The reviewer hopes this book will be widely read and enjoyed, and that it will be followed by other volumes telling even more of the fascinating story of Soviet mathematics. It should also be followed in a few years by an update, so that we can know if this great accumulation of talent will have survived the economic and political crisis that is just now robbing it of many of its most brilliant stars (see the article, ''To guard the future of Soviet mathematics,'' by A. M. Vershik, O. Ya. Viro, and L. A. Bokut' in Vol. 14 (1992) of *The Mathematical Intelligencer*).

Creating Modern Probability. Its Mathematics, Physics and Philosophy in Historical Perspective. By Jan von Plato. Cambridge/New York/Melbourne (Cambridge Univ. Press). 1994. 323 pp.

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Aside from the role probabilistic concepts play in modern science, the history of the axiomatic foundation of probability theory is interesting from at least two more points of view. Probability as it is understood nowadays, probability in the sense of Kolmogorov (see [3]), is not easy to grasp, since the definition of probability as a normalized measure on a σ-algebra of ''events'' is not a very obvious one. Furthermore, the discussion of different concepts of probability might help in understanding the philosophy and role of ''applied mathematics.'' So the exploration of the creation of axiomatic probability should be interesting not only for historians of science but also for people concerned with didactics of mathematics and for those concerned with philosophical questions.

Nevertheless, up to now, this history has only been written in fragments. The cover text of Jan von Plato's *Creating Modern Probability*, which tells us that this ''is the only book to chart the history and development of modern probability theory,'' is true. Von Plato has given a first thoughtful account of the creation of ''Modern Probability.''

Kolmogorov's axioms constitute a theory of probability remarkable in two ways. First of all, it is a formalistic approach. Probability is defined in an abstract way, without any interpretation concerning the meaning of this construction (at least in the axioms). It thus becomes a mathematically deep subject. This depth is the second characteristic feature of ''Modern Probability'': only from 1900 on did people start to handle the infinite in a nontrivial way. This concerns infinite sets of elementary events as well as elementary events that are infinitary themselves, necessary in the theory of stochastic processes.

Investigating the story of probability which is modern in this sense, von Plato has one main interest: ''No one had pursued the background of modern probability in any detail, so that I felt free to let my own particular interests act as my guide. As a result, the emphasis here is on foundational questions'' (p. ix). This explains why most of the more philosophical discussions are careful and well grounded,

* It is a pleasure to thank Ila Patel and Erhard Scholz (Wuppertal) for their support.

while the reader interested in the historical line of development is sometimes left alone with the task of connecting the large number of presented facts and forming his own conclusions.

Von Plato covers the time from about 1850 (first uses of probabilistic ideas in statistical mechanics) up to the 1960s (Kolmogorov's research in random sequences). Following his line of investigation, we have to distinguish three main parts of the book:

(*i*) *The Establishment of Connections between Pure Mathematics—Especially the Theory of Measure and Integration—and Probability Theory from 1900 On* (Chap. 2)

In 1900, the first measure-theoretic approaches to probability appeared in the works of two Swedish scholars, namely Torsten Bróden and Anders Wiman. They solved a problem concerning the distribution of integers in a continued fraction for randomly chosen irrationals, posed by the astronomer Hugo Gyldén, (implicitly) using measure theory. Moreover, 1900 is the year of David Hilbert's famous Paris lecture on *Mathematische Probleme.*

Von Plato carefully discusses the problem of continued fractions, mentions Hilbert's sixth problem (the axiomatization of probability), and gives some hints at early attempts to solve it, especially the dissertations of Rudolf Laemmel (Zürich 1904) and Ugo Broggi (Göttingen 1907).

