Eur. Phys. J. C (2017) 77:140
DOI 10.1140/epjc/s10052-017-4706-z

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Non-zero θ_{13} and leptonic CP phase with A_4 symmetry

M. Sruthilaya, R. Mohanta[a]

School of Physics, University of Hyderabad, Hyderabad 500 046, India

Received: 11 October 2016 / Accepted: 19 February 2017

Abstract We consider a model based on A_4 symmetry to explain the phenomenon of neutrino mixing. The spontaneous symmetry breaking of A_4 symmetry leads to a co-bimaximal mixing matrix at leading order. We consider the effect of higher order corrections in neutrino sector and find that the mixing angles thus obtained, come well within the 3σ ranges of their experimental values. We study the implications of this formalism on the other phenomenological observables, such as CP violating phase, Jarlskog invariant and the effective Majorana mass $|M_{ee}|$. We also obtain the branching ratio of the lepton flavour violating decay $\mu \to e\gamma$ in the context of this model and find that it can be less than its present experimental upper bound.

1 Introduction

Neutrinos are the least interacting entities among the standard model particles and exist in three flavours (electron neutrino, muon neutrino and tau neutrino). They change their flavour as they propagate and this phenomenon is known as neutrino oscillation which occurs since the flavour eigenstates of neutrinos are mixture of mass eigenstates. The mixing is described by PMNS matrix [1,2], which can be parameterized in terms of three mixing angles and three CP violating phases as

$$V_{\rm PMNS} = U_{\rm PMNS}.P_\nu$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_\nu, \quad (1)$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$, θ_{12}, θ_{23} and θ_{13} are the three mixing angles, $\delta_{\rm CP}$ is the Dirac phase and the other two Majorana phases come in P_ν

$$P_\nu = {\rm diag}(e^{i\rho}, e^{i\sigma}, 1).$$

Neutrino oscillation experiments gained a lot of interest as a probe to neutrino mixing and mass spectrum since the oscillation probability depends on mixing angles, Dirac CP phase and the mass square differences (Δm^2_{21} and Δm^2_{23}). Results from earlier experiments indicated that θ_{13} is very small, can be zero and the lepton mixing is very close to TBM (tri-bimaximal mixing) see also [3–12], which predicts $\sin\theta_{13} = 0$, $\sin^2\theta_{23} = 1/2$ and $\tan^2\theta_{12} = 1/2$. This made it possible to explain the neutrino mixing as TBM type, with small deviation due to perturbation in the charged-lepton or neutrino sector. There are many models which explain TBM mixing pattern on the basis of A_4 symmetry [13] with a certain set of Higgs scalars and vacuum alignments. Recent experimental observations of moderately large θ_{13} [14,15], made neutrino mixing a little far from TBM type, but close to co-bimaximal mixing which predicts non-zero θ_{13} ($\theta_{13} \neq 0$, $\theta_{23} = \pi/4$, $\delta_{\rm CP} = \pm\pi/2$) [16]. Supersymmetric models based on A_4 family symmetry, combined with the generalized CP symmetry [17], can also predict trimaximal (TM) lepton mixing, (in which either only the first column or only the second column of the lepton mixing matrix is assumed to take the TBM form), together with either zero CP violation or $\delta_{\rm CP} = \pm\pi/2$. Also models based on S_4 family symmetry and generalized CP symmetry [18] predict trimaximal lepton mixing and the Dirac CP is predicted to be either conserved or maximally broken. In Ref. [19], a minimal extension of the simplest A_4 model has been considered, which not only can induce non-zero θ_{13} value, consistent with the recent observations, but also can correlate the CP violation in neutrino oscillation with the octant of the atmospheric mixing angle θ_{23}. In this paper, we would like to consider a model based on A_4 symmetry which gives co-bimaximal mixing in neutrino sector at leading order. To accommodate deviations in mixing angles to make them compatible with the experimental results, we include a perturbation in neutrino sector due to higher order corrections, which can be represented as five-dimensional operators. The best-fit values and 3σ ranges

[a] e-mail: rmsp@uohyd.ernet.in

Table 1 The best-fit values and the 3σ ranges of the neutrino oscillation parameters from Ref. [20]

Mixing parameters	Best fit values	3σ Range
$\sin^2\theta_{12}$	0.323	$0.278 \to 0.375$
$\sin^2\theta_{23}$ (NH)	0.567	$0.393 \to 0.643$
$\sin^2\theta_{23}$ (IH)	0.573	$0.403 \to 0.640$
$\sin^2\theta_{13}$ (NH)	0.0226	$0.0190 \to 0.0262$
$\sin^2\theta_{13}$ (IH)	0.0229	$0.0193 \to 0.0265$
δ_{CP} (NH)	1.41π	$(0 \to 2\pi)$
δ_{CP} (IH)	1.48π	$(0 \to 2\pi)$
$\Delta m^2_{21}/10^{-5}$ eV^2	7.60	$7.11 \to 8.18$
$\Delta m^2_{31}/10^{-3}$ eV^2(NH)	2.48	$2.30 \to 2.65$
$\Delta m^2_{31}/10^{-3}$ eV^2(IH)	−2.38	$-2.54 \to -2.20$

of neutrino oscillation parameters taken from Ref. [20] are given in Table 1.

The paper is organized as follows. The details of our model is presented in Sect. 2. In Sects. 3 and 4, we discuss the vacuum alignment and lepton flavour violating muon decay $\mu \to e\gamma$ in the context of the model. In Sect. 5, we describe the higher order corrections in neutrino sector and we conclude our discussion in Sect. 6.

2 The model

The model is based on A_4 group [21], which is the group of even permutation of four objects and is the smallest non-Abelian discrete group with triplet irreducible representation. It has four irreducible representations: 1, 1′, 1″ and 3, with the multiplication rule

$$3 \times 3 = 1 + 1' + 1'' + 3 + 3. \tag{2}$$

As we know, A_4 allows the charged-lepton mass matrix to be diagonalized by the Cabibbo–Wolfenstein matrix [22]

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \tag{3}$$

where $\omega = e^{2\pi i/3} = -1/2 + i\sqrt{3}/2$.

In this work, our discussion is limited to the leptonic sector. The particle content of the model includes, in addition to standard model fermions (i.e., the lepton doublets l_{iL} and charged lepton singlets l_{iR}), three right-handed neutrinos (ν_{iR}), four Higgs doublets (ϕ_i, ϕ_0) and three Higgs singlets (χ_i). They belong to four irreducible representations of A_4 as given in Table 2.

Here A_4 symmetry is accompanied by an additional $U(1)_X$ symmetry as discussed in Ref. [13], which prevents the existence of Yukawa interactions of the form $\bar{l}_{iL}\nu_{iR}\tilde{\phi}_i$ and

Table 2 Particle content of the model along with their quantum numbers

	$SU(2)_L$	$U(1)_Y$	A_4
l_{iL}	2	−1	3
l_{1R}			1
l_{2R}	1	−2	1′
l_{3R}			1″
ν_{iR}	1	0	3
ϕ_i	2	1	3
ϕ_0	2	1	1
χ_i (real gauge singlet)	1	0	3

$\bar{l}_{iL}l_{iR}\phi_0$ as l_{iL}, l_{iR}, $\tilde{\phi}_0$ have quantum number $X = 1$, and all other fields have $X = 0$. The phenomenologically disallowed Nambu-Goldstone boson does not arise in this case as $U(1)_X$ symmetry does not break spontaneously but explicitly. Thus, the Yukawa Lagrangian for the leptonic sector is given as [23]

$$\begin{aligned}\mathcal{L} = &-\left\{\left[\lambda_1\left(\bar{l}_{iL}\phi_i\right)l_{1R}\right] + \left[\lambda_2\left(\bar{l}_{iL}\phi_i\right)''l_{2R}\right] + \left[\lambda_3\left(\bar{l}_{iL}\phi_i\right)'l_{3R}\right]\right\} \\ &-\left\{\lambda_0\left[\left(\bar{l}_{iL}\nu_{iR}\right)\tilde{\phi}_0\right] + \frac{1}{2}\left[M\left(\bar{\nu}_{iR}\hat{\nu}_{iR}\right)\right] + \lambda_\chi\left[\left(\bar{\nu}_{iR}\hat{\nu}_{iR}\right)_3\chi_i\right]\right\} \\ &+\text{h.c.},\end{aligned} \tag{4}$$

where $\hat{\nu}_{iR}$ are antiparticles of ν_{iR} and $(\bar{l}_{iL}\phi_i)'$, $(\bar{l}_{iL}\phi_i)''$ and $\left(\bar{\nu}_{iR}\hat{\nu}_{iR}\right)_3$ are $1'$, $1''$ and triplet representations of A_4 respectively. As the scalars ϕ_i, ϕ_0 and χ_i get vacuum expectation values v_i, v_0 and ω_i respectively, the above Lagrangian becomes

$$\mathcal{L} = -\bar{l}_L M_l l_R - \bar{\nu}_L M_D \nu_R - \frac{1}{2}\bar{\nu}_R M_R \hat{\nu}_R + \text{h.c}, \tag{5}$$

where M_l, M_D and M_R are charged-lepton, Dirac neutrino and right-handed neutrino mass matrices and have the forms

$$M_l = \begin{pmatrix} \lambda_1 v_1 & \lambda_2 v_1 & \lambda_3 v_1 \\ \lambda_1 v_2 & \lambda_2 v_2 \omega^2 & \lambda_3 v_2 \omega \\ \lambda_1 v_3 & \lambda_2 v_3 \omega & \lambda_3 v_3 \omega^2 \end{pmatrix}, \tag{6}$$

$$M_D = \lambda_0 v_0 I, \tag{7}$$

where I is the identity matrix, and

$$M_R = \begin{pmatrix} M & \lambda_\chi\omega_3 & \lambda_\chi\omega_2 \\ \lambda_\chi\omega_3 & M & \lambda_\chi\omega_1 \\ \lambda_\chi\omega_2 & \lambda_\chi\omega_1 & M \end{pmatrix}. \tag{8}$$

For the vacuum alignment $v_i = v$, the charged lepton sector can be diagonalized by the transformation:

$$U_\omega \cdot M_l \cdot I = \begin{pmatrix} \sqrt{3}v\lambda_1 & 0 & 0 \\ 0 & \sqrt{3}v\lambda_2 & 0 \\ 0 & 0 & \sqrt{3}v\lambda_3 \end{pmatrix}, \tag{9}$$

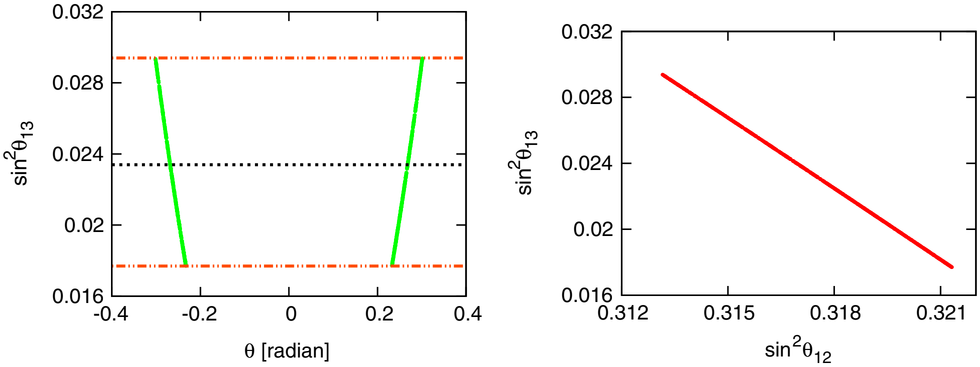

Fig. 1 Variation of $\sin^2\theta_{13}$ with θ (*left panel*) and the correlation plots between $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$ (*right panel*). The *black dashed line* in the *left panel* denotes the central value of $\sin^2\theta_{13}$ and the *red dot-dashed lines* represent the corresponding 3σ values

where U_ω is the Cabibbo–Wolfenstein matrix given in Eq. (3). The light neutrino mass is given by the type-I seesaw formula

$$M_\nu = -M_D^T \cdot M_R^{-1} \cdot M_D. \tag{10}$$

Since M_D is proportional to an identity matrix, the neutrino mixing matrix will be the one which diagonalizes the right-handed neutrino mass matrix M_R. The Majorana mass matrix M_R can be parameterized as

$$M_R = \begin{pmatrix} A & C & D \\ C & A & B \\ D & B & A \end{pmatrix}, \tag{11}$$

in a basis where charged-lepton mass matrix is not diagonal. However, in the charged lepton mass diagonal basis $M_R^d = U_\omega^\dagger \cdot M_R \cdot U_\omega^*$ and can be diagonalized by tri-bimaximal (TBM) mixing matrix for $D = C = 0$, which we don't need as it gives vanishing θ_{13}. Even if these conditions are not satisfied some of the off-diagonal elements of M_R become zero in TBM basis and one can go to the TBM basis through the transformation

$$M_R' = U_T^\dagger \cdot M_R \cdot U_T^* = \begin{pmatrix} A+B & \frac{1}{\sqrt{2}}(D+C) & 0 \\ \frac{1}{\sqrt{2}}(D+C) & A & \frac{i}{\sqrt{2}}(D-C) \\ 0 & \frac{i}{\sqrt{2}}(D-C) & B-A \end{pmatrix}, \tag{12}$$

where

$$U_T = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{-i}{\sqrt{2}} \end{pmatrix}. \tag{13}$$

