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The QCD critical end point in the SU(3) Nambu–Jona-Lasinio model

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Abstract

We study the chiral phase transition at finite temperature T and baryonic chemical potential μ_B within the framework of the SU(3) Nambu–Jona-Lasinio (NJL) model. The QCD critical end point (CEP) and the critical line at finite T and μ_B are investigated: the study of physical quantities, such as the baryon number susceptibility and the specific heat in the vicinity the CEP, will provide relevant information concerning the order of the phase transition. The class of the CEP is determined by calculating the critical exponents of those quantities.

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Keywords: NJL model; Phase transition; Chiral symmetry; Finite temperature; Chemical potential

1. Introduction

It is commonly accepted that the vacuum of quantum chromodynamics (QCD) undergoes a phase transition to the quark gluon plasma (QGP) at high temperature and/or quark chemical potential. Such a new state of matter is experimentally studied in on-going heavy-ion collisions at CERN, Brookhaven and JIRN [1].

The discussion about the existence of a tricritical point (TCP) or a CEP is also a topic of recent interest. As is well known, a TCP separates the first-order transition at high chemical potential from the second-order transition at high temperatures. If the second-order transition is replaced by a smooth crossover, a CEP which separates the two lines is found. The existence of the CEP in QCD was suggested at the end of the eighties [2,3], and its properties have been studied since then (for a review see Ref. [4,5]).

The most recent lattice results with $N_f = 2 + 1$ staggered quarks of physical masses indicate the location of the CEP at $T^{\text{CEP}} = 162 \pm 2$ MeV, $\mu^{\text{CEP}} = 360 \pm 40$ MeV [6], however its exact location is not yet known (it depends strongly of the mass of the strange quark). At the CEP the phase transition is of second order, belonging to the three-dimensional Ising universality class, and this kind of phase transitions is characterized by long-wavelength fluctuations of the order parameter.

The possible signatures of the CEP in heavy-ion collisions have been studied in detail in [4,7,8]. In heavy-ion collision experiments, fluctuations of T and μ_B, can be found, respectively, in event-by-event fluctuations of p_T spectra or in event-by-event fluctuations in the baryon number to pion ratio [7,9]. The event-by-event fluctuations of T can be related to the heat capacity: if the specific heat C diverges, as the CEP is approached from either the left or the right, the fluctuations of T are suppressed. On the other hand, the event-by-event fluctuations of μ_B can be related to the baryon number susceptibility, χ_B, which is also divergent. This im-

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plies that fluctuations of μ_B are also suppressed at the critical point.[1]

As pointed out in Ref. [10], the critical region around the CEP is not pointlike but has a richer structure. The critical region is defined as the region where the mean field theory of phase transitions breaks down and nontrivial critical exponents emerge. The size of this critical region is important for future searches for the CEP in heavy-ion collisions [11].

It is also expected that the strange quark should have an important effect on the position of the TCP and the CEP. At finite T and zero μ_B, in the limiting case with $m_u = m_d = 0$ and infinite strange quark mass m_s, the chiral phase transition is likely to be of second order and the static critical behavior is expected to belong to the universality class of the Heisenberg $O(4)$ model in three dimensions [12]. When m_s is finite and less than some critical value $m_s^{\rm crit}$, the second-order transition becomes of first order. This leads to a tricritical point in the $T - m_s$ plane [13].

Some studies have been done in the SU(2) sector [10,14] but less attention has been given to the effects of the strange quark [15]. In this Letter we aim at investigating the chiral phase transition and the CEP in quark matter with strange quarks.

2. Model calculations

We perform our calculations in the framework of the three-flavor NJL model [16–19], including the determinantal 't Hooft interaction that breaks the $U_A(1)$ symmetry, which Lagrangian reads

$$\mathcal{L} = \bar{q}(i\partial\cdot\gamma - \hat{m})q + \frac{g_S}{2}\sum_{a=0}^{8}\left[\left(\bar{q}\lambda^a q\right)^2 + \left(\bar{q}(i\gamma_5)\lambda^a q\right)^2\right] + g_D\left[\det\left[\bar{q}(1+\gamma_5)q\right] + \det\left[\bar{q}(1-\gamma_5)q\right]\right]. \quad (1)$$

Here $q = (u, d, s)$ is the quark field with three flavors, $N_f = 3$, and three colors, $N_c = 3$. $\hat{m} = \mathrm{diag}(m_u, m_d, m_s)$ is the current quark mass matrix and λ^a are the Gell–Mann matrices, $a = 0, 1, \ldots, 8$, $\lambda^0 = \sqrt{\frac{2}{3}}\mathbf{I}$. The three momentum integrals are regularized by the cutoff Λ, and a standard set of parameters [17,20] given by $\Lambda = 602.3$ MeV, $g_S\Lambda^2 = 3.67$, $g_D\Lambda^5 = -12.36$, $m_u = m_d = 5.5$ MeV and $m_s = 140.7$ MeV allow to reproduce the following vacuum observables: $M_\pi = 135.0$ MeV, $M_K = 497.7$ MeV, $f_\pi = 92.4$ MeV and $M_{\eta'} = 960.8$ MeV. With this set of parameters we also obtain $\langle\bar{q}_u q_u\rangle = \langle\bar{q}_d q_d\rangle = -(241.9\ \mathrm{MeV})^3$ and $\langle\bar{q}_s q_s\rangle = -(257.7\ \mathrm{MeV})^3$, for the quark condensates and $M_u = M_d = 367.7$ MeV and $M_s = 549.5$ MeV, for the constituent quark masses.

The fundamental relation is provided by the baryonic thermodynamic potential

$$\Omega(\mu_i, T) = E - TS - \sum_{i=u,d,s}\mu_i N_i, \quad (2)$$

from which the relevant equations of state for the entropy S, the pressure P and the particle number N_i can be calculated as usually (the expressions are given in Section IV of Ref. [20]). So we take the temperature T, the volume V and the chemical potential of the i-quark (μ_i) as the full independent state variables, and a grand canonical approach is applied to our model of strong interacting matter. It can simulate either a region in the interior of a neutron star, or a hot and dense fireball created in a heavy-ion collision. Since electrons and positrons are not involved in the strong interaction, we impose the condition $\mu_e = 0$. So we naturally get the chemical equilibrium condition $\mu_u = \mu_d = \mu_s = \mu_B$ that is used along the work. This condition is only valid around the CEP, where the temperature is expected to be high enough to avoid color condensation, or at small chemical potential where this phenomena is not yet present. The baryon number susceptibility is the response of the baryon number density $\rho_B(T, \mu_i)$ to an infinitesimal variation of the quark chemical potential μ_i [21]:

$$\chi_B = \frac{1}{3}\sum_{i=u,d,s}\left(\frac{\partial\rho_i}{\partial\mu_i}\right)_T. \quad (3)$$

Here, the quark density is $\rho_i = N_i/V = \frac{N_c}{\pi^2}\int p^2\,dp\,(n_i(\mu_i, T) - \bar{n}_i(\mu_i, T))$. Other relevant observable, in the context of possible signatures for chiral symmetry restoration in the hadron–quark transition and for transition from the hadronic matter to the QGP [21–23], is the specific heat which is defined by

$$C = \frac{T}{V}\left(\frac{\partial S}{\partial T}\right)_{N_i} = \frac{T}{V}\left[\left(\frac{\partial S}{\partial T}\right)_{\mu_i} - \frac{[(\partial N_i/\partial T)_{\mu_i}]^2}{(\partial N_i/\partial\mu_i)_T}\right], \quad (4)$$

where we have transformed the derivative $(\partial S/\partial T)_{N_i}$ using the formula of the Jacobian. In fact, we work in the grand canonical ensemble where (T, V, μ_i) are the set of natural independent variables (still holding N_i and V fixed).

3. The critical end point of QCD

To study the influence of explicit chiral symmetry breaking on the location of the critical points we vary the current quark masses m_i keeping the other model parameters. The phase diagram for the SU(3) NJL model is presented in Fig. 1 as a function of μ_B and T, and considering different cases for the current quark masses. The main motivation is the discussion of the critical behavior of the system, starting with the location of the CEP. Using physical values of the quark masses [18,24]: $m_u = m_d = 5.5$ MeV, $m_s = 140.7$ MeV, this point is localized at $T^{\rm CEP} = 67.7$ MeV and $\mu_B^{\rm CEP} = 318.4$ MeV ($\rho_B^{\rm CEP} = 1.68\rho_0$). We also verified that, contrarily to what happens in the chiral limit for the SU(2) sector where the TCP is found, the NJL model in SU(3), also in the chiral limit ($m_u = m_d = m_s = 0$),

[1] The divergence of χ_B is directly related to an anomaly in the event-by-event fluctuation of baryon number: in a heavy-ion collision experiment it is expected that the event-by-event fluctuation of the proton number is relatively enhanced for collisions which have passed in the vicinity of the CEP or the TCP. It is also expected an increase in event-by-event fluctuations at low p_T near the CEP [4,7].

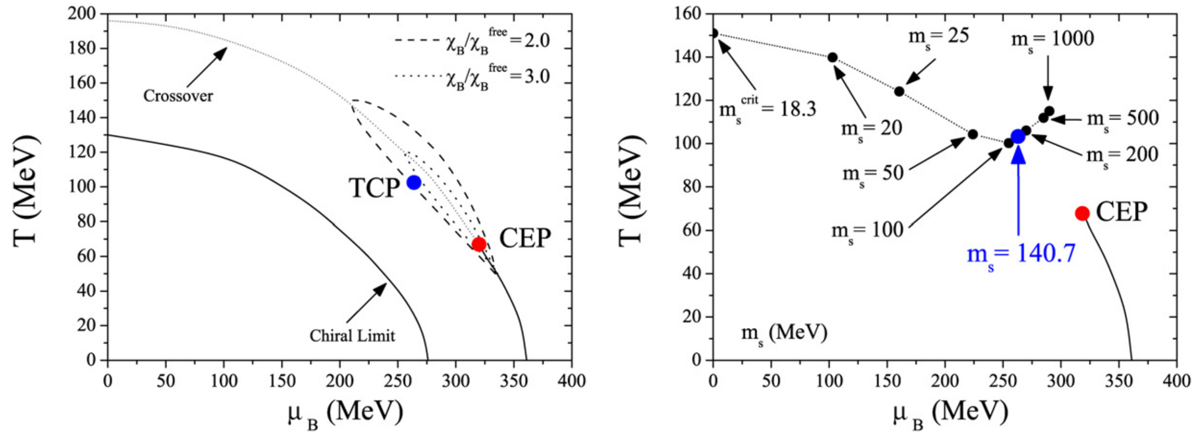

Fig. 1. Left panel: the phase diagram in the SU(3) NJL model. The solid line represents the first-order phase transition. The size of the critical region is also plotted for $\chi_B/\chi_B^{\rm free} = 2(3)$. Right panel: the phase diagram and the "line" of TCPs for $m_u = m_d = 0$ and different values of m_s (the dotted lines are just drawn to guide the eye).

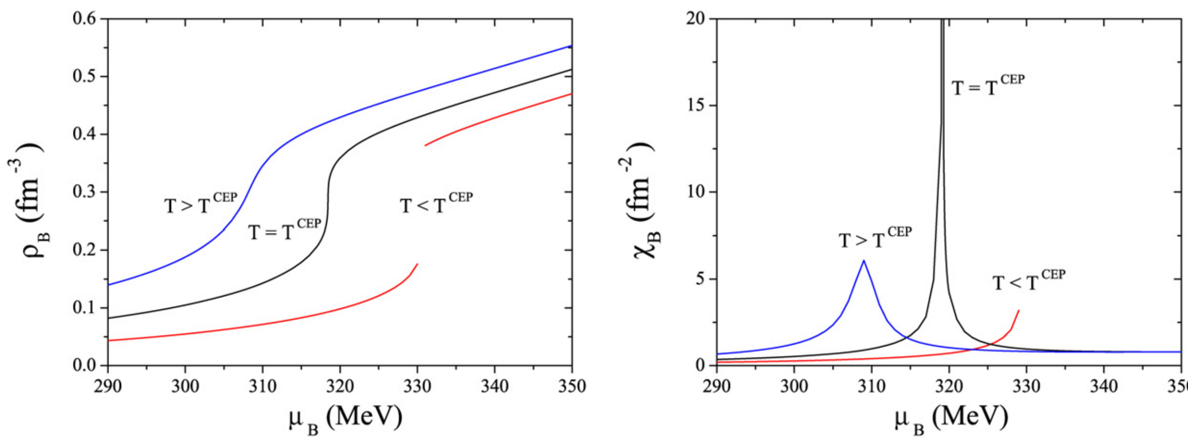

Fig. 2. Baryonic density (left panel) and baryon number susceptibility (right panel) as function of μ_B for different temperatures around the CEP: $T^{\rm CEP} = 67.7$ MeV and $T = T^{\rm CEP} \pm 10$ MeV.

