A PVDF sensor for the in-situ measurement of stress intensity factors during fatigue crack growth

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Abstract

Several analytical and numerical studies of inverse analysis are performed to verify the feasibility and accuracy of the proposed K-sensor. At first, the application to cracks in sheets under in-plane stresses is investigated and compared with the analytical solution for the Griffith's crack under mixed mode. It was found that the convergence radius, where the electrodes have to be placed, must be smaller than half of the crack length, which is sufficient for real cracks of several millimeters. The obtained accuracy of crack tip location and \((K_1,k_2)-factors\) is better than 1%. Second, the technique is applied to cracks in thin-walled plates of Kirchhoff type under bending and torsion moments. In this case, the plate intensity factors \((k_3,k_4)\) are of interest. Again, the inverse identification procedure is studied by synthetic analytical and numerical solutions of simple crack configurations. Due to the assumptions of the Kirchhoff plate model, the sensors have to be placed outside a radius of 3 times plate thickness \(h\). The obtained accuracy in position and intensity factors is quite sufficient as well. The practical realization of the \(K\)-factor sensor requires good electric signal measurement and amplification. Its experimental testing on components is ongoing work.

Keywords: \(K\)-factor sensor; PVDF; fatigue crack growth; structural health monitoring;

1. Introduction

The duration and reliability of engineering structures under deterministic or stochastic alternating loads is often limited by subcritical fatigue crack growth. To ensure the guaranteed minimum service life of machines and plants the growth rates of observed or expected fatigue cracks must be known. Since mostly stochastic load collectives and complex geometries are encountered, predictions based on pure calculation are problematic. In particular, civil engi-
engineering and aircraft structures as well as wind energy plants are exposed to complex load collectives. Computational loading analyses are inexact due to uncertainties in the load assumptions. Therefore, expensive inspections or monitoring of highly loaded structural components with respect to cracks are regularly required.

For this purpose, a new type of crack sensor has been developed, which is particularly suitable for monitoring cracks in plate and shell structures under conditions of linear elastic fracture mechanics. Here, the fatigue crack growth occurs stress controlled, so that the crack paths usually exhibit large radii of curvature and the mode I crack opening mechanism dominates. This article gives a brief overview on the sensor concept and demonstrates the feasibility and potential of the developed sensor. The authors are looking for potential partners who are willing to bring the developed sensor concept in application. In particular, the use in the field of aerospace is sought.

2. Measuring principle of the PVDF sensor

Let’s consider an arbitrarily loaded cracked plate of isotropic material. The task is to determine the stress intensity factors \( (K_I, K_{II}) \) as well as crack tip position. To this end a piezoelectric film made of PVDF with measuring electrodes is attached to the surface of the cracked plate. The deformation of the underlying structure is completely transmitted to the film (Fig. 1a). The films are available in any dimensions and thicknesses of 9-110 \( \mu \)m, including e.g. copper or gold electrodes. The measured electrical signals depend on the position of the measuring points in relation to the crack position and the load situation. From the measured signals, the \( K \)-factors and the crack tip location can be determined by solving the inverse boundary value problem. To ensure its solvability more electrodes have be taken into account than unknown variables.

![Figure 1. (a) PVDF film on cracked structure, (b) polarization of PVDF film with electrodes, (c) structure of PVDF \( \beta \) in modification.](image)

Polyvinylidene fluoride (PVDF) is efficiently applicable as actuator or sensor. The chemical basic molecule \(-\text{(CH}_2\text{CF}_2)\text{n}-\) exhibits a strong electronegativity of the fluorine compared to the carbon [1]. Thereby, the CF\(_2\) dipoles are aligned perpendicularly to the molecular chain axis (Fig. 1c), attaining a maximum of polarization in the \( \beta \)-modification of PVDF. Macroscopic piezoelectric properties of the polymeric material are obtained by the polarization process, where in most cases a mechanical stretching superimposes an electric field in the thickness direction. The piezoelectric and dielectric properties of PVDF are comparable to those of ceramic piezoelectrics, the mechanical properties are much different, though. Due to the anisotropy in the 1-2-plane caused by production the material is orthotropic. Concerning mechanical properties, the polymeric PVDF material has significant advantages compared to piezoelectric ceramics. The small elastic modulus and the extremely low mass of the sensor material open up new application fields. In contrast to brittle piezoelectric ceramics, PVDF is mechanically and chemically compatible with polymer to be integrated into fibre reinforced laminates. Due to simple adhesive attachment and the high flexibility of the layer (even across the crack gap) an application on any curved surface is feasible. The temperature range for an application of PVDF films as actuators or sensors is between -70°C and +90°C. However, PVDF exhibits strong pyroelectric properties that must not be neglected. The material constants of PVDF can be found e. g. in [2]. Special design of the coating of the polymer layer allows for the construction of complex sensor arrays accomplishing a high spatial resolution.

