Journal of King Saud University – Engineering Sciences (2014) 26, 69–81

King Saud University

Journal of King Saud University – Engineering Sciences

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ORIGINAL ARTICLE

Gravitation search algorithm: Application to the optimal IIR filter design

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Received 1 March 2012; accepted 18 December 2012
Available online 26 December 2012

KEYWORDS

IIR filter;
GSA;
Evolutionary optimization techniques;
Stability

Abstract This paper presents a global heuristic search optimization technique known as Gravitation Search Algorithm (GSA) for the design of 8th order Infinite Impulse Response (IIR), low pass (LP), high pass (HP), band pass (BP) and band stop (BS) filters considering various non-linear characteristics of the filter design problems. This paper also adopts a novel fitness function in order to improve the stop band attenuation to a great extent. In GSA, law of gravity and mass interactions among different particles are adopted for handling the non-linear IIR filter design optimization problem. In this optimization technique, searcher agents are the collection of masses and interactions among them are governed by the Newtonian gravity and the laws of motion. The performances of the GSA based IIR filter designs have proven to be superior as compared to those obtained by real coded genetic algorithm (RGA) and standard Particle Swarm Optimization (PSO). Extensive simulation results affirm that the proposed approach using GSA outperforms over its counterparts not only in terms of quality output, i.e., sharpness at cut-off, smaller pass band ripple, higher stop band attenuation, but also the fastest convergence speed with assured stability.

1. Introduction

Signal processing is the subject of pinnacle importance in the blossoming field of science and technology. A signal is the carrier of information and signal processing deals with the analysis and processing of signals to extract or modify the information present in them. Requirement of signal processing is of prime importance due to noise entrapment or impairment of direct usage of information within the signal. Signals are broadly classified as analog and digital ones. Analog signals are continuous functions of time and analog signal processing (ASP) is used for the analysis of such kind of signals for analog systems. On the other hand, digital signal processing (DSP) is useful for handling discrete time signal for digital systems in which filtering holds a significant position. Application of the filter is not only widