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Optical Binding Force Between Two Chiral Spheres by An Incident On-axis Gaussian Beam

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Abstract

According to the electromagnetic scattering of two spheres, the incident on-axis Gaussian beam is expanded in terms of spherical vector wave functions (SVWFs), and the beam shape coefficients are obtained by applying the localized approximation method. Using the addition theorem, the interaction scattering fields of two chiral spheres and the internal fields are also expanded in terms of SVWFs. Based on the continuous tangential boundary conditions, the scattered field coefficients are derived analytically. Utilizing the Maxwell's stress tensor integration technique, the optical binding force between two chiral spheres is formulated explicitly. Numerical simulations of the binding force are carried out. The effects of the beam width and the radius of the sphere on the force are analyzed. The numerical results are compared with the results from references.

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Keywords: Gaussian beam； chiral sphere； binding force

1. Introduction

Radiation pressure is perhaps the most commonly known manifestation of optical forces. When an electromagnetic wave is reflected off an object, momentum is imparted to the object such that the total momentum of the system (wave+object) is conserved. When two or more particles are present, the multiple scattering between the objects can, under certain conditions, lead to optically bound states. This is often referred to as optical binding, and it was first discovered by Burns *et al.* on a system of two plastic spheres in water in 1989[1]. The optical

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binding phenomenon has found wide application not only in fundamental physics but also in fields as diverse as physical chemistry, cell biology and nanotechnology. The OB force was observed[1] that when two or more particles are present in an optical field, their motion becomes coupled due to interactions between the induced electric currents and scattered fields.

Many scholars have analysed OB force quantitatively in theory. Chaumet et.al calculated the force when the incident wave is a plane wave or wave beam by using the coupled dipole method (CDM)[2-4]. Jack investigated the OB force of a two-sphere cluster utilizing the Maxwell Stress Tensor and Multiple scattering approach (MS-MST) under the illumination of a plane wave[5]. Kawano investigated the force of an array of polystyrene micronsized spheres in a dual-beam trap by using the Maxwell Stress Tensor and Generalized Multiple Technique (MST-GMT)[6]. The T-Matrix approach is also a very effective method and has been applied to the study of this problem by some researchers[7]. Zhang et.al investigated the OB force of two nanorods under plane wave illumination[8, 9].

However, the literature referred to focused on isotropic particles, and the reports on chiral particles are extremely scarce in the literature. The scattering of a large chiral sphere has been investigated in [10]. In this paper, based on MST and Generalized Lorenz-Mie Theory (GLMT), we investigate the OB force between two isotropic chiral spheres illuminated by an on-axis Gaussian beam. The effects of the beam width, radius of the sphere on the OB force are analyzed.

2. Theory formulations

2.1. Expansion of total incident fields and scattered fields

Considering two identical isotropic chiral spheres of radius a and global coordinate system $Oxyz$, the axes of the coordinate systems $O_1x_1y_1z_1$ and $O_2x_2y_2z_2$ are respectively coincident and parallel with the axes of coordinate system $Oxyz$. The coordinates of the centers of sphere 1 and sphere 2 are $(0,0,2)$ and $(0,0,0)$. As shown in Fig.1, the spheres are illuminated by an x-polarized on-axis Gaussian beam that propagates in z-direction, and the center of the beam waist is located at $(0,0,0)$.

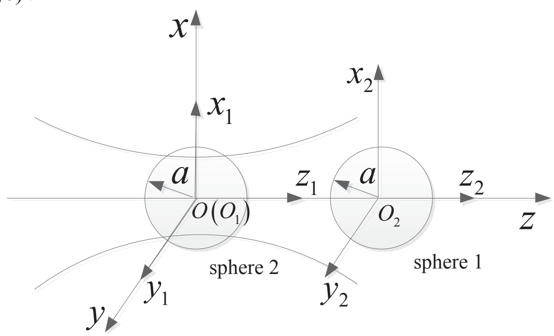

Fig. 1. Scattering of on-axis Gaussian beam from bisphere

In terms of SVWFs[11], the incident Gaussian beam can be expanded in the coordinate system $Oxyz$ as follows:

$$\begin{aligned}\mathbf{E}^i &= -\sum_{n=1}^{\infty}\sum_{m=-n}^{n} iE_{mn}\left[f_{mn}^i\mathbf{N}_{mn}^{(1)} + g_{mn}^i\mathbf{M}_{mn}^{(1)}\right]\\ \mathbf{H}^i &= -\frac{k_0}{\omega\mu_0}\sum_{n=1}^{\infty}\sum_{m=-n}^{n} E_{mn}\left[g_{mn}^i\mathbf{N}_{mn}^{(1)} + f_{mn}^i\mathbf{M}_{mn}^{(1)}\right]\end{aligned} \tag{1}$$

Here, $k_0 = 2\pi/\lambda$ and λ is the surrounding medium wavelength, μ_0 denotes the permeability of the surrounding medium, and the normalization factor E_{mn} and the incident coefficients f_{mn}^i and g_{mn}^i are defined as:

$$E_{mn} = i^n \sqrt{\frac{(2n+1)(n-m)!}{n(n+1)(n+m)!}} \tag{2}$$

$$f^i_{jmn} = E_0 i \sum_{s=-n}^{n} \rho(s,m,n) C_{ns} g^s_{n,TM}{}'' / E_{mn},\ g^i_{jmn} = -E_0 \sum_{s=-n}^{n} \rho(s,m,n) C_{ns} g^s_{n,TE}{}'' / E_{mn} \tag{3}$$

where $g^s_{n,TM}{}''$ and $g^s_{n,TE}{}''$ are the beam shape coefficients obtained by applying the local approximation of GLMT:

$$\begin{bmatrix} g^s_{n,TM}{}'' \\ ig^s_{n,TE}{}'' \end{bmatrix} = (-1)^{s-1} K_{ns} \psi'' e^{ik_0 z''} \frac{1}{2} \left[e^{i(s-1)\varphi''} J_{s-1}\left(2\frac{\bar{Q}''\rho''\rho_n}{w_0^2}\right) \pm e^{i(s+1)\varphi''} J_{s+1}\left(2\frac{\bar{Q}''\rho''\rho_n}{w_0^2}\right) \right] \tag{4}$$

where

$$\begin{gathered} \psi'' = i\bar{Q}'' \exp(-i\bar{Q}''\rho''^2 / w_0^2) \exp(-i\bar{Q}''(n+0.5)^2 / k_0^2 w_0^2), \bar{Q}'' = (i - 2z'' / (k_0 w_0^2))^{-1} \\ \rho'' = \sqrt{x''^2 + y''^2}, \rho_n = (n+0.5)/k_0,\ \varphi'' = \arctan(x''/y''),\quad k_0 = 2\pi/\lambda \end{gathered} \tag{5}$$

$$C_{ns} = \begin{cases} i^{n-1} \dfrac{2n+1}{n(n+1)} & s \ge 0 \\ (-1)^{|s|} \dfrac{(n+|s|)!}{(n-|s|)!} i^{n-1} \dfrac{2n+1}{n(n+1)} & s < 0 \end{cases},\quad K_{ns} = \begin{cases} (-i)^{|s|} \dfrac{i}{(n+0.5)^{|s|-1}}, & s \ne 0 \\ \dfrac{n(n+1)}{n+0.5}, & s = 0 \end{cases} \tag{6}$$

The scattered fields of sphere 1 can also be expanded with the SVWFs as follows:

$$\begin{aligned} \mathbf{E}^{s1} &= \sum_{n=1}^{\infty} \sum_{m=-n}^{n} iE_{mn} \left[a^s_{mn} \mathbf{N}^{(3)}_{mn} + b^s_{mn} \mathbf{M}^{(3)}_{mn} \right] \\ \mathbf{H}^{s1} &= \frac{k_0}{\omega\mu_0} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} E_{mn} \left[b^s_{mn} \mathbf{N}^{(3)}_{mn} + a^s_{mn} \mathbf{M}^{(3)}_{mn} \right] \end{aligned} \tag{7}$$

For sphere 1, the total incident fields include two parts[12]: the initial incident field $\mathbf{E}^i, \mathbf{H}^i$ and the scattered field of sphere 2 $\mathbf{E}^{s2}, \mathbf{H}^{s2}$ which can be written as

$$\mathbf{E}^{it} = \mathbf{E}^i + \mathbf{E}^{s2} \quad , \quad \mathbf{H}^{it} = \mathbf{H}^i + \mathbf{H}^{s2} \tag{8}$$