His remarks in regard to Felix Hausdorff's *Grundzüge der Mengenlehre* (1914) exemplify how the reader is expected to find his own conclusion: in the *Grundzüge,* probability is taken up as an example or application of measure theory. Von Plato concludes that Hausdorff's "book was for a long time the standard reference for set theory; therefore the connection between probability and measure theory can be considered well established in the mathematical literature by 1914" (p. 35). Then he states—without any explanation—that the "applicability of Lebesgue's powerful integration theory was also obvious to many" (p. 35). So naturally there arises an important historiographical question: "Why did success not come at once, instead of twenty or thirty years later" (p. 35)? Von Plato's answer: ". . . the mathematical requirements of applied work on probability are not usually on any level that would require proper use of measure theory. What is more, measure theoretic probability proper is needed only for handling infinite sets and sequences of events. Outside the strictly infinitistic domain, to say that probability is a normalized measure is just to say that probabilities are numbers between 0 and 1 that have to be added in a special way" (p. 35–36). Nevertheless, we find more information in later chapters of the book, where we are taught how important the "infinitistic domain" was from the point of view of applications–especially for applications in physics, extensively discussed in the twenties, the years preceding Kolmogorov's success.

One more topic of chapter 2 has to be mentioned: von Plato's examination of the contributions of the French mathematician Emile Borel, known for the "denumerable probabilities" and Borel's strong law of large numbers. Carefully analyzing Borel's philosophical position, mainly his constructivistic ideas, he brings the reader to understand the meaning of Borel's approach. Borel did *not* pursue measure theory in the unit interval. Von Plato reveals several misunderstandings concerning Borel, the newest one Jack Barone and Albert Novikoff's opinion that Borel employed measure theory without realizing it (see [1]).

I only want to list the other topics of chapter 2: Strong laws of large numbers, the equidistribution problem of reals mod(1) and a discussion of Herman Weyl's views on causality. Just to mention what might have been discussed as well: the development of modern measure theory itself. Here von Plato only repeats a statement of Kolmogorov [3, III] that the abstract kind of measure used in axiomatic probability theory was first given by Frechet in 1915 (p. 27).

(*ii*) *Physical Aspects* (Chaps. 3–5)

What kinds of influences from the development of physics do we have to distinguish?

Jan von Plato discusses "Probability in statistical physics" (Chap. 3), "Quantum mechanical probability and indeterminism" (Chap. 4), and "Classical embeddings of probability and chance" (Chap. 5), the last chapter being concerned with attempts to explain the existence of probability and chance along the lines of classical physics. The key term worth mentioning here is "dynamical systems," and the most important names of scientists engaged in this field are Henri Poincare, Marian v. Smoluchowski, and Eberhard Hopf.

The most interesting chapter is on statistical physics—the longest chapter of the whole book, demanding about a quarter of the total space. Von Plato does not stress this topic that much without reason as "The theory of stochastic processes was the most profound thing statistical physics gave to probability theory" (p. 12). Stochastic processes come along with infinite-dimensional spaces of elementary events and with nondenumerable state spaces, hence with just the conceptual difficulties which were first treated successfully in "modern" probability. So "from these technical and conceptual points of view, modern probability owes more to classical statistical mechanics than to quantum theory" (p. 18).

For me, the discussion of the importance of statistical physics for probability theory is one of the highlights of the book: Nobody else has presented this important insight so clearly. The key role of statistical mechanics for the theory of probability has too long been underrated, especially compared to the role of quantum theory.

The discussion of the history of quantum mechanics (from the probabilistic point of view) begins with the ideas of Max Planck (1900) and ends with Heisenberg's *Unschärferelation* (1927), but to my mind it does not become clear what this has to do with the foundation of probability theory. The only thing von Plato has to say about this is that from "the point of view of probability theory, the most interesting aspects of quantum mechanics are its indeterminism and its phenomenon of interference" (p. 162).

Before going on to the discussion of the last part of von Plato's book, let me note some questions that could have been discussed in this context as well. What about cultural influences on the scientific conception of the world as discussed by

Paul Forman [2], who only appears in a footnote (p. 161), though much more could be said about his thoughts? What about such influences on the discussions about probability? What about the influences coming from outside physics, such as problems connected with automatic telephone exchanges, problems from biology and the social sciences?

