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Transversity Form Factors and Generalized Parton Distributions of the Pion in Chiral Quark Models

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Abstract The transversity Generalized Parton Distributions (tGPDs) and related transversity form factors of the pion are evaluated in chiral quark models, both local (Nambu–Jona–Lasinio) and nonlocal, involving a momentum-dependent quark mass. The obtained tGPDs satisfy all a priori formal requirements, such as the proper support, normalization, and polynomiality. We evaluate generalized transversity form factors accessible from the recent lattice QCD calculations. These form factors, after the necessary QCD evolution, agree very well with the lattice data, confirming the fact that the spontaneously broken chiral symmetry governs the structure of the pion also in the case of the transversity observables.

This presentation is based on our two recent papers [13, 29], where more results and details may be found. The tGPDs are the least-explored Generalized Parton Distributions (for extensive reviews see [3, 6, 31] and references therein). For the quark sector they correspond to the hadronic matrix elements of the tensor quark bilinears, $\bar{q}(x)\sigma^{\mu\nu}q(0)$, and are the maximum-helicity chirally odd objects. Information on these rather elusive entities comes from the recent lattice determination [7] of the pion transversity form factors (tFFs), defined via

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moments of tGPDs in the Bjorken x variable. That way the lattice calculations provide a direct path to verify the underlying models for quantities which are very difficult to be accessed experimentally.

Our analysis consist of two distinct parts: (1) the model determination of tFFs and tGPDs of the pion and (2) the QCD evolution. For the first part and within a non perturbative setup we apply chiral quark models which have proven to be very useful in the determination of the soft matrix elements entering various high-energy processes [1, 2, 5, 9–12, 15–17, 21, 22, 25–28, 30, 32, 33, 36, 40, 42–48, 50–52, 54, 55]. We use the standard local Nambu–Jona–Lasinio (NJL) model with the Pauli–Villars regularization [49] and two versions of the nonlocal models, where the quark mass depends on the virtuality: the instanton-motivated model [23] and the Holdom–Terning–Verbeek (HTV) model [35] (these variants differ in the form of the quark-pion vertex).

The second ingredient is the QCD evolution where renormalization improved radiative and perturbative gluonic corrections are added to the low energy matrix element. The scale where the quark model calculation is carried out can be identified with the help of the momentum fraction carried by the quarks. According to phenomenological extractions [34, 53] as well as lattice calculations [4], the valence quarks carry about 50% of the total momentum at the scale $\mu = 2\text{GeV}$. In quark models, having no explicit gluon degrees of freedom, the valence quarks carry 100% of the momentum. This allows us to fix the quark model scale, μ_0 , such that upon evolution to 2 GeV the fraction drops to $47 \pm 2\%$. The result of the LO DGLAP evolution is $\mu_0 = 313_{-10}^{+20}$ MeV. This perturbative renormalization scale is unexpectedly and rather uncomfortably low. Yet, the prescription has been favorably and independently confirmed by comparing to a variety of other high-energy data or lattice calculations (see [12] and references therein). Moreover, the NLO DGLAP modifications have been shown to yield moderate corrections as well [22]. To summarize, “*our approach = chiral quark models + QCD evolution*”.

The pion u -quark tFFs, $B_{ni}^{\pi,u}(t)$, can be defined in a manifestly covariant way (see, e.g., [24]) with the help of two auxiliary four-vectors, a and b , satisfying $a^2 = (ab) = 0$ and $b^2 \neq 0$. Then

$$\langle \pi^+(p') | \bar{u}(0) i \sigma^{\mu\nu} a_\mu b_\nu (i \overleftrightarrow{D} a)^{n-1} u(0) | \pi^+(p) \rangle = (a \cdot P)^{n-1} \frac{[a \cdot p b \cdot p']}{m_\pi} \sum_{\substack{i=0, \\ \text{even}}}^{n-1} (2\xi)^i B_{ni}^{\pi,u}(t), \quad (1)$$

with the skewness parameter defined as $\xi = -a \cdot q / (2a \cdot P)$ (we use the so-called symmetric kinematics). The symbol $\overleftrightarrow{D}^\beta = \overleftarrow{\partial}^\beta - i g A^\beta$ denotes the QCD covariant derivative, and $\overleftarrow{\partial}^\beta = \frac{1}{2}(\overrightarrow{\partial}^\beta - \overleftarrow{\partial}^\beta)$. Further, p' and p are the initial and final pion momenta, $P = \frac{1}{2}(p' + p)$, $q = p' - p$, and $t = -q^2$. The factor $1/m_\pi$ is introduced by convention in order to have dimensionless form factors. Finally, the bracket $[\dots]$ denotes the antisymmetrization in the vectors a and b . The tFFs defined in (1) apply to the u -quarks, while the tFFs for the d -quarks follow from the isospin symmetry, $B_{ni}^{\pi,d}(t) = (-1)^n B_{ni}^{\pi,u}(t)$. The definition of the corresponding tGPD is [3]

$$\langle \pi^+(p') | \bar{u}(-a) i \sigma^{\mu\nu} a_\mu b_\nu u(a) | \pi^+(p) \rangle = \frac{[a \cdot p b \cdot p']}{m_\pi} \int_{-1}^1 dX e^{-iX P \cdot a} E_T^{\pi,u}(X, \xi, t), \quad (2)$$

where we do not write explicitly the gauge link operator needed to keep the color gauge invariance. The tFFs are related to the Mellin moments of the tGPD,

$$\int_{-1}^1 dX X^{n-1} E_T^{\pi,u}(X, \xi, t) = \sum_{\substack{i=0, \\ \text{even}}}^{n-1} (2\xi)^i B_{ni}^{\pi,u}(t), \quad (3)$$

displaying the polynomiality property. Thus, the information carried by tGPDs is contained in (infinitely many) tFFs. Some of them ($n = 1, 2$) have been calculated on Euclidean lattices [7].

The details concerning the chiral quark models and parameters used have been presented in [13, 29]. The calculation of the tFFs and tGPDs is made through the use of standard techniques at the one-quark-loop level, which corresponds to the large- N_c limit with confinement neglected.

Next, we describe the LO DGLAP-ERBL evolution. For the case of tFFs the procedure is very straightforward, as it gives a triangular-matrix multiplicative structure (see, e.g., [8]). Explicitly, with the short-hand notation $B_{ni} = B_{ni}^\pi(t; \mu)$ and $B_{ni}^0 = B_{ni}^\pi(t; \mu_0)$, we have

$$\begin{aligned}
B_{10} &= L_1 B_{10}^0, \quad B_{32} = \frac{1}{5}(L_1 - L_3)B_{10}^0 + L_3 B_{32}^0, \\
&\dots \\
B_{20} &= L_2 B_{20}^0, \quad B_{42} = \frac{3}{7}(L_2 - L_4)B_{20}^0 + L_4 B_{42}^0, \\
&\dots \\
B_{30} &= L_3 B_{30}^0, \quad B_{52} = \frac{2}{3}(L_3 - L_5)B_{30}^0 + L_5 B_{52}^0, \\
&\dots
\end{aligned} \tag{4}$$

We define

$$L_n = \left(\frac{\alpha(\mu)}{\alpha(\mu_0)} \right)^{\gamma_n^T / (2\beta_0)}. \tag{5}$$

The anomalous dimensions in the transversity channel are given by $\gamma_n^T = \frac{32}{3} \sum_{k=1}^n 1/k - 8$, $\beta_0 = \frac{11}{3} N_c - \frac{2}{3} N_f$, and the running coupling constant is $\alpha(\mu) = 4\pi / [\beta_0 \log(\mu^2 / \Lambda_{\text{QCD}}^2)]$, with $\Lambda_{\text{QCD}} = 226 \text{ MeV}$ for $N_c = N_f = 3$.

