Pulsar kicks from majoron emission

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Abstract

We show that majoron emission from a hot nascent neutron star can be anisotropic in the presence of a strong magnetic field. If majorons carry a non-negligible fraction of the supernova energy, the resulting recoil velocity of a neutron star can explain the observed velocities of pulsars.

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1. Introduction

Pulsar velocities present a long-standing puzzle [1]. The distribution of pulsar velocities is non-Gaussian, with an average velocity 250–500 km/s [2,3]. As many as 15% of pulsars appear to have velocities greater than 1000 km/s [3]. Pulsars are magnetized rotating neutron stars born in supernova explosions of ordinary stars, and so one expects these high velocities to originate in the supernova explosions. However, a pure hydrodynamical asymmetry does not seem to be sufficient to account for such high velocities. According to advanced 3-dimensional calculations, pulsar velocities from an asymmetric collapse cannot exceed 200 km/s [4]. The origin of pulsar velocities remains a tantalizing puzzle.

While only 1% of the gravitational energy goes into the supernova explosion, a much greater energy pool is in neutrinos that take away 99% of the initial energy. An anisotropy in the neutrino emission as small as a few per cent is sufficient to explain the observed pulsar velocities. The neutrinos are produced in weak processes whose rates depend on the angle between the neutrino momentum and the electron spin. Inside a hot neutron star, the electrons are polarized by a strong magnetic field. Hence, the neutrinos can be produced with a considerable anisotropy. It was suggested that the weak interactions alone could lead to an anisotropic flux of neutrinos which could explain the pulsar kicks [5]. However, this asymmetry is quickly erased by scattering of the neutrinos on their way out of the neutron star. In fact, one can show that, in an approximate thermal and chemical equilibrium, an anisotropy in the production or scattering ampli-
tudes cannot generate an anisotropy in the neutrino flux [6].

There are two ways to evade this no-go theorem [6]. One is to consider an ordinary neutrino outside its neutrinosphere, where it is not in thermal equilibrium. For example, conversions from one neutrino type to another between their respective neutrinospheres, in the region where one of them is trapped but the other one is free-streaming, could explain the pulsar kicks [7]. However, present constraints on the neutrino masses do not allow the resonant neutrino oscillations to take place at densities around the neutrinospheres, and so this mechanism does not work.

Another possibility is that there is a new particle, whose interaction with matter is even weaker than those of neutrinos. Such a particle could be produced out of equilibrium, and the no-go theorem of Ref. [6] does not apply. It has been proposed that an asym- out of equilibrium, and the no-go theorem of Ref. [6] does not apply. It has been proposed that an asym-

Majorons, $\Phi$, are massless pseudo-scalar particles [9] which, to a good approximation, have interactions only with neutrinos described by the Lagrangian

$$\mathcal{L}_{\text{int}} = \frac{\Phi}{2} (g_{\alpha\beta} v^T_{\alpha} v_{\beta} + g^*_{\alpha\beta} v^*_{\alpha} v_{\beta}).$$

The role of the majoron emission in the supernova cooling process has been studied extensively [10,11]. Inside a supernova core neutrinos have an effective potential given by

$$\mathcal{L}_{\text{eff}} = -v^T_{\alpha} V_{\alpha\beta} v_{\beta},$$

where $V_{\alpha\beta} = \text{diag}(V_e, V_\mu, V_\tau)$ and

$$V_e = \sqrt{2} G_F n_B (Y_e + 2 Y_{\nu_e} - n_e/2),$$

$$V_\mu = V_\tau = \sqrt{2} G_F n_B (Y_{\nu_\mu} - n_\mu/2).$$

Here, $Y_i = (n_i - n_\beta)/n_B$ and $n_B$ is the baryon density. We note that, for the values of the majoron couplings we consider, the terms in the potential due to the majoron exchange [12] are negligible; in other words, $|g_{\alpha\beta}|^2 n_B Y_i/T^2 \ll V_e, V_\mu$.

Because of the nonzero effective potential, the dispersion relations of neutrinos and antineutrinos inside the core are different, making processes such as $\nu\nu \rightarrow \Phi$ and $\bar{\nu} \rightarrow \nu\Phi$ kinematically possible. These processes give rise to a majoron flux, which can transfer some energy, $E_\Phi$, from the core. Obviously, $E_\Phi$ cannot be as high as the total supernova energy, $E_{\text{total}} = 1.5 - 4.5 \times 10^{53}$ erg. This is because neutrinos form supernova 1987A have been observed, and this observation implies that at least a third of $E_{\text{total}}$ was emitted in neutrinos. Based on this observation, one can derive strong bounds on the couplings [10,11]:

$$g_{ee} < 4 \times 10^{-7}, \quad g_{\mu\mu}, g_{\tau\tau} < 10^{-6}. \quad (5)$$