The scattered fields of sphere 2 have the same form as Eq. (7). Based on the addition theorem of SVWFs[13, 14], the total incident field of sphere 1 can be derived as follows:

$$\begin{aligned} \mathbf{E}^{it} &= -\sum_{n=1}^{\infty} \sum_{m=-n}^{n} iE_{mn} \left[a^{it}_{mn} \mathbf{N}^{(1)}_{mn}(k_0 r_1, \theta_1, \phi_1) + b^{it}_{mn} \mathbf{M}^{(1)}_{mn}(k_0 r_1, \theta_1, \phi_1) \right] \\ \mathbf{H}^{it} &= -\frac{k_0}{\omega\mu_0} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} E_{mn} \left[b^{it}_{mn} \mathbf{N}^{(1)}_{mn}(k_0 r_1, \theta_1, \phi_1) + a^{it}_{mn} \mathbf{M}^{(1)}_{mn}(k_0 r_1, \theta_1, \phi_1) \right] \end{aligned} \tag{9}$$

The corresponding coefficients are [12]:

$$a_{mn}^{it} = f_{mn}^{i} + a_{mn}^{s}(l,j) = f_{mn}^{i} - \sum_{\upsilon=1}^{\infty}\sum_{\mu=-\upsilon}^{\upsilon}\left[a_{\mu\upsilon}^{s}A_{mn}^{\mu\upsilon\prime}(2,1) + b_{\mu\upsilon}^{s}B_{mn}^{\mu\upsilon\prime}(2,1)\right]$$
$$b_{mn}^{it} = g_{mn}^{i} + b_{mn}^{s}(l,j) = g_{mn}^{i} - \sum_{\upsilon=1}^{\infty}\sum_{\mu=-\upsilon}^{\upsilon}\left[a_{\mu\upsilon}^{s}B_{mn}^{\mu\upsilon\prime}(2,1) + b_{\mu\upsilon}^{s}A_{mn}^{\mu\upsilon\prime}(2,1)\right] \tag{10}$$

Where f_{mn}^{i} and g_{mn}^{i} are initial incident coefficients, $A_{mn}^{\mu\upsilon}$ and $B_{mn}^{\mu\upsilon}$ are the so-called addition theorem coefficients.

2.2. Expansion of internal field in a chiral sphere

The media of the chiral sphere can be describes by the following constitutive relations[15]:

$$\mathbf{D} = \varepsilon_c\mathbf{E} + i\kappa\sqrt{\varepsilon_0\mu_0}\mathbf{H},\ \mathbf{B} = -i\kappa\sqrt{\varepsilon_0\mu_0}\mathbf{E} + \mu_c\mathbf{H} \tag{11}$$

where ε_c , μ_c , and κ are permittivity, permeability, and chirality parameter of the sphere, respectively; ε_0 and μ_0 denote the permittivity and permeability of free space, respectively. An electromagnetic wave with angular frequency ω in chiral media is always decomposed into two modes: the right-handed circularly polarized (RCP) wave with wave number $k_R = \omega\left(\sqrt{\mu_c\varepsilon_c} + \kappa\sqrt{\mu_0\varepsilon_0}\right)$ and the left-handed circularly polarized (LCP) wave with wave number $k_L = \omega\left(\sqrt{\mu_c\varepsilon_c} - \kappa\sqrt{\mu_0\varepsilon_0}\right)$

Based on Bohren's method[16], the expansions of the internal field of a chiral sphere can be written as[10]:

$$\mathbf{E} = \sum_{n=1}^{\infty}\sum_{m=-n}^{n}\left[A_{mn}\mathbf{M}_{mn}^{(1)}(\mathbf{r},k_R) + A_{mn}\mathbf{N}_{mn}^{(1)}(\mathbf{r},k_R) + B_{mn}\mathbf{M}_{mn}^{(1)}(\mathbf{r},k_L) - B_{mn}\mathbf{N}_{mn}^{(1)}(\mathbf{r},k_L)\right]$$
$$\mathbf{H} = Q\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\left[A_{mn}\mathbf{N}_{mn}^{(1)}(\mathbf{r},k_R) + A_{mn}\mathbf{M}_{mn}^{(1)}(\mathbf{r},k_R) + B_{mn}\mathbf{N}_{mn}^{(1)}(\mathbf{r},k_L) - B_{mn}\mathbf{M}_{mn}^{(1)}(\mathbf{r},k_L)\right] \tag{12}$$

where $Q = -i\sqrt{\varepsilon_c / \mu_c}$. A_{mn} and B_{mn} represent the unknown expansion coefficients of the internal field.