The piezoelectric material behaviour is described in Voigt matrix notation by:

\[
\sigma = c^e \varepsilon - e^E , \quad D = e\varepsilon + x^e E ,
\]

where \( \sigma \) stresses, \( \varepsilon \) strains, \( E \) electric field, \( D \) electric displacement field, \( c^e \) elastic stiffness constants, \( e \) piezoelectric constants and \( x^e \) dielectric constants. The polarization is in the thickness direction \( x_3 \) of the film (Fig. 1b).
Depending on which measurement method was chosen (charge measurement \( E_3 = 0 \), potential measurement \( D_3 = 0 \) and under assumption of a plane stress state \( \sigma_{33} = 0 \) for a very thin film, one obtains the electrical voltage at the electrode for charge measurement [3,4]:

\[
U = -\frac{A}{C_f} (C_{11} e_{11} + C_{22} e_{22}), \quad C_{11} = (\frac{E_3}{\varepsilon_{33}} e_{33} - c_{13}^E e_{33})/\varepsilon_{33}, \quad C_{22} = (\frac{E_3}{\varepsilon_{33}} e_{33} - c_{13}^E e_{33})/\varepsilon_{33},
\]

with the parameters \( A \) electrode surface, \( C_f \) capacitance of the capacitor (charge amplifier). For potential measurement circuit [5,6] holds:

\[
U = -k_u h (C_{11} e_{11} + C_{22} e_{22}), \quad C_{11} = \frac{c_{11}^E}{e_{33}^2 + \kappa_{33}^E c_{33}^E}, \quad C_{22} = \frac{c_{22}^E}{e_{33}^2 + \kappa_{33}^E c_{33}^E},
\]

where \( k_u \) is signal amplification factor, \( h \) thickness of the PVDF film.

3. Assessment of the crack tip stress state

Due to the adhesive bonding of the sensor film to the structure it can be ideally assumed that the strains are transmitted from the surface of the specimen to the film without sliding. The unknown location of the crack tip in relation to the film needs introducing an auxiliary local coordinate system, while a global coordinate system is on the crack tip (Fig. 2). The transformation of the strains and coordinates between the two systems is required.

Using the charge measuring method (see (2)) and assuming a plane stress state \( (\sigma_{11}, \sigma_{22}, \sigma_{12}) \) at the surface of the specimen, one obtains by Hooke’s law the measured voltage at point \( i \):

\[
U_i = C_0 (\sigma_{11} f_i + \sigma_{22} f_{ii} + \sigma_{12} f_{i1i}),
\]

with

\[
f_1 = \cos^2 \beta (C_{11} - \nu C_{22}) + \sin^2 \beta (C_{22} - \nu C_{11}), \quad f_{ii} = \sin^2 \beta (C_{11} - \nu C_{22}) + \cos^2 \beta (C_{22} - \nu C_{11}),
\]

\[
f_{i1i} = (1 + \nu) (C_{11} - C_{22}) \sin(2\beta), \quad C_0 = -A/E C_f.
\]

To correlate the measured electrical signals with the stress intensity factors, the stresses \( (\sigma_{11}, \sigma_{22}, \sigma_{12}) \) in the crack tip region have to be expressed as eigenfunctions, where the coefficients of first term are coupled with stress intensity factors. The shape of eigenfunctions depends on the chosen theory (in-plane stress theory [7], bending and twisting of KIRCHHOFF plate [8], bending of MINDLIN plate, ...). For example Williams derived the eigenfunctions for in-plane stresses in a cracked plate, which can be written in following form [9]:

\[
\sigma_{11}(r, \varphi) = \sum_{n=1}^{\infty} \frac{r^{-1}}{n} \left\{ a_n M_{11}^{(n)} + b_n N_{11}^{(n)} \right\}, \quad \sigma_{22}(r, \varphi) = \sum_{n=1}^{\infty} \frac{r^{-1}}{n} \left\{ a_n M_{22}^{(n)} + b_n N_{22}^{(n)} \right\},
\]

\[
\sigma_{12}(r, \varphi) = \sum_{n=1}^{\infty} \frac{r^{-1}}{n} \left\{ a_n M_{12}^{(n)} + b_n N_{12}^{(n)} \right\},
\]

