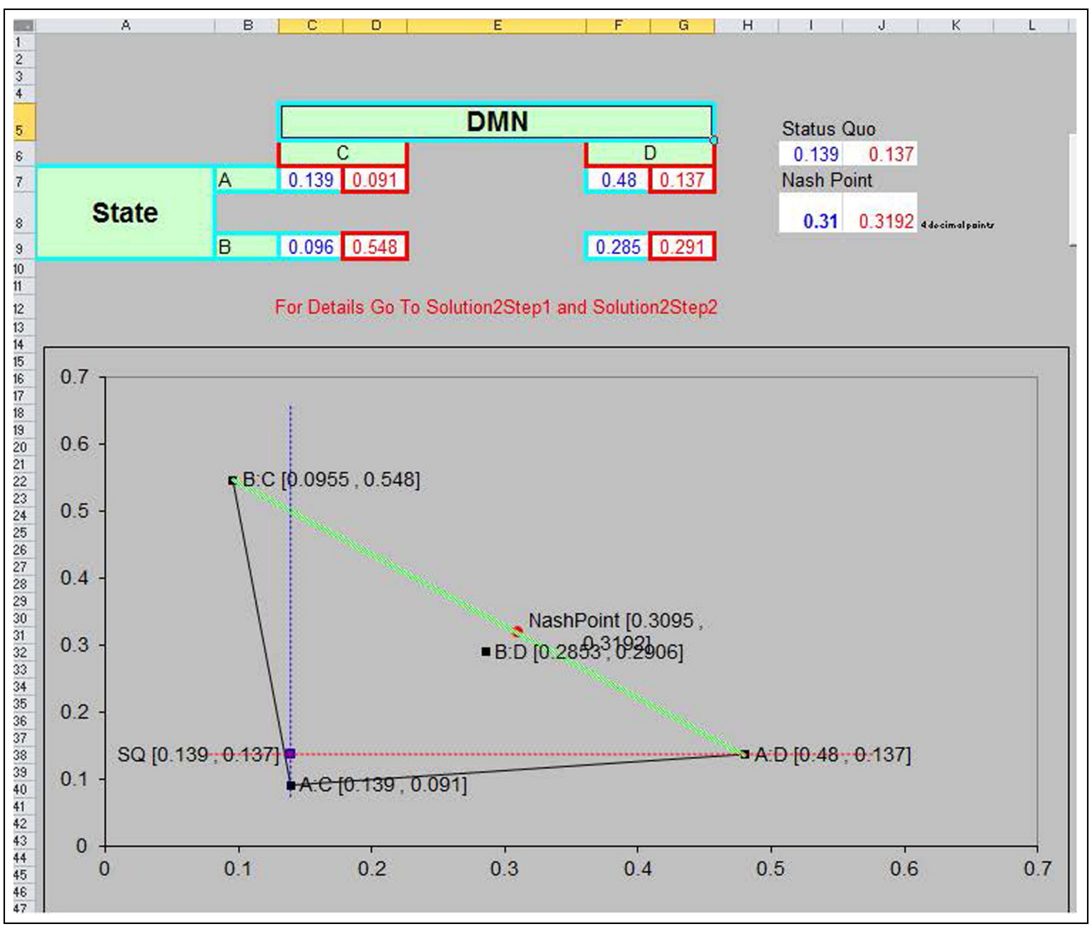

Figure 8. Nash arbitration solver by Feix.[30]

Conducting AHP analysis greatly improved the accuracy of the equalizing strategy of the State, and reinforced our conclusions.

Further extension

We decomposed both kinetic and non-kinetic strategies into actual actions. We decomposed the kinetic strategy into actions {A1, A2, A3} defined as: A1 = disrupt network, A2 = prosecute or fine, A3 = legislation. We decomposed the non-kinetic strategy into actions {B1, B2} defined as: B1 = social outcry, C2 = illuminate the network. This yielded 10 strategies and we put them in order of priority:

B1C, B2C, B1D, B2D, A2C, A2D, A3C, A3D, A1C, A1D

Using the AHP template from before, we obtained cardinal values for these 10 strategies for our two players.