does not exhibit a TCP[2]: chiral symmetry is restored via a first-order transition for all baryonic chemical potentials and temperatures (see left panel of Fig. 1). This pattern of chiral symmetry restoration remains for $m_u = m_d = 0$ and $m_s < m_s^{\rm crit}$ [25]. In our model we found $m_s^{\rm crit} = 18.3$ MeV for $m_u = m_d = 0$. When $m_s \geqslant m_s^{\rm crit}$, at $\mu_B = 0$, the transition is of second order and, as μ_B increases, the line of the second-order phase transition will end in a first-order line at the TCP. Several TCPs are plotted for different values of m_s in the right panel of Fig. 1. As m_s increases, the value of T for this "line" of TCPs decreases as μ_B increases getting closer to the CEP and, when $m_s = 140.7$ MeV, it starts to move away from the CEP. The TCP for $m_s = 140.7$ MeV is the closest to the CEP and is located at $\mu_B^{\rm TCP} = 265.9$ MeV and $T^{\rm TCP} = 100.5$ MeV. If we choose $m_u = m_d \neq 0$, instead of second-order transition we have a smooth crossover for all the values of m_s and the "line" of TCPs becomes a "line" of CEPs.

4. Behavior of χ_B and C in the vicinity of the CEP and their critical exponents

A bound to the size of the critical region around the CEP can be found by calculating the baryon number susceptibility, the specific heat and their critical behaviors. If the critical region of the CEP is small, it is expected that most of the fluctuations associated with the CEP will come from the mean field region around the critical region [10].

In the left panel of Fig. 2 is plotted the baryon number density for three different temperatures around the CEP. For temperatures below $T^{\rm CEP}$ we have a first-order phase transition and, consequently, χ_B has a discontinuity (right panel of Fig. 2). For $T = T^{\rm CEP}$ the slope of the baryon number density tends to infinity at $\mu_B = \mu_B^{\rm CEP}$ which implies a diverging susceptibility (this behavior was found in [10,14] using different models in the SU(2) sector). For temperatures above $T^{\rm CEP}$, in

[2] Both situations are in agreement with what is expected: the chiral phase transition at the chiral limit is of second order for $N_f = 2$ and first order for $N_f \geqslant 3$ [12].

the crossover region, the discontinuity of χ_B disappears at the transition line, and the density changes gradually in a continuous way as we can see in the right panel of Fig. 2. A similar behavior is found for the specific heat for three different chemical potentials around the CEP, as we can observe from Fig. 3.

As we have already seen, several thermodynamic quantities diverge at the CEP. In order to make this statement more precise, we will focus on the values of a set of indices, the so-called critical exponents, which describe the behavior near the critical point of various quantities of interest (in our case ϵ and α are the critical exponents of χ_B and C, respectively). The motivation for this study arises from fundamental phase transition considerations, and thus transcend any particular system. These critical exponents will be determined by finding two directions, temperature-like and magnetic-field-like, in the $T - \mu_B$ plane near the CEP, because, as pointed out in [26], the form of the divergence depends on the route which is chosen to approach the critical end point.

Starting with the baryon number susceptibility, if the path chosen is asymptotically parallel to the first-order transition line, the divergence of χ_B scales with an exponent γ_B. In the mean-field approximation it is expected $\gamma_B = 1$ for this path. For any other path not parallel to the first-order line, the divergence scales with the exponent $\epsilon = 1 - 1/\delta$. Once in the mean-field approximation $\delta = 3$, we will have $\epsilon = 2/3$ and $\gamma_B > \epsilon$ is verified. The last condition is responsible for the elongation of the critical region, χ_B being enhanced in the direction parallel to the first-order transition line (see Fig. 1). To estimate the critical region around the CEP we can calculate the dimensionless ratio $\chi_B/\chi_B^{\text{free}}$ where χ_B^{free} is obtained taking the chiral limit $m_u = m_d = m_s = 0$. Left panel of Fig. 1 shows a contour plot for two fixed ratios ($\chi_B/\chi_B^{\text{free}} = 2.0(3.0)$) in the phase diagram around the CEP where we confirm the elongation, in the direction parallel to the first-order transition line, of the region where χ_B is enhanced.