2.3. The scattering coefficients

Substituting Eqs. (7), (9), and (12) into the boundary conditions at the spherical interface yields the relationship between a_{mn}^{s}, b_{mn}^{s} and a_{mn}^{it}, b_{mn}^{it} as follows:

$$a_{mn}^{s} = A_n^{sa}a_{mn}^{it} + A_n^{sb}b_{mn}^{it},\ b_{mn}^{s} = B_n^{sa}a_{mn}^{it} + B_n^{sb}b_{mn}^{it} \tag{13}$$

$$A_n^{sa} = \frac{\psi_n(x_0)}{\xi_n(x_0)}\frac{\dfrac{D_n^{(1)}(x_1)-\eta_r D_n^{(1)}(x_0)}{\eta_r D_n^{(1)}(x_1)-D_n^{(3)}(x_0)} + \dfrac{D_n^{(1)}(x_2)-\eta_r D_n^{(1)}(x_0)}{\eta_r D_n^{(1)}(x_2)-D_n^{(3)}(x_0)}}{\dfrac{\eta_r D_n^{(3)}(x_0)-D_n^{(1)}(x_1)}{\eta_r D_n^{(1)}(x_1)-D_n^{(3)}(x_0)} + \dfrac{\eta_r D_n^{(3)}(x_0)-D_n^{(1)}(x_2)}{\eta_r D_n^{(1)}(x_2)-D_n^{(3)}(x_0)}},\ A_n^{sb} = \frac{\psi_n(x_0)}{\xi_n(x_0)}\frac{\dfrac{\eta_r D_n^{(1)}(x_1)-D_n^{(1)}(x_0)}{\eta_r D_n^{(1)}(x_1)-D_n^{(3)}(x_0)} + \dfrac{\eta_r D_n^{(1)}(x_2)-D_n^{(1)}(x_0)}{\eta_r D_n^{(1)}(x_2)-D_n^{(3)}(x_0)}}{\dfrac{\eta_r D_n^{(3)}(x_0)-D_n^{(1)}(x_1)}{\eta_r D_n^{(1)}(x_1)-D_n^{(3)}(x_0)} + \dfrac{\eta_r D_n^{(3)}(x_0)-D_n^{(1)}(x_2)}{\eta_r D_n^{(1)}(x_2)-D_n^{(3)}(x_0)}} \tag{14}$$

$$B_n^{sa} = A_n^{sb}\ ,\ B_n^{sb} = \frac{\psi_n(x_0)}{\xi_n(x_0)}\frac{\dfrac{\eta_r D_n^{(1)}(x_1)-D_n^{(1)}(x_0)}{D_n^{(1)}(x_1)-\eta_r D_n^{(3)}(x_0)} + \dfrac{\eta_r D_n^{(1)}(x_2)-D_n^{(1)}(x_0)}{D_n^{(1)}(x_2)-\eta_r D_n^{(3)}(x_0)}}{\dfrac{D_n^{(3)}(x_0)-\eta_r D_n^{(1)}(x_1)}{D_n^{(1)}(x_1)-\eta_r D_n^{(3)}(x_0)} + \dfrac{D_n^{(3)}(x_0)-\eta_r D_n^{(1)}(x_2)}{D_n^{(1)}(x_2)-\eta_r D_n^{(3)}(x_0)}} \tag{15}$$

2.4. Derivation of binding force

For sphere 1, the binding force exerted on the particle can be expressed as[16]:

$$\mathbf{F}_{binding} = \frac{1}{2}\mathrm{Re}\int_0^{2\pi}\int_0^{\pi}\left[\varepsilon E_r \mathbf{E}_1 + H_r \mathbf{H}_1 - (\frac{1}{2}\varepsilon E_1^2 + \frac{1}{2}H_1^2)\hat{e}_r\right] r^2 \sin\theta\cos\phi d\theta d\phi \tag{16}$$

where $\mathbf{E}_1 = \mathbf{E}^{it} + \mathbf{E}^{s1}$ and $\mathbf{H}_1 = \mathbf{H}^{it} + \mathbf{H}^{s1}$ are the total external fields of sphere 1. Substituting $\mathbf{E}_1$ and $\mathbf{H}_1$ into Eq. (16) , the axial and transverse binding force can be expressed as following:

