# CMS ridge effect at LHC as a manifestation of bremsstrahlung of gluons off quarks accelerated in a strong color field 

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#### Abstract

The recently reported effect of long-range nearside angular correlations at LHC occurs for large multiplicities of particles with $1 \mathrm{GeV}<p_{T}<3 \mathrm{GeV}$. To understand the effect several possibilities have been discussed. In the letter we propose a simple qualitative mechanism which corresponds to gluon bremsstrahlung of quarks moving with acceleration defined by the string tension. The smallness of azimuthal angle difference $\Delta \phi$ along with large $\Delta \eta$ at large multiplicities in this interval of $p_{T}$ are natural in this mechanism. The mechanism predicts also bremsstrahlung photons with mean values of $p_{T} \approx 2.9$ and 0.72 GeV .


In paper [1] the effect in proton-proton collisions at LHC is reported for existence of a ridge in the plot of data for two-particle correlations versus pseudo-rapidity difference $\Delta \eta$ and azimuthal angle $\Delta \phi$ plane. This ridge means essential excess of events with $\Delta \phi$ close to zero and large $\Delta \eta$. It is important to emphasize that the effect is observed under condition that the accompanying charged particles have multiplicity $>100$ and are each situated in restricted region of transverse momentum $1 \mathrm{GeV}<p_{T}<3 \mathrm{GeV}$.

The result already causes discussion devoted to possible interpretation of the data [2-14]. In [2] the possibilities of interpretation of the effect in terms of quark-antiquark strings are discussed. The first point is that a string is formed being stretched close to the direction of $p p$ collision and thus might decay with $\Delta \phi \approx 0$ and $\approx \pi$. However, it is emphasized in [2] that there is no reason, why the effect is observed only for very high multiplicity $N>100$, while multiple production of hadrons via string decomposition leads to randomization and thus simulations show no ridge.

In the present letter we would try to explain why even for high multiplicity the correlation persists when we take into

[^0]account radiation of gluons in the process of QCD quark-anti-quark string formation in proton-proton collision at very high energies. As a matter of fact we mean that in the course of collision a quark is moving with acceleration in the strong color field which in particular may be described in terms of formation of a string between either quark from the first proton and anti-quark from the second one or vice versa. However we do not use some particular string model (see e.g. [15]). We qualitatively assume a formation of some extended object of cigar-like shape inside which quark and anti-quark interact with each other due to strong color field which accelerate both of them. We shall use simple qualitative considerations based on the classical theory of radiation which is described e.g. in book [16] and then we perform quasi-classical estimates.

Each quark (anti-quark) moves considerable time in a very strong color field which we describe in terms of the string tension $A$. The direction of the acceleration is evidently close to the direction of momenta of the colliding protons. In this case an acceleration has almost the same direction as the velocity and thus we may use the well-known classical expression for dipole electromagnetic radiation of electric charge $e$ moving with acceleration being parallel to velocity of the motion [16]
$\frac{d E}{d t}=\frac{2 \alpha w^{2}}{3}$,
where $w$ is an acceleration and $\alpha$ is the fine structure constant. We take initial expression (1) in an accompanying reference frame. For strongly interacting quarks we exchange (1) for the relation
$\frac{d E}{d t}=\frac{\alpha_{s}}{9}\left(\frac{A^{2}}{m}\right)^{2}$,
where acceleration $w=A^{2} / m$ with $A$ and $m$ being the string tension and a light quark mass. We also change $\alpha \rightarrow \alpha_{s}$ with the evident color factor. In view to make estimates we
take the following values for fundamental quantities entering (2)
$m_{u}=2.5 \mathrm{MeV} ; \quad m_{d}=5 \mathrm{MeV} ; \quad A=420 \mathrm{MeV} ;$
here the light quark masses are chosen to be in the middle of interval of their possible values: $1.7 \mathrm{MeV}<m_{u}<3.3 \mathrm{MeV}$; 4.1 $\mathrm{MeV}<m_{d}<5.8 \mathrm{MeV}$ [17].

So quarks are moving with acceleration and thus radiate gluons. Let us obtain a simple quasi-classical estimate of the mean energy of radiated gluons in an accompanying reference frame. In view of this we rewrite expression (2) in the form
$\frac{\Delta E}{\Delta t}=\frac{\alpha_{s}}{9}\left(\frac{A^{2}}{m}\right)^{2}$.
Now to obtain the quasi-classical estimates we use the well-known uncertainty relation
$\Delta E \Delta t=1$.
Finally we have for the mean energy of a gluon
$\Delta E=\sqrt{\frac{\alpha_{s}}{9}} \frac{A^{2}}{m}$.
Then we use the standard one loop expression for $\alpha_{s}$ at scale $\Delta E$
$\alpha_{S}(\Delta E)=\frac{12 \pi}{\left(33-2 N_{f}\right) \ln \left(\frac{\Delta E^{2}}{\Lambda_{\mathrm{QCD}}^{2}}\right)}$.
We have for the one loop expression (7) with $N_{f}=4$, $\Lambda_{\mathrm{QCD}} \approx 190 \mathrm{MeV} .{ }^{1}$ Then with this result the solution of relations (6), (7) under conditions (3) gives us the following estimates for radiation off quarks $u$ and $d$ :
$\Delta E_{u} \approx 11.2 \mathrm{GeV} ; \quad \Delta E_{d} \approx 5.6 \mathrm{GeV}$.
The result (8) gives an estimate for mean energies of the bremsstrahlung gluons in an accompanying reference frame.

First of all let us consider an explanation of large differences in pseudo-rapidity $\Delta \eta$ along with small differences in azimuthal angle $\Delta \phi$. Here we are to take into account both quarks constituting the extended object (a cigar). Namely let "the cigar" be produced with some overall momentum $k$ while its position remains being (almost) parallel to the line of $p p$ collision. Such situation is presented in Fig. 1. Then velocities of quarks are not parallel to the direction of

[^1]

Fig. 1 The string moving with momentum $k$ from the point of collision of two protons, $\psi_{1}, \psi_{2}$ are angles in (9) and $q_{1}, q_{2}$ are momenta of the quarks
acceleration, but constitute some angles $\psi_{1}, \psi_{2}$ with this direction in laboratory reference frame. When a velocity and an acceleration are not parallel $\mathbf{v w}=v w \cos \psi$ and there are two accelerated quarks we have the following angular distribution [16]:
$\frac{d E}{d t^{\prime}}=\frac{\alpha_{s}}{24 \pi}\left(\frac{A^{2}}{m}\right)^{2}$

$$
\times\left(\Phi\left(\psi_{1}, \theta, \phi, v_{1}\right)+\Phi\left(\psi_{2}, \theta, \phi, v_{2}\right)\right) d \Omega
$$

$\Phi(\psi, \theta, \phi, v)=\frac{X+v^{2} Y}{Z^{5}}$
$X=\sin ^{2} \theta-2 v \sin \psi \sin \theta \cos \phi$
$Y=\cos ^{2} \theta \sin ^{2} \psi+\sin ^{2} \theta \sin ^{2} \psi \cos ^{2} \phi$
$Z=1-v(\cos \psi \cos \theta+\sin \psi \sin \theta \cos \phi) ;$
where $t^{\prime}$ is a time with account of a retardation [16], $\psi_{1}, \psi_{2}$ are, respectively, angles for the first and the second quark. Small $\Delta \phi$ along with the wide spread in pseudo-rapidity in the effect [1] is connected with the same sign of $\sin \psi$, because quarks are directed to one side from the line of collision (see Fig. 1). Then the distribution in polar angle $\theta$ has two maxima divided by some significant interval $\Delta \theta$, while distribution in azimuthal angle $\phi$ is again close to zero. After integration of (9) by $\phi$ and $\theta$ correspondingly we have these distributions. The situation is illustrated in Figs. 2 and 3, in which we present normalized distribution in rapidity $\eta$ and normalized angular distribution in $\phi$ in $\phi$ for $\psi_{1}=0.1, \psi_{2}=\pi-0.1, v_{1}=v_{2}=$ 0.999 .
$\frac{d E(\eta)}{d t^{\prime}}=\frac{\alpha_{s}}{24 \pi}\left(\frac{A^{2}}{m}\right)^{2} \Phi(\eta) \frac{d \eta}{\cosh ^{2} \eta} ;$
$\Phi(\eta)=\int_{-\pi}^{\pi} \Phi_{12}\left(\psi_{i}, v_{i}, \theta, \phi\right)_{\cos \theta=f(\eta)} d \phi ;$
$f(\eta)=\frac{\sinh \eta}{\cosh \eta} ;$
$\frac{d E(\phi)}{d t^{\prime}}=\frac{\alpha_{s}}{24 \pi}\left(\frac{A^{2}}{m}\right)^{2} \Phi(\phi) d \phi ;$


Fig. 2 Behavior of $\Phi_{N}(\eta), v=0.999, \psi_{1}=0.1, \psi_{2}=\pi-0.1$, for $-5<\eta<5$


Fig. 3 Behavior of $\Phi_{N}(\phi), v=0.999, \psi_{1}=0.1, \psi_{2}=\pi-0.1$, for $-\pi<\phi<\pi$
$\Phi(\phi)=\int_{0}^{\pi} \Phi_{12}\left(\psi_{i}, v_{i}, \theta, \phi\right) \sin \theta d \theta ;$

$$
\begin{align*}
\Phi_{12}\left(\psi_{i}, v_{i}, \theta, \phi\right)= & \Phi\left(\psi_{1}, \theta, \phi, v_{1}\right) \\
& +\Phi\left(\psi_{2}, \theta, \phi, v_{2}\right) \tag{11}
\end{align*}
$$

From Figs. 2, 3 we see that $\Delta \eta$ may be quite significant while $\Delta \phi$ is small. One should note that the peaks in Figs. 2 and 3 become narrower with increasing of speed and with increasing of $\psi$. We emphasize that the effect of a peak around $\phi=0$ is connected with transverse movement of "the cigar" as is illustrated in Fig. 1. The more is transverse momentum $k$, i.e. angles $\psi_{i}$, the narrower becomes the distribution in $\phi$. Provided transverse momentum $k$ being very small the distribution in $\phi$ becomes practically isotropic.

Now let us consider properties of gluon radiation of a single quark. For this purpose we approximately assume the same direction of the velocity and of the acceleration in the
actual reference frame of LHC. Angular distribution is described by the following expression [16]:

$$
\begin{align*}
& \begin{aligned}
\frac{d E}{d t^{\prime}} & =\frac{\alpha_{s}}{24 \pi}\left(\frac{A^{2}}{m}\right)^{2} \frac{\sin ^{2} \theta^{\prime}}{\left(1-v \cos \theta^{\prime}\right)^{5}} d \Omega \\
& =\frac{\alpha_{s}}{24 \pi}\left(\frac{A^{2}}{m}\right)^{2} \Phi_{0}\left(\theta^{\prime}\right) d \Omega
\end{aligned} \\
& \Phi_{0}\left(\theta^{\prime}\right)=\Phi\left(0, \theta^{\prime}, \phi, v\right) \tag{12}
\end{align*}
$$

where $v$ is a velocity of a quark, $\theta^{\prime}$ is a polar angle and $d \Omega=\sin \theta^{\prime} d \theta^{\prime} d \phi$. Using the angular distribution of the radiation (12) we estimate the mean $p_{T}$ of the radiated gluon as
$\left\langle p_{T}^{g}\right\rangle=\frac{\Delta E}{\sqrt{1-v^{2}}} \frac{I_{1}}{I_{2}} ; \quad I_{2}=\int \Phi_{0}\left(\theta^{\prime}\right) d \Omega ;$
$I_{1}=\int \Phi_{0}\left(\theta^{\prime}\right) A\left(v, \theta^{\prime}\right) \sin \theta^{\prime} d \Omega ;$
$A\left(v, \theta^{\prime}\right)=1+\frac{\cos \theta^{\prime}\left(1-v^{2}\right)-v \sin ^{2} \theta^{\prime}}{1-v^{2} \cos ^{2} \theta^{\prime}} ;$
where $\Phi_{0}\left(\theta^{\prime}\right)$ is defined in (12). Calculating the integrals in (13) with the aid of the following relation valid for $v \rightarrow 1$ and $\rho>\frac{\mu}{2}$ :
$\int_{0}^{\pi} \frac{\sin ^{\mu-1} \theta d \theta}{(1-v \cos \theta)^{\rho}}=\frac{2^{\mu-\rho} \Gamma\left(\frac{\mu}{2}\right) \Gamma\left(\frac{1}{2}\right) \Gamma(2 \rho-\mu)}{\left(1-v^{2}\right)^{\rho-\mu / 2} \Gamma(v) \Gamma\left(\frac{1-\mu}{2}+\rho\right)}$,
we obtain for quark $u$ and $d$, respectively, with $v \rightarrow 1$,
$\left\langle p_{T}^{g}(u)\right\rangle=\frac{9 \pi \Delta E_{u}}{32} \approx 9.9 \mathrm{GeV} ;$
$\left\langle p_{T}^{g}(d)\right\rangle=\frac{9 \pi \Delta E_{d}}{32} \approx 4.95 \mathrm{GeV}$.
We have to bear in mind also that in the process of hadronization a gluon give few ordinary hadrons. We assume that a radiating quark moves together with the proton to which it belongs. Then we estimate the multiplicity for gluon energy in an accompanying reference frame (8) by the following expression valid in the region of few GeV for charged multiplicity [18]:

$$
\begin{align*}
\left\langle N_{c h}\right\rangle & =a+b \ln \sqrt{s} \\
a & =-0.43 \pm 0.09 ; \quad b=2.75 \pm 0.06 \tag{15}
\end{align*}
$$

Neutral particles have to be also taken into account. In view to estimate the total multiplicity we multiply expression (15) by $\frac{3}{2}$. Then we estimate $\sqrt{s}=\sqrt{2 \Delta E M_{p}+M_{p}^{2}}$ and corresponding mean multiplicity

$$
\begin{align*}
u: & \sqrt{s}=4.15 \mathrm{GeV} ; & \langle N\rangle=5.2  \tag{16}\\
d: & \sqrt{s}=3.37 \mathrm{GeV} ; & \langle N\rangle=4.3 .
\end{align*}
$$

We take values (16) with spread $\pm 2$ and thus obtain estimate for transverse momenta of hadrons $p_{T}=p_{T}^{g} / N$
$u: \quad 1.3 \mathrm{GeV}<p_{T}<3.0 \mathrm{GeV}$;
$d: \quad 0.8 \mathrm{GeV}<p_{T}<2.0 \mathrm{GeV}$.
Estimates (17) just correspond to the interval of the ridge effect [1]. Of course, by changing light quark masses in their allowable regions we can move boundaries in (17). However, the order of magnitude of the effect remains the same.

Next point of our interpretation is that gluons are flying in the narrow cone in the directions of a quark and the average angular spread for the multiple gluon radiation is estimated to be
$\langle\Delta \bar{\theta}\rangle \simeq \frac{\left\langle p_{T}^{g}\right\rangle \sqrt{N_{g}}}{\left\langle E_{g}\right\rangle N_{g}} ;$
where $N_{g}$ is the multiplicity of bremsstrahlung gluons in the event. Obtaining (18) we take into account that average transverse momentum squared for $N_{g}$ produced gluons
$\left\langle p_{T}^{2}\left(N_{g}\right)\right\rangle=\left\langle p_{T}^{g}\right\rangle^{2} N_{g}$
due to statistical nature of the multiple radiation. We take for transverse momentum of a gluon $p_{T}^{g}$ estimates (14) and $\left\langle E_{g}\right\rangle$ is a mean energy of a gluon. Dispersion of polar angle $\Delta \bar{\theta}$ leads to dispersions of the angles $\psi_{i}$ in the distributions (10), (11). In this connection a significant increase of $\Delta \bar{\theta}$ leads to widening of distribution in $\phi$ (11). From (18) we see that for small multiplicity of gluons $\Delta \bar{\theta}$ increases and this explains why the effect disappears in this case. For estimation of the real experimental situation [1] we replace the denominator in (18) by the energy of partons' collision
$\langle\Delta \bar{\theta}\rangle \simeq \frac{2\left\langle p_{T}^{g}\right\rangle \sqrt{N_{g}}}{\sqrt{x_{1} x_{2} s}}$,
where $x_{1}, x_{2}$ are values of $x$ for quark in the first proton and the anti-quark in the second one. Number of radiated gluons $N_{g}$ depends on angle $\psi$ and velocity $v$. Using again formulas from [16] we have the following estimate:
$N_{g}=\frac{\sqrt{x_{1} x_{2} s}}{2 \Delta E \sqrt{1+\frac{\sin ^{2} \psi}{\left(1-v^{2}\right)}}}$.
For example with $\psi=0.1$ and $v=0.999$, average $\Delta E=$ $\left(\Delta E_{u}+\Delta E_{d}\right) / 2=8.4 \mathrm{GeV}, \sqrt{s}=7 \mathrm{TeV}[1]$ and with average of the product $\left\langle x_{1} x_{2}\right\rangle \approx 0.01$ (see, e.g. [19] and references therein) we have $N_{g} \simeq 17$. Now in our interpretation one bremsstrahlung gluon gives average number of charged hadrons $N_{c h} \approx 3.2$. Bearing in mind that our quasi-classical estimate corresponds to non-coherent production of gluons, with $N_{g} \approx 17$ we estimate total number of charged particles
produced by a quark $N_{c h}^{q}=54$ that gives just multiplicity $\geq 100$ for two radiating quarks. Provided one takes into account bremsstrahlung of gluons off all valence quarks moving in the strong color field the multiplicity of charged particles can only increase, which corresponds to results [1] for lower limit of multiplicity $N_{c h}^{q} \geq 100$. So our mechanism does not contradict the real experimental situation [1].