For the baryon number susceptibility we will start with a path parallel to the μ_B-axis in the (T, μ_B)-plane, from lower μ_B towards the critical $\mu_B^{\text{CEP}} = 318.4$ MeV, at fixed temperature $T^{\text{CEP}} = 67.7$ MeV. In Fig. 4 we plot χ_B as a function of μ_B close to the CEP. Using a linear logarithmic fit

$$\ln \chi_B = -\epsilon \ln \left|\mu_B - \mu_B^{\text{CEP}}\right| + c_1, \tag{5}$$

where the term c_1 is independent of μ_B, we obtain $\epsilon = 0.67 \pm 0.01$, which is consistent with the mean field theory prediction $\epsilon = 2/3$.

Once there is no reason why the critical exponent should be equal for both regions, below and above μ_B^{CEP}, we also study the baryon number susceptibility from higher μ_B towards the critical μ_B^{CEP}. The logarithmic fit used now is $\ln \chi_B = -\epsilon' \ln |\mu_B - \mu_B^{\text{CEP}}| + c_1'$. Our result shows that $\epsilon' = 0.68 \pm 0.01$ which is very near the value of ϵ. This means that the size of the region we observe is approximately the same independently of the direction we choose for the path parallel to the μ_B-axis.

Paying attention to the specific heat around the CEP, we have used a path parallel to the T-axis in the (T, μ_B)-plane from lower (higher) T towards the critical $T^{\text{CEP}} = 67.7$ MeV at fixed

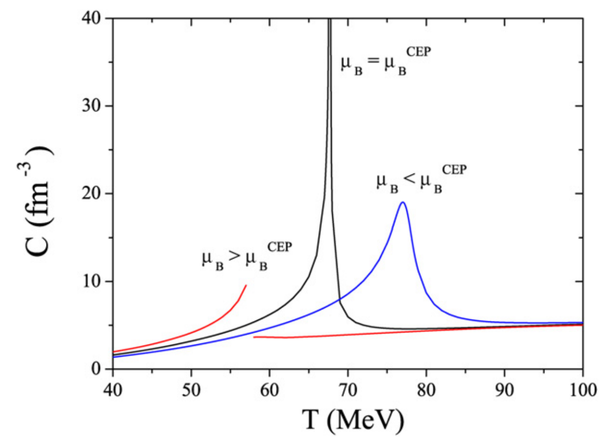

Fig. 3. Specific heat as a function of T for different values of μ_B around the CEP ($\mu_B^{\text{CEP}} = 318.4$ MeV and $\mu_B = \mu_B^{\text{CEP}} \pm 10$ MeV).

$\mu_B^{\text{CEP}} = 318.4$ MeV. In Fig. 4 we plot C as a function of T close to the CEP in a logarithmic scale. Now, we see that for the region $T < T^{\text{CEP}}$ we have a nontrivial critical exponent $\alpha = 0.61 \pm 0.01$.[3] This value of α is not in agreement with what was suggested by universality arguments in [10]: it is expected that χ_B and C should be essentially the same near the TCP and the CEP which implies $\alpha = \epsilon = 2/3$. One possible interpretation of this result could be the effect of the hidden TCP on the CEP, as it was already seen in Refs. [10,14], and which influence could be stronger in the specific heat rather than in the baryon number susceptibility. However, this explanation is not valid in the framework of the NJL model once, at the TCP, the value of α is already not consistent with the respective mean field value.

To support the last statement, let us analyze the behavior of the specific heat around the TCP when $m_s = 140.7$ MeV ($T^{\text{TCP}} = 100.5$ MeV, $\mu_B^{\text{TCP}} = 265.9$), the nearest TCP from the CEP (see right panel of Fig. 1). Using a path parallel to the T-axis from lower T towards the critical T^{TCP}, at fixed chemical potential μ_B^{TCP}, we find $\alpha = 0.45 \pm 0.01$. This result, in spite of being close, is not in agreement with the respective mean field value ($\alpha = 1/2$). We notice that the inconsistency with the mean field values only occurs for the specific heat. In fact, it is found that the critical exponent for χ_B (γ_B once we are in the TCP) whose value, $\gamma_B = 0.50 \pm 0.02$, is in agreement with the respective mean field value ($\gamma_B = 1/2$).