With the condition $D = -C$, M_R' becomes

$$\begin{pmatrix} A+B & 0 & 0 \\ 0 & A & i\sqrt{2}D \\ 0 & i\sqrt{2}D & B-A \end{pmatrix}, \tag{14}$$

which can be diagonalized by U_R, having the form

$$U_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & is \\ 0 & is & c \end{pmatrix}, \tag{15}$$

where s and c stand for $\sin\theta$ and $\cos\theta$ respectively and satisfy the relation

$$\frac{cs}{c^2 - s^2} = \frac{\sqrt{2}D}{B} = \frac{\sqrt{2}\omega_2}{\omega_1}. \tag{16}$$

It should be noted that, this ratio should be real, since $\omega_{1,2}$ are VEV of real scalar fields χ_i. The condition $C = -D$ can be realized with the vacuum alignment $\langle\chi_i\rangle = (\omega_1, \omega_2, -\omega_2)$ [24]. Thus, the lepton mixing matrix becomes

$$U = U_\omega \cdot U_T \cdot U_R, \tag{17}$$

which basically known as co-bimaximal mixing matrix and predicts the mixing angles and CP violating Dirac phase as $\theta_{13} \neq 0$, $\theta_{23} = \pi/4$ and $\delta_{\rm CP} = \pm\pi/2$. Also, the mixing angles θ_{12} and θ_{13} are not independent and one can express $\sin^2\theta_{12}$ in terms of $\sin^2\theta_{13}$ as

$$\sin^2\theta_{12} = \frac{1 - 3\sin^2\theta_{13}}{3(1 - \sin^2\theta_{13})}, \quad \text{with } \sin\theta_{13} = \frac{s}{\sqrt{3}}. \tag{18}$$

To illustrate these results, we show in Fig. 1 the variation of $\sin^2\theta_{13}$ with θ (left panel) and the correlation plot between $\sin^2\theta_{13}$ and $\sin^2\theta_{12}$ (right panel). From the figure it can be seen that the observed values of solar (θ_{12}) and reactor (θ_{13}) mixing angles can be accommodated in this model.

3 Vacuum alignment

The complete scalar potential is given by

$$V = V(\phi_i) + V(\chi_i) + V(\phi_0) + V(\phi_i\chi_i) + V(\phi_i\phi_0) \tag{19}$$

with

$$V(\phi_i) = \mu_{\phi_i}^2 \sum_j \phi_j^\dagger \phi_j + \frac{\lambda_1^{\phi_i}}{2}\left(\sum_j \phi_j^\dagger \phi_j\right)^2 + \lambda_2^{\phi_i}(\phi_1^\dagger\phi_1 + \omega\phi_2^\dagger\phi_2 + \omega^2\phi_3^\dagger\phi_3)(\phi_1^\dagger\phi_1 + \omega^2\phi_2^\dagger\phi_2 + \omega\phi_3^\dagger\phi_3) + \lambda_3^{\phi_i}[(\phi_2^\dagger\phi_3)(\phi_3^\dagger\phi_2) + (\phi_3^\dagger\phi_1)(\phi_1^\dagger\phi_3) + (\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)] + \left\{\frac{\lambda_4^{\phi_i}}{2}\left[\left(\phi_2^\dagger\phi_3\right)^2 + \left(\phi_3^\dagger\phi_1\right)^2 + \left(\phi_1^\dagger\phi_2\right)^2\right] + \text{h.c.}\right\},$$

$$V(\chi_i) = \mu_{\chi_i}^2 \sum_j \chi_j\chi_j + \delta^{\chi_i}\chi_1\chi_2\chi_3 + \lambda_1^{\chi_i}\left(\sum_j \chi_j\chi_j\right)^2 + \lambda_2^{\chi_i}(\chi_1\chi_1 + \omega\chi_2\chi_2 + \omega^2\chi_3\chi_3)(\chi_1\chi_1 + \omega^2\chi_2\chi_2 + \omega\chi_3\chi_3) + \lambda_3^{\chi_i}[(\chi_2\chi_3)^2 + (\chi_3\chi_1)^2 + (\chi_1\chi_2)^2],$$

$$V(\phi_0) = \mu_{\phi_0}^2\phi_0^\dagger\phi_0 + \lambda_1^{\phi_0}\left(\phi_0^\dagger\phi_0\right)^2,$$

$$V(\phi_i\chi_i) = \delta^{\phi_i\chi_i}(\phi_2^\dagger\phi_3\chi_1 + \phi_3^\dagger\phi_1\chi_2 + \phi_1^\dagger\phi_2\chi_3) + \lambda_1^{\phi_i\chi_i}\sum_{j,k}\phi_j^\dagger\phi_j\chi_k\chi_k + \lambda_2^{\phi_i\chi_i}(\phi_1^\dagger\phi_1 + \omega\phi_2^\dagger\phi_2 + \omega^2\phi_3^\dagger\phi_3)(\chi_1\chi_1 + \omega^2\chi_2\chi_2 + \omega\chi_3\chi_3) + \lambda_3^{\phi_i\chi_i}(\phi_2^\dagger\phi_3\chi_2\chi_3 + \phi_3^\dagger\phi_1\chi_3\chi_1 + \phi_1^\dagger\phi_2\chi_1\chi_2) + \text{h.c.}, \quad (20)$$

$$V(\phi_i\phi_0) = \lambda_1^{\phi_i\phi_0}\left(\sum_j \phi_j^\dagger\phi_j\right)\phi_0^\dagger\phi_0 + \lambda_2^{\phi_i\phi_0}\left(\sum_j \phi_j^\dagger\phi_0\,\phi_0^\dagger\phi_j\right) + [\lambda_3^{\phi_i\phi_0}(\phi_1^\dagger\phi_0\phi_2^\dagger\phi_3 + \phi_2^\dagger\phi_0\phi_3^\dagger\phi_1 + \phi_3^\dagger\phi_0\phi_1^\dagger\phi_2) + \lambda_4^{\phi_i\phi_0}(\phi_1^\dagger\phi_0\phi_3^\dagger\phi_2 + \phi_2^\dagger\phi_0\phi_1^\dagger\phi_3 + \phi_3^\dagger\phi_0\phi_2^\dagger\phi_1) + \text{h.c.}], \quad (21)$$

$$V(\chi_i\phi_0) = \lambda^{\phi_0\chi_i}\left(\sum_j \chi_j\chi_j\right)\phi_0^\dagger\phi_0. \quad (22)$$

The last term in Eq. (21) breaks $U(1)_X$ symmetry explicitly and removes Goldstone boson which occurs due to the spontaneous breaking of $U(1)_X$ symmetry. In this model, we have the vacuum alignment $\langle\phi_0\rangle = u$, $\langle\phi_i\rangle = (v, v, v)$, and $\langle\chi_i\rangle = (w_1, w_2, -w_2)$ which is a possible minimum of scalar potential for $V(\phi_i\chi_i) = 0$. A vanishing $V(\phi_i\chi_i)$ can be achieved in the limit χ_i decouples from rest of the field as mentioned in Ref. [13]. The decoupling of χ_i requires $\lambda_\chi \to 0$, $\lambda^{\phi_0\chi_i} \to 0$. To generate an acceptable neutrino mass spectrum λ_χ has to be nonzero but can be small. A small but nonzero λ_χ will generate a sufficiently small $V(\phi_i\chi_i)$ which will be too small to alter vacuum alignment considerably. In this limit the minimization condition on u is given by

$$\mu_{\phi_0}^2 u + 2\lambda_1^{\phi_0}(u^*u)u + \lambda_1^{\phi_i\phi_0}(|v_1|^2 + |v_2|^2 + |v_3|^2)u + \lambda_2^{\phi_i\phi_0}\left(\sum_{j,k} v_j^* v_k\right)u + \lambda_3^{\phi_i\phi_0 *}[v_1v_2v_3^* + v_2v_3v_1^* + v_3v_1v_2^*] + \lambda_4^{\phi_i\phi_0 *}[v_1v_3v_2^* + v_2v_1v_3^* + v_3v_2v_1^*] = 0$$

The above equation has a solution

$$u = \frac{\lambda_3^{\phi_i\phi_0 *}[v_1v_2v_3^* + v_2v_3v_1^* + v_3v_1v_2^*] + \lambda_4^{\phi_i\phi_0 *}[v_1v_3v_2^* + v_2v_1v_3^* + v_3v_2v_1^*]}{\mu_{\phi_0}^2 + (\lambda_1^{\phi_i\phi_0} + \lambda_2^{\phi_i\phi_0})(\sum_j |v_j|^2)} \quad (23)$$

for $|u|^2 \ll |v_i|^2$.

(a) Thus, for this case, i.e., for $|u|^2 \ll |v_i|^2$ minimization conditions on v_i are given as

$$\frac{\partial V}{\partial v_i^*} = \mu_{\phi_i}^2 v_i + \lambda_1^{\phi_i} v_i \sum_j |v_j|^2 + \lambda_2^{\phi_i} v_i\left(2|v_i|^2 - \sum_{j\neq i}|v_j|^2\right) + \lambda_3^{\phi_i} v_i\left(\sum_{j\neq i}|v_j|^2\right) + \lambda_4^{\phi_i} v_i^* \sum_{j\neq i} v_j^2 = 0. \quad (24)$$

Considering $\lambda_4^{\phi_i}$ as real, one can get the solution

$$v_i = v = \sqrt{\frac{-\mu_{\phi_i}^2}{3\lambda_1^{\phi_i} + 2\left(\lambda_3^{\phi_i} + \lambda_4^{\phi_i}\right)}}, \quad (25)$$

which is allowed.

(b) Minimization conditions on w_i is given by

$$\frac{\partial V}{\partial w_1} = 2\left[\mu_{\chi_i}^2 + \lambda_2^{\chi_i\prime}\left(w_2^2 + w_3^2\right)\right]w_1 + \delta^{\chi_i}w_2w_3 + 4\lambda_1^{\chi_i\prime}w_1^3 = 0, \quad (26)$$

$$\frac{\partial V}{\partial w_2} = 2\left[\mu_{\chi_i}^2 + \lambda_2^{\chi_i\prime}\left(w_1^2 + w_3^2\right)\right]w_2 + \delta^{\chi_i}w_1w_3 + 4\lambda_1^{\chi_i\prime}w_2^3 = 0, \quad (27)$$

$$\frac{\partial V}{\partial w_3} = 2\left[\mu^2_{\chi_i} + \lambda_2^{\chi_i\prime}\left(w_2^2 + w_1^2\right)\right] w_3 + \delta^{\chi_i} w_2 w_1 + 4\lambda_1^{\chi_i\prime} w_3^3 = 0, \quad (28)$$

one of the solutions of above set of equations is $w_1 \neq 0$, $w_3 = -w_2 \neq 0$, which is the vacuum alignment condition for $\langle\chi_i\rangle$.