The two lowest tFFs available from the lattice data, $B_{10}^{\pi,u}$ and $B_{20}^{\pi,u}$, evolve multiplicatively as follows:

$$B_{n0}^{\pi,u}(t; \mu) = B_{n0}^{\pi,u}(t; \mu_0) \left(\frac{\alpha(\mu)}{\alpha(\mu_0)} \right)^{\gamma_n^T / (2\beta_0)}, \tag{6}$$

which numerically gives

$$B_{10}^{\pi,u}(t; 2 \text{ GeV}) = 0.75 B_{10}^{\pi,u}(t; \mu_0), \quad B_{20}^{\pi,u}(t; 2 \text{ GeV}) = 0.43 B_{20}^{\pi,u}(t; \mu_0). \tag{7}$$

Note a stronger reduction for B_{20} compared to B_{10} as the result of the evolution. In the chiral limit and at $t = 0$

$$B_{10}^{\pi,u}(t=0; \mu_0) / m_\pi = \frac{N_c M}{4\pi^2 f_\pi^2}, \quad \frac{B_{20}^{\pi,u}(t=0; \mu)}{B_{10}^{\pi,u}(t=0; \mu)} = \frac{1}{3} \left(\frac{\alpha(\mu)}{\alpha(\mu_0)} \right)^{8/27}. \tag{8}$$

The results of the model calculation followed by evolution are presented in Fig. 1. We note a very good agreement with the data [7] for the NJL model, as well as for the non-local HTV model. The agreement for the instanton model is worse, which shows that the calculation can be used to discriminate between various approaches.

Next, we show the results for the full tGPD. Having explored the t -dependence in tFFs, here we set $t = 0$ and focus on the X behavior for fixed $\xi = 1/3$ or $\xi = 0$. Since the evolution is different for the symmetric and antisymmetric combinations, we explore the isovector and isoscalar tGPDs:

$$\begin{aligned}
E_T^{\pi,I=1}(X, \xi, Q^2) &\equiv E_T^{\pi,S}(X, \xi, Q^2) = E_T^\pi(X, \xi, Q^2) + E_T^\pi(-X, \xi, Q^2), \\
E_T^{\pi,I=0}(X, \xi, Q^2) &\equiv E_T^{\pi,A}(X, \xi, Q^2) = E_T^\pi(X, \xi, Q^2) - E_T^\pi(-X, \xi, Q^2).
\end{aligned}$$

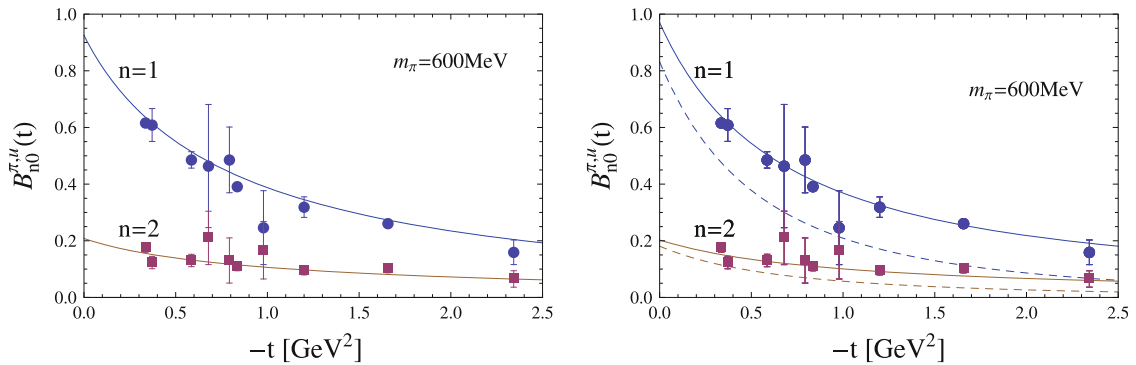


Fig. 1 The transversity form factors of the pion, $B_{10}^{\pi,u}(t)$ and $B_{20}^{\pi,u}(t)$, evaluated at $m_\pi = 600 \text{ MeV}$ in the local NJL model (*left panel*) and in nonlocal models (*right panel*, *solid line* HTV model, *dashed line* instanton model) and compared to the lattice data [7]