$$\begin{aligned} F_x + iF_y = \frac{n_0 P_0}{\pi c k_0^2 w_0{}^2}\sum_{n=1}^{\infty}\sum_{m=-n}^{n} & [\sqrt{(n-m)(n+m+1)} N_{mn}^{-1} N_{m+1n}^{-1} (a_{mn}^{i} b_{m+1n}^{s*} \\ & + b_{mn}^{i} a_{m+1n}^{s*} + a_{mn}^{s} a_{m+1n}^{i*} + a_{mn}^{s} b_{m+1n}^{i*} + 2a_{mn}^{s} b_{m+1n}^{s*} + 2b_{mn}^{s} a_{m+1n}^{s*}) \\ & - i\sqrt{\frac{(n-m-1)(n-m)}{(2n-1)(2n+1)}}(n-1)(n+1) N_{mn}^{-1} N_{m+1n-1}^{-1} (a_{mn}^{i} a_{m+1n-1}^{s*} \\ & + b_{mn}^{i} b_{m+1n-1}^{s*} + a_{mn}^{s} a_{m+1n-1}^{i*} + b_{mn}^{s} b_{m+1n-1}^{i*} + 2a_{mn}^{s} a_{m+1n-1}^{s*} + 2b_{mn}^{s} b_{m+1n-1}^{s*}) \\ & - i\sqrt{\frac{(n+m+1)(n+m+2)}{(2n+1)(2n+3)}} n(n+2) N_{mn}^{-1} N_{m+1n+1}^{-1} (a_{mn}^{i} a_{m+1n+1}^{s*} + \\ & b_{mn}^{i} b_{m+1n+1}^{s*} + a_{mn}^{s} a_{m+1n+1}^{i*} + b_{mn}^{s} b_{m+1n+1}^{i*} + 2a_{mn}^{s} a_{m+1n+1}^{s*} + 2b_{mn}^{s} b_{m+1n+1}^{s*})] \end{aligned} \tag{17}$$

$$\begin{aligned} F_z = \frac{2n_0 P_0}{\pi c k_0^2 w_0{}^2}\mathrm{Re}\sum_{n=1}^{\infty}\sum_{m=-n}^{n} & [in(n+2)\sqrt{\frac{(n-m+1)(n+m+1)}{(2n+1)(2n+3)}} N_{mn}^{-1} N_{mn+1}^{-1} (a_{mn+1}^{i} a_{mn}^{s*} + b_{mn+1}^{s} a_{mn}^{i*} \\ & + b_{mn+1}^{i} b_{mn}^{s*} + b_{mn+1}^{s} b_{mn}^{i*} + 2a_{mn+1}^{s} a_{mn}^{s*} + 2b_{mn+1}^{s} b_{mn}^{s*}) - mN_{mn}^{-2}(a_{mn}^{i} b_{mn}^{s*} + b_{mn}^{i} a_{mn}^{s*} + 2a_{mn}^{s} b_{mn}^{s*})] \end{aligned} \tag{18}$$

where $P_0 = k\pi w_0^2 E_0{}^2/(4\omega\mu_0)$ is the power of the incident beam.

3. Numerical results and discussion

In order to verify the correctness of our theory and codes, Fig. 1(a) shows the comparison of the axial binding force between our numerical results and those in [15]. The black curve represents the axial radiation force of a chiral sphere. Since the interaction between spheres can be ignored when the distance between them are large enough. Therefore for the red curve, we put the two spheres at (0,0,0) and (0,0,2000) on z-axis. The parameters used in Fig.1(a) are as follows: the refractive index of the sphere is $n = 1.54$;the refractive index of the surrounding medium is $n_0 = 1.33$;the radius of the spheres are $a = 0.5\mu\mathrm{m}$; the power of the incident Gaussian beam is $P = 0.1\mathrm{W}$; the wavelength of the beam in vacuum is $\lambda = 0.488\mu\mathrm{m}$; and the beam waist radius is $\omega_0 = 0.5\mu\mathrm{m}$. They are in good agreement, confirming the validity of the theory and codes in this paper.