4 Effect of additional higgs doublets on lepton flavour violating decay $\mu \to e\gamma$

Since $|\,u\,|^2 \ll v^2$ one can neglect the mixing between ϕ_i and ϕ_0 and the mass-squared matrices in the Re$[\phi_i^0]$, Im$[\phi_i^0]$, and $\phi_i^\pm$ bases have the same form [25].

$$M^2 = \begin{pmatrix} a & b & b \\ b & a & b \\ b & b & a \end{pmatrix}, \quad (29)$$

where $a = 2(\lambda_1^{\phi_i} + 2\lambda_2^{\phi_i})v^2, -4\lambda_4^{\phi_i}v^2, -2(\lambda_3^{\phi_i} + \lambda_4^{\phi_i})v^2$, and $b = 2(\lambda_1^{\phi_i} - \lambda_2^{\phi_i} + \lambda_3^{\phi_i} + \lambda_4^{\phi_i})v^2, 2\lambda_4^{\phi_i}v^2, (\lambda_3^{\phi_i} + \lambda_4^{\phi_i})v^2$ for Re$[\phi_i^0]$, Im$[\phi_i^0]$, and $\phi_i^\pm$ respectively. Hence, there are three linear combinations of ϕ_is, $\phi = \frac{1}{\sqrt{3}}\left(\phi_1 + \phi_2 + \phi_3\right)$, $\phi' = \frac{1}{\sqrt{3}}\left(\phi_1 + \omega\phi_2 + \omega^2\phi_3\right)$, and $\phi'' = \frac{1}{\sqrt{3}}\left(\phi_1 + \omega^2\phi_2 + \omega\phi_3\right)$ with vacuum expectation values $\sqrt{3}v$, 0, and 0 respectively. The Higgs doublet ϕ with mass-squared eigenvalues $(3\lambda_1^{\phi_i} + 2\lambda_3^{\phi_i} + 2\lambda_4^{\phi_i})v^2, 0, 0$ for Re$[\phi^0]$, Im$[\phi^0]$ and $\phi^\pm$ can be identified as standard model Higgs doublet which gives masses to charged leptons. One can see this by expressing Yukawa interactions of ϕ_is with leptons in charged lepton mass diagonal basis

$$\begin{aligned}\mathcal{L} = &\left(\frac{m_e}{\sqrt{3}v}\overline{(\nu_e, e)}_L e_R + \frac{m_\mu}{\sqrt{3}v}\overline{(\nu_\mu, \mu)}_L \mu_R + \frac{m_\tau}{\sqrt{3}v}\overline{(\nu_\tau, \tau)}_L \tau_R\right)\phi \\ &+\left(\frac{m_e}{\sqrt{3}v}\overline{(\nu_\mu, \mu)}_L e_R + \frac{m_\mu}{\sqrt{3}v}\overline{(\nu_\tau, \tau)}_L \mu_R + \frac{m_\tau}{\sqrt{3}v}\overline{(\nu_e, e)}_L \tau_R\right)\phi' \\ &+\left(\frac{m_e}{\sqrt{3}v}\overline{(\nu_\tau, \tau)}_L e_R + \frac{m_\mu}{\sqrt{3}v}\overline{(\nu_e, e)}_L \mu_R + \frac{m_\tau}{\sqrt{3}v}\overline{(\nu_\mu, \mu)}_L \tau_R\right)\phi''\end{aligned} \quad (30)$$

The Higgs doublets ϕ' and ϕ'' contributes to flavour violating decays such as $\mu \to e\gamma$. The prominent contribution comes from ϕ' and the branching ratio is given by [25],

$$\text{Br}(\mu \to e\gamma) = \frac{9}{32\pi^2} m_\tau^4 \left(\frac{M_R^2 - M_I^2}{M_R^2 M_I^2}\right)^2 \left(\frac{v_0^2}{3v^2}\right)^2 \quad (31)$$

where $M_R^2 = 2(3\lambda_2^{\phi_i} - \lambda_3^{\phi_i} - \lambda_4^{\phi_i})v^2$, $M_I^2 = -6\lambda_4^{\phi_i}v^2$ are mass-squared eigenvalues of $\frac{1}{\sqrt{3}}(\text{Re}[\phi_1] + \omega\text{Re}[\phi_2] + \omega^2\text{Re}[\phi_3])$ and $\frac{1}{\sqrt{3}}(\text{Im}[\phi_1] + \omega\text{Im}[\phi_2] + \omega^2\text{Im}[\phi_3])$ respectively and $v_0^2 = (1/2\sqrt{2}G_F)$. The predicted branching ratio will be below the experimental upper limit Br$(\mu \to e\gamma) < 4.2 \times 10^{-13}$ [26] for

$$\left(\frac{M_R^2 - M_I^2}{M_R^2 M_I^2}\right)^{\frac{1}{2}} < 1.56 \times 10^{-3}\ \text{GeV}^{-1}. \quad (32)$$

5 Perturbation in neutrino sector

In this section, we will consider the perturbations to mass matrices due to higher order corrections. Prominent corrections come from five-dimensional operator $\lambda_{ij}\bar{\nu}_{iR}\hat{\nu}_{jR}\chi_i\chi_j$ which modifies right-handed neutrino mass matrix. Charged lepton and Dirac neutrino masses also receive corrections from $\lambda'_{jk}\bar{l}_{il}\phi_i l_{jR}\chi_i$ and $\lambda'_{jk}\bar{l}_{il}\tilde{\phi}_0\nu_{jR}\chi_i$ respectively, and here we are neglecting those corrections since they allow the mixing of χ_i with other fields.

All elements of Majorana mass matrix M_R receive corrections which is proportional to $\omega_1^2 + \omega_2^2$ for diagonal elements and $\omega_1\omega_2$ for off diagonal elements. Since $0.04 < (\omega_2/\omega_1) < 0.22$, obtained from Eq. (16), using the allowed value of $s = \sqrt{3}\sin\theta_{13}$, we neglect corrections to off- diagonal elements.

$$\delta M_R \simeq \begin{pmatrix} \lambda_{11}\omega_1^2 & 0 & 0 \\ 0 & \lambda_{22}\omega_1^2 & 0 \\ 0 & 0 & \lambda_{33}\omega_1^2 \end{pmatrix}. \quad (33)$$

These corrections will modify the light neutrino mass matrix and the inverse of modified light neutrino mass matrix in TBM basis can be parameterized as

$$M_\nu^{-1} = \begin{pmatrix} B + A & 0 & 0 \\ 0 & A & i\sqrt{2}D \\ 0 & i\sqrt{2}D & B - A \end{pmatrix} + \begin{pmatrix} \frac{1}{2}\left(\lambda_{22} + \lambda_{33}\right) & 0 & \frac{i}{2}\left(\lambda_{33} - \lambda_{22}\right) \\ 0 & \lambda_{11} & 0 \\ \frac{i}{2}\left(\lambda_{33} - \lambda_{22}\right) & 0 & \frac{-1}{2}\left(\lambda_{33} + \lambda_{22}\right) \end{pmatrix}\omega_1^2. \quad (34)$$

Hence, in the charged lepton diagonal basis light neutrino mass matrix can be diagonalized by

$$U = U_\omega \cdot U_T \cdot U_R \cdot U_{13}, \quad (35)$$

where

$$U_{13} = \begin{pmatrix} c' & 0 & s'e^{-i\phi} \\ 0 & 1 & 0 \\ -s'e^{i\phi} & 0 & c' \end{pmatrix}. \quad (36)$$

with $s' = \sin\theta'$ and $c' = \cos\theta'$.

