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Transverse-momentum resummation for heavy-quark hadroproduction

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Abstract

We consider the production of a pair of heavy quarks ($Q\bar{Q}$) in hadronic collisions. When the transverse momentum q_T of the heavy-quark pair is much smaller than its invariant mass, the QCD perturbative expansion is affected by large logarithmic terms that must be resummed to all orders. This behavior is well known from the simpler case of hadroproduction of colorless high-mass systems, such as vector or Higgs boson(s). In the case of $Q\bar{Q}$ production, the final-state heavy quarks carry color charge and are responsible for additional soft radiation (through direct emission and interferences with initial-state radiation) that complicates the evaluation of the logarithmically-enhanced terms in the small-q_T region. We present the all-order resummation structure of the logarithmic contributions, which includes color flow evolution factors due to soft wide-angle radiation. Resummation is performed at the completely differential level with respect to the kinematical variables of the produced heavy quarks. Soft-parton radiation produces azimuthal correlations that are fully taken into account by the resummation formalism. These azimuthal correlations are entangled with those that are produced by initial-state collinear radiation. We present explicit analytical results up to next-to-leading order and next-to-next-to-leading logarithmic accuracy.

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1. Introduction

We consider the inclusive production of a $Q\bar{Q}$ pair of heavy quarks (Q) in hadron–hadron collisions. The bulk of the cross section is produced in the kinematical region where the transverse momentum q_T of the $Q\bar{Q}$ pair is smaller than the mass m of the heavy quark. In this paper we are interested in the small-q_T region, namely, the region where $q_T \ll m$ (including the limit $q_T \to 0$). From the phenomenological point of view, the most relevant process is the production of a pair of top–antitop ($t\bar{t}$) quarks [1], because of its topical importance in the context of both Standard Model (SM) and beyond-SM physics. In our theoretical study at the formal level, we consider a generic pair of heavy quarks.

The q_T cross section of the $Q\bar{Q}$ pair is computable in QCD perturbation theory [2], provided m is much larger than the QCD scale Λ_{QCD}. The cross section is obtained by convoluting the parton densities of the colliding hadrons with the partonic cross sections, which are evaluated as power series expansion in the QCD coupling α_{S}. In the small-q_T region the perturbative expansion is badly behaved, since the size of the perturbative coefficients is enhanced by powers of $\ln q_T$. A reliable theoretical calculation requires the all-order resummation of these logarithmically-enhanced terms. This type of perturbative behavior is well known [3–5] from the simpler case of hadroproduction of a high-mass lepton pair through the Drell–Yan (DY) mechanism. In the case of the DY process the all-order resummation of the $\ln q_T$ terms is fully understood [3–6]. At the level of leading-logarithmic (LL) contributions, the extension of resummation from the DY process to the heavy-quark process is relatively straightforward, and it was first discussed long ago in Ref. [7] (related studies were presented in Ref. [8]). Beyond the LL level, the structure of $\ln q_T$ terms for the heavy-quark process is definitely different (the main physical differences are discussed below) from that of the DY process, and this difference implies very relevant theoretical complications. The all-order resummation for the heavy-quark process has been discussed only very recently by H.X. Zhu et al. in Refs. [9,10]. The analysis of Refs. [9,10] is limited to the study of the q_T cross section after integration over the azimuthal angles of the produced heavy quarks. In this paper we illustrate the results of our independent study of transverse-momentum resummation for $Q\bar{Q}$ production. We present our all-order resummation formalism for $Q\bar{Q}$ production, and we perform the resummation up to the next-to-next-to-leading logarithmic (NNLL) level, by explicitly including *all* the contributions up to the next-to-leading order (NLO) in the perturbative expansion. Our formalism and results are valid at the fully-differential level with respect to the kinematics of the produced heavy quarks. In particular, we consider the explicit dependence on the azimuthal angles of the heavy quarks and we have full control, at the resummed level, of the ensuing *azimuthal correlations* in the small-q_T region. In the case of the azimuthally-averaged q_T cross section we find agreement with the NNLL results of Refs. [9,10].

The DY lepton-pair production is a specific process of a general class of hard-scattering processes in which the produced high-mass system F in the final state is formed by a set of colorless (i.e., non-strongly interacting) particles (e.g., F can be a lepton pair, or a photon pair, or one or more vector bosons or Higgs bosons). Transverse-momentum resummation for the q_T distribution of F is fully understood for this entire class of processes (i.e., independently of the specific particle content of the system F). Indeed, transverse-momentum resummation for these processes has an all-order *universal* (process-independent) structure [11,6,12–14], which has been explicitly worked out [14–17] at NNLL accuracy and the next-to-next-to-leading order (NNLO) in the perturbative expansion. This universality structure eventually originates from the underlying physical mechanism that produces the q_T broadening of the system F at small q_T: the transverse momentum of F is produced by (soft and collinear) QCD radiation from the initial-state colliding partons. The $Q\bar{Q}$ production process definitely belongs to a different class of processes, since the produced final-state heavy quarks carry color charge and, therefore, they act as additional source of QCD radiation. The q_T of the $Q\bar{Q}$ pair depends on initial-state radiation, on final-state radiation and on quantum (and color flow) interferences between radiation from the initial and final states. These physical differences between $Q\bar{Q}$ production and the production of a colorless system F lead to very relevant technical and conceptual complications in the context of transverse-momentum resummation for $Q\bar{Q}$ production. An important issue regards the presence of possible contributions from factorization-breaking effects of collinear radiation [18–21]. Other complications, which already arise in the context of threshold resummation for the $Q\bar{Q}$ total cross section [22–26], regard the effect of non-abelian color correlations produced by initial-state and final-state interferences. Additional important complications and effects, which are specific of transverse-momentum resummation, regard the azimuthal-angle distribution of the $Q\bar{Q}$ pair. In the case of the DY process, q_T resummation has no effect on the azimuthal correlation between the produced leptons, since the q_T broadening of the lepton pair is entirely due to QCD radiation from the initial-state ($q\bar{q}$) partons. In contrast, the q_T of the $Q\bar{Q}$ pair is also due to radiation from Q and $\bar{Q}$ separately, and this leads to q_T-dependent azimuthal correlations. The main features of $Q\bar{Q}$ production that we have just highlighted will be briefly recalled in the presentation of our resummation results.

The paper is organized as follows. In Section 2 we introduce our notation and we illustrate our all-order resummation formalism. In Section 3 we present and discuss the explicit form of the resummation coefficients up to NLO and NNLL accuracy. Our results are summarized in Section 4.

2. All-order resummation

We consider the inclusive hard-scattering process

$$h_1(P_1) + h_2(P_2) \to Q(p_3) + \bar{Q}(p_4) + X, \tag{1}$$

where the collision of the two hadrons h_1 and h_2 with momenta P_1 and P_2 produces the $Q\bar{Q}$ pair, and X denotes the accompanying final-state radiation. The hadron momenta P_1 and P_2 are treated in the massless approximation ($P_1^2 = P_2^2 \simeq 0$). The heavy quarks have momenta p_3 and p_4, and the total four-momentum of the $Q\bar{Q}$ pair is $q^\mu = p_3^\mu + p_4^\mu$. In a reference frame where the colliding hadrons are back-to-back, the total momentum q^μ is fully specified by its invariant mass M ($M^2 = q^2$), rapidity y ($y = \frac{1}{2}\ln\frac{q\cdot P_2}{q\cdot P_1}$) and transverse-momentum vector $\mathbf{q_T}$. Analogously, the momentum p_j^μ ($j = 3, 4$) of the heavy quark is specified by the heavy-quark mass m ($p_3^2 = p_4^2 = m^2$), rapidity y_j and transverse-momentum vector $\mathbf{p_T}_j$. The two-dimensional transverse-momentum vectors $\mathbf{q_T}$, $\mathbf{p_T}_3$ and $\mathbf{p_T}_4$ have azimuthal angles ϕ_q, ϕ_3 and ϕ_4.

The kinematics of the observed heavy quarks is fully determined by the their total momentum q and by two additional and *independent* kinematical variables that specify the angular distribution of Q and $\bar{Q}$ with respect to the momentum q of the $Q\bar{Q}$ pair. These two additional kinematical variables are generically denoted as $\boldsymbol{\Omega} = \{\Omega_A, \Omega_B\}$ (correspondingly, we define $d\boldsymbol{\Omega} = d\Omega_A d\Omega_B$). For instance, we can use $\boldsymbol{\Omega} = \{y_3, \phi_3\}$ or any other equivalent pairs of kinematical variables (e.g., $y_3 \to y_3 - y$, $\phi_3 \to \phi_4$ and so forth). We thus consider the most general fully-differential cross section

$$\frac{d\sigma(P_1, P_2; \mathbf{q_T}, M, y, \boldsymbol{\Omega})}{d^2\mathbf{q_T}\, dM^2\, dy\, d\boldsymbol{\Omega}} \tag{2}$$

for the inclusive-production process in Eq. (1). Note that the cross section in Eq. (2) and the corresponding q_T resummation formula can be straightforwardly integrated with respect to one or more of the final-state variables $\{\Omega_A, \Omega_B, y, \phi_q, M\}$, thus leading to results for observables that are more inclusive than the differential cross section in Eq. (2).

The hadronic cross section in Eq. (2) is computable within QCD by convoluting partonic cross sections with the scale-dependent parton distributions $f_{a/h}(x, \mu^2)$ ($a = q_f, \bar{q}_f, g$ is the label of the massless partons) of the colliding hadrons. The partonic cross sections are expressed as a power series expansion in α_S. At the leading order (LO) in the perturbative expansion, the partonic cross sections are proportional to α_S^2 and there are only two contributing partonic processes, namely, the quark–antiquark ($q\bar{q}$) annihilation process $q_f\bar{q}_f \to Q\bar{Q}$ and the gluon fusion process $gg \to Q\bar{Q}$. In both LO processes, the q_T dependence of the partonic cross section (and of the ensuing hadronic cross section) is simply proportional to $\delta^{(2)}(\mathbf{q_T})$, because of transverse-momentum conservation. At higher perturbative orders, the partonic cross sections receive contributions from elastic ($c\bar{c} \to Q\bar{Q}$) and inelastic ($a\,b \to Q\bar{Q} + X$) partonic processes. The q_T dependence of the partonic cross section includes contributions that are 'singular' in the limit $q_T \to 0$: these singular contributions are proportional to $\alpha_S^{n+2}\delta^{(2)}(\mathbf{q_T})$ or to logarithmic terms of the type $\alpha_S^{n+2}\frac{1}{q_T^2}\ln^k(M^2/q_T^2)$ with $k \le 2n-1$ (more precisely, the logarithmic terms are expressed in terms of singular, though integrable over q_T, 'plus'-distributions). We thus decompose the cross section in Eq. (2) as follows:

$$d\sigma = d\sigma^{(\text{sing})} + d\sigma^{(\text{reg})}, \tag{3}$$

where the component $d\sigma^{(\text{sing})}$ embodies *all* the singular terms in the limit $q_T \to 0$, whereas $d\sigma^{(\text{reg})}$ includes the remaining non-singular terms. In this paper we deal with the all-order evaluation and resummation of the small-q_T singular terms in $d\sigma^{(\text{sing})}$. At fixed value of q_T, the cross section depends on the mass scales M and m. We use M to set the scale of the $\ln q_T$ terms, and the remaining dependence on the two mass scales is controlled by the dimensionless ratio $2m/M$ or, equivalently, by the relative velocity v of Q and $\bar{Q}$,

$$v = \sqrt{1 - \frac{m^4}{(p_3 \cdot p_4)^2}} = \sqrt{1 - \left(\frac{2m^2}{M^2 - 2m^2}\right)^2}. \tag{4}$$

In our resummation treatment at small q_T, the mass scales M and m are considered to be parametrically of the same order. In two particular regions, namely, the threshold region where $2m/M \to 1$ (or $v \to 0$) and the high-mass region where $2m/M \to 0$ (or $v \to 1$), the size of the coefficients of the $\ln q_T$ terms can be enhanced, and accurate quantitative predictions may require additional resummation of the dependence on $2m/M$ (or v). Note, however, that our treatment of the small-q_T dependence is valid in the *entire* region $q_T \ll M$ (and not only in the subregion $q_T \ll m$). In other words, in our treatment of the small-q_T region, the decomposition in Eq. (3) is such that we have $d\sigma^{(\text{reg})}/d\sigma^{(\text{sing})} = \mathcal{O}(q_T/M)$ (modulo logarithmic corrections) order-by-order in the perturbative QCD expansion (note that $\mathcal{O}(q_T/M) \ll \mathcal{O}(q_T/m)$ if $m \ll M$).

Our discussion of the decomposition in Eq. (3) can be expressed in a more formal way. We consider the order-by-order perturbative expansion of the q_T cross section $d\sigma$ and we write $d\sigma = \sum_n d\sigma^{(n)}$, where $d\sigma^{(n)}$ is the contribution at the n-th perturbative order in α_S. Analogous perturbative expansions apply to $d\sigma^{(\text{sing})}$ and $d\sigma^{(\text{reg})}$ in terms of the n-th order contributions $d\sigma^{(\text{sing})(n)}$ and $d\sigma^{(\text{reg})(n)}$, and we have $d\sigma^{(n)} = d\sigma^{(\text{sing})(n)} + d\sigma^{(\text{reg})(n)}$. The regular component of the q_T cross section is thus specified by requiring that the integration of $d\sigma^{(\text{reg})}/d^2\mathbf{q_T}$ over the range $0 \le q_T \le Q_0$ leads to a finite result that, at each fixed order in α_S, *vanishes* in the limit $Q_0 \to 0$. We have

$$\int_0^{Q_0^2} dq_T^2 \frac{d\sigma^{(\text{reg})(n)}}{d^2\mathbf{q_T}\, dM^2\, dy\, d\boldsymbol{\Omega}} = \mathcal{O}(Q_0/M), \quad Q_0 \to 0,$$

and we note that the right-hand side is power suppressed through the ratio Q_0/M. This requirement on $d\sigma^{(\text{reg})}$ uniquely specifies all the singular terms of $d\sigma$ that are included in $d\sigma^{(\text{sing})}$, although there is still some freedom on how non-singular terms (i.e., terms leading to corrections of $\mathcal{O}(Q_0/M)$ in the limit $Q_0 \to 0$) are split between $d\sigma^{(\text{reg})(n)}$ and $d\sigma^{(\text{sing})(n)}$. In the following we present an explicit all-order expression of $d\sigma^{(\text{sing})}$ (see Eq. (5)). This expression can systematically be expanded in powers of α_S thus leading to the explicit expression of $d\sigma^{(\text{sing})(n)}$. The explicit expression of $d\sigma^{(\text{sing})(n)}$ then uniquely determines $d\sigma^{(\text{reg})(n)}$ in terms of the complete perturbative expression of the q_T cross section (i.e., we have $d\sigma^{(\text{reg})(n)} = d\sigma^{(n)} - d\sigma^{(\text{sing})(n)}$). More detailed discussions on the decomposition in Eq. (3) and on its perturbative expansion can be found in Refs. [13,27].

We illustrate the method that we have used to derive our resummation results for $d\sigma^{(\text{sing})}$. More details and additional results will be presented in forthcoming studies. We carry out our analysis of the singular terms in the small-q_T region by working in impact parameter ($\mathbf{b}$) space and, thus, we first perform the Fourier transformation of $d\sigma^{(\text{sing})}/d^2\mathbf{q_T}$ with respect to $\mathbf{q_T}$ at fixed $\mathbf{b}$. The final results for $d\sigma^{(\text{sing})}/d^2\mathbf{q_T}$ are then eventually recovered by performing the inverse Fourier transformation from $\mathbf{b}$ space to $\mathbf{q_T}$ space (see Eq. (5)). In $\mathbf{b}$ space the singular terms are proportional to power of $\ln(Mb)$ ($q_T \ll M$ corresponds to $bM \gg 1$). These $\ln(Mb)$ terms are produced by the radiation of soft and collinear partons (i.e., partons with low transverse momentum k_T, say, with $k_T \ll M$) in the inclusive final state X of the inelastic partonic processes $a\,b \to Q\bar{Q} + X$. Soft and collinear radiation is treated by using the universal (process-independent) all-order factorization formulae [28–32,19,33] of QCD scattering amplitudes. Soft/collinear factorization at the amplitude (and squared amplitude) level is not spoiled by kinematical effects at the cross section level, since we are working in $\mathbf{b}$ space (in the small-q_T limit, the kinematics of the q_T cross section is exactly factorized [4] by the Fourier transformation to $\mathbf{b}$ space). Therefore, the $\ln(Mb)$ terms are explicitly computed by the phase space integration (in $\mathbf{b}$ space) of the soft/collinear factors. The application of the known explicit expressions [34,29,35,30,31,36] of soft/collinear factorization formulae allows us to compute the structure of $d\sigma^{(\text{sing})}$ up to NNLO and NNLL accuracy. The method that we have just described is completely analogous (as applied in the NNLL + NLO computation of Ref. [37] and outlined to all orders in Ref. [14]) to the method that is applicable to transverse-momentum resummation for the production of a system F of colorless particles. The differences between the production of F and $Q\bar{Q}$ production are due to the non-abelian color charge of the produced heavy quarks. The complications that arise from these differences are basically related to soft radiation at wide angles with respect to the direction of the colliding partons. As a consequence, the structure of $d\sigma^{(\text{sing})}$ for $Q\bar{Q}$ production definitely *differs* (and the differences already appear at the NLO) from that of transverse-momentum resummation for the production of a colorless system F. Beyond the NNLL + NNLO level of perturbative accuracy, non-abelian soft wide-angle interactions of absorptive origin produce violation of strict factorization for space-like collinear radiation [19]. Therefore, the all-order formula of $d\sigma^{(\text{sing})}$ that is presented below is based on some assumptions about possible contributions that can arise from factorization-breaking effects of collinear radiation [18–21]. In particular, we assume that infrared divergences produced by inclusive parton radiation at transverse momentum $k_T \ll 1/b$ are either canceled or customarily factorized in the parton distributions $f_{a/h}(x, 1/b^2)$ evolved up to the scale $\mu \sim 1/b$. Moreover, our resummed result for $d\sigma^{(\mathrm{sing})}$ includes only the possible soft/collinear correlation structures that we have explicitly uncovered up to NNLL + NNLO. These issues certainly deserve further and future investigations. We remark that we have full control of the all-order structure of $d\sigma^{(\mathrm{sing})}$ up to NNLL + NNLO accuracy. Possible additional structures are likely to be absent till very high perturbative orders [18–20,38].

In the following we use parton densities $f_{a/h}(x, \mu^2)$ as defined in the $\overline{\mathrm{MS}}$ factorization scheme. The running coupling $\alpha_S(\mu^2)$ denotes the renormalized QCD coupling in the $\overline{\mathrm{MS}}$ renormalization scheme with decoupling of the heavy quark Q [39] (e.g., in the case of $t\bar{t}$ production, $\alpha_S(\mu^2)$ is the $\overline{\mathrm{MS}}$ coupling in the 5-flavor scheme), and m is the renormalized pole mass of the heavy quark Q. Obviously our explicit results can be straightforwardly expressed in different factorization/renormalization schemes by applying the corresponding scheme transformation relations (e.g., the pole mass m can be replaced by the $\overline{\mathrm{MS}}$ running mass $m(\mu^2)$). To present the resummation results for $Q\bar{Q}$ production we closely follow the formulation of transverse-momentum resummation for the production of a colorless system F, and we use the same notation as in Refs. [13,14] (more details about the notation can be found therein). This presentation allows us to clearly identify and highlight the structural differences that arise in the context of $Q\bar{Q}$ production.

Our results for the singular component $d\sigma^{(\mathrm{sing})}$ of the $Q\bar{Q}$ production cross section are given by the following all-order resummation formula:

$$\frac{d\sigma^{(\mathrm{sing})}(P_1, P_2; \mathbf{q_T}, M, y, \boldsymbol{\Omega})}{d^2\mathbf{q_T}\, dM^2\, dy\, d\boldsymbol{\Omega}} = \frac{M^2}{2P_1 \cdot P_2} \sum_{c=q,\bar{q},g} \left[d\sigma^{(0)}_{c\bar{c}}\right] \int \frac{d^2\mathbf{b}}{(2\pi)^2}\, e^{i\mathbf{b}\cdot\mathbf{q_T}}\, S_c(M, b)$$
$$\times \sum_{a_1,a_2} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} \left[(\mathbf{H}\,\boldsymbol{\Delta})C_1 C_2\right]_{c\bar{c};a_1a_2} f_{a_1/h_1}\big(x_1/z_1, b_0^2/b^2\big)\, f_{a_2/h_2}\big(x_2/z_2, b_0^2/b^2\big), \tag{5}$$

where $b_0 = 2e^{-\gamma_E}$ ($\gamma_E = 0.5772\ldots$ is the Euler number) is a numerical coefficient, and the kinematical variables x_1 and x_2 are

$$x_1 = \frac{M}{\sqrt{2P_1 \cdot P_2}}\, e^{+y}, \qquad x_2 = \frac{M}{\sqrt{2P_1 \cdot P_2}}\, e^{-y}. \tag{6}$$

The right-hand side of Eq. (5) involves the (inverse) Fourier transformation with respect to the impact parameter $\mathbf{b}$ and two convolutions over the longitudinal-momentum fractions z_1 and z_2. The parton densities $f_{a_i/h_i}(x, \mu^2)$ of the colliding hadrons are evaluated at the scale $\mu = b_0/b$, which depends on the impact parameter. The factor that is denoted by the symbol $[d\sigma^{(0)}_{c\bar{c}}]$ refers to the *partonic* elastic-production process $c\bar{c} \to Q\bar{Q}$ of the $Q\bar{Q}$ pair,

$$c(p_1) + \bar{c}(p_2) \to Q(p_3) + \bar{Q}(p_4), \quad c = q, \bar{q}, g, \tag{7}$$

with

$$p_i = x_i P_i, \quad i = 1, 2, \tag{8}$$

where P_i $(i = 1, 2)$ are the momenta of the colliding hadrons (see Eq. (1)) and x_i $(i = 1, 2)$ are the momentum fractions in Eq. (6). Making the symbolic notation explicit, the symbol $[d\sigma^{(0)}_{c\bar{c}}]$ is related the LO cross section $d\hat{\sigma}^{(0)}$ for $Q\bar{Q}$ production by the partonic process in Eq. (7), and we have

$$\left[d\sigma^{(0)}_{c\bar{c}}\right] = \alpha_S^2\left(M^2\right)\frac{d\hat{\sigma}^{(0)}_{c\bar{c}\to Q\bar{Q}}(p_1, p_2; p_3, p_4)}{M^2\, d\boldsymbol{\Omega}}. \tag{9}$$

QCD radiative correction are embodied in the factors S_c and $[(\mathbf{H}\,\boldsymbol{\Delta})C_1C_2]$ on the right-hand side of Eq. (5). The expression in Eq. (5) involves the sum of two types of contributions, which correspond to the LO partonic channels: the contribution of the $q\bar{q}$ annihilation channel ($c = q, \bar{q}$) and the contribution of the gluon fusion channel ($c = g$). In each of these channels, the structure of Eq. (5) is *apparently* similar to the structure of transverse-momentum resummation for the production of a colorless system F [11,6,12–14] (see Eq. (6) of Ref. [14] for direct comparison). The important *differences* that occur in the case of $Q\bar{Q}$ production are hidden in the symbolic notation of the factor $[(\mathbf{H}\,\boldsymbol{\Delta})C_1C_2]$ and, more specifically, they are due to the factor $\boldsymbol{\Delta}$ that is related to the accompanying *soft*-parton radiation in $Q\bar{Q}$ production. In the case of production of a colorless system F, the factor $\boldsymbol{\Delta}$ is absent (i.e. $\boldsymbol{\Delta} = 1$).

The expression of the symbolic factor $[(\mathbf{H}\,\boldsymbol{\Delta})C_1C_2]$ for the $q\bar{q}$ annihilation channel is

$$\left[(\mathbf{H}\,\boldsymbol{\Delta})C_1C_2\right]_{c\bar{c};a_1a_2} = (\mathbf{H}\,\boldsymbol{\Delta})_{c\bar{c}}\, C_{ca_1}\left(z_1; \alpha_S\left(b_0^2/b^2\right)\right) C_{\bar{c}\,a_2}\left(z_2; \alpha_S\left(b_0^2/b^2\right)\right) \quad (c = q, \bar{q}), \tag{10}$$

whereas for the gluon fusion channel ($c = g$) we have

$$\begin{aligned}&\left[(\mathbf{H}\,\boldsymbol{\Delta})C_1C_2\right]_{gg;a_1a_2}\\ &\quad = (\mathbf{H}\,\boldsymbol{\Delta})_{gg;\mu_1\nu_1,\mu_2\nu_2} C^{\mu_1\nu_1}_{ga_1}\left(z_1; p_1, p_2, \mathbf{b}; \alpha_S\left(b_0^2/b^2\right)\right) C^{\mu_2\nu_2}_{ga_2}\left(z_2; p_1, p_2, \mathbf{b}; \alpha_S\left(b_0^2/b^2\right)\right).\end{aligned} \tag{11}$$

The functions C_{ca} and $C^{\mu\nu}_{ga}$ are described below. The factors $(\mathbf{H}\,\boldsymbol{\Delta})$ in Eqs. (10) and (11) depend on $\mathbf{b}$, M and on the kinematical variables of the partonic process in Eq. (7) (this dependence is not explicitly denoted in Eqs. (10) and (11)). Eq. (11) includes the sum over the repeated indices $\{\mu_i, \nu_i\}$, which refer to the Lorentz indices of the colliding gluons $g(p_i)$ $(i = 1, 2)$ in Eq. (7). In Eqs. (10) and (11) we use the shorthand notation $(\mathbf{H}\,\boldsymbol{\Delta})$ for the contribution of the factors $\mathbf{H}$ and $\boldsymbol{\Delta}$, since these factors embody a non-trivial dependence on the color structure (and color indices) of the partonic process in Eq. (7). To take into account the color dependence, we use the color space formalism of Ref. [40]: the color-index dependence of the scattering amplitude $\mathcal{M}$ of the process in Eq. (7) is represented by a vector $|\mathcal{M}\rangle$ in color space, and color matrices are represented by color operators acting onto $|\mathcal{M}\rangle$. Using the color space formalism, we can write the explicit representation of $(\mathbf{H}\,\boldsymbol{\Delta})$. In the case of the $q\bar{q}$ annihilation channel, we have

$$(\mathbf{H}\,\boldsymbol{\Delta})_{c\bar{c}} = \frac{\langle\widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}}|\boldsymbol{\Delta}|\widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}}\rangle}{\alpha_S^2(M^2)|\mathcal{M}^{(0)}_{c\bar{c}\to Q\bar{Q}}(p_1, p_2; p_3, p_4)|^2} \quad (c = q, \bar{q}), \tag{12}$$

where the 'hard-virtual' amplitude $\widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}}$ is directly related to the infrared-finite part of the all-order (virtual) scattering amplitude $\mathcal{M}_{c\bar{c}\to Q\bar{Q}}$ of the partonic process in Eq. (7), and

$\mathcal{M}^{(0)}_{c\bar{c}\to Q\bar{Q}}$ is the tree-level (LO) contribution to $\mathcal{M}_{c\bar{c}\to Q\bar{Q}}$ ($|\mathcal{M}^{(0)}_{c\bar{c}\to Q\bar{Q}}|^2$ is the squared amplitude summed over the colors and spins of the partons $c, \bar{c}, Q, \bar{Q}$). The relation between $\mathcal{M}$ and $\widetilde{\mathcal{M}}$ is given in Eq. (26). The analogue of Eq. (12) in the gluon fusion channel is

$$(\mathbf{H}\,\boldsymbol{\Delta})_{gg;\mu_1\nu_1,\mu_2\nu_2} = \frac{\langle \widetilde{\mathcal{M}}^{\nu_1'\nu_2'}_{gg\to Q\bar{Q}}|\boldsymbol{\Delta}|\widetilde{\mathcal{M}}^{\mu_1'\mu_2'}_{gg\to Q\bar{Q}}\rangle d_{\mu_1'\mu_1} d_{\nu_1'\nu_1} d_{\mu_2'\mu_2} d_{\nu_2'\nu_2}}{\alpha_S^2(M^2)|\mathcal{M}^{(0)}_{gg\to Q\bar{Q}}(p_1, p_2; p_3, p_4)|^2}, \tag{13}$$

where $\{\mu_i', \nu_i'\}$ $(i = 1, 2)$ are exactly (see Eq. (26)) the gluon Lorentz indices of the scattering amplitude $\mathcal{M}_{gg\to Q\bar{Q}}(p_1, p_2; p_3, p_4)$, and $d^{\mu\nu} = d^{\mu\nu}(p_1, p_2)$ is the following polarization tensor,

$$d^{\mu\nu}(p_1, p_2) = -g^{\mu\nu} + \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{p_1 \cdot p_2}, \tag{14}$$

which projects onto the Lorentz indices in the transverse plane. The soft-parton factor $\boldsymbol{\Delta}$ depends on color matrices, and it acts as a color space operator in Eqs. (12) and (13). We can also introduce a color space operator $\mathbf{H}$ through the definition $\alpha_S^2\,|\mathcal{M}^{(0)}|^2\,\mathbf{H} = |\widetilde{\mathcal{M}}\rangle\langle\widetilde{\mathcal{M}}|$. Therefore, according to Eqs. (12) and (13), the shorthand notation $(\mathbf{H}\,\boldsymbol{\Delta})$ is equivalent to $(\mathbf{H}\,\boldsymbol{\Delta}) = \mathrm{Tr}[\mathbf{H}\,\boldsymbol{\Delta}]$, where 'Tr' exactly denotes the color space trace of the color operator $\mathbf{H}\,\boldsymbol{\Delta}$.

We now illustrate the structural form of the resummation formulae in Eqs. (5), (10)–(13), and the differences between $Q\bar{Q}$ production and the production of a colorless system F. The hard factor $\mathbf{H}$ is independent of the impact parameter $\mathbf{b}$, and it depends on the scattering amplitude $\mathcal{M}_{c\bar{c}\to Q\bar{Q}}$. An analogous process-dependent hard factor (which depends on the scattering amplitude of the process $c\bar{c} \to F$) [14] appears for the production of a colorless system F. The functions C_{ca} [12] and $C^{\mu\nu}_{ga}$ [13] in Eqs. (10) and (11) are *universal* (they are process-independent and only depend on the parton indices), and they are computable as power series expansions in $\alpha_S(b_0^2/b^2)$. These functions originate from initial-state collinear radiation of partons with typical transverse momentum $k_T \sim 1/b$. The function $S_c(M, b)$ in Eq. (5) is the Sudakov form factor [11], and it is also *universal*. Thus, for instance, the $q\bar{q}$ annihilation channel functions S_q and C_{qa} also contribute to transverse-momentum resummation for the DY process [6], whereas the gluon fusion channel functions S_g and $C^{\mu\nu}_{ga}$ also contribute in the case of Higgs boson production [13]. The Sudakov form factor $S_c(M, b)$ resums logarithmic terms $\alpha_S^n \ln^k(Mb)$, starting from the LL contributions (those with $k = 2n$) to the q_T cross section. The Sudakov form factor is due to QCD radiation from the initial-state partons c and $\bar{c}$ in the process of Eq. (7) and, more precisely, it is produced by soft and flavor-conserving collinear radiation with typical transverse momentum k_T in the range $1/b \lesssim k_T \lesssim M$. The factor $\boldsymbol{\Delta}$ in Eqs. (5), (10)–(13) is specific of $Q\bar{Q}$ production ($\boldsymbol{\Delta} = 1$ for the production of a colorless system F), and it is due to QCD radiation of soft non-collinear (at wide angles with respect to the direction of the initial-state partons) partons from the underlying subprocess $c\bar{c} \to Q\bar{Q}$. Therefore, $\boldsymbol{\Delta}$ embodies the effect of soft radiation from the $Q\bar{Q}$ final state and from initial-state and final-state interferences. As in the case of the Sudakov form factor, the soft radiation contribution to $\boldsymbol{\Delta}$ involves the transverse-momentum range $1/b \lesssim k_T \lesssim M$. Therefore, $\boldsymbol{\Delta}$ resums additional logarithmic terms $\alpha_S^n \ln^k(Mb)$ (see Eq. (15)), although the dominant contributions to $\boldsymbol{\Delta}$ are of next-to-leading-logarithmic (NLL) type, since they are produced by non-collinear radiation. Moreover, soft-parton radiation at the scale $k_T \sim 1/b$ has a 'special' physical role, since it is eventually responsible for azimuthal correlations (see Eqs. (15) and (18)).

The soft-parton factor $\boldsymbol{\Delta}$ depends on the impact parameter $\mathbf{b}$, on M and on the kinematics of the partonic process in Eq. (7). To explicitly denote the kinematical dependence (which is in turn related to the two angular variables $\boldsymbol{\Omega}$ of the q_T cross section), we use the rapidity difference $y_{34} = y_3 - y_4$ between $Q(p_3)$ and $\bar{Q}(p_4)$ and the azimuthal angle ϕ_3 of the quark $Q(p_3)$ (the dependence on $2m/M$ is not explicitly denoted in the following). The all-order structure of $\boldsymbol{\Delta}$ is

$$\boldsymbol{\Delta}(\mathbf{b}, M; y_{34}, \phi_3) = \mathbf{V}^{\dagger}(b, M; y_{34})\mathbf{D}\big(\alpha_S\big(b_0^2/b^2\big); \phi_{3b}, y_{34}\big)\mathbf{V}(b, M; y_{34}), \tag{15}$$

where

$$\mathbf{V}(b, M; y_{34}) = \overline{P}_q \exp\left\{-\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\boldsymbol{\Gamma}_t\big(\alpha_S\big(q^2\big); y_{34}\big)\right\}, \tag{16}$$

$$\boldsymbol{\Gamma}_t(\alpha_S; y_{34}) = \frac{\alpha_S}{\pi}\boldsymbol{\Gamma}_t^{(1)}(y_{34}) + \left(\frac{\alpha_S}{\pi}\right)^2 \boldsymbol{\Gamma}_t^{(2)}(y_{34}) + \sum_{n=3}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n \boldsymbol{\Gamma}_t^{(n)}(y_{34}), \tag{17}$$

$$\mathbf{D}(\alpha_S; \phi_{3b}, y_{34}) = 1 + \frac{\alpha_S}{\pi}\mathbf{D}^{(1)}(\phi_{3b}, y_{34}) + \sum_{n=2}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n \mathbf{D}^{(n)}(\phi_{3b}, y_{34}). \tag{18}$$

The color operator (matrix) $\boldsymbol{\Gamma}_t$ is the soft anomalous dimension matrix that is specific of transverse-momentum resummation for $Q\bar{Q}$ production. This quantity is computable order-by-order in α_S as in Eq. (17). The *evolution* factor $\mathbf{V}$ in Eq. (16) is obtained by the exponentiation of the integral of the soft anomalous dimension. The integral is performed over the transverse-momentum scale q^2 of the QCD running coupling, and the symbol $\overline{P}_q$ in Eq. (16) denotes the anti path-ordering of the exponential matrix with respect to the integration variable q^2. The evolution operator $\mathbf{V}$ explicitly resums logarithmic terms $\alpha_S^n(M^2)\ln^k(Mb)$ (with $k \le n$) through the integration over q^2. Soft-parton radiation from the process $c\bar{c} \to Q\bar{Q}$ produces non-abelian color correlations that are embodied in the soft anomalous dimension matrix. The structure of $\mathbf{V}$ is *typical* of the resummation of soft-gluon logarithmic contributions in QCD multiparton hard-scattering processes [41,42]. Operators that are analogous to $\mathbf{V}$ arise in the context of threshold resummation for the $Q\bar{Q}$ total cross section [22–26]. The color operator $\mathbf{D}$ in Eq. (15) is computable as a powers series expansion in $\alpha_S(b_0^2/b^2)$ (see Eq. (18)). This operator does not explicitly depend on the hard scale M^2, and its dependence on the scale b^2 is due to the running coupling $\alpha_S(b_0^2/b^2)$. Therefore, the operator $\mathbf{D}$ effectively resums $\ln(Mb)$ contributions to $\boldsymbol{\Delta}$ by using the renormalization group evolution of $\alpha_S(\mu^2)$ to express $\alpha_S(b_0^2/b^2)$ in terms of $\alpha_S(M^2)$ and $\ln(M^2b^2)$.

An important point about the structure of the soft factor $\boldsymbol{\Delta}$ in Eq. (15) regards its dependence on the rapidity and azimuth kinematical variables of the $Q\bar{Q}$ pair. Both $\boldsymbol{\Gamma}_t$ and $\mathbf{D}$ depend on y_{34} and this produces an ensuing dependence of the operators $\mathbf{V}$ and $\boldsymbol{\Delta}$. The azimuthal dependence is *specific* of transverse-momentum resummation. In particular, we remark that $\boldsymbol{\Gamma}_t$ and, thus, the evolution operator $\mathbf{V}$ do not depend on azimuthal angles. In contrast, the operator $\mathbf{D}$ does depend on ϕ_3 and, more importantly, it depends on $\phi_{3b} = \phi_3 - \phi_b$, where ϕ_b is the azimuth of the two-dimensional impact parameter vector $\mathbf{b}$. Inserting this dependence on ϕ_{3b} in the resummation formula (5) and performing the inverse Fourier transformation from $\mathbf{b}$ space to $\mathbf{q_T}$ space, we obtain an ensuing dependence of the $\mathbf{q_T}$ cross section on $\phi_3 - \phi_q$ (where ϕ_q is the azimuthal angle of $\mathbf{q_T}$). In other words, the resummation formula (5) leads to q_T-dependent *azimuthal correlations* of the produced $Q\bar{Q}$ pair in the small-q_T region. These azimuthal correlations are produced by the dynamics of soft-parton radiation, and they are entirely embodied in the soft-parton factor $\mathbf{D}$ of Eq. (15). The ϕ_{3b} dependence occurs in $\mathbf{D}$, at the characteristic scale $1/b$, and does not occur in the evolution operator $\mathbf{V}$: this fact has a definite physical origin in the distinction between real and virtual radiative contributions. Virtual radiation involves soft partons with transverse momentum k_T in the entire range $k_T \lesssim M$, while real radiation is due to partons with $k_T \lesssim q_T \sim 1/b$. The dynamics of $\mathbf{V}$ is essentially driven by soft *virtual* partons, which cannot produce azimuthal correlations. Real radiation plays a 'minimal' role in $\mathbf{V}$: it simply produces the cancellation of virtual terms (and the ensuing infrared divergences) in the region $k_T \lesssim 1/b$, thus leading to remaining contributions from the region $1/b \lesssim k_T \lesssim M$ (see the limit of integrations over $q \sim k_T$ in Eq. (16)). Azimuthal correlations are instead necessarily produced by *real* radiation, which first occur at scale $k_T \sim q_T \sim 1/b$: these correlations are thus 'trapped' in the soft factor $\mathbf{D}(\alpha_S(b_0^2/b^2))$, at the corresponding scale $1/b$.

As first pointed out in Ref. [12], the structure of transverse-momentum resummation is invariant under a class of renormalization group transformations, named resummation-scheme transformations. This symmetry permits a redefinition of the individual resummation factors in such a way that their total contribution to the q_T cross section is left unchanged. In particular, we can consider a resummation-scheme transformation that changes (redefines) the separate factors $\mathbf{H}$, $\mathbf{V}$ and $\mathbf{D}$ in such a way that $(\mathbf{H}\,\boldsymbol{\Delta})$ (i.e., Eqs. (12) and (13)) is invariant. Such a transformation can introduce an arbitrary ϕ_{3b} dependence of the redefined factors $\mathbf{H}$, $\mathbf{V}$, $\mathbf{D}$. Our key point about the structure of the azimuthal correlations in Eq. (15) is that there are necessarily schemes in which the dependence on ϕ_b is absent from $\mathbf{H}$ and $\mathbf{V}$, and it is entirely embodied in $\mathbf{D}$. This key point eventually follows from our previous discussion on the physical origin of the soft-parton azimuthal correlations. In particular, we can define the factor $\mathbf{D}$ in Eq. (15) in such a way that it gives a trivial contribution after azimuthal average over $\mathbf{b}$. Thus, the soft factor $\mathbf{D}$ can fulfills the property

$$\big\langle \mathbf{D}(\alpha_S; \phi_{3b}, y_{34})\big\rangle_{\text{av.}} = 1, \tag{19}$$

where the symbol $\langle \ldots \rangle_{\text{av.}}$ denotes the azimuthal average over the angle ϕ_b of the impact parameter vector $\mathbf{b}$.

We note that the transverse-momentum resummation formula (5) has an additional source of azimuthal correlations. These additional azimuthal correlations are due to the $\mathbf{b}$ dependence of the function $C_{ga}^{\mu\nu}$ that contributes to Eq. (11). The two sources of azimuthal correlations have a definitely different physical origin. The azimuthal correlations produced by $C_{ga}^{\mu\nu}$ originate from initial-state collinear radiation [13], while those produced by $\mathbf{D}$ originate from soft radiation in the processes, such as $Q\bar{Q}$ production, with final-state colored partons. This difference is manifest in the $q\bar{q}$ annihilation channel, where we find soft-parton azimuthal correlations (produced by $\mathbf{D}$) without accompanying azimuthal correlations of collinear origin (see Eq. (10)).

The gluon collinear function $C_{ga}^{\mu\nu}$ of Eq. (11) has the following all-order form [13]:

$$C_{ga}^{\mu\nu}(z; p_1, p_2, \mathbf{b}; \alpha_S) = d^{\mu\nu}(p_1, p_2) C_{ga}(z; \alpha_S) + D^{\mu\nu}(p_1, p_2; \mathbf{b}) G_{ga}(z; \alpha_S), \tag{20}$$

where $d^{\mu\nu}$ is given in Eq. (14),

$$D^{\mu\nu}(p_1, p_2; \mathbf{b}) = d^{\mu\nu}(p_1, p_2) - 2\frac{b^\mu\, b^\nu}{\mathbf{b^2}}, \tag{21}$$

and $b^\mu = (0, \mathbf{b}, 0)$ is the two-dimensional impact parameter vector in the four-dimensional notation ($b^\mu b_\mu = -\mathbf{b^2}$). The perturbative expansion of C_{ga} ($a = q, \bar{q}, g$) starts at $\mathcal{O}(1)$ ($C_{ga}(z; \alpha_S) = \delta(1-z)\delta_{ga} + \mathcal{O}(\alpha_S)$), analogously to the collinear functions C_{qa} and $C_{\bar{q}\,a}$ in Eq. (10), whereas the expansion of the gluonic function G_{ga} starts at $\mathcal{O}(\alpha_S)$. From Eq. (20) we see that the dependence of $C_{ga}^{\mu\nu}$ on the azimuthal angle ϕ_b of $\mathbf{b}$ is entirely embodied in the Lorentz tensor

$D^{\mu\nu}$ of Eq. (21): therefore, this azimuthal dependence is uniquely specified at arbitrary perturbative orders in α_S. This specific azimuthal dependence is a consequence [13] of the fact that gluonic collinear radiation is intrinsically spin-polarized and its spin-polarization structure is uniquely specified (see, e.g., Eq. (50) in Ref. [13]) by helicity conservation rules. The contribution of the gluon fusion channel is the sole source of azimuthal correlations [43,13] in transverse-momentum resummation for the production of a colorless system F. The azimuthal dependence of $C_{ga}^{\mu\nu}$ produces a definite structure of azimuthal correlations with respect to the azimuthal angle ϕ_q of the transverse momentum $\mathbf{q_T}$. As shown in Ref. [13], the small-$\mathbf{q_T}$ resummed cross section for the production of a colorless system F through gluon fusion leads to azimuthal correlations that are expressed in terms of a linear combination of *only* four Fourier harmonics $(\cos(2\phi_q), \sin(2\phi_q), \cos(4\phi_q), \sin(4\phi_q))$.

In the case of q_T resummation for $Q\bar{Q}$ production, the azimuthal dependence is present in both the $q\bar{q}$ annihilation channel and the gluon fusion channel. In both channels, the ϕ_b dependence of the resummation formula (5) is embodied in the resummation factors at scale b_0^2/b^2, which are (see Eqs. (10), (11) and (15))

$$\mathbf{D} C_{ca_1} C_{\bar{c}a_2} \quad (c = q, \bar{q}), \tag{22}$$

$$\mathbf{D} C_{ga_1}^{\mu_1\nu_1} C_{ga_2}^{\mu_2\nu_2}, \tag{23}$$

where we have omitted the argument of the various factors to shorten the notation. As we have just recalled, the azimuthal dependence of the collinear function $C_{ga}^{\mu\nu}$ is relatively simple and it is uniquely specified to all perturbative orders. In contrast, the ϕ_b dependence of $\mathbf{D}$ is determined by the process-dependent dynamics of soft-parton radiation in $Q\bar{Q}$ production: this dependence is definitely cumbersome already at the first perturbative order (see Eq. (36)), and it receives additional contributions to each subsequent order. Therefore, the ensuing azimuthal correlations of the q_T cross section depend on Fourier harmonics of any degrees. In particular, in the gluon fusion channel (see Eq. (23)), the azimuthal dependence originating from soft-parton radiation is entangled with the azimuthal dependence of collinear origin: the complete azimuthal dependence is determined by a non-trivial interplay of color (soft) and spin (collinear) correlations.

The resummation formula (5) can be straightforwardly averaged over the azimuth ϕ_q of $\mathbf{q_T}$. The resummation formula for the azimuthally-averaged q_T cross section is obtained from Eq. (5) through two simple replacements: the integrand factor $e^{i\mathbf{b}\cdot\mathbf{q_T}}$ is replaced by the 0-th order Bessel function $J_0(bq_T)$ and the factors in Eqs. (22) and (23) are replaced by their azimuthal average over ϕ_b. Performing the azimuthal average over ϕ_b, we have

$$\langle \mathbf{D} C_{ca_1} C_{\bar{c}a_2} \rangle_{\text{av.}} = C_{ca_1} C_{\bar{c}a_2} \quad (c = q, \bar{q}), \tag{24}$$

$$\big\langle \mathbf{D} C_{ga_1}^{\mu_1\nu_1} C_{ga_2}^{\mu_2\nu_2} \big\rangle_{\text{av.}} \neq \big\langle C_{ga_1}^{\mu_1\nu_1} C_{ga_2}^{\mu_2\nu_2} \big\rangle_{\text{av.}}. \tag{25}$$

Owing to the property in Eq. (19), the effect of the soft-parton factor $\mathbf{D}$ disappears from the right-hand side of Eq. (24): therefore, in the $q\bar{q}$ annihilation channel, soft wide-angle radiation contributes to the azimuthally-averaged q_T cross section only through the evolution factor $\mathbf{V}^\dagger\mathbf{V}$ from Eq. (15). Despite the property in Eq. (19), however, in the gluon fusion channel we have the inequality in Eq. (25) (owing to Eq. (19) and the fact that $G_{ga} = \mathcal{O}(\alpha_S)$, the inequality is due to contributions at $\mathcal{O}(\alpha_S^2)$). Therefore, the soft factor $\mathbf{D}$ still gives a non-trivial effect to the azimuthally-averaged q_T cross section through the contribution of the gluon fusion channel. This effect is proportional to the factor $\langle \mathbf{D} C_{ga_1}^{\mu_1\nu_1} C_{ga_2}^{\mu_2\nu_2} \rangle_{\text{av.}}$, which originates from the entangled soft/collinear azimuthal dependence of the q_T resummation formula (5).

The contribution of the hard factor $\mathbf{H}$ to Eqs. (12) and (13) is independent of $\mathbf{b}$, it depends on the hard scale M and it is entirely specified by the hard-virtual amplitude $\widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}}$. The auxiliary amplitude $\widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}}$ is related to the scattering amplitude $\mathcal{M}_{c\bar{c}\to Q\bar{Q}}$ by the following all-order factorization formula:

$$|\widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}}(p_1,p_2;p_3,p_4)\rangle = \left[1-\widetilde{\mathbf{I}}_{c\bar{c}\to Q\bar{Q}}\left(\alpha_S(M^2),\epsilon\right)\right]|\mathcal{M}_{c\bar{c}\to Q\bar{Q}}(p_1,p_2;p_3,p_4)\rangle, \tag{26}$$

where

$$\widetilde{\mathbf{I}}_{c\bar{c}\to Q\bar{Q}}\left(\alpha_S(M^2),\epsilon\right) = \frac{\alpha_S(\mu_R^2)}{2\pi}\widetilde{\mathbf{I}}^{(1)}_{c\bar{c}\to Q\bar{Q}}\left(\epsilon, M^2/\mu_R^2\right) + \sum_{n=2}^{\infty}\left(\frac{\alpha_S(\mu_R^2)}{2\pi}\right)^n \widetilde{\mathbf{I}}^{(n)}_{c\bar{c}\to Q\bar{Q}}\left(\epsilon, M^2/\mu_R^2\right), \tag{27}$$

and μ_R is the renormalization scale. The function $\widetilde{\mathbf{I}}_{c\bar{c}\to Q\bar{Q}}(\alpha_S,\epsilon)$ also depends on the momenta p_i ($i\le 4$), although this dependence is not explicitly denoted in its argument. The structure of Eq. (26) is analogous [14] to that of the hard-virtual amplitudes of transverse-momentum resummation for the production of colorless systems F. The main technical difference regards the color treatment and, thus, the 'subtraction' operator $\widetilde{\mathbf{I}}_{c\bar{c}\to Q\bar{Q}}$ is a color operator acting onto the color vector $|\mathcal{M}_{c\bar{c}\to Q\bar{Q}}\rangle$.

The all-order (virtual) amplitude of the process $c\bar{c}\to Q\bar{Q}$ has ultraviolet (UV) and infrared (IR) divergences. We consider their regularization by analytic continuation in $d=4-2\epsilon$ space–time dimensions, and we use the customary scheme of conventional dimensional regularization (CDR). The quantity $\mathcal{M}_{c\bar{c}\to Q\bar{Q}}(p_1,p_2;p_3,p_4)\equiv\mathcal{M}_{c\bar{c}\to Q\bar{Q}}(\{p_i\})$ in the right-hand side of Eq. (26) is the renormalized on-shell scattering amplitude [44,45], and it has the perturbative expansion

$$\mathcal{M}_{c\bar{c}\to Q\bar{Q}}\left(\{p_i\}\right) = \alpha_S\left(\mu_R^2\right)\mu_R^{2\epsilon}\left[\mathcal{M}^{(0)}_{c\bar{c}\to Q\bar{Q}}\left(\{p_i\}\right)+\sum_{n=1}^{\infty}\left(\frac{\alpha_S(\mu_R^2)}{2\pi}\right)^n\mathcal{M}^{(n)}_{c\bar{c}\to Q\bar{Q}}\left(\{p_i\};\mu_R\right)\right]. \tag{28}$$

The perturbative expansion of $\widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}}$ is completely analogous to that in Eq. (28), with $\widetilde{\mathcal{M}}^{(0)}_{c\bar{c}\to Q\bar{Q}}=\mathcal{M}^{(0)}_{c\bar{c}\to Q\bar{Q}}$ and the replacement $\mathcal{M}^{(n)}_{c\bar{c}\to Q\bar{Q}}\to\widetilde{\mathcal{M}}^{(n)}_{c\bar{c}\to Q\bar{Q}}$ ($n\ge 1$). Using Eq. (26), we can readily obtain $\widetilde{\mathcal{M}}^{(n)}_{c\bar{c}\to Q\bar{Q}}$ as a function of $\mathcal{M}^{(k)}_{c\bar{c}\to Q\bar{Q}}$ and $\widetilde{\mathbf{I}}^{(k)}_{c\bar{c}\to Q\bar{Q}}$ with $k\le n$. For instance, at the NLO level we have

$$\widetilde{\mathcal{M}}^{(1)}_{c\bar{c}\to Q\bar{Q}} = \mathcal{M}^{(1)}_{c\bar{c}\to Q\bar{Q}} - \widetilde{\mathbf{I}}^{(1)}_{c\bar{c}\to Q\bar{Q}}\mathcal{M}^{(0)}_{c\bar{c}\to Q\bar{Q}}. \tag{29}$$

The renormalized virtual amplitude $\mathcal{M}_{c\bar{c}\to Q\bar{Q}}$ still has IR divergences in the form of $1/\epsilon$ poles. The subtraction operator $\widetilde{\mathbf{I}}_{c\bar{c}\to Q\bar{Q}}(\alpha_S,\epsilon)$, which originates from real emission contributions to the q_T cross section, also contains IR divergences. More precisely, it exactly includes the IR divergent terms that are necessary to cancel the IR divergences of the amplitude $\mathcal{M}_{c\bar{c}\to Q\bar{Q}}$, and it includes additional IR finite terms that are specific of the q_T cross section in Eq. (5). Therefore, the hard-virtual amplitude $\widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}}$ can be safely computed in the limit $\epsilon\to 0$. The expressions of $(\mathbf{H}\,\mathbf{\Delta})$ in Eqs. (12) and (13) have to be evaluated by setting $\epsilon=0$ in $\widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}}$, although the four-dimensional limit $\epsilon \to 0$ is not explicitly denoted in the right-hand side of those equations. We note that the *all-order* factors $\mathcal{M}$, $\widetilde{\mathbf{I}}$ and, hence, $\widetilde{\mathcal{M}}$ are renormalization-group invariant quantities (i.e., they are independent of μ_R). Their dependence on μ_R only appears throughout the fixed-order truncation of the perturbative series in powers of $\alpha_S(\mu_R^2)$ (see Eqs. (27), (28) and (29)). We also remark that the operator $\widetilde{\mathbf{I}}_{c\bar{c}\to Q\bar{Q}}$ is completely independent of the spin of the four external hard partons of the process $c\bar{c} \to Q\bar{Q}$. In particular, the gluon Lorentz indices $\{\mu_i', \nu_i'\}$ $(i = 1, 2)$ of $\widetilde{\mathcal{M}}_{gg\to Q\bar{Q}}$ in Eq. (13) are exactly those of the corresponding amplitude $\mathcal{M}_{gg\to Q\bar{Q}}$ in the right-hand side of Eq. (26).

In the region of very small values of q_T, $q_T \lesssim \Lambda$ (Λ is the QCD scale) or, equivalently, at very large values of b ($b\Lambda \gtrsim 1$), the perturbative computation of the q_T cross section has to be supplemented with non-perturbative corrections. Non-perturbative contributions are embodied in transverse-momentum-dependent (TMD) parton densities [11,46,47] that can be used to express the q_T cross section in the small-q_T region through TMD factorization (see Ref. [48] and references therein). In the context of TMD factorization, roughly speaking, the factor $\sqrt{S_c(M,b)}C(\alpha_S(b_0^2/b^2)) \otimes f(b_0^2/b^2)$ (here C denotes the collinear functions in Eqs. (10) and (11), and the symbol '$\otimes$' denotes the convolution with respect to the momentum fraction z) of the resummation formula (5) arises from the TMD parton density [15,49] in the region $b\Lambda \lesssim 1$. In the case of production of a colorless system F, the resummation formula (5) has no other b-dependent factors. In the case of $Q\bar{Q}$ production, the presence in Eq. (5) of one additional b-dependent factor, the soft-parton factor $\mathbf{\Delta}$, is consistent with a breakdown (in weak form) [50] of TMD factorization. In the production processes of strongly interacting systems (such as $Q\bar{Q}$ pairs), TMD parton densities have to be supplemented with additional and process-dependent non-perturbative factors [51]. As we have previously discussed, the breakdown (in strong form) [18] of TMD factorization can have connection with high-order structures in transverse-momentum resummation.

In the framework of TMD factorization, azimuthal correlations in heavy-quark production processes at small q_T have been discussed in Ref. [52]. The azimuthal dependence that is explicitly worked out in Ref. [52] arises from TMD factorization and, therefore, it is consistent with the azimuthal dependence (see Ref. [13] and the discussion below Eq. (21)) driven by the gluon collinear function $C_{ga}^{\mu\nu}$ of the resummation formula (5). The complete structure of azimuthal correlations in Eq. (5) receives additional contributions from the soft-parton factor $\mathbf{\Delta}$ (see Eqs. (22)–(23) and the accompanying discussion). Since $\mathbf{\Delta}$ is related to TMD factorization breaking effects, these (color-charge-dependent) azimuthal correlations cannot originate from process-independent TMD parton densities (see also Section V of the second paper in Ref. [52]).

3. Explicit results for the resummation coefficients

In this section we present our explicit analytic results for the resummed cross section in Eq. (5) up to NLO and NNLL accuracy. To this purpose we can exploit the knowledge of the universal (process-independent) factors S_c, C_{ca} and $C_{ga}^{\mu\nu}$ up to NNLL + NNLO. The Sudakov form factor $S_c(M, b)$ has an all-order representation [11] (see, e.g., Eq. (8) in Ref. [14]) that is fully specified by two perturbative functions $A_c(\alpha_S)$ and $B_c(\alpha_S)$. The corresponding perturbative coefficients $A_c^{(1)}$, $B_c^{(1)}$, $A_c^{(2)}$ [46,47], $B_c^{(2)}$ [53,37] and $A_c^{(3)}$ [15] are explicitly known, and they determine $S_c(M, b)$ up to NNLL accuracy. The partonic collinear functions C_{ca} $(c = q, \bar{q})$ and $C_{ga}^{\mu\nu}$ in Eqs. (10) and (11) are known [54,55,16,17] up to $\mathcal{O}(\alpha_S^2)$ (i.e., NNLO). The two computations in Refs. [16] and [17] are fully independent and they lead to results in full agreement. As we have already recalled, the determination of the individual (separate) factors of the resummation formula (5) requires the specification of a resummation scheme [12]. The collinear functions of Ref. [17], which refer to transverse-momentum resummation according to the formulation of Ref. [15], are eventually related to our functions C_{ca} and $C_{ga}^{\mu\nu}$ [16] throughout a transformation of resummation scheme. In the following, to present our results, we explicitly consider the 'hard scheme' used in Ref. [14]. The expressions of the universal factors S_c, C_{ca} and $C_{ga}^{\mu\nu}$ in the hard scheme can be found in Ref. [14]. The remaining perturbative ingredients of the $Q\bar{Q}$ resummation formula (5) are the hard factor $\mathbf{H}$ (i.e., the subtraction operator $\widetilde{\mathbf{I}}_{c\bar{c}\to Q\bar{Q}}$), the soft evolution factor $\mathbf{V}$ (i.e., the soft anomalous dimension $\mathbf{\Gamma}_t$) and the soft azimuthal-correlation factor $\mathbf{D}$. We have computed $\widetilde{\mathbf{I}}_{c\bar{c}\to Q\bar{Q}}$, $\mathbf{\Gamma}_t$ and $\mathbf{D}$ at $\mathcal{O}(\alpha_S)$, and we have determined $\mathbf{\Gamma}_t$ at $\mathcal{O}(\alpha_S^2)$ by relating it to the $\mathcal{O}(\alpha_S^2)$ computation [56–58] of the IR anomalous dimension of the scattering amplitude $\mathcal{M}_{c\bar{c}\to Q\bar{Q}}$: these results complete the evaluation of the $Q\bar{Q}$ resummation formula (5) up to NNLL + NLO. Using the hard scheme, the results of our computation are presented below (see Eqs. (30), (33), (36) and (40)).

The color operators $\widetilde{\mathbf{I}}_{c\bar{c}\to Q\bar{Q}}$, $\mathbf{\Gamma}_t$ and $\mathbf{D}$ depend on the color charges $(\mathbf{T}_i)^a$ ($a = 1,\dots, N_c^2-1$ is the color index of the radiated gluon) of the four ($i \le 4$) radiating partons $c, \bar{c}, Q, \bar{Q}$. Using the color space formalism of Ref. [40], the color charge $(\mathbf{T}_i)^a$ is a color matrix in either the fundamental (if i is a quark) or adjoint (if i is a gluon) representation of $SU(N_c)$ in QCD with N_c colors. Note that the color flow of the process $c\bar{c} \to Q\bar{Q}$ is treated as 'outgoing', so that $\mathbf{T}_3$ and $\mathbf{T}_4$ are the color charges of $Q(p_3)$ and $\bar{Q}(p_4)$, while $\mathbf{T}_1$ and $\mathbf{T}_2$ are the color charges of the anti-partons $\bar{c}(-p_1)$ and $c(-p_2)$ in Eq. (7). According to this notation, color conservation implies $\sum_{i=1}^4 \mathbf{T}_i|\dots\rangle = 0$, where $|\dots\rangle$ is a color-singlet state vector, such as $|\mathcal{M}_{c\bar{c}\to Q\bar{Q}}\rangle$ or $|\widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}}\rangle$. We also define $\mathbf{T}_i\cdot\mathbf{T}_j \equiv (\mathbf{T}_i)^a(\mathbf{T}_j)^a$ and, in particular, $\mathbf{T}_i^2$ is a c-number term (more precisely, $\mathbf{T}_i^2$ is a multiple of the unit matrix in color space) given by the Casimir factor (C_F or C_A) of the corresponding representation of $SU(N_c)$. We have $\mathbf{T}_1^2 = \mathbf{T}_2^2 = C_F = (N_c^2-1)/(2N_c)$ in the $q\bar{q}$ annihilation channel, $\mathbf{T}_1^2 = \mathbf{T}_2^2 = C_A = N_c$ in the gluon fusion channel, whereas $\mathbf{T}_3^2 = \mathbf{T}_4^2 = C_F$. Considering the kinematics of the process $c\bar{c}\to Q\bar{Q}$ in Eq. (7), four-momentum conservation leads to the relations $y_3 - y = y - y_4 = y_{34}/2$, $\mathbf{p}_{\mathbf{T}3}^2 = \mathbf{p}_{\mathbf{T}4}^2 \equiv \mathbf{p}_{\mathbf{T}}^2$ and the heavy-quark transverse mass $m_T = \sqrt{m^2 + \mathbf{p}_{\mathbf{T}}^2}$ is related to y_{34} by using $M = 2m_T\cosh(y_{34}/2)$. Using these kinematical relations the operators $\widetilde{\mathbf{I}}_{c\bar{c}\to Q\bar{Q}}$, $\mathbf{\Gamma}_t$ and $\mathbf{D}$ can eventually be expressed in term of the two independent variables y_{34} and $2m/M$ (or, equivalently, the relative velocity v in Eq. (4)). As already discussed, $\mathbf{D}$ additionally depends on the relative azimuthal angle ϕ_{3b} (or, equivalently, ϕ_{4b}).