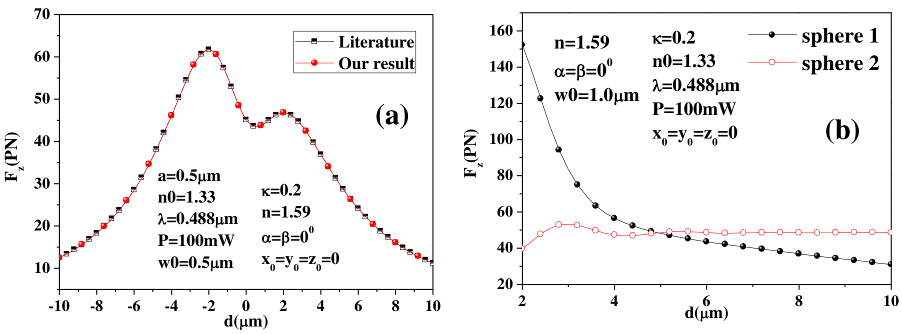

Fig. 2. Axial binding force. (a)Comparison with literature; (b) F_z exerted on sphere 1 and sphere 2

Fig.1(b) shows the axial binding force exerted on sphere 1 and sphere 2. It is clear that when $d < 5\mu\text{m}$, force of sphere 1 is larger than that of sphere 2; and when $d > 5\mu\text{m}$, force of sphere 2 is larger than that of sphere 1. This is because the center of the incident beam and the center of sphere 2 are superposable.

As the spheres are located on the z-axis, they experience a zero transverse binding force. Fig.3 shows axial binding force F_z of sphere 1 when sphere 2 stay fixedly and sphere 1 varies from 2 to 10 on z-axis. Fig. 3(a) shows F_z versus inter-particle separation d for different beam waist radii, assuming the spheres with radius $a = 0.5\mu\text{m}$, the chirality parameter $\kappa = 0.2$ and refractive index $n = 1.59$ in a surrounding medium with refractive index $n_0 = 1.33$. The beam center of the incident Gaussian with wavelength $\lambda = 0.488\mu\text{m}$ in vacuum and power $P = 0.1\text{W}$ is located at the origin. The beam waist radius ω_0 is $0.5\mu\text{m}$, $1.0\mu\text{m}$ and $1.5\mu\text{m}$, respectively. As shown in Fig.3(a), as the distance between the two spheres increases from $2\mu\text{m}$ to $10\mu\text{m}$, F_z decreases in general. And with the increase of ω_0, Fig. 3(b) shows F_z versus inter-particle separation d for different radii of sphere. The radius of sphere is $0.3\mu\text{m}$, $0.5\mu\text{m}$ and $0.6\mu\text{m}$, respectively. It is clear that F_z decreases when the radius increases. This is because when the radius increases, the two spheres become closer.

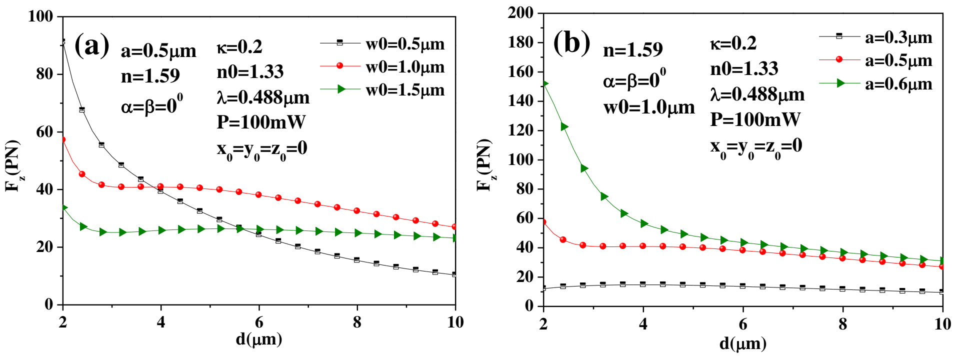

Fig. 3. Effects of beam waist radius and sphere size on axial binding force

4. Conclusion

In this paper, the initial incident fields and the scattered fields of a sphere are transformed into the total incident fields of another sphere in a chiral bisphere system. The specific form of total incident fields are given. Based on the far-field approximation, the analytical expressions of the binding force exerted on one of the two spheres are obtained by an incident on-axis Gaussian beam. We calculated the OB force exerted on both sphere 1 and sphere 2. The effects of the beam waist radius and radius of sphere on the force are numerically analyzed. The works in this paper indicate that the method we used here is reasonable and feasible, which is helpful to further research on binding force between more complex particles.

Acknowledgements

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