To obtain mixing angles we compare lepton mixing matrix U (35) with PMNS matrix (1), i.e.,

$$U = U_{\text{PMNS}}. \quad (37)$$

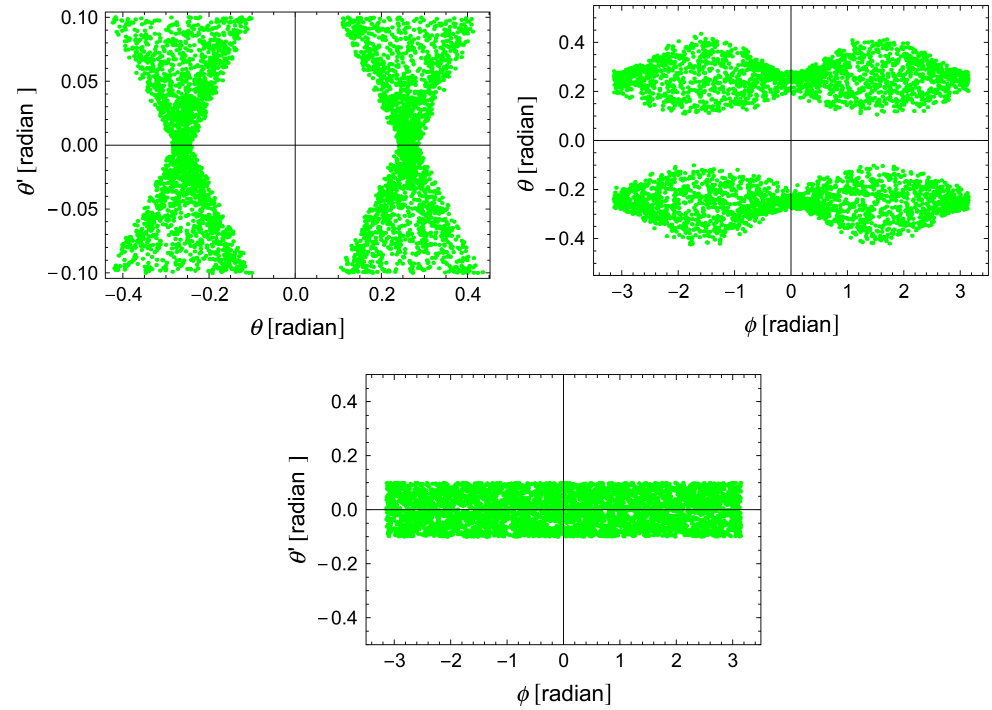

Fig. 2 Allowed parameter space in $\theta' - \theta$ (*left panel*), $\theta - \phi$ (*right panel*) and $\theta' - \phi$ planes compatible with the observed data

The mixing angles $\sin^2\theta_{12}$, $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$ are related to the elements of U as

$$\sin^2\theta_{12} = \frac{|U_{12}|^2}{1-|U_{13}|^2}, \quad \sin^2\theta_{23} = \frac{|U_{23}|^2}{1-|U_{13}|^2},$$
$$\sin^2\theta_{13} = |U_{13}|^2, \tag{38}$$

where U_{ij} is the ijth element of the lepton mixing matrix U. Now using Eqs. (3), (13), (35) and (38), we obtain

$$\sin^2\theta_{13} = \frac{1}{3}[2s'^2 - 2\sqrt{2}sc's'\sin\phi + s^2c'^2], \tag{39}$$

$$\sin^2\theta_{12} = \frac{1-s^2}{3-(2s'^2 - 2\sqrt{2}sc's'\sin\phi + s^2c'^2)}, \tag{40}$$

$$\sin^2\theta_{23} = \frac{1}{2} + \frac{\sqrt{3}cc's'\cos\phi}{3-(2s'^2 - 2\sqrt{2}sc's'\sin\phi + s^2c'^2)}, \tag{41}$$

Another important parameter is $J_{\rm CP}$, the Jarlskog invariant, which is a measure of CP violation, is found to have the value in this model as

$$J_{\rm CP} = {\rm Im}[U_{11}U_{22}U_{21}^*U_{12}^*]$$
$$= \frac{c}{6\sqrt{3}}[\sqrt{2}sc'^2 - (1+c^2)c's'\sin\phi - \sqrt{2}ss'^2]. \tag{42}$$

In standard parametrization, the value of $J_{\rm CP}$ is

$$J_{\rm CP} = \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta_{\rm CP}. \tag{43}$$

Comparing Eqs. (42) and (43), we obtain

$$\sin\delta_{\rm CP} = \frac{\sqrt{2}s(c'^2 - s'^2) - c's'(1+c^2)\sin\phi}{\sqrt{X'(2-X'+s^2)\left(1-\frac{Y'^2}{(3-X')^2}\right)}}, \tag{44}$$

where

$$X' = \left[2s'^2 - 2\sqrt{2}sc's'\sin\phi + s^2c'^2\right],$$
$$Y' = 2\sqrt{3}cc's'\cos\phi. \tag{45}$$

To show that the model predicts the mixing angles compatible with the observed data,we obtain the allowed parameter space compatible with the 3σ range of the observed data by varying the parameters s between $[-1, 1]$, s' between $[-0.1, 0.1]$ and ϕ between $[-\pi, \pi]$, we show the allowed parameter space in various planes in Fig. 2. Using these allowed values of different parameters, we show the correlation plots between $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ (left panel), $\sin^2\theta_{13}$ and $\sin^2\theta_{12}$ (right panel) and between $\sin^2\theta_{13}$ and $\delta_{\rm CP}/J_{CP}$ (bottom panel) in Fig. 3. From these plots it can be seen that by including higher order correction to right handed neutrino mass matrix, it is possible to accommodate the observed data.