However, the data from SN1987a are not precise enough to rule out the possibility that $E_\Phi$ was a non-negligible fraction of $E_{\text{total}}$. Let us define

$$x \equiv E_\Phi/E_{\text{total}},$$

and let us assume that the emission of majorons is anisotropic, with an asymmetry $\epsilon$ of a few percent. Then the overall anisotropy is $\epsilon x$. If this quantity is of the order of $10^{-2}$, the anisotropic emission will give the neutron star a recoil consistent with the observed pulsar velocities. We will show that the neutron star’s magnetic field can cause such an asymmetry.

Let us examine whether the majorons are trapped. Inside a supernova core, the processes $\Phi \rightarrow \nu\nu$ and $\nu\Phi \rightarrow \bar{\nu}$ are kinematically allowed. Indeed, if the couplings are very large ($g > 10^{-5}$), the majorons are trapped inside the core so they cannot transfer a significant amount of energy to the outside [10]. Thus, the bounds from supernova cooling exclude only a small window in the coupling constant values. In this Letter, we will concentrate on the coupling constant values that saturate the bounds in Eq. (5). For such small values of the couplings, the mean free path of $\nu\Phi \rightarrow \nu$ is two orders of magnitude larger than the radius of the supernova core [11]. The majoron decay length is even larger. As a result, one can assume that the majorons leave the core without undergoing any interaction or decay. Also as it is discussed in [11], for the values of coupling satisfying the upper bounds (5), the four particle interactions involving majorons, such as $\Phi \nu \rightarrow \Phi \nu$, $\nu\nu \rightarrow \Phi \Phi$, etc., are negligible.

Now let us assume that there is a uniform strong magnetic field in the core along the $\hat{z}$-direction: $\vec{B} = |\vec{B}| \hat{z}$. In the presence of such a magnetic field, the medium is polarized [13], and the average spin of electrons is

$$\langle \lambda_e \rangle = -\frac{e \vec{B}}{2} \left( \frac{3}{\pi^2} \right)^{1/3} n_e^{-2/3}.$$
As a result, the effective potential of neutrinos receives a new contribution, $\delta V$ [13]:

$$\delta V = -\sqrt{2} G_F Y_e n_B (\lambda_e) \cos \theta \text{ diag} \left( \frac{3}{2}, \frac{1}{2}, \frac{1}{2} \right), \quad (8)$$

where $\theta$ is the angle between the neutrino momentum and the direction of the polarization. Since the effective potential of the neutrinos depends on the direction of their momentum, the rates of the processes $\nu\nu \rightarrow \Phi$ and $\bar{\nu} \rightarrow \nu \Phi$ will also depend on the direction. The emission of majorons produced in these three-particle processes is strongly correlated with the direction of the initial neutrinos [11]. Therefore, the majoron emission will be anisotropic.

We stress that in all our discussion we neglect the neutrino magnetic moment, which is very small in the Standard Model with massive neutrinos. The magnetic field affects the neutrinos only indirectly, through polarizing the electrons in the medium. If some new physics makes the neutrino magnetic moment non-negligible, it may have implications for the pulsar kicks [14].

The rest of this Letter is organized as follows. In Section 2, we evaluate the momentum that the process $\nu_e \nu_e \rightarrow \Phi$ can exert on the neutron star in terms of the total energy transferred to majorons. In Section 3, we perform the same analysis for the processes $\nu_\mu \nu_\mu \rightarrow \Phi$ and $\bar{\nu}_e \rightarrow \nu_\mu \nu_\mu$. In Section 4, we summarize our conclusions and discuss the effects of a realistic configuration of the magnetic field, which is probably not a pure dipole.

2. Effects of $\nu_e \nu_e \rightarrow \Phi$

During the first few seconds after the core collapse, inside the inner core ($r < 10$ km), the electron neutrinos are degenerate; $\mu_{\nu_e} \sim 100$–200 MeV and $T \sim 10$–40 MeV [15]. Right after the core bounce $V_e$ is positive, which makes the process $\nu_e \rightarrow \bar{\nu}_e \Phi$ kinematically allowed. However, after about one second $V_e$ becomes negative and instead of $\nu_e$-decay, $\nu_e \nu_e \rightarrow \Phi$ becomes the source for the production of $\Phi$. As discussed in Ref. [11], the time during which $V_e$ is positive is too short to be important for energy depletion (or momentum transfer). Thus, we concentrate on the time when $V_e < 0$.

