Comment on the Core Conductor Model

Dear Sir:

In their recent paper (1), Clark and Plonsey treat the two-dimensional problem of a cylindrical axon in a volume conductor of conductivity σ. The interior of the axon is considered to be a passive conductor of conductivity σ_i, and expressions are developed for the longitudinal currents inside and outside the axon in terms of the potentials on the inner and outer surfaces of the membrane, respectively. Available data, on the other hand, involve not these two potentials, ϕ_s^o and ϕ_s^i separately, but rather the transmembrane potential

$$\phi_m(z) = \phi_s^i(z) - \phi_s^o(z).$$

To relate their results to available data, Clark and Plonsey construct reasonable "synthetic" potentials, ϕ_s^o and ϕ_s^i chosen so that ϕ_m is also reasonable. This note is to point out that by using an additional constraint, expressions can be developed involving $\phi_m(z)$ directly. The notation of the original paper will be retained.

Let i_m^o and i_m^i be the transmembrane currents per unit length at the outer and inner surfaces of the membrane, respectively. Then from equations (24) and (30) of Clark and Plonsey

$$i_m^o = \sigma a \int_{-\infty}^{\infty} |k| \; F^0(k) \frac{K_1(|k| \; a)}{K_0(|k| \; a)} e^{-ikz} \, dk = \frac{\partial I_l^o}{\partial z}$$

$$i_m^i = \sigma_i a \int_{-\infty}^{\infty} |k| \; F^i(k) \frac{I_1(|k| \; a)}{I_0(|k| \; a)} e^{-ikz} \, dz = -\frac{\partial I_l^i}{\partial z}.$$

From equations (1) and (2) of Clark and Plonsey,

$$i_m^o = i_m^i = i_m.$$

Therefore

$$\sigma F^0(k) \frac{K_1(|k| \; a)}{K_0(|k| \; a)} = \sigma_i F^i(k) \frac{I_1(|k| \; a)}{I_0(|k| \; a)}.$$

By definition

$$\phi_m(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [F^i(k) - F^0(k)] e^{-ikz} \, dk = \frac{1}{2\pi} \int_{-\infty}^{\infty} F^0(k) \alpha(|k| \; a) e^{-ikz} \, dk,$$

where (my definition)

$$\alpha(|k| \; a) \equiv \frac{\sigma K_1(|k| \; a) I_0(|k| \; a)}{\sigma_i K_0(|k| \; a) I_1(|k| \; a)} - 1.$$

Then

$$\Phi_m(k) \equiv \int_{-\infty}^{\infty} \phi_m(z) e^{ikz} \, dk = F^0(k) \alpha(|k| \; a)$$

and within an arbitrary constant,

$$\phi^0(\rho, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\Phi_m(k)}{\alpha(|k|\ a)} \frac{K_0(|k|\ \rho)}{K_0(|k|\ a)} e^{-ikz}\ dk.$$

A similar expression can be obtained for $\phi^i(\rho, z)$. The longitudinal currents and the transmembrane current can then be given in terms of the transmembrane voltage. The last result is

$$i_m(z) = \sigma a \int_{-\infty}^{\infty} |k| \frac{\Phi_m(k)\, K_1(|k|\ a)}{\alpha(|k|\ a)\, K_0(|k|\ a)} e^{-ikz}\ dk.$$

Note that by comparison the cable equations give the result that i_m is proportional to $\partial^2\phi_m/\partial z^2$. It would be interesting to compare the two results.

This work was supported in part by the Joint Services Electronics Program (Contract DA36-039-AMC-0300(E)), the National Institutes of Health (Grant 2 PO1 MH-04737-06), the National Science Foundation (Grant GK-835), the National Aeronautics and Space Administration (Grant NsG-496), and in part by Public Health Service Fellowship 7F3-GM24, 286-01A1 from the Institute of General Medical Sciences.

Received for publication 2 May 1966.

DAVID B. GESELOWITZ
Center for Communication Sciences,
Research Laboratory of Electronics
Massachusetts Institute of Technology
Cambridge

REFERENCE

1. CLARK, J., and PLONSEY, R., *Biophysic. J.*, 1966, **6**, 95.