In this model, light neutrinos acquire Majorana masses through Type-I seesaw which indicates neutrinos are of Majorana type. Majorana nature of neutrinos predicts the existence of neutrino-less double beta decay ($0\nu\beta\beta$), which is a process where two neutrons inside a nucleus convert into two protons without emitting neutrinos, i.e., $(A, Z) \rightarrow$

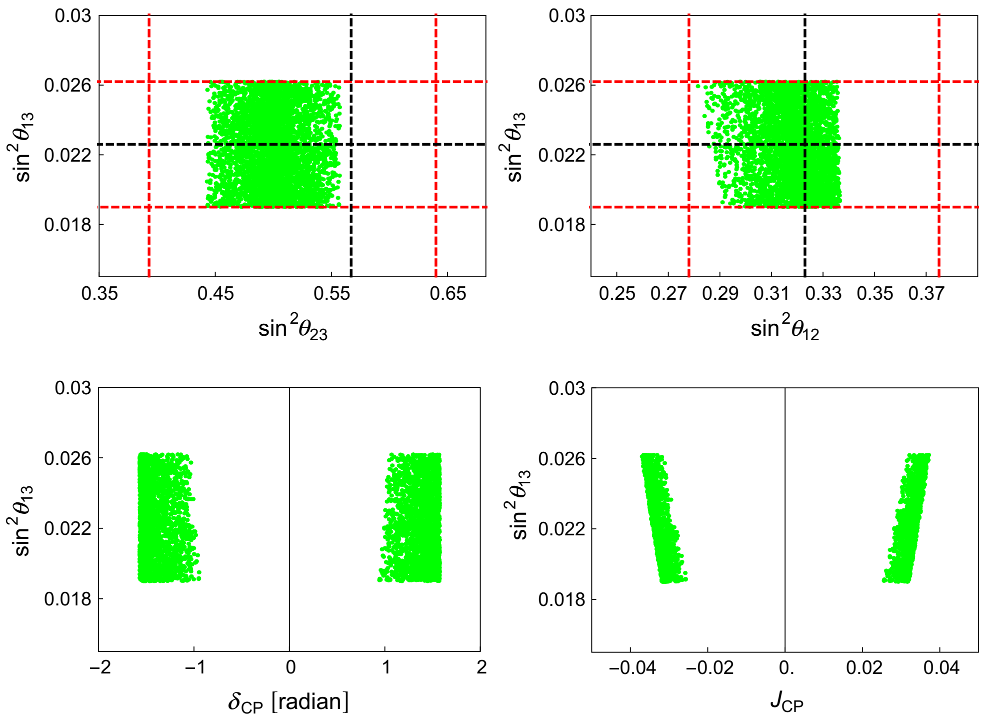

Fig. 3 Correlation plots between $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ (*left panel*), and $\sin^2\theta_{13}$ and $\sin^2\theta_{12}$ (*right panel*) and between $\sin^2\theta_{13}$ and δ_{CP}/J_{CP} (*bottom panel*) including the corrections

$(A, Z+2)+2e$. Several experiments like KamLAND-Ze [27], EXO [28] and GERDA [29] are searching for the neutrino-less double beta decay. These experiments put upper bound on $|M_{ee}|$, the (1, 1) element of neutrino mass matrix, since the half-life of $0\nu\beta\beta$ decay is proportional to $|M_{ee}|^2$. The expression for $|M_{ee}|$ in the flavor basis is

$$|M_{ee}| = |U_{11}^2 m_1 + U_{12}^2 m_2 + U_{13}^2 m_3|, \quad (46)$$

where m_1, m_2, and m_3 are light neutrino masses and U_{1j}'s are elements of first row of the lepton mixing matrix U, which are given as

$$U_{11} = \frac{2}{\sqrt{6}}c' - \frac{i}{\sqrt{3}}ss'e^{i\phi},$$
$$U_{12} = \frac{1}{\sqrt{3}}c,$$
$$U_{13} = \frac{2}{\sqrt{6}}s'e^{-i\phi} + \frac{i}{\sqrt{3}}sc' \,. \quad (47)$$

The lowest upper bound on $|M_{ee}|$ is 0.22 eV came from GERDA phase-I data. Here we study the variation of $|M_{ee}|$ with the lightest neutrino mass m_1 (m_3), in the case of normal (inverted) hierarchy as shown in Fig. 4. In our calculation we have used the relations

$$m_2 = \sqrt{m_1^2 + \Delta m_{21}^2},$$
$$m_3 = \sqrt{m_1^2 + \Delta m_{31}^2}, \quad (48)$$

for normal hierarchy and

$$m_1 = \sqrt{m_3^2 + \Delta m_{13}^2},$$
$$m_2 = \sqrt{m_3^2 + \Delta m_{13}^2 + \Delta m_{21}^2}, \quad (49)$$

for inverted hierarchy, and obtained upper limit on m_1 (m_3) as 0.071 (0.065) eV taking into account the cosmological upper bound on $\Sigma_i m_i$ as 0.23 eV [30]. Another observable is the kinetic electron neutrino mass in beta decay (m_e), which is probed in direct search for neutrino masses, can be expressed as

$$m_e = \sqrt{|U_{11}|^2 m_1^2 + |U_{12}|^2 m_2^2 + |U_{13}|^2 m_3^2}. \quad (50)$$

In the right panel of Fig. 4, we show the variation of m_e with the lightest neutrino mass m_1 (m_3) for normal hierarchy (inverted hierarchy) case, and the upper limit on m_e is found to be 0.07 (0.08) eV.

6 Conclusions

We consider a model based on A_4 symmetry, which gives co-bimaximal form ($\theta_{23} = \pi/4$, $\delta_{CP} = \pm\pi/2$ and $\theta_{13} \neq 0$) for the leading order neutrino mixing matrix. There are four Higgs doublets ϕ_0, and ϕ_i, for $i = 1, 2, 3$ in this model. One of the three linear combinations (ϕ) of ϕ_i behaves exactly as standard model Higgs doublet while neutral component of