Nonlinear programming approach for two or more strategies for each player

For games with two players and more than two strategies each, we present the nonlinear optimization approach by Barron.[32] Consider a two person game with a payoff matrix as before. We separated the payoff matrix into two matrices **M** and **N** for players I and II. We solved the following nonlinear optimization formulation in expanded form in Equation (2):

$$\text{maximize} \sum_{i=1}^{n}\sum_{j=1}^{m} x_i a_{ij} y_j + \sum_{i=1}^{n}\sum_{j=1}^{m} x_i b_{ij} y_j + -p - q$$

subject to

$$\begin{aligned} &\sum_{j=1}^{m} a_{ij} y_j \le p,\ i = 1, 2, \ \ldots, n, \\ &\sum_{i=1}^{n} x_i b_{ij} \le q,\ j = 1, 2, \ \ldots, m, \end{aligned} \tag{2}$$

$$\sum_{i=1}^{n} x_i = \sum_{j=1}^{m} y_j = 1$$
$$x_i \ge 0,\ y_j \ge 0$$

We solved this using the computer algebra system Maple (version 15). We defined the following matrices: *M, N, X,* and *Y*.

$$M = \begin{bmatrix} 0.084 & 0.183 \\ 0.084 & 0.183 \\ 0.028 & 0.0648 \\ 0.07 & 0.162 \\ 0.056 & 0.1296 \end{bmatrix}$$

$$N = \begin{bmatrix} 0.0445 & 0.495 \\ 0.0445 & 0.0495 \\ 0.0924 & 0.0696 \\ 0.231 & 0.174 \\ 0.1848 & 0.132 \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

We set up the nonlinear optimization problem:

$$\begin{aligned} \text{maximize}\, Z = {} & 0.1285 y_1 x_1 + 0.1285 y_1 x_2 + 0.1204 y_1 x_3 \\ & + 0.301 y_1 x_4 + 0.2408 y_1 x_5 + 0.678 y_2 x_1 + 0.2325 y_2 x_2 \\ & + 0.1344 y_2 x_3 + 0.336 y_2 x_4 + 0.2616 y_2 x_5 - p - q \end{aligned}$$

subject to

$$y_1 + y_2 = 1$$
$$x_1 + x_2 + x_3 + x_4 + x_5 = 1$$
$$0.028 y_1 + 0.0648 y_2 < p$$
$$0.056 y_1 + 0.1296 y_2 < p$$
$$0.07 y_1 + 0.162 y_2 < p$$
$$0.084 y_1 + 0.183 y_2 < p$$
$$0.0445 x_1 + 0.0445 x_2 + 0.0924 x_3 + 0.231 x_4 + 0.1848 x_5 < q$$
$$0.495 x_1 + 0.0495 x_2 + 0.0696 x_3 + 0.174 x_4 + 0.132 x_5 < q$$

Non-negativity

Our solution using either the NLP solver or the QP solver in Maple is:

$$Z = -0.001129 \text{ when}$$

$$\begin{aligned} &p = 0.084,\ q = 0.0595,\ x_1 = 0,\ x_2 = .92,\ x_3 = 0, \\ &x_4 = 0.8,\ x_5 = 0,\ y_1 = 1, \text{ and } y_2 = 0. \end{aligned}$$

The solution indicated that the State should use a random equalizing strategy 92% of the time to illuminate the network and 8% of the time to find the network. The DMN maintains central control 100% of the time.

5. Conclusions

Although no significant changes resulted from MADM analysis, the quality of the findings in both 'Social Network Analysis of the Dark Money Network' and 'Insight Into the Dark Money Network' (both unpublished) were improved. After the TOPSIS analysis of the DMN, it was clear that to only look at the out-degree centrality as the important metric did not show the full picture. By analyzing where organizations ranked among multiple metrics, TOPSIS revealed new insights that were analyzed in order to improve the dark money study. AHP provided the proper interval scaling to game theory, and the results relative to using kinetic or non-kinetic strategies make stronger recommendations since the utilities, rather than ordinal ranking, are used. In addition, being able to recommend that about 85% of your effort should go to non-kinetic solutions is a more specific recommendation. The quality of the analysis has been greatly improved.

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