The first-order term $\widetilde{\mathbf{I}}^{(1)}$ of the subtraction operator $\widetilde{\mathbf{I}}_{c\bar{c}\to Q\bar{Q}}$ in Eqs. (26) and (27) has the following form:

$$\widetilde{\mathbf{I}}^{(1)}_{c\bar{c}\to Q\bar{Q}}\left(\epsilon, \frac{M^2}{\mu_R^2}\right) = -\frac{1}{2}\left(\frac{M^2}{\mu_R^2}\right)^{-\epsilon}\left\{\left(\frac{1}{\epsilon^2} + i\pi\frac{1}{\epsilon} - \frac{\pi^2}{12}\right)\left(\mathbf{T}_1^2 + \mathbf{T}_2^2\right) + \frac{2}{\epsilon}\gamma_c - \frac{4}{\epsilon}\mathbf{\Gamma}_t^{(1)}(y_{34}) + \mathbf{F}_t^{(1)}(y_{34})\right\}. \tag{30}$$

The flavor-dependent coefficients γ_c ($c = q, \bar{q}, g$) originate from collinear radiation: the explicit values of these coefficients are $\gamma_q = \gamma_{\bar{q}} = 3C_F/2$ and $\gamma_g = (11C_A - 2N_f)/6$, and N_f is the number of flavors of massless quarks (e.g., $N_f = 5$ in the case of $t\bar{t}$ production). The IR finite contribution $\mathbf{F}_t^{(1)}$ to Eq. (30) is

$$\mathbf{F}_t^{(1)}(y_{34}) = \left(\mathbf{T}_3^2 + \mathbf{T}_4^2\right)\ln\left(\frac{m_T^2}{m^2}\right) + (\mathbf{T}_3 + \mathbf{T}_4)^2\,\mathrm{Li}_2\left(-\frac{\mathbf{p}_{\mathbf{T}}^2}{m^2}\right) + \mathbf{T}_3\cdot\mathbf{T}_4\frac{1}{v}L_{34}, \tag{31}$$

where the function L_{34} is

$$L_{34} = \ln\left(\frac{1+v}{1-v}\right)\ln\left(\frac{m_T^2}{m^2}\right) - 2\,\mathrm{Li}_2\left(\frac{2v}{1+v}\right) - \frac{1}{4}\ln^2\left(\frac{1+v}{1-v}\right)$$
$$+ 2\left[\mathrm{Li}_2\left(1-\sqrt{\frac{1-v}{1+v}}\,e^{y_{34}}\right) + \mathrm{Li}_2\left(1-\sqrt{\frac{1-v}{1+v}}\,e^{-y_{34}}\right) + \frac{1}{2}\,y_{34}^2\right] \tag{32}$$

and Li_2 is the customary dilogarithm function, $\mathrm{Li}_2(z) = -\int_0^z \frac{dt}{t}\ln(1-t)$.

The color operator $\mathbf{\Gamma}_t^{(1)}(y_{34})$ in the right-hand side of Eq. (30) is exactly equal to the first-order term of the soft anomalous dimension in Eq. (17), and its explicit form is

$$\mathbf{\Gamma}_t^{(1)}(y_{34}) = -\frac{1}{4}\Bigg\{\left(\mathbf{T}_3^2+\mathbf{T}_4^2\right)(1-i\pi) + \sum_{\substack{i=1,2\\ j=3,4}} \mathbf{T}_i\cdot\mathbf{T}_j \ln\frac{(2p_i\cdot p_j)^2}{M^2m^2}$$
$$+ 2\mathbf{T}_3\cdot\mathbf{T}_4\left[\frac{1}{2v}\ln\left(\frac{1+v}{1-v}\right) - i\pi\left(\frac{1}{v}+1\right)\right]\Bigg\}. \tag{33}$$

We note that the second term in the right-hand side of Eq. (33) can be rewritten as

$$\sum_{\substack{i=1,2\\ j=3,4}} \mathbf{T}_i\cdot\mathbf{T}_j \ln\frac{(2p_i\cdot p_j)^2}{M^2m^2} = (\mathbf{T}_3+\mathbf{T}_4)^2\ln\left(\frac{m_T^2}{m^2}\right) - (\mathbf{T}_1-\mathbf{T}_2)\cdot(\mathbf{T}_3-\mathbf{T}_4)\,y_{34}, \tag{34}$$

where we have simply used color conservation and kinematical relations.

The expression of $\widetilde{\mathbf{I}}^{(1)}_{c\bar{c}\to Q\bar{Q}}$ in Eq. (30) contains IR divergent terms in the form of double and single poles $1/\epsilon^2$ and $1/\epsilon$. We have explicitly checked that these IR divergent terms are exactly those that control the factorized IR structure [59] of general one-loop scattering amplitudes with massive external partons. This directly proves that the one-loop hard-virtual amplitude $\widetilde{\mathcal{M}}^{(1)}_{c\bar{c}\to Q\bar{Q}}$ in Eq. (29) is IR finite in the limit $\epsilon \to 0$. The right-hand side of Eq. (30) also contains IR finite contributions. As previously discussed (see, e.g., the first paragraph of this section), these IR finite contributions depend on the specification of the resummation scheme. The explicit expression in the right-hand side of Eq. (30) is specific of the hard scheme [14], supplemented with the property in Eq. (19). Since $\widetilde{\mathbf{I}}^{(1)}_{c\bar{c}\to Q\bar{Q}}$ does not depend on $\mathbf{b}$, this scheme choice uniquely determines how IR finite contributions are split between $\widetilde{\mathbf{I}}^{(1)}_{c\bar{c}\to Q\bar{Q}}$ and $\mathbf{D}^{(1)}$.

The soft-parton operator $\mathbf{D}$ in Eq. (18) also depends on the relative azimuthal angle ϕ_{3b} (or, equivalently, ϕ_{4b}). The expression of the first-order term $\mathbf{D}^{(1)}$ is quite involved. To shorten the notation we define the auxiliary variable c_{3b},

$$c_{3b} = \frac{\sqrt{\mathbf{p}_\mathbf{T}^2}}{m}\cos(\phi_{3b}) = -\frac{\sqrt{\mathbf{p}_\mathbf{T}^2}}{m}\cos(\phi_{4b}). \tag{35}$$

We obtain the following result:

$$\mathbf{D}^{(1)}(\phi_{3b}, y_{34}) = \left(\mathbf{T}_3^2+\mathbf{T}_4^2\right)\left[\frac{c_{3b}\,\mathrm{arcsinh}(c_{3b})}{\sqrt{1+c_{3b}{}^2}} - \frac{1}{2}\ln\left(\frac{m_T^2}{m^2}\right)\right]$$

$$
\begin{aligned}
&- (\mathbf{T}_3 + \mathbf{T}_4)^2 \left(\operatorname{arcsinh}^2(c_{3b}) + \frac{1}{2} \mathrm{Li}_2\left(-\frac{\mathbf{p}_\mathbf{T}^2}{m^2} \right) \right) \\
&+ \frac{1}{2v} \mathbf{T}_3 \cdot \mathbf{T}_4 \left(L_{34}^\varphi - L_{34} \right),
\end{aligned} \tag{36}
$$

where L_{34} is given in Eq. (32). The function L_{34}^φ is

$$
L_{34}^\varphi = \mathrm{Sign}(c_{3b}) \left[L_\xi \left(\xi(c_{3b}, \alpha_{34}), \alpha_{34} \right) - L_\xi \left(\xi(-c_{3b}, \alpha_{34}), \alpha_{34} \right) \right] \tag{37}
$$

with

$$
\begin{aligned}
L_\xi(\xi, \alpha) = {}& \frac{1}{2} \ln^2 \frac{\xi(1+\xi)}{\alpha + \xi} - \ln^2 \frac{\xi}{\alpha + \xi} \\
&+ 2 \left[\mathrm{Li}_2(-\xi) - \mathrm{Li}_2\left(\frac{\alpha + \xi}{\alpha - 1} \right) + \ln(\alpha + \xi) \ln(1 - \alpha) \right]
\end{aligned} \tag{38}
$$

and

$$
\xi(c, \alpha) = \left(c + \sqrt{1 + c^2} \right) \left(c + \sqrt{\alpha + c^2} \right), \qquad \alpha_{34} = \frac{2\sqrt{1 - v^2}}{1 - \sqrt{1 - v^2}} c_{3b}^2. \tag{39}
$$

By simple inspection of Eq. (36), we can observe that the azimuthal dependence of $\mathbf{D}^{(1)}$ is quite complex and entangled with the color correlation factor $\mathbf{T}_3 \cdot \mathbf{T}_4$: this is a consequence of its dynamical origin from the specific angular pattern of soft-gluon radiation in $Q\bar{Q}$ production. We note that the expression in Eq. (36) has a vanishing azimuthal average (i.e., $\langle \mathbf{D}^{(1)}(\phi_{3b}, y_{34}) \rangle_{\mathrm{av.}} = 0$) and, therefore, the property in Eq. (19) is fulfilled.

The first-order term $\mathbf{\Gamma}_t^{(1)}$ (see Eq. (33)) of the soft anomalous dimension controls (through Eq. (16)) q_T resummation up to NLL accuracy. The second-order term $\mathbf{\Gamma}_t^{(2)}$ of the soft anomalous dimension in Eq. (17) is also necessary to determine the NNLL contributions. Both $\mathbf{\Gamma}_t^{(1)}$ and $\mathbf{\Gamma}_t^{(2)}$ are related to the IR singularities of the virtual scattering amplitude $\mathcal{M}_{c\bar{c} \to Q\bar{Q}}$, which are explicitly known at one-loop [59] and two-loop [56–58] order: exploiting this knowledge, we have determined $\mathbf{\Gamma}_t^{(2)}$. We obtain the result

$$
\begin{aligned}
\mathbf{\Gamma}_t(\alpha_S; y_{34}) = {}& \frac{1}{2} \mathbf{\Gamma}^{\mathrm{sub.}}_{c\bar{c} \to Q\bar{Q}}(\alpha_S; y_{34}) - \left(\frac{\alpha_S}{\pi} \right)^2 \frac{1}{4} \left(\left[\mathbf{\Gamma}_t^{(1)}(y_{34}), \mathbf{F}_t^{(1)}(y_{34}) \right] + \pi \beta_0 \mathbf{F}_t^{(1)}(y_{34}) \right) \\
&+ \mathcal{O}\left(\alpha_S^3 \right),
\end{aligned} \tag{40}
$$

where $12\pi\beta_0 = 11N_c - 2N_f$, and $\mathbf{F}_t^{(1)}$ and $\mathbf{\Gamma}_t^{(1)}$ are given in Eqs. (31) and (33). The 'subtracted' anomalous dimension $\mathbf{\Gamma}^{\mathrm{sub.}}_{c\bar{c} \to Q\bar{Q}}$ is directly related to the IR anomalous dimension of Ref. [58] (as explained below). The perturbative expansion of the right-hand side of Eq. (40) includes both the first-order and second-order terms $\mathbf{\Gamma}_t^{(1)}$ and $\boldsymbol{\Gamma}_t^{(2)}$ (obviously, $\mathbf{\Gamma}^{\mathrm{sub.}}_{c\bar{c} \to Q\bar{Q}} = 2(\alpha_S/\pi)\mathbf{\Gamma}_t^{(1)} + \mathcal{O}(\alpha_S^2)$), while terms at $\mathcal{O}(\alpha_S^3)$ and beyond are neglected.

