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# Note

## A Short Proof of Chvátal's Watchman Theorem

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This note contains a short proof of Chvátal's Watchman Theorem using the existence of a three-coloring of a triangulated polygon.

In 1975 Chvátal [1], proved the following result:

THEOREM. *If $S$ is a polygon with $n$ vertices, then there is a set $T$ of at most $n/3$ points of $S$ such that for any point $p$ of $S$ there is a point $q$ of $T$ with the segment $pq$ lying entirely in $S$.*

If we think of $S$ as a museum, with paintings on the walls, then the theorem gives a bound on the number of stationary watchmen required to guard every part of the museum. We present a simple proof.

*Proof.* Triangulate $S$ so that no new vertices are added. Every such triangulation has a coloring with three colors $a$, $b$, $c$. Let $T_k$ be the set of vertices colored $a$, and assume that $|T_a| \leqslant |T_b| \leqslant |T_c|$. Choosing $T = T_a$ implies $|T| \leqslant n/3$. Finally, every point $q$ of $S$ lies in some triangle of $S$, and every triangle of $S$ has a point $p$ of $T$ on it. Since triangles are convex, we have $pq \subset S$.

## REFERENCE

1. V. CHVÁTAL, A combinatorial theorem in plane geometry, *J. Combinatorial Theory B* **18** (1975), 39–41.



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