(*iii*) *"The Final Stage, 1919–1933"* (Chaps. 6–8)

In this "final stage," there were several mathematicians who gave different—and controversially discussed—answers to the question "What is probability?" The last part of the book under discussion is devoted to the works of the three most important of them: to the frequentist probabilities of Richard von Mises (Chap. 6), to Andrei Kolmogorov's measure theoretic approach (Chap. 7), and to Bruno de Finetti, suggesting subjective probabilities (Chap. 8) instead of the two aforementioned proposals of objective probabilities.

The work of Richard von Mises is discussed in only 19 pages, so we find a good introduction to his idea about probability—the theory of random sequences ("Kollektivs"), a tool to connect mathematics with statistical physics—and the discussions it provoked until its extinction in the 1940s. Some explanations seem to be a bit confused: "By 1919 von Mises was ready to introduce randomness as a basic concept in the theory of probability" (p. 183). So according to von Plato the "question naturally arises, whether that randomness is compatible with the supposedly deterministic classical dynamics that still was the accepted basis of physics" (p. 189). This suggests that von Mises made physics probabilistic because it had to fit in with his probability theory, a suggestion explicitly stated in the introduction of the book ("Randomness, according to von Mises, is incompatible with mechanical motion, hence the need for a purely probabilistic physics," p. 13). Reading Mises' works carefully (see especially [4]), one gets the strong impression that it should be the other way round: as the old deterministic mechanical approach in some cases becomes idle, there is the need for a probabilistic approach. Only those situations in physics that do not allow calculation (because of a complicated irregular behaviour) have to be tackled probabilistically, so probability only makes sense if some irregularity is involved—hence the need for a purely random probability theory. . . .

Be that as it may, the passage on Kolmogorov is another highlight of the book: Next to remarks on Kolmogorov's philosophy of mathematics (especially his intuitionism) we find a longer discussion of his work on random processes. These motivated an important line of development towards measure theoretic probability, uniting ideas stemming from statistical mechanics (and leading to elementary events in the space Ω^R) with those about strong laws of large numbers, "'probability 1' theorems about the space Ω^N" (p. 23), as discussed in chapter 2. Thus the weight of infinite product measures—one of the real novelties of the *Grundbegriffe* and maybe the most substantial reason for their success—becomes clear immediately. Von Plato's analysis of this work was overdue for a long time, and his sensitive discussion of "The curious reappraisal of von Mises' theory"(Chap. 7.5) helps to close the circle drawn around modern objectivistic probability.

The book closes with a supplement about Nicole Oresme (c. 1325–1382) and the ergodicity of rotations, a highly useful bibliography of 28 pages (mainly original works, only some secondary sources), an index of names, and one of subjects.

Hence we find a large array of topics covered on these 323 pages, some described in the literature for the first time. A concluding chapter might have been helpful. Nevertheless for anybody interested in the history of modern probability theory this book will serve as a suitable introduction. The extensive bibliography will enable one to compose a personal reading list. So indeed the cover text gives us the right impression: "The principal audience for the book comprises philosophers and historians of science, mathematicians concerned with probability and statistics, and physicists."

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La réforme de la dynamique, De corporum concursu (1678) et autres textes inédits. By G. W. Leibniz. Edition, introduction, French translations and commentaries by Michel Fichant. Paris (Librairie philosophique J. Vrin). 1994. 445 pp. 230 FF.

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L'intérêt d'une édition de textes de Leibniz écrits en 1678 sur les chocs n'est pas à démontrer. Ces textes répondent à de nombreux questionnements d'historiens sur l'évolution de la dynamique leibnizienne entre les premiers travaux de 1671 et les textes ultérieurs de 1686, mieux connus.

Le *De corporum concursu* édité par Michel Fichant montre que Leibniz est déjà en possession de la notion de "force vive" comme force fondamentale à la fin de ce travail. Les lois des chocs écrites en 1671 pour un "pur état de nature," sans ressort et sans loi de conservation, laissent place en 1678 à des lois générales pour des chocs élastiques déduites des conservations respectives de la "force vive," de la direction du mouvement du centre de gravité, et de la vitesse relative des corps. Leibniz est alors en mesure d'exprimer la force totale des corps comme la somme d'une force de percussion qui ne dépend que de la vitesse relative, et d'une