As we have previously discussed, the evolution operator $\mathbf{V}$ and, thus, $\mathbf{\Gamma}_t$ are essentially determined by virtual soft-parton radiation through the cancellation mechanism of the IR singularities of the scattering amplitude $\mathcal{M}_{c\bar{c} \to Q\bar{Q}}$. This origin in manifest in Eq. (30), where $\boldsymbol{\Gamma}_t^{(1)}$ enters as coefficient of the single pole $1/\epsilon$. In particular, setting $\boldsymbol{\Gamma}_t^{(1)} = 0$ in Eq. (30), the IR divergences of the subtraction operator $\widetilde{\mathbf{I}}^{(1)}_{c\bar{c} \to Q\bar{Q}}$ would be exactly equal to those of the analogous subtraction operator [14] for the production of a colorless system F. This means that $\mathbf{\Gamma}_t^{(1)}$ controls the IR divergences due to soft wide-angle radiation in the process $c\bar{c} \to Q\bar{Q}$. This origin of $\mathbf{\Gamma}_t$ remains valid at higher perturbative orders, and it leads to the contribution $\mathbf{\Gamma}^{\text{sub.}}_{c\bar{c}\to Q\bar{Q}}$ in Eq. (40). The subtracted anomalous dimension $\mathbf{\Gamma}^{\text{sub.}}_{c\bar{c}\to Q\bar{Q}}$ is given by the following relation:

$$\mathbf{\Gamma}^{\text{sub.}}_{c\bar{c}\to Q\bar{Q}}(\alpha_S; y_{34}) = \mathbf{\Gamma}(\mu) - \left[\frac{1}{2}\big(\mathbf{T}_1^2 + \mathbf{T}_2^2\big)\gamma_{\text{cusp}}(\alpha_S)\left(\ln\frac{M^2}{\mu^2} - i\pi\right) + 2\gamma^c(\alpha_S)\right], \tag{41}$$

where the terms on the right-hand side are written by exactly using the notation of Eq. (5) of Ref. [58]. The term $\mathbf{\Gamma}(\mu)$ is the anomalous-dimension matrix that controls the IR divergences of the scattering amplitude $\mathcal{M}_{c\bar{c}\to Q\bar{Q}}$, while the square-bracket term on the right-hand side of Eq. (41) is the corresponding expression of $\mathbf{\Gamma}(\mu)$ for a generic process $c\bar{c} \to F$ (where the system F is colorless). The square-bracket term is the contribution of soft and collinear radiation from the colliding partons c and $\bar{c}$. In Eq. (41), this contribution is subtracted from $\mathbf{\Gamma}(\mu)$, so that $\mathbf{\Gamma}^{\text{sub.}}_{c\bar{c}\to Q\bar{Q}}$ embodies the remaining IR effects due to soft wide-angle radiation in the process $c\bar{c} \to Q\bar{Q}$. We note that the subtraction in Eq. (41) exactly corresponds to the splitting procedure used in Eq. (57) of Ref. [10] to introduce the anomalous dimension $\boldsymbol{\gamma}^h_{i\bar{i}}$: therefore, we have $\mathbf{\Gamma}^{\text{sub.}}_{c\bar{c}\to Q\bar{Q}} = \boldsymbol{\gamma}^h_{c\bar{c}}$. The expression of $\boldsymbol{\Gamma}(\mu)$ at $\mathcal{O}(\alpha_S^2)$ is computed and explicitly given in Ref. [58]. This expression (which is too long to be reported here) straightforwardly leads to the $\mathcal{O}(\alpha_S^2)$ term of $\mathbf{\Gamma}^{\text{sub.}}_{c\bar{c}\to Q\bar{Q}}$ in Eq. (41) and to the ensuing contribution in Eq. (40). The additional contribution to $\mathbf{\Gamma}_t$ in Eq. (40) is proportional to $\mathbf{F}_t^{(1)}$, and it is due to the corresponding IR finite contribution to $\widetilde{\mathbf{I}}^{(1)}_{c\bar{c}\to Q\bar{Q}}$ in Eq. (30). Both contributions eventually originate from the property in Eq. (19) of the soft-parton factor $\mathbf{D}$.

We note that the first-order term $\mathbf{\Gamma}_t^{(1)}$ of the soft anomalous dimension includes (see Eq. (33)) an absorptive (antihermitian) term of the type $\mathbf{\Gamma}^{(1)}_{(C)} \propto i\mathbf{T}_3 \cdot \mathbf{T}_4$ (it is due to the non-abelian QCD analogue of the QED Coulomb phase) that involves color correlations between two partons. Owing to its antihermitian character, $\mathbf{\Gamma}^{(1)}_{(C)}$ gives a vanishing contribution (see the factors in Eqs. (12), (13) and (15)) to the $Q\bar{Q}$ cross section at the NLO. Nonetheless, $\mathbf{\Gamma}^{(1)}_{(C)}$ does contribute to the singular component of the q_T cross section at higher perturbative orders. A related comment applies to a term, $\mathbf{\Gamma}^{(2)}_{(3)} \propto f^{abc}\mathbf{T}_1^a\,\mathbf{T}_3^b\,\mathbf{T}_4^c$, that contributes to the second-order anomalous dimension $\mathbf{\Gamma}_t^{(2)}$. The triple color correlation term $\mathbf{\Gamma}^{(2)}_{(3)}$ originates from the commutator $[\mathbf{\Gamma}_t^{(1)}, \mathbf{F}_t^{(1)}]$ in the right-hand side of Eq. (40) and from a corresponding term in $\mathbf{\Gamma}^{\text{sub.}}_{c\bar{c}\to Q\bar{Q}}$ (see Eq. (41) and the expression of $\mathbf{\Gamma}(\mu)$ in Eq. (5) of Ref. [58]). In the computation of the $Q\bar{Q}$ cross section (see the factors in Eqs. (12), (13) and (15)), $\mathbf{\Gamma}^{(2)}_{(3)}$ gives a vanishing contribution at the NNLO [25,58,60]. This follows from the fact that the tree-level amplitude $\mathcal{M}^{(0)}_{c\bar{c}\to Q\bar{Q}} = \widetilde{\mathcal{M}}^{(0)}_{c\bar{c}\to Q\bar{Q}}$ is real and, therefore, $\langle\widetilde{\mathcal{M}}^{(0)}_{c\bar{c}\to Q\bar{Q}}|\,\mathbf{\Gamma}^{(2)}_{(3)}|\widetilde{\mathcal{M}}^{(0)}_{c\bar{c}\to Q\bar{Q}}\rangle = 0$ [20,60]. At higher perturbative orders the resummation factors in Eqs. (12), (13) and (15) include additional absorptive terms (e.g., the one-loop amplitude $\widetilde{\mathcal{M}}^{(1)}_{c\bar{c}\to Q\bar{Q}}$ is not purely real) and, therefore, $\mathbf{\Gamma}^{(2)}_{(3)}$ gives non-vanishing contributions to the q_T cross section beyond the NNLO level (see the related discussion in the Note Added of Ref. [19]).

Transverse-momentum resummation for $Q\bar{Q}$ production has been studied in Refs. [9,10]. The framework developed in Refs. [9,10] is an extension of the SCET formulation of q_T resummation that was presented in Ref. [15] for the cases of DY and Higgs boson production. The authors of Refs. [9,10] consider the azimuthally-averaged q_T cross section and present results at the NLO and NNLL accuracy. We have performed a comparison between those results and our results, and we find full agreement. The comparison poses no difficulties since, as we have discussed, we can straightforwardly obtain the azimuthally-averaged q_T cross section by integrating the resummation formula (5). In particular, at NLO and NNLL accuracy, we can simply set $\mathbf{\Delta} = \mathbf{V}^\dagger \mathbf{V}$ (i.e., $\mathbf{D} = 1$) in Eq. (5) (this follows from Eq. (24) and from the fact the inequality in Eq. (25) is due to terms of $\mathcal{O}(\alpha_S^2)$, which start to contribute at the NNLO and beyond NNLL accuracy). We note that the various (hard, soft, collinear) resummation factors in our Eq. (5) and those in Ref. [10] are separately different, since they correspond to the use of different resummation schemes.

As discussed in Section 2, the results presented in this section are obtained by using soft/collinear factorization formulae [34,29,35,30,31,36], and they can be extended to the complete NNLO level through the evaluation of the $\mathcal{O}(\alpha_S^2)$-terms $\mathbf{D}^{(2)}$ and $\widetilde{\mathbf{I}}^{(2)}$ in Eqs. (18) and (27). The extension, which does not require further conceptual steps, is certainly complex from the computational viewpoint.

4. Summary

In this paper we have considered the transverse-momentum distribution of a heavy-quark pair produced in hadronic collisions. As in the case of simpler processes, such as the hadroproduction of a system of non-strongly interacting particles, the perturbative QCD computation of the q_T cross section is affected by large logarithmic terms that need be resummed to all perturbative orders. We have discussed the new issues that arise in the case of heavy-quark production, and we have presented our all-order resummation formula (see Eq. (5)) for the logarithmically-enhanced contributions. The main differences with respect to the production of colorless systems is the appearance of the soft factor $\mathbf{\Delta}$ (see Eq. (15)) that is due to soft-parton radiation at large angles with respect to the direction of the colliding hadrons (partons). The factor $\mathbf{\Delta}$ embodies the effect of soft radiation from the heavy-quark final state and from initial-state and final-state interferences. The dynamics of soft-parton radiation produces color-dependent azimuthal correlations in the small-q_T region. This azimuthal dependence is fully taken into account by the resummation formula and it is embodied in the soft-parton factor $\mathbf{\Delta}$: the dependence is controlled by the color operator $\mathbf{D}$ and it is factorized with respect to the color (soft) evolution factor $\mathbf{V}$ (see Eq. (15)). We have shown how the azimuthal correlations of soft-parton origin are entangled with the azimuthal dependence due to gluonic collinear radiation (see Eq. (23)), and we have discussed the ensuing effect on the azimuthally-averaged q_T cross section. We have presented the explicit results of the perturbative coefficients of the resummation formula up to NLO and NNLL accuracy (see Eqs. (30), (33), (36) and (40)).

Transverse-momentum resummation for heavy-quark production is important for phenomenological applications through resummed calculations [10], especially for the production of top-quark pairs. Given the huge amount of top-quark pairs that have been produced at the LHC in its first run, and the even higher number of $t\bar{t}$ events that are expected at $\sqrt{s} = 13$ (14) TeV, the possibility of relying on accurate computations of the transverse-momentum spectrum of the $t\bar{t}$ pair down to the low-q_T region is very relevant for physics studies within and beyond the SM.

We point out that the q_T resummation formalism for $Q\bar{Q}$ production has implications not only for resummed calculations but also for fixed-order computations up to NNLO. The q_T subtraction formalism [54] is an efficient method to perform fully-exclusive NNLO computations of hard-scattering processes, and it is based on the knowledge of the small-q_T limit of the transverse-momentum cross section of the corresponding process. In the case of the production of colorless systems, thanks to the complete understanding of the all-order structure of the large logarithmic terms, the method is fully developed up to NNLO. The resummation formula presented in Eq. (5) makes possible to apply the q_T subtraction formalism also to heavy-quark production at NNLO, once the explicit results of the resummation factors at the corresponding order will be available.

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