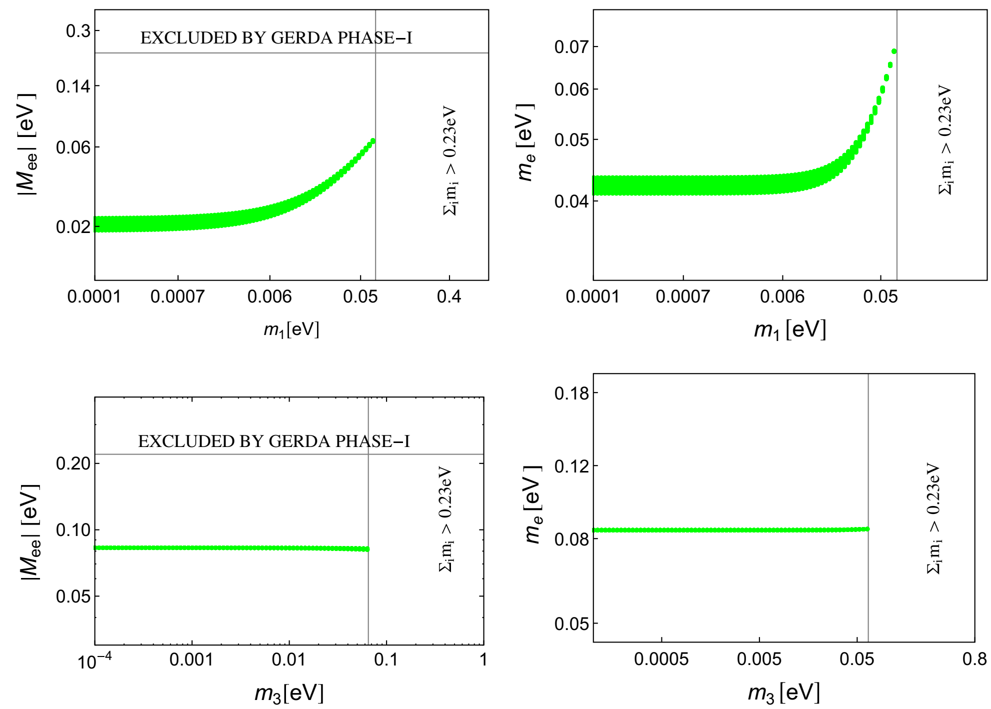

Fig. 4 Variation of $|M_{ee}|$ with the lightest neutrino mass m_1 (m_3) (*left panel*) and m_e vs. m_1 (m_3) in the *right panel* for the normal mass hierarchy (inverted mass hierarchy) case

the other two (ϕ', ϕ'') contribute to the lepton flavour violating decays such as $\mu \to e\gamma$. We have considered higher order corrections in neutrino sector coming from five-dimensional operators after spontaneous breaking of A_4 symmetry. The mixing angles, thus obtained are found to be within the 3σ ranges of their experimental values. The CP violating phase δ_{CP} is found to be around the region $\pm\pi/2$, and the upper limit on the Jarlskog invariant is $\mathcal{O}(10^{-2})$. We also studied the variation of the effective neutrino mass $|M_{ee}|$ with the lightest neutrino mass m_1 (m_3) in the case of normal (inverted) hierarchy and found its value to be lower than the experimental upper limit for all allowed values of m_1 (m_3).

Acknowledgements SM would like to thank University Grants Commission for financial support. The work of RM was partly supported by the Science and Engineering Research Board (SERB), Government of India through Grant No. SB/S2/HEP-017/2013.

References

1. B. Pontecorvo, Sov. Phys. JETP **7**, 172 (1958)
2. Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. **28**, 870 (1962)
3. P.F. Harrison, D.H. Perkins, W.G. Scott, Phys. Lett. B **458**, 79 (1999)
4. P.F. Harrison, D.H. Perkins, W.G. Scott, Phys. Lett. B **530**, 167 (2002)
5. Z.Z. Xing, Phys. Lett. B **533**, 85 (2002)
6. P.F. Harrison, W.G. Scott, Phys. Lett. B **535**, 163 (2002)
7. P.F. Harrison, W.G. Scott, Phys. Lett. B **557**, 76 (2003)
8. X.-G. He, A. Zee, Phys. Lett. B **560**, 87 (2003)
9. L. Wolfenstein, Phys. Rev. D **D18**, 958 (1978)
10. Y. Yamanaka, H. Sugawara, S. Pakvasa, Phys. Rev. D **25**, 1895 (1982)
11. Y. Yamanaka, H. Sugawara, S. Pakvasa, Phys. Rev. D **29**, 2135(E) (1984)
12. N. Li, B.-Q. Ma, Phys. Rev. D **71**, 017302 (2005). arXiv:hep-ph/041212
13. X.-G. He, Y.-Y. Keum, R.R. Volkas, JHEP **0604**, 39 (2006). arXiv:hep-ph/0601001
14. F.P. An et al., Daya Bay Collaboration, Chin. Phys. C **37**, 011001 (2013). arXiv:1210.6327
15. J.K. Ahn et al., RENO Collaboration, Phys. Rev. Lett. **108**, 191802 (2012). arXiv:1204.0626
16. Ernest Ma, Phys. Lett. B **752**, 198 (2016). arXiv:1510.02501
17. G.-J. Ding, S.F. King, A.J. Stuart, JHEP **12**, 006 (2013). arXiv:1307.4213
18. C.-C. Li, G.-J. Ding, Nucl. Phys. B **881**, 206 (2014). arXiv:1312.4401
19. D.V. Ferero, S. Morisi, J.C. Ramao, J.W.F. Valle, Phys. Rev. D **88**, 016003 (2013)
20. D.V. Forero, M. Tortola, J.W.F. Valle, Phys. Rev. D **90**, 093006 (2014). arXiv: 1405.7540
21. E. Ma, G. Rajasekaran, Phys. Rev. D **64**, 113012 (2001)

22. E. Ma, D. Wegman, Phys. Rev. Lett. **107**, 061803 (2011). arXiv:1106.4269
23. W. Grimus, Phys. Part. Nucl. **42**, 566 (2011)
24. E. Ma, Phys. Rev. D **70**, 031901 (2004). arXiv:hep-ph/0404199
25. Ernest Ma, G. Rajasekaran, Phys. Rev. D **64**, 113012 (2001). arXiv:hep-ph/0106291
26. A.M. Baldini et al., MEG Collaboration, Eur. Phys. J. C **76**, 434 (2016)
27. K. Asakura et al., KamLAND-Zen Collaboration. Nucl. Phys. A **946**, 171 (2016)
28. M. Auger et al., EXO-200 Collaboration, Phys. Rev. Lett. **109**, 032505 (2012)
29. M. Agostini et al., GERDA Collaboration, Phys. Rev. Lett. **111**(12), 122503 (2013). arXiv:1307.4720
30. P.A.R. Ade et al., Planck Collaboration, Astron. Astrophys. **594**, A13 (2016). arXiv:1502.01589