Experimental investigation and modeling of the tension behavior of polycarbonate with temperature effects from low to high strain rates

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The effects of strain rate and temperature on the tension stress–strain responses of polycarbonate are experimentally investigated over a wide range of strain rates (0.001–1700 s⁻¹) and temperatures (0–120 °C). A modified split Hopkinson tension bar is used for high-rate uniaxial tension tests. Experimental results indicate that the stress–strain responses of polycarbonate at high strain rates exhibit the non-linear characteristics including the obvious yielding and strain softening. The tension behavior is strongly dependent on the strain rate and temperature. The values of yield stress and strain at yield present a dramatic increase at higher strain rates and decrease with the increase in temperature. Moreover, there exists a significant rate-sensitivity transition in the polycarbonate tension yield behavior. Based on the experimental investigation, a physically based three-dimensional elastoplastic constitutive model for the finite deformation of glassy polymers is used to characterize the rate-temperature dependent yield and post-yield behavior of polycarbonate when subjected to tension loading. The model results are shown close to the experimental data within the investigated strain-rate and temperature ranges.

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1. Introduction

Amorphous polymers have been extensively utilized as structural materials due to their excellent combination of mechanical property, production cost and weight. Polycarbonate can supply the good impact resistance as well as the transparency. Thus, it has a variety of engineering applications such as aircraft, automotive and security shielding components for impact protection. High strain rates are commonly encountered during the impact events such as projectile striking or blast loading and the polycarbonate will deform at high strain rates under the extreme temperature environments at actual usage. Consequently, an accurate understanding and modeling of the impact responses of polycarbonate at various temperatures is of great importance for scientific research and engineering structural design.

The split Hopkinson bar is an effective tool for investigating the dynamic behavior of materials at high strain rates (Gray, 2000). With the technical development of split Hopkinson pressure bar (SHPB) testing (Gray and Blumenthal, 2000), it becomes possible to evaluate the rate dependence of the stress–strain behavior of polymers at high strain rates up to 10³ s⁻¹, which is two orders of magnitude higher than those commonly performed under quasi-static and moderate strain-rate loadings from 10⁻⁴ to 10¹ s⁻¹. A great deal of attention has been given to the dynamic stress equilibrium, the constant strain-rate loading and the high signal-to-noise ratio of transmitted pulses for polymeric materials due to their properties of low density, stiffness and strength. Some experimental modifications such as the pulse shaping technique and the use of low-impedance aluminum, titanium or polymeric bars are proposed to perform the SHPB testing for low impedance polymers (Zhao et al., 1997; Chen et al., 2002; Zhao, 2003; Song and Chen, 2004; Khan and Farrokh, 2006). Also, the optimal specimen thickness and lubricant needs to be found to obtain correct response of polymers from SHPB experiments (Dioh et al., 1993; Cady et al., 2003; Trautmann et al., 2005). Numerous studies have investigated the effects of strain rate and temperature on the compressive behavior of polycarbonate (Rietsch and Bouette, 1990; Siviour et al., 2005; Mulliken and Boyce, 2006; Richeton et al., 2006; Omar et al., 2011). It is well known that polycarbonate exhibits an obvious elastic–plastic stress–strain behavior including linear elasticity, nonlinear elasticity, yielding, post-yield strain softening and strain hardening when subjected to high-rate compressive loading. Moreover, a dramatic increase of yield stress may be observed at high strain rates. Investigations on the relationship between the yield stress and logarithm strain rate in a wide strain-rate region indicate that there exists a transition
threshold of rate sensitivity for yield behavior (Mulliken and Boyce, 2006), which is dependent on temperature (Richeton et al., 2006).

For the rate-dependent nonlinear elastic stress–strain behavior of polymers, an extensive number of rheological theories incorporating linear viscoelasticity and nonlinear viscoelasticity have been proposed in the differential or integral forms. The series and parallel connections of linear and nonlinear springs and dashpots are developed to simulate the viscoelastic responses of polymers (Krempl and Khan, 2003; Ward and Sweeney, 2004; Khan et al., 2006; Shim and Mohr, 2011). Based on the widely accepted knowledge that the dominant mechanism during the plastic deformation of glassy polymers is a thermally activated process of molecular movements, analytical models are proposed for capturing the transition characteristic of yield behavior in the moderate strain-rate domain for strain rates less than $10^3 \text{s}^{-1}$ (Ree and Eyring, 1955; Bauwens-Crowet, 1973). For glassy polymers, the yield behavior is related to the two thermally activated processes, $\alpha$ and $\beta$, which are the primary factors dominated at low and high strain rates up to the rate in the order of $10^4 \text{s}^{-1}$ (Mulliken and Boyce, 2006). Furthermore, the modeling of yield-rate relation incorporating the temperature influence is extended over a wide range of temperatures across the glass transition to the rubbery state (Richeton et al., 2007). In the case of modeling the viscoplastic responses of glassy polymers, phenomenological models and physically based models have been introduced to describe the finite deformation behavior of glassy polymers (Boyce et al., 1988; Arruda and Boyce, 1999a; Wu and Van der Giessen, 1993; Arruda et al., 1995; Frank and Brockman, 2001; Anand and Gurtin, 2003; Wu and Buckley, 2004; Ghorbel, 2008). According to the concept that the plastic deformation is related to the macromolecular structure as well as the motion of the polymer segments and the alignment of the long-chain polymer molecules, Mulliken and Boyce (2006) developed an elastic–viscoplastic model by taking account of both $\alpha$ and $\beta$ molecular processes to encompass high-rate compressive loading conditions. Following the thermodynamic framework and based on the principle of virtual power, Anand and co-workers developed the thermo-mechanically coupled theories to model the strain-rate and temperature dependent large-deformation responses of amorphous polymers in a temperature range which spans their glass transition temperature (Anand et al., 2009; Sri-vastava et al., 2010). Although differing in detail, the aforementioned elastic–viscoplastic constitutive models have the ability to predict the finite strain deformation including linear elasticity, yielding, post-yielding and thermo-mechanical coupling behavior of amorphous polymers within a large strain-rate and temperature range.

Owing to the fact that there exist significant differences in tensile and compressive deformation behavior of polymers under high strain-rate loadings (Chen et al., 2002), it is therefore necessary to study the dynamic tension behavior of polycarbonate at various temperatures as a supplement of previous researches. In particular, tension tests can give us more deformation and damage information for glassy polymers such as crazing (Estavez et al., 2000; James et al., 2012) and necking (Li and Buckley, 2010; Uchida and Tada, 2011). However, due to the testing technique difficulty, very little research has been reported on the tension behavior of polymers at high strain rates (Cheng and Chen, 2003; Sarva and Boyce, 2007; Yin and Wang, 2010). The purpose of the present paper is to investigate the effects of strain rate and temperature on the tension responses of polycarbonate. A modified experimental technique based on the split Hopkinson tension bar (SHTB) was introduced to perform the high rate tension testing on polycarbonate at various temperatures. Experimental technique for detecting strain signals at a high signal-to-noise ratio was employed to capture the weak transmitted signal. The shape of the incident stress pulse was controlled in order to obtain the reliable stress–strain responses. Based on the experimental investigation, a rate and temperature dependent elastoplastic constitutive model was extended to describe the strain-rate and temperature dependent tension stress–strain responses of polycarbonate.

2. Experimental details

2.1. Material

The material used in the present study was polycarbonate, purchased as extruded sheets (3 mm in thickness) of Lexan® MR10 from GE plastics. The mass density of polycarbonate is $1.2 \times 10^3 \text{kg/m}^3$. The glass transition temperature is 140°C. Specimens were machined directly from sheet stock using a miling machine and then kept at room temperature for more than three days prior to testing.

2.2. High strain-rate uniaxial tension tests at different temperatures

High strain-rate tension tests at engineering strain rates of $10^2$–$10^3 \text{s}^{-1}$ were carried out using the split Hopkinson tension bar technique for obtaining the engineering stress – engineering strain relation of polycarbonate.

The technical differences between the SHTB and SHPB are the generation of the incident stress pulse and the specimen connection to the incident/transmitted bars. The SHTB setup used in the current study is schematically shown in Fig. 1 together with a Lagrange $X$-$t$ diagram for illustrating the wave propagation in the bars. The pulse generating system includes a rotating disk with two pairs of impact hammers, an impact block, a prefixed metal bar and a connector. Also the top view of the pulse generating system is shown in Fig. 1. The incident stress pulse is initiated by the impact of the hammer fixed on the high-speed rotating disk on the impact block, which causes the prefixed metal bar connected to the block and the incident bar to deform until fracture. The amplitude of the incident stress pulse depends upon the diameter, $d_p$, of the prefixed metal bar. Correspondingly, a high diameter results in higher strain rates. The rise time and the duration of the incident stress pulse are dependent on the impact velocity and the length, $l_p$, of the prefixed metal bar. The maximum strain of the specimen is proportional to the duration of the incident stress pulse. The aim of this technique is to generate the incident stress pulse by making use of the plastic flow of prefixed metal bar and filter out the high-frequency components of the loading wave arising from the direct impact between the hammer and the block. The prefixed metal bar actually acts as a low pass filter and the oscillation in the incident pulse can be filtered a great deal.

The incident strain $\varepsilon_i(t)$, reflected strain $\varepsilon_r(t)$, and transmitted strain $\varepsilon_t(t)$ were recorded as functions of time $t$ using strain gages G1 and G2 attached to the bars at two locations, respectively. According to the one-dimensional elastic stress wave theory (Zukas et al., 1982), the engineering stress, strain and strain rate in the specimen can be obtained from the strain gage measurements as:

\begin{equation}
\sigma_{i}(t) = \frac{P_1(t) + P_2(t)}{2A_s} = \frac{EA}{2A_s} \left[ \varepsilon_i(t) + \varepsilon_r(t) + \varepsilon_t(t) \right]
\end{equation}

\begin{equation}
\varepsilon_i(t) = \frac{u_1(t) - u_2(t)}{2} = \frac{C_0}{l_p} \int_0^t [\varepsilon_i(\tau) - \varepsilon_r(\tau) - \varepsilon_t(\tau)] d\tau
\end{equation}

\begin{equation}
\dot{\varepsilon}_r(t) = \frac{C_0}{l_p} [\varepsilon_i(t) - \varepsilon_r(t) - \varepsilon_t(t)]
\end{equation}

where $P_1(t)$, $P_2(t)$ and $u_1(t)$, $u_2(t)$ are the forces and displacements on the right end of the incident bar and on the left end of the
transmitted bar, respectively. $E$ and $A$ are the Young’s modulus and the cross-sectional area of the incident/transmitted bars. $C_0$ is the longitudinal wave velocity in the tension bars. $A_i$ and $l_i$ are the cross-sectional area and gage length of the specimen, respectively. The specimen stress-time and strain–time profiles are combined to obtain the dynamic engineering stress–strain response.

The experimental principle of such a split Hopkinson bar assumes that there exists a state of stress equilibrium and uniform deformation in the specimen during the process of impact loading. The conditions of stress equilibrium and homogeneous deformation in the specimen and constant strain-rate state during the loading should be satisfied to obtain the reliable results of stress–strain responses of materials in a Hopkinson bar test. However, the equilibrium time for polycarbonate is much longer than the metal specimen due to its low longitudinal wave speed. Therefore the matching relation between the shape of the incident stress pulse and the specimen dimension should be found to meet the requirement of stress equilibrium and constant strain-rate loading state in the specimen; for example, adjusting the impact velocity of the impact hammer to increase the rise time of the incident pulse. Stress equilibrium means the forces on the two ends of specimen are equal during loading, namely $P_1(t) = P_2(t)$, so the relation $\dot{\varepsilon}_i(t) + \dot{\varepsilon}_r(t) = \dot{\varepsilon}_t(t)$ can be deduced when the incident and transmitted bars are made out of the identical material and have the same cross-sectional area. Then the strain rate history in the specimen will be given as following:

$$\dot{\varepsilon}_t(t) = \frac{2C_0}{l_i} (\varepsilon_i(t) - \varepsilon_r(t)) = -\frac{2C_0}{l_v} \varepsilon_r(t)$$  (4)

It is seen in Eq. (4) that the reflected signal measured from the gage on the incident bar indicates the strain-rate history in the specimen. The amplitude of the reflected signal is proportional to the strain rate in the specimen. The shape of the reflected strain signal depends on the waveforms of the incident pulse and the transmitted pulse which is determined by the specimen material’s response. A flat platform in the reflected wave means a constant strain-rate state in the specimen.

The dumb-bell shaped flat-plate specimen was used in the split Hopkinson tension bar testing to make the connection to the bars and eliminate the stress concentration, as is shown in Fig. 2. The specimen was bonded to the incident/transmitted bars by means of adhesion. Compared with the commonly used threaded connection of cylindrical specimen in split Hopkinson tension tests, adhesive connection to the bars can prevent the interference on the wave propagation in the bars owing to the existence of the thread clearance. Such interference may result in non-constitutive oscillations in the measured stress–strain curves, which do not represent the true material responses. Generally, the length-to-width ration of the tension specimen used in SHTB testing is larger than that used in SHPB testing. As for the determination of the specimen gage length, it is apparent that the longer the gage length of specimen, the larger the longitude inertia is in the specimen, which leads to a stress oscillation on the stress-time curve and much more time to reach the stress equilibrium. On the other hand, the gage length should be long enough to ensure the stress and strain uniformity in the testing section of the specimen under a unidirectional stress state. Also, the gage width of the specimen...
needs to satisfy such uniformity requirement. According to the shape of the transmitted signal obtained from the preliminary trial high-rate tests, the specimen dimension should be determined as accurately as possible to meet the requirement of experimental principle and reach the constant strain-rate state in the specimen.

Compared with the metals, polycarbonate is a low-impedance, low-strength material. When the incident stress pulse travels to the incident bar/specimen interface, most of the incident pulse is reflected into the incident bar, which leads to a weak transmitted strain signal in the transmitted bar. To increase the amplitude of the transmitted signal, the semiconductor strain gages which have a high gage factor were mounted on the surface of the transmitted bar instead of the conventional strain gage to monitor the transmitted stress pulse. Furthermore, the gage signals were gathered using a low-noise wide-band amplifier with frequency response up to 2 MHz and recorded using a high resolution digital oscilloscope Nicolet Integra 40.

In the present work, the prefixed metal bar was made of aluminum alloy. Both incident and transmitted bars were fabricated of steel with a diameter of 14 mm. The incident bar was 3000 mm long while the transmitted bar was 2500 mm. The specimen thickness, \(T\), was 3 mm. The gage length, \(l_s\), width, \(W_s\), and fillet radius, \(R\), of the specimen were 8, 4 and 2 mm, respectively. The width of the connection part, \(W\), was 14 mm and the total length of the specimen, \(l\), was 68 mm. A liquid-nitrogen cooled environmental chamber and a resistance-wire heated temperature chamber were used to create the testing temperatures below and above the room temperature, respectively.

2.3. Quasi-static and moderate-rate uniaxial tension tests

Quasi-static tension tests at strain rates of \(10^{-3}\) and \(10^{-2}\) s\(^{-1}\) were performed on a MTS810 servo-hydraulic system to obtain the stress–strain relations using a constant engineering strain-rate control mode. The tensile deformation was measured using the mechanical-type extensometer. Low and high temperature chambers were used to investigate the temperature dependence of tension behavior for polycarbonate. Moderate rate tension tests within the rate range of \(10^{-1}\)–\(10^{1}\) s\(^{-1}\) were performed on the moderate strain-rate testing set-up and the specimen deformation was measured using the optical extensometer based on the position sensitive detector technique. The detailed description of this setup is presented in the work of Wu et al. (2004). The specimen geometry used in quasi-static and moderate strain-rate tension tests was similar to that used in the high-rate tests except that the gage length was long enough to avoid the end effects. The dimensions of the specimen were of 62 mm in gage length, 10 mm in width and 3 mm in thickness. Similar to the calculation of engineering strain in the high-rate tension tests, the engineering strain was measured over the entire gage length of the specimen in the quasi-static and moderate-rate tension tests.

3. Experimental results and discussion

Fig. 3 shows the typical strain gage signals recorded on the incident/transmitted bars. A clear transmitted pulse with high signal-to-noise ratio was obtained. The raw incident stress pulse is smooth, so the numerical smoothing of experimental data is not necessary, which enhances the accuracy of experimental stress–strain results. The reflected pulse has a nearly flat plateau part referring to the constant strain-rate loading condition. In order to evaluate the strain-rate states in dynamic tension tests, the strain-rate and stress history traces in the specimen deduced from the gage signals are presented in Figs. 4 and 5. In Fig. 4, the waveform of strain rate is observed to be almost flat, which indicates that a nearly constant strain-rate loading can be achieved. Comparing the measured results of stress history and strain rate history, it can be noticed that the major deformation in the specimen occurs at a nearly constant strain rate.

Quasi-static and dynamic uniaxial tensile experiments were conducted at the strain rates of 0.001, 0.01 s\(^{-1}\) and 370, 800,
1700 s\(^{-1}\), respectively. The test temperatures were 0, 20, 80 and 120 °C below the glass transition temperature. Also, the moderate strain-rate experiments at rates of 0.5, 3.1, 10 and 32 s\(^{-1}\) were performed at the room temperature. Fig. 6 illustrates the tensile engineering stress – engineering strain responses of polycarbonate as a function of strain rates at different temperatures. The responses of the material are clearly shown to be strongly dependent on the strain rate at all testing temperatures. An obvious stress drop occurs following a plastic flow platform when the stress reaches the peak point. In the case of quasi-static loading, the typical tensile response of polycarbonate includes the linear elastic and nonlinear elastic homogeneous deformation stages and the strain-softening and plastic-flow inhomogeneous deformation stages corresponding to the shear-band forming, necking generation and propagation, which can be captured within the gage section of the specimen through the naked-eye observation or the full-field digital image technique (Lu and Ravi-chandar, 1999; Parsons et al., 2004). Also, the elastic stage and the yield-like behavior can be found in dynamic stress–strain curves. In considering the deformation of polymers, the yield stress is generally defined as the true stress at the peak point on the stress–strain curves. The values of yield stress and strain at yield at high strain rates increase apparently than those under quasi-static loading, but they change slightly at different high strain rates investigated in the present paper. Moreover, the post-yield behavior of polycarbonate is sensitive to the strain rate and the level of flow stress increases with increasing strain rates. The slope of the stress drop is dependent on the strain rate and temperature, but its value varies non-monotonically with strain rate and temperature. Compared with the isothermal quasi-static loading process, it is worth noticing that the deformation process at high strain rates can be considered as an adiabatic process because the heat generated in the specimen has no sufficient time to dissipate. Although existing the adiabatic temperature rise in the specimen, the tensile responses at high strain rates are similar to quasi-static responses. Due to the limitation of the duration of the incident stress pulse which is restricted by the length of the prefixed metal bar, the total measurable uniaxial strains at high strain rates are smaller in magnitude than that under quasi-static loading. However, the plastic flow platform after the strain softening is captured except the loading case at strain rate of 370 s\(^{-1}\) and temperature of 0 °C. Similar results can be found in the moderate rate tests due to the limitation of the maximum measurement range of the optical extensometer. In the case of the temperature dependence of the tension behavior, it is apparent that the yield and post-yield behavior of polycarbonate is sensitive to the temperature. The values of yield stress and strain at yield decrease with the increase of temperature. In addition, there is a change in initial Young’s modulus with in temperature. It is noticed that the tension stress–strain response characteristics of polycarbonate at various strain rates shown in Fig. 6 is similar to compression behavior presented in the literatures (Mulliken and Boyce, 2006; Richeton et al., 2006). Compared to constitutive response under homogeneous compression, there exists the inhomogeneous deformation in the post-yield response such as neck initiation and propagation in the tensile tests.

To clearly understand the influence of strain rate on the tension properties of polycarbonate, the true yield stress, which is converted from the engineering values using a constant volume

Fig. 6. Engineering stress – engineering strain curves at different strain rates and at temperatures of (a) 0 °C, (b) 20 °C, (c) 80 °C and (d) 120 °C.
assumption, plotted against the strain rate at different temperatures is shown in Fig. 7. It can be seen that the tension yield stress of polycarbonate are sensitive to the strain rate and temperature. The values of yield stress at high strain rates and room temperature are much greater than that under quasi-static loading, which presents a considerable strain-rate strengthening phenomenon. Similar trend of marked strain-rate effect on the yield properties can be observed at temperatures 0, 80, and 120 °C. On the other hand, the yield stress increases in a nonlinear form with the logarithm strain rate under room temperature loading conditions, which is comparable to the data in the literature (Mulliken and Boyce, 1986; Richeton et al., 2006) although differing in values due to tension and compression loadings. In the case of the temperature sensitivity, the yield stress increases dramatically when the temperature decreases. Moreover, comparing the values of yield stress under quasi-static and high-rate loading, it is found that the effect of strain rate on the yield stress exhibits increasing trend when the testing temperature increases.

4. Modeling of tension responses

The experimental tension stress–strain responses at various strain rates and temperatures indicate that polycarbonate deforms in a nonlinear manner including elasticity, yield, strain softening and strain hardening. Also, the yield-rate behavior shows a nonlinear relation within a wide range of strain rates. The similar deformation behavior can be found for polycarbonate under compression loading conditions. Boyce et al. (1988) assumed that the plastic resistance to flow of glassy polymers may be decomposed into two parts: the intermolecular resistance to segment rotation and the entropic resistance to molecular alignment. Furthermore, Mulliken and Boyce (2006) proposed that the intermolecular deformation resistance may be decomposed into contributions from the primary (ξ) process and the most significant secondary process (β) for considering the nonlinear relationship between compression yield stress and the logarithm of strain rate from low to high strain rates. Richeton et al. (2006, 2007) modified the viscoplastic flow rule using the nonlinear cooperative model for the yield stress of glassy polymers to accommodate larger range of material responses including high strain rate compression and rubbery region. Following the works of Boyce et al. (1988), Arruda and Boyce (1993b) and Richeton et al. (2006), the molecular theory incorporating the chain movement mechanism was combined with the thermally activated mechanism of the yield behavior to build an elastic–viscoplastic constitutive model for describing the strain-rate and temperature dependent tension responses of polycarbonate. The one-dimensional rheological depiction of the constitutive model is presented in Fig. 8. There are three components in the model: a linear elastic spring, a viscoplastic dashpot and a nonlinear Langevin spring. The linear elastic spring and viscoplastic dashpot in series represents the intermolecular resistance to chain-segment rotation and the Langevin spring represents the entropic resistance to chain alignment. The viscoplastic dashpot controlled by the flow law describes the post-yield viscoplastic flow and strain softening.

4.1. Kinematic description of finite strain

The finite deformation of an elastoplastic material is generally described by the deformation gradient tensor, which can be multiplicatively decomposed into elastic, plastic, and rotation tensors as follows by introducing an intermediate “relaxed” configuration between the initial undeformed configuration and the current configuration.

$$ F = F^e F^p $$

Assuming that the plastic flow is incompressible, we can obtain $\det F^p = 1$. Following polar decomposition, the deformation gradient tensors can be further decomposed in terms of the stretch tensors and the rotation tensors. For example, $F^e$ may be decomposed as

$$ F^e = V^e R^e $$

The velocity gradient, $L$, can be decomposed into elastic and plastic components as

$$ L = FF^{-1} = F^e F^{-1} + F^p F^{-1} $$

where $L^p$ is the plastic velocity gradient in the relaxed configuration with a zero stress state. This tensor is given by

$$ L^p = F^p F^{-1} $$

The plastic velocity gradient tensor may be expressed as the sum of its skew-symmetric and symmetric parts as

$$ L^p = W^p + D^p $$

where $W^p$ and $D^p$ are the rates of plastic spin and plastic stretching, respectively. For numerical simplicity, plastic flow is assumed to be irrotational and then $W^p$ is frequently set to zero (Boyce et al., 1989).

The rate of plastic stretching, $D^p$, is constitutively formulated with plastic shear strain rate $\gamma^p$ to be (Boyce et al., 1988)

$$ D^p = \dot{\gamma}^p \frac{1}{\sqrt{2\pi}} $$
where $\mathbf{T}^e$ is the deviatoric stress in element A as shown in Fig. 8. The effective equivalent shear stress which activates the plastic flow, $\tau$, is given by

$$\tau = \left[ \frac{1}{2} \mathbf{T}^e \mathbf{T}^e \right]^{1/2} \quad (11)$$

4.2. Constitutive description

Corresponding to the components of the model shown in Fig. 8, the total Cauchy stress acting on material, $\mathbf{T}$, can be separated to two parts as following

$$\mathbf{T} = \mathbf{T}^s + \mathbf{T}^p \quad (12)$$

where $\mathbf{T}^s$ is the Cauchy stress of element A describing the intermolecular contribution to the material stress state. $\mathbf{T}^p$ is the Cauchy stress of element B representing the entropic resistance related to the stretch level.

In element A, the stress depends on the linear elastic spring following the Hooke’s law

$$\mathbf{T}^s = \frac{1}{J} \mathbf{C}^e (\ln \mathbf{V}) \quad (13)$$

where $J$ is the elastic volume change, $\mathbf{C}^e$ is fourth-order modulus tensor, $\ln \mathbf{V}$ is the Hencky strain and $J = \det \mathbf{F}$.

In element B, the stress is controlled by nonlinear Langevin spring, which can be described by the Arruda–Boyce eight-chain model (Arruda and Boyce, 1993b) as following

$$\mathbf{T}^p = \frac{C_k(\theta)}{3} \sqrt{\frac{N(\theta)}{\lambda_{\text{chain}}}} L^{-1} \left[ \lambda_{\text{chain}} \sqrt{N(\theta)} \right] \left[ \mathbf{B} - \frac{\lambda_{\text{chain}}}{\lambda_{\text{chain}}} \mathbf{I} \right] \quad (14)$$

where $C_k(\theta)$ is the rubbery modulus and $N$ is the number of rigid chain links between entanglements. Both of them are dependent on the absolute temperature $\theta$. $\mathbf{B} = (\det \mathbf{F})^{1/2} \mathbf{F}^T$ is the isochoric left Cauchy–Green tensor and $\lambda_{\text{chain}} = \sqrt{\frac{C_k(\theta)}{C_2}}$ is the stretch on each chain in the eight-chain network. $L$ is the Langevin function defined as $L(x) = \coth x − 1/x$. The rubbery modulus is related to the thermally temperature-dependent evolving chain density, $n(\theta)$, as follows

$$C_k(\theta) = n(\theta) k \theta \quad (15)$$

where $k$ is Boltzmann’s constant. The evolution of the chain density with temperature is modeled as (Raha and Bowden, 1972; Arruda et al., 1995)

$$n(\theta) = b − a \exp \left( -\frac{E_a}{k \theta} \right) \quad (16)$$

where $E_a$ is the thermal dissociation energy. Model parameters $a$ and $b$ correspond to the networks which undergo thermal dissociation and non-dissociating networks, respectively. The evolution of $n(\theta)$ and $N(\theta)$ follows the relationship

$$n(\theta) N(\theta) = \text{Const} \quad (17)$$

in order that the total number of rigid links and hence, the mass of the physics system, is conserved. Consequently, the thermally evolving number of rigid chain links can be deduced according to the values of $n$ and $N$ at a certain temperature such as $n(293 \text{ K})$ and $N(293 \text{ K})$.

The yield behavior of polymers is normally considered as a thermally activated process. Flow law corresponding to the viscoplastic dashpot in element A provides a way of incorporating the strain rate and temperature dependent plastic deformation. The plastic shear strain rate, $\dot{\gamma}_p$, is given by (Fotheringham and Cherry, 1978; Richeton et al., 2006)

$$\dot{\gamma}_p = \dot{\gamma}_0 \exp \left( -\frac{\Delta H_p}{k \theta} \right) \sinh \left[ \frac{(\tau - \tau_p) \cdot \dot{V}}{2k \theta} \right] \quad (18)$$

where $\dot{\gamma}_0$ is the pre-exponential shear rate factor, $\Delta H_p$ is the activation energy, $m$ is the material parameter describing the cooperative nature of the polymer chain segments, $V$ is the shear activation volume, and $\tau_i$ is the evolving internal shear stress.

From the polymer chains transformation explanation, the strain softening is assumed as the chains rearrange from isotropic state to alignment state which is more stable under tension loading. During the rearranging process, the internal shear stress decreases with the plastic deformation increasing until chains reach the more stable structure. The internal shear stress evolution process can be formulated as (Boyce et al., 1988; Richeton et al., 2007)

$$\dot{\tau}_i = h \cdot \left( 1 - \frac{\tau_i}{\tau_m} \right) \cdot \dot{\gamma}_p \quad (19)$$

where $h$ is the softening slope. The initial value of the internal shear stress, $\tau_m$, is defined as $\tau_m = m_0 - n_0 \theta$. The stress referring to the preferred structural state of the material, $\tau_m$, is dependent on temperature. Here, a simple linear relation between $\tau_m$ and temperature is adopted to be $\tau_m = m_3 - n_2 \theta$.

4.3. Finite element modeling

Numerical simulations were conducted to better understand the tensile deformation responses of polycarbonate specimen during various temperature and strain-rate loadings. The aforementioned constitutive model was implemented numerically into a commercial finite element code, ABAQUS/Explicit, through a user-defined material subroutine. The connection parts of the specimen were supposed to be in perfect contact with the bars during the experiment. For simplicity, the incident and transmitted bars were not included in the dynamic numerical simulation. Hence only the entire specimen was modeled in the three-dimensional numerical simulation. The specimen geometry used in the numerical simulations was identical to that in the tensile tests. The specimen was represented by 8-node brick element, as is shown in Fig. 9. One end of the connection part of the specimen

![Fig. 9. Mesh geometry of the tension specimen for dynamic loading.](https://example.com/image)
was fixed and the velocity boundary conditions were applied to the other end of the specimen connection part. The simulations were performed at constant engineering strain rates corresponding to the actual tension tests. For dynamic loading, the material density used in the simulations was set as 1.0\( \rho_{PC} \) (\( \rho_{PC} = 1.2 \times 10^3 \text{ kg/m}^3 \)). The wave propagation in the specimen was captured during the initial period of the loading and the stress equilibrium was obtained due to the wave traveling back and forth inside the specimen several times. The engineering stress was defined as the tensile reaction forces divided by the original cross-sectional area in the middle of the specimen. To keep consistent with the actual experimental measurement conditions, the engineering strain was obtained from the relative displacements between the two ends of the specimen gage section.

The material constants and model parameters used in the FEM simulation were determined by experimental measurement and curve fitting. The values of Young's modulus were measured directly from the experimental stress–strain curves within the linear deformation region. The Poisson ratio for polycarbonate was taken as a constant, \( \nu = 0.38 \), which is independent of strain rate and temperature. The experimentally measured values of yield stress under room temperature loading conditions were used to solve for the initial values of parameters \( \gamma_0 \), \( \Delta H_e \), \( V \) and \( m \) of the viscoplastic dashpot. Then the yield stress data at low and high temperatures were correlated to obtain the appropriate parameter values of the dashpot through a nonlinear regression method. Similarly, the value of internal shear stress at yield, \( \tau_s \), was obtained from Eq. (18) using the yield data at different strain rates and then was extended to incorporate the temperature effect using the yield data at four temperatures. Finally the values of \( m_1 \) and \( n_1 \) corresponding to the initial value of the internal shear stress were determined with a linear correlation to the absolute temperature \( \theta \). The remaining unknown quantities for the strain softening were parameters \( h \) and \( \tau_{PS} \). For simplicity, the softening slope, \( h \), was taken to be a constant in all simulation cases. These two parameters were solved simultaneously by fitting the model to the experimental stress–strain data over the region dominated by strain

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Summary of material parameters in the elastic–viscoplastic model.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_0 ) at 293 K (MPa)</td>
<td>18</td>
</tr>
<tr>
<td>( N ) at 293 K</td>
<td>4.84</td>
</tr>
<tr>
<td>( a ) (1/m(^3))</td>
<td>( 7.20 \times 10^{10} )</td>
</tr>
<tr>
<td>( b ) (1/m(^3))</td>
<td>( 7.24 \times 10^{10} )</td>
</tr>
<tr>
<td>( E_a ) (J)</td>
<td>( 4.94 \times 10^{-23} )</td>
</tr>
<tr>
<td>( \gamma_0 ) (s(^{-1}))</td>
<td>67000</td>
</tr>
<tr>
<td>( \Delta H_e ) (kJ/mol)</td>
<td>46</td>
</tr>
<tr>
<td>( V ) (m(^3))</td>
<td>( 6.8 \times 10^{-28} )</td>
</tr>
<tr>
<td>( m )</td>
<td>6.66</td>
</tr>
<tr>
<td>( h ) (MPa/K)</td>
<td>200</td>
</tr>
<tr>
<td>( m_1 ) (MPa)</td>
<td>69.7</td>
</tr>
<tr>
<td>( m_2 ) (MPa)</td>
<td>1.23</td>
</tr>
<tr>
<td>( n_1 ) (MPa/K)</td>
<td>0.14</td>
</tr>
<tr>
<td>( n_2 ) (MPa/K)</td>
<td>0.0025</td>
</tr>
</tbody>
</table>

Fig. 10. Comparison of the model results and the experimental stress–strain behavior in uniaxial tension.
softening. Once the values of $\tau_{ps}$ at four testing temperatures were determined, the parameters $m_2$ and $n_2$ were calculated by performing a linear fitting to the absolute temperature $\theta$. The material parameters corresponding to the nonlinear Langevin spring were determined by correlating model simulations to the experimental stress–strain curves at all strain-rate and temperature loadings in order to achieve optimal agreement between the simulated and experimental stress–strain responses of polycarbonate. The final values of the constitutive parameters used in the three-dimensional finite element simulation are outlined in Table 1.

5. Model results and discussion

The comparison of the numerically calculated results of engineering stress and engineering strain and the experimental data for the uniaxial tension responses at different strain rates and temperatures are presented in Fig. 10. The model results show that the constitutive model used here is capable of capturing the typical features of the tension deformation process of polycarbonate including the elastic stage, the yield peak, the subsequent post-yield softening stage and the flow platform. The model is shown to have the ability to describe the rate and temperature dependent behavior of polycarbonate under tension loading within a wide range of strain rates and temperatures. It is noticed that the nonlinear elastic regime prior to yield of tension behavior cannot be captured in the simulation because the linear elastic spring is used in the rheological model as shown in Fig. 8. The highest amount of deviation for strain at yield between the simulated and experimental data is less than 5%, which is relatively small compared to the large plastic strain. On the other hand, there exist some discrepancies between the model results and the experiment data at large axial strains in the post-yield portion of the stress–strain curves at high strain rates. This might be due to the thermal softening generated from the adiabatic temperature rise in the specimen at high strain rates, which is not accounted for in the present model. Such adiabatic heating has been found to play an important role in the deformation process and the thermo-mechanical coupling needs to be considered in the constitutive model. However, additional tests must be undertaken to identify the thermo-mechanical transfer parameters correctly, for example the transient temperature measurements during high rate loading (Rittel, 1999; Li and Lambros, 2001; Bjerkea et al., 2002) and the high-rate temperature jump tests (Nemat-Nasser et al., 2001) which are based on the dynamic recovery experimental technique (Nemat-Nasser et al., 1991).

To demonstrate the capability of the flow law used in the present model for capturing the strain rate and temperature dependence of the tension yield stress, the comparison between the experimental data and the numerical results is shown in Fig. 7. In the simulations, the value of yield stress increases nonlinearly with the increase of logarithm strain rate as observed in the experimental tension behavior of polycarbonate.

6. Conclusions

The rate and temperature dependent tension responses of polycarbonate were investigated within a wide range of strain rates up to 1700 s$^{-1}$ and temperatures up to 120 °C below the glass transition temperature. The high strain rate tension tests were carried out using a modified split Hopkinson tension bar. Compared to the tension behavior under quasi-static loading, the polycarbonate exhibits the similar deformation characteristics such as nonlinear elasticity, yielding and strain softening when deformed under high rates of loading. Moreover, the experimental results show that the tension behavior of polycarbonate is sensitive to strain rate and the values of yield stress and strain at yield increase dramatically with increasing strain rate. Such strain-rate strengthening phenomenon can be observed at all investigated temperatures. The influence of temperature on the tension behavior of polycarbonate is obvious under both quasi-static and high rate loading conditions. Based on the experimental investigation, a finite strain elastic–viscoplastic constitutive model combined with the finite element analysis was used to describe the rate-temperature dependent tension behavior of polycarbonate. In rheological description, the constitutive model contains a linear elastic spring and a viscoplastic dashpot in series together with a nonlinear Langevin spring in parallel to represent the intermolecular resistance to chain-segment rotation and the entropic resistance to chain alignment, respectively. The post-yield softening is described by the evolution of the temperature dependent internal stress. The correlations from the model were close to the observed experimental responses, which indicate that the model presented here is capable of capturing the major features of deformation behavior of amorphous glassy polymers subjected to tension loading within a wide range of strain rates and temperatures. However, the nonlinear elasticity and the thermal softening during the high rate deformation need to be incorporated into the model to accurately describe the tension behavior of glassy polymers over the whole deformation region.

Acknowledgments

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References


2547