However, our qualitative considerations do not take into account possible correlations between gluons, which may lead to decreasing of the estimate under discussion.

Now with $N_{g}=17, \sqrt{s}=7 \mathrm{TeV}$, average $\left\langle p_{T}^{g}\right\rangle=7.4 \mathrm{GeV}$ and $\left\langle x_{1} x_{2}\right\rangle=0.01$ we have from (19)
$\langle\Delta \bar{\theta}\rangle \approx 0.09 ;$
This angular spread (21) actually gives widening of distributions (10), (11) in $\eta$ and $\phi$. The resulting $\Delta \phi$ is to be obtained by simultaneous account of (21) and of $\Phi(\phi)$ width (11). Let us also draw attention to widening of the ridge with $\sqrt{s}$ decreasing. E.g. for $\sqrt{s}=0.9 \mathrm{TeV}$ in accordance with (19) $\langle\Delta \bar{\theta}\rangle=0.8$, which means vanishing of the effect because the distribution (11) becomes very broad.

The last remarks could be considered as an explanation of essential energy dependence of the effect [1]. Indeed, the effect means bump in $\Delta \phi$ distribution near zero with width $\simeq 0.6$. With decreasing of energy we have two sources of widening of this bump. The first one is connected with $\langle\Delta \bar{\theta}\rangle$ and the second one is given by widening of distribution (11) with velocity $v$ decreasing (see Fig. 3). We have already noted that for $\sqrt{s}=0.9 \mathrm{TeV}$ the first source alone is enough for vanishing of the effect. For $\sqrt{s}=2.36 \mathrm{TeV}$, for which in [1] the effect is also absent one needs to take into account both sources and this is to be done in realistic simulations.

Thus, one can conclude the simple mechanism of gluon bremsstrahlung off quarks moving in a strong color field describes qualitatively the CMS ridge effect. Of course, a real situation could be much more involved. In particular, other color configurations, as was pointed out in various studies (see, for example, [12]), may play a significant role. Our consideration based on simple quasi-classical estimations shows that constituting string configurations may lead to basic features of the ridge effect, namely, correlations in particular kinematic region at very high multiplicities. Obviously, in order to show more accurate properties of proposed mechanism one should elaborate in more detail corresponding model and develop corresponding event generator to perform more realistic simulations.

Let us note that the accelerated quarks radiate photons as well. The same quasi-classical estimation gives for radiated photons two values of mean $p_{T}$ for two values of (anti-)quark charge (shown in brackets)
$\left(\frac{2 e}{3}\right): \quad p_{T} \approx 2.9 \mathrm{GeV} \times \frac{2.5}{m_{u}(\mathrm{MeV})} ;$
$\left(\frac{e}{3}\right): \quad p_{T} \approx 0.72 \mathrm{GeV} \times \frac{5}{m_{d}(\mathrm{MeV})}$.
It seems to be interesting to check these predictions with CMS data in the region of the ridge. In case of confirmation of the effect, measurement of $p_{T}$ of the photons may give useful information on current masses of light quarks $m_{u}, m_{d}$. Let us recall that for the moment these parameters are known with considerable uncertainty.

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[^1]:    ${ }^{1}$ We normalize $\alpha_{s}$ at the point of $\tau$-lepton mass due to better precision of data here.