Consider two electron neutrinos with momenta $p_{1} = |\vec{p}_{1}|(1, \sin \theta_{1}, 0, \cos \theta_{1})$,

$$p_{2} = |\vec{p}_{2}|(1, \sin \theta_{2} \cos \phi, \sin \theta_{2} \sin \phi, \cos \theta_{2}).$$

The cross-section of $\nu_e \nu_e \rightarrow \Phi$ is given by [11]

$$\sigma = \frac{2\pi g_{ee}^2}{4p_{1}^2 p_{2}^2|v_{1} - v_{2}|} (p_{1} + p_{2})$$

$$\times [2V_e + \delta V_1 + \delta V_2] \delta (\cos \theta - \cos \theta_{0}). \quad (9)$$

where $\cos \theta_{0} = \vec{p}_{1} \cdot \vec{p}_{2}/(|\vec{p}_{1}| |\vec{p}_{2}|)$ and

$$\cos \theta_{0} = 1 + \frac{p_{1} + p_{2}}{p_{1} p_{2}} (2V_e + \delta V_1 + \delta V_2). \quad (10)$$

Note that $\delta V_1$ and $\delta V_2$ depend on the directions of $\vec{p}_{1}$ and $\vec{p}_{2}$. Integrating over all possible momenta of the neutrinos, we find that the neutrinos inside a volume $dV$ during time $d\tau$, transfer a momentum to the core which can be estimated as

$$d\Phi = \frac{7\sqrt{2}}{24} G F n_e (\lambda_e) |g_{ee}|^2 (\mu_{\nu_e})^2 dV d\tau. \quad (11)$$

Of course, the process $\nu_e \nu_e \rightarrow \Phi$ speeds up the deleptonization process and, therefore, the duration of the neutrino emission becomes shorter. However, for $g_{ee} < 4 \times 10^{-7}$, $\Gamma (\nu_e \nu_e \rightarrow \Phi) \ll \Gamma (\nu_e \nu_e \rightarrow \nu_e \nu_e)$, and we expect that the $\beta$-equilibrium is maintained, and the overall evolution of the density profiles is similar to the case without majoron emission [15].

Since we do not know the value of $|g_{ee}|$, it is convenient to write the total momentum transferred to the core in terms of the energy taken away by majorons, $E_{\Phi} = x E_{\text{total}}$:

$$\int d\Phi \approx \frac{\sqrt{2} G F n_e E_{\text{total}} x (\lambda_e)}{2(|V_e|)}$$

$$= -\frac{\sqrt{2} G F E_{\text{total}} x e}{4(|V_e|)} \left( \frac{3n_e}{\pi^4} \right)^{1/3} |\vec{B}|, \quad (12)$$

where $|V_e|$ is the average of $|V_e|$ over time and volume that the process $\nu_e \nu_e \rightarrow \Phi$ takes place. The value of $|V_e|$ changes with time because of the loss of the electron lepton number through neutrino and majoron emission. The initial value of $V_e$ is about 3 eV, which corresponds to $Y_e \approx 0.4$, $Y_{\nu_e} \approx 0.03$ [16,17]. In the absence of majoron emission, this quantity evolves and becomes negative on the time scale of few seconds.
The majoron emission can, of course, affect the evolution of the matter potential. Calculating the exact time-dependence of $|V_e|$ is beyond the scope of this Letter. Here we take a representative value $\langle |V_e| \rangle \approx 0.5$ eV [15–17].

To give the star of mass $M_\star$ a velocity of $v$, this mechanism requires a magnetic field of the order of

$$|\vec{B}| = \frac{M_\star}{1.4M_\odot} \frac{v}{500 \text{ km/s}} \frac{3 \times 10^{53} \text{ erg} \langle |V_e| \rangle}{E_{\text{total}}} \frac{0.5 \text{ eV}}{n_e} \left( \frac{0.05 \text{ fm}^{-3}}{n_e} \right)^{1/3} \frac{0.5}{x} \times 3 \times 10^{16} \text{ G}. \quad (13)$$

Little is known about the magnetic fields in the core of a hot neutron star at birth. Observations show that magnetic fields at the surface of an average radio pulsar millions of years after birth are of the order of $10^{12}$ G. However, some of the observed neutron stars appear to have surface magnetic fields as high as $10^{15}$ G [18]. It is reasonable to assume that the field in the core of a neutron star is stronger than it is on the surface. It is also likely that the magnetic field inside a typical neutron star grows to $\sim 10^{16}$ G or higher during the first seconds after the onset of a supernova explosion due to a dynamo action [19]. Analysis of the dynamo in the linear regime show exponential growth of the field on time scales $\sim 0.1$ s up to a saturation field of the order of $\sim 10^{16}$ G [20], beyond which the evolution of the field becomes non-linear. We are not aware of any detailed calculation of the neutron star dynamo that would describe the field’s non-linear evolution beyond saturation and the resulting global structure of magnetic field. The magnetic field subsequently evolves and decays during the later stages of neutron star cooling. An assumption that all neutron stars have strong interior magnetic fields at birth is not in contradiction with any data.