Nevertheless we observe that the values of α in the TCP and in the CEP are consistent within the NJL model. We also stress that the universality arguments are so general that they give no quantitative results and, due to the lack of information from the lattice simulations, they should be confronted with model calculations. The eventual difference between the values of the C and χ_B critical exponents can be interesting in heavy-ion collisions experiments. Finally, for the region $T > T^{\text{CEP}}$ the critical exponent is $\alpha' = 0.67 \pm 0.01$ which is compatible with the value of ϵ'. This means that the specific heat is sensitive to the way we approach the CEP.

[3] We use the linear logarithmic fit $\ln C = -\alpha \ln |T - T^{\text{CEP}}| + c_2$ where the term c_2 is independent of T.

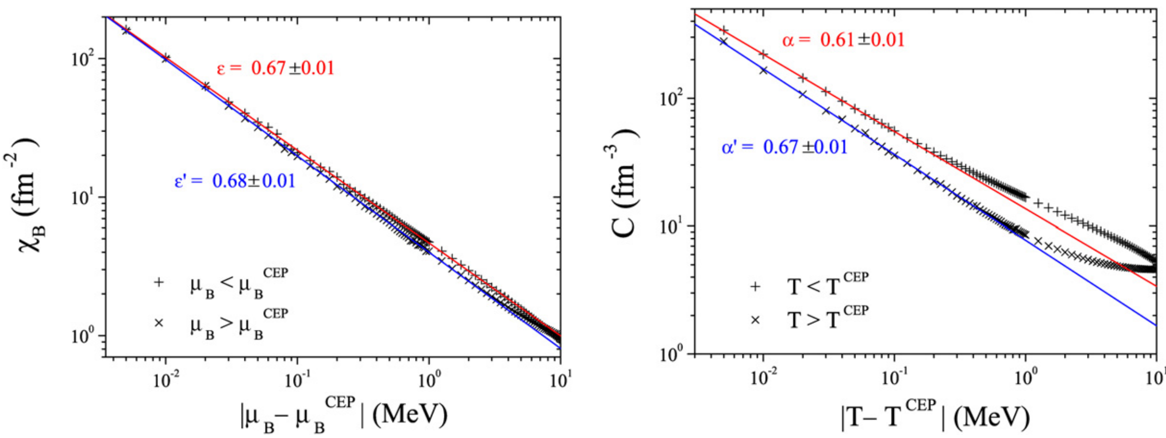

Fig. 4. Left panel: baryon number susceptibility as a function of $|\mu_B - \mu_B^{\mathrm{CEP}}|$ at fixed temperature $T^{\mathrm{CEP}} = 67.7$ MeV. Right panel: specific heat as a function of $|T - T^{\mathrm{CEP}}|$ at fixed chemical potential $\mu_B^{\mathrm{CEP}} = 318.4$ MeV.

5. Summary and conclusions

We have analyzed the phase diagram in the SU(3) NJL which reproduces the essential features of QCD: a first-order phase transition for low temperatures and the existence of the CEP. Contrarily to what happens in the SU(2) sector, in the chiral limit ($m_u = m_d = m_s = 0$) we do not find a TCP in the NJL model, which agrees with what is expected: the chiral phase transition at $m_i = 0$ is of second order for $N_f = 2$ and first order for $N_f \geqslant 3$. When $m_u = m_d = 0$ and $m_s > m_s^{\mathrm{crit}}$ ($m_s^{\mathrm{crit}} = 18.3$ MeV) the transition is of second order ending in a first-order line at the TCP. As m_s increases we have a "line" of TCPs. For $m_u = m_d \neq 0$ there is a crossover for all the values of m_s and the "line" of TCPs becomes a "line" of CEPs. The location of the CEP depends strongly of the strange quark mass. Around the CEP we have studied the baryon number susceptibility and the specific heat which are related with event-by-event fluctuations of μ_B or T in heavy-ion collisions. In the NJL model, for χ_B, we conclude that the obtained critical exponents are consistent with the mean field values $\epsilon = \epsilon' = 2/3$ (the NJL model only produces mean field behaviors). From our study of the critical exponent for the specific heat, we conclude that α is different from ϵ. More relevant information about the CEP can be obtained from the spectral functions and the isentropic trajectories in its vicinity. This work is in progress.