with

\[
M_{11}^{(n)} = \frac{n}{2} \left\{ \left[ 2 + (-1)^n + \frac{n}{2} \right] \cos \left( \frac{n}{2} - 1 \right) \varphi - \left( \frac{n}{2} - 1 \right) \cos \left( \frac{n}{2} - 3 \right) \varphi \right\},
\]

\[
N_{11}^{(n)} = \frac{n}{2} \left\{ \left[ -2 + (-1)^n - \frac{n}{2} \right] \sin \left( \frac{n}{2} - 1 \right) \varphi + \left( \frac{n}{2} - 1 \right) \sin \left( \frac{n}{2} - 3 \right) \varphi \right\},
\]

\[
M_{22}^{(n)} = \frac{n}{2} \left\{ \left[ 2 - (-1)^n - \frac{n}{2} \right] \cos \left( \frac{n}{2} - 1 \right) \varphi + \left( \frac{n}{2} - 1 \right) \cos \left( \frac{n}{2} - 3 \right) \varphi \right\},
\]

\[
N_{22}^{(n)} = \frac{n}{2} \left\{ \left[ -2 - (-1)^n + \frac{n}{2} \right] \sin \left( \frac{n}{2} - 1 \right) \varphi - \left( \frac{n}{2} - 1 \right) \sin \left( \frac{n}{2} - 3 \right) \varphi \right\},
\]

\[
M_{12}^{(n)} = \frac{n}{2} \left\{ \left( \frac{n}{2} - 1 \right) \sin \left( \frac{n}{2} - 3 \right) \varphi - \left( \frac{n}{2} + (-1)^n \right) \sin \left( \frac{n}{2} - 1 \right) \varphi \right\},
\]

\[
N_{12}^{(n)} = \frac{n}{2} \left\{ \left( \frac{n}{2} - 1 \right) \cos \left( \frac{n}{2} - 3 \right) \varphi - \left( \frac{n}{2} - (-1)^n \right) \cos \left( \frac{n}{2} - 1 \right) \varphi \right\},
\]

where \( a_n \) and \( b_n \) are coefficients of the \( n \)-th eigenfunction of the crack field solution. Their values for \( n = 1,2 \) correspond to:

\[
K_i = a_1 \sqrt{2 \pi}, \quad K_{ii} = b_1 \sqrt{2 \pi}, \quad T = 4a_2.
\]
As will be explained later, the consideration of higher order terms from (5) is required for placing electrodes in greater distances to the crack tip.

The Eq. (4) with (5) gives the relationship between the measured voltages at the position \((r, \varphi)\) and the load situation at the crack tip. If the loading situation at the crack tip and crack tip position in respect to the sensor position are to be determined based on these measured signals the solution of an inverse boundary value problem is required. The measuring points have to be sufficiently close to the crack tip in order not to leave the range of validity of series expansion for finite \(n\). The range of convergence of WILLIAMS series can be demonstrated for different \(n\) on an example of the Griffith crack (i) [3] or a crack in infinite bended plate (ii) [10]. The areas in which the error is not greater than 1% due to the finite numbers of the WILLIAMS series in relation to the analytical solution are shown in Fig. 3a for case (i) and Fig. 3b for (ii). For example in the area \(R = a/2\) around the crack tip the approximation with 7 terms satisfied sufficiently well the 1% error criterion. Furthermore is characteristic for (i) and (ii) that the even terms of the series with \(n > 2\) are null (e.g. \(a_4 = b_4 = a_6 = b_6 = \cdots = 0\)).
4. Concept testing on GRIFFITH crack

The measurement method is tested on an example of the GRIFFITH crack. We assume an infinite plate made of aluminium alloy AW 7075 with \( v = 0.33 \) and \( E = 72 \, \text{GPa} \) with a crack of length \( 2a = 40 \, \text{mm} \) as depicted in Fig. 4. The PVDF film is applied to the surface of the plate and provides the electrical signals for the evaluation \( (G_f = 0.1 \, \text{pF}) \). In this example the measuring electrodes are placed inside the region of \( R = a/2 \). Hence from Fig. 3a it is seen that only 7 terms of WILLIAMS series have to be taking into account. The number of measuring points is specified to 16 with electrode area \( A = 0.25 \, \text{mm}^2 \). Instead of actual sensor signals, the electrical voltages emanating at the electrodes are artificially generated from analytical solution of the GRIFFITH crack, and served as input values for the testing (see Fig. 4).