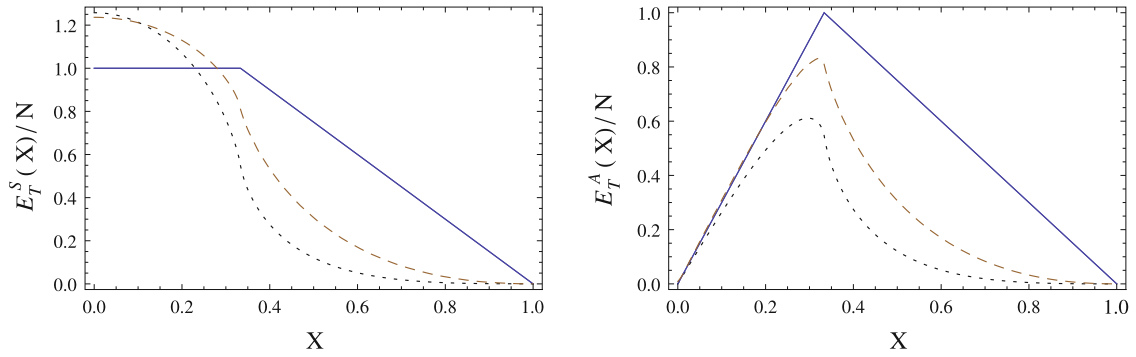


Fig. 2 The symmetric (*left panel*) and antisymmetric (*right panel*) tGPDs of the pion at $t = 0$ and $\xi = 1/3$, evaluated in the NJL model in the chiral limit at the quark-model scale $\mu_0 = 313$ MeV (*solid lines*) and evolved to the scales $\mu = 2$ GeV (*dashed lines*) and 1 TeV (*dotted lines*)

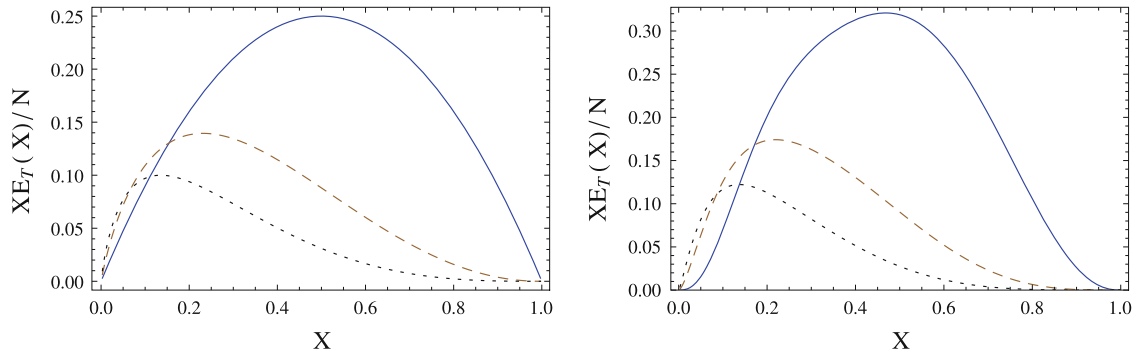


Fig. 3 The tGPD of the pion at $t = 0$ and $\xi = 0$, evaluated in the chiral limit in the NJL model (*left panel*) and in the instanton model (*right panel*) at the quark-model scale $\mu_0 = 313$ MeV (*solid lines*) and evolved to the scales $\mu = 2$ GeV (*dashed lines*) and 1 TeV (*dotted lines*)

The evolution has been carried out with the method involving the Gegenbauer moments [37–39,41]. The results for $\xi = 1/3$ in the NJL model are shown in Fig. 2. The curves are conventionally normalized in such a way that at the quark model scale

$$\int_0^1 dX E_T^{\pi,S}(X, \xi, t = 0; \mu_0)/N = \frac{1 + \xi}{2}. \quad (9)$$

We note the decreasing, shifting to lower X , effect of the QCD evolution. In Fig. 3 we compare the result in the NJL model (*left panel*) and the nonlocal instanton model (*right panel*). Except for different end-point behavior (see [29] for details), the results are qualitatively very similar.

In conclusion we wish to stress that the absolute predictions for the multiplicatively evolved B_{10} and B_{20} agree remarkably well with the lattice results, supporting the assumptions of numerous calculations following the same “chiral-quark-model + QCD evolution” scheme. It would also be interesting to access the tGPDs directly through the use of the transverse-lattice techniques [14,18–20].

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