A smaller magnetic field would result in a smaller asymmetry in the majoron emission. However, some numerical calculations show that small seed asymmetries may get amplified through hydrodynamic instabilities [21]. Thus, even if the magnetic field is much smaller than $10^{16}$ G, the majoron emission can result in a substantial pulsar kick.

We conclude that, if the majorons carry away a substantial fraction of the released energy, they can give the pulsar a recoil velocity that is high enough to explain the data.

3. Effects of $v_\mu v_\mu \rightarrow \Phi$ and $\bar{v}_\mu \rightarrow v_\mu \Phi$

The distributions of $v_\mu v_\mu \rightarrow \Phi$ and $\bar{v}_\mu \rightarrow v_\mu \Phi$ in a supernova core are thermal; however, the densities of these neutrinos are substantially lower than that of $v_e$: $\mu_{v_\mu} = \mu_{v_\tau} = 0$ and $T \ll \mu_{v_e}$. For the evolution of a neutron star, $v_\mu$ and $v_\tau$ are approximately equivalent. So, hereafter we collectively call them $v_\mu$ to avoid repetition. In a supernova core, $V_\mu$ is negative and as a result, the two processes $v_\mu v_\mu \rightarrow \Phi$ and $\bar{v}_\mu \rightarrow v_\mu \Phi$ can occur. In analogy with the $v_e v_e \rightarrow \Phi$ case, one can show that in the presence of a strong magnetic field a net momentum will be imparted to the supernova core given by

$$\int d\vec{P} = \frac{\sqrt{2} GF n_e E_{\text{total}} x}{6|V_\mu|} \langle \bar{\lambda}_e \rangle = -\frac{\sqrt{2} GF E_{\text{total}} x e}{12|V_\mu|} \left( \frac{3n_e}{\pi^4} \right)^{1/3} |\vec{B}| \hat{z}. \quad (14)$$

Again, if $x \geq 0.1$ and $|\vec{B}| \sim 10^{16}$ G, neutron stars can gain high enough velocities.

4. Discussions and conclusions

In this Letter we have shown that despite the strong bounds on the majoron couplings to neutrinos, an asymmetric emission of majorons can explain the high velocities of pulsars, provided that a substantial fraction of the binding energy of the star is emitted in the form of majorons ($E_\Phi / E_{\text{total}} \geq 0.1$). The asymmetric emission can be caused by a magnetic field of order of $10^{16}$ G in the supernova core. Such high magnetic fields are quite possible in a supernova core [19].

The bulk of majorons are produced deep inside the core, where the structure of the magnetic field is unknown. Surface magnetic fields are measured at much later times, when the neutron star is very cold. One does not expect a significant correlation between the field inside the core during the first seconds of a supernova explosion and the field on the surface of a cold neutron star that emerges from this explosion. As a result, we do not expect a correlation between the pulsar velocity and its observed magnetic field (the so-called $B$–$v$ correlation). However, since the duration of majoron emission is much longer than the period

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and $T \ll \mu_{v_e}$. For the evolution of a neutron star, $v_\mu$ and $v_\tau$ are approximately equivalent. So, hereafter we collectively call them $v_\mu$ to avoid repetition. In a supernova core, $V_\mu$ is negative and as a result, the two processes $v_\mu v_\mu \rightarrow \Phi$ and $\bar{v}_\mu \rightarrow v_\mu \Phi$ can occur. In analogy with the $v_e v_e \rightarrow \Phi$ case, one can show that in the presence of a strong magnetic field a net momentum will be imparted to the supernova core given by

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of rotation of an average neutron star at birth, one expects a correlation between the axis of rotation and the direction of the pulsar velocity [22,23]. The direction of the rotational axis is known for only a few pulsars, and for these pulsars it seems to be coincident with the pulsar velocity. However, more data would be needed to reach a definite conclusion.

As was suggested by Spruit and Phinney [22], the mechanism responsible for the large pulsar velocities can also cause large angular momenta of pulsars. The emission of majorons can give rise to a large angular momentum, provided that the magnetic field is not rotationally symmetric. The dynamo mechanism [19,20] can generate an off-centered dipole component if the convection at intermediate depths is faster than in the center. The latter is, indeed, likely because the negative entropy and lepton number gradients necessary for convection can develop in the outer regions, cooled by the neutrino emission.

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