The solution of the non-linear equation system (see Eq. (4), (5)) can be carried out e.g. with LEVENBERG-MARQUARDT algorithm or principal axis algorithm. The GRIFFITH crack is under mixed mode load \( (\sigma = 80 \, \text{MPa}, \tau = 80 \, \text{MPa}) \). The PVDF film is inclined located in respect to the crack coordinate system with \( x_0 = -4 \, \text{mm}, y_0 = -6 \, \text{mm} \) and \( \beta = -10^\circ \). Some abbreviations are introduced: RS – reference solution, IV – initial values, solution of LEVENBERG-MARQUARDT algorithm (LM) and principal axis algorithm (PA). The solution of the inverse problem is presented in Table 1. From Table 1 it can be seen that the film location with respect to crack position and both stress intensity factors could be calculated very well. An essential precondition for determining correct solution was the choice of suitable initial values (IV), which is not always possible.

<table>
<thead>
<tr>
<th>( x_0 ) (mm)</th>
<th>RF</th>
<th>IV</th>
<th>LM</th>
<th>IV</th>
<th>PA</th>
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<td>-4.00</td>
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<td>-3.99</td>
<td>-1.00</td>
<td>-3.99</td>
<td></td>
</tr>
<tr>
<td>( y_0 ) (mm)</td>
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<td>-5.00</td>
<td>-6.00</td>
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<td>-6.00</td>
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<tr>
<td>( \beta ) [(^\circ)]</td>
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<td>-6.00</td>
<td>-9.90</td>
<td>-1.00</td>
<td>-9.90</td>
</tr>
<tr>
<td>( K_I ) [MPa](\sqrt{\text{mm}})</td>
<td>634.13</td>
<td>400.00</td>
<td>635.78</td>
<td>400.00</td>
<td>635.47</td>
</tr>
<tr>
<td>( K_{II} ) [MPa](\sqrt{\text{mm}})</td>
<td>634.13</td>
<td>400.00</td>
<td>631.40</td>
<td>400.00</td>
<td>631.39</td>
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<tr>
<td>( T ) [MPa]</td>
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<td>-50.00</td>
<td>-80.64</td>
<td>-50.00</td>
<td>-80.56</td>
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<tr>
<td>( a_3 ) [MPa]</td>
<td>3.16</td>
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<td>3.16</td>
<td>1.00</td>
<td>3.16</td>
</tr>
<tr>
<td>( b_3 ) [MPa]</td>
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<td>-3.09</td>
<td>-1.00</td>
<td>-3.09</td>
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<tr>
<td>( a_{5} \times 10^{-2} ) [MPa]</td>
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<td>-1.00</td>
<td>-1.91</td>
<td>-1.00</td>
<td>-1.90</td>
</tr>
<tr>
<td>( b_{5} \times 10^{-2} ) [MPa]</td>
<td>1.97</td>
<td>1.00</td>
<td>1.74</td>
<td>1.00</td>
<td>1.75</td>
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<tr>
<td>( a_{7} \times 10^{-4} ) [MPa]</td>
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<td>1.00</td>
<td>1.83</td>
<td>1.00</td>
<td>1.84</td>
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<tr>
<td>( b_{7} \times 10^{-4} ) [MPa]</td>
<td>-2.47</td>
<td>-1.00</td>
<td>-1.39</td>
<td>-1.00</td>
<td>-1.40</td>
</tr>
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</table>

Measured electrical voltages in [mV]

<table>
<thead>
<tr>
<th>( \sigma = 80 , \text{MPa}, \tau = 80 , \text{MPa} )</th>
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<tr>
<td>103.30</td>
</tr>
<tr>
<td>241.01</td>
</tr>
<tr>
<td>-152.45</td>
</tr>
<tr>
<td>-34.69</td>
</tr>
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</table>

Figure 4. Crack in infinite plate with 16 electrodes.
5. Summary and outlook

The strains on the surface of a cracked thin-walled structure, resulting from a mechanical loading, cause deformations at the attached piezoelectric PVDF film. Thus, on its surface, charges are generated which are measured at discrete locations using an array of electrodes. The presented approach allows a simultaneous calculation of stress intensity factors and crack tip position. Regarding the arrangement of the measuring electrodes, the convergence radius of the WILLIAMS series expansion was determined. The measuring electrodes must be placed inside the convergence radius. Because of the limited number of practical series terms, the radius should be chosen in general even smaller. Test calculations on the Griffith crack verified the general suitability of the measurement method. In this case, the solution of the inverse problem provides very good results. An accuracy of 1% is achieved concerning the crack position and the value of stress intensity factors. For an application of the method to cracks in real components and structures appropriate PVDF sensors could be manufactured.

At present experiments are performed on a tensile specimen made of aluminum alloy AW 7075 with an edge crack and an attached sensor with 49 electrodes (Fig. 5). The most challenging problem is the performance and reliability of the charge amplifier device.

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References