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References

[1] A.N. Sissakian, A.S. Sorin, V.D. Toneev, nucl-th/0608032.
[2] M. Asakawa, K. Yazaki, Nucl. Phys. A 504 (1989) 668.
[3] A. Barducci, et al., Phys. Lett. B 231 (1989) 463;
A. Barducci, et al., Phys. Rev. D 41 (1990) 1610;
A. Barducci, et al., Phys. Rev. D 49 (1994) 426.
[4] M.A. Stephanov, Prog. Theor. Phys. Suppl. 153 (2004) 139;
M.A. Stephanov, Int. J. Mod. Phys. A 20 (2005) 4387.
[5] R. Casalbuoni, hep-ph/0610179.
[6] Z. Fodor, S.D. Katz, JHEP 0404 (2004) 050.
[7] M. Stephanov, K. Rajagopal, E. Shuryak, Phys. Rev. Lett. 81 (1998) 4816.
[8] M. Stephanov, K. Rajagopal, E. Shuryak, Phys. Rev. D 60 (1999) 114028.
[9] Y. Hatta, M.A. Stephanov, Phys. Rev. Lett. 91 (2003) 102003.
[10] Y. Hatta, T. Ikeda, Phys. Rev. D 67 (2003) 014028.
[11] C. Nonaka, M. Asakawa, Phys. Rev. C 71 (2005) 044904.
[12] R.D. Pisarski, F. Wilczek, Phys. Rev. D 29 (1984) 338.
[13] F. Wilczek, Int. J. Mod. Phys. A 7 (1992) 3911;
K. Rajagopal, F. Wilczek, Nucl. Phys. B 399 (1993) 395;
S. Gavin, A. Gocksch, R. Pisarski, Phys. Rev. D 49 (1994) 3079.
[14] B.-J. Schaefer, J. Wambach, hep-ph/0603256.
[15] K. Rajagopal, Nucl. Phys. A 661 (1999) 150;
J. Berges, K. Rajagopal, Nucl. Phys. B 538 (1999) 215.
[16] Y. Nambu, G. Jona-Lasinio, Phys. Rev. 122 (1961) 345;
Y. Nambu, G. Jona-Lasinio, Phys. Rev. 124 (1961) 246.
[17] S.P. Klevansky, Rev. Mod. Phys. 64 (1992) 649;
T. Hatsuda, T. Kunihiro, Phys. Rep. 247 (1994) 221.
[18] P. Rehberg, S.P. Klevansky, J. Hüfner, Phys. Rev. C 53 (1996) 410.
[19] M. Buballa, Phys. Rep. 407 (2005) 205.
[20] P. Costa, M.C. Ruivo, Y.L. Kalinovsky, C.A. de Sousa, Phys. Rev. C 70 (2004) 025204.
[21] L.D. McLerran, Phys. Rev. D 36 (1987) 3291.
[22] M. Asakawa, U. Heinz, B. Müller, Phys. Rev. Lett. 85 (2000) 2072.
[23] J.P. Blaizot, E. Iancu, A. Rebhan, Phys. Lett. B 523 (2001) 143.
[24] P. Costa, M.C. Ruivo, C.A. de Sousa, Y.L. Kalinovsky, Phys. Rev. D 70 (2004) 116013;
P. Costa, M.C. Ruivo, C.A. de Sousa, Y.L. Kalinovsky, Phys. Rev. D 71 (2005) 116002;
P. Costa, M.C. Ruivo, Yu.L. Kalinovsky, Phys. Lett. B 560 (2003) 171;
P. Costa, M.C. Ruivo, Yu.L. Kalinovsky, Phys. Lett. B 577 (2003) 129.
[25] S.D.H. Hsu, M. Schwetz, Phys. Lett. B 432 (1998) 203.
[26] R.B. Griffiths, J. Wheeler, Phys. Rev. A 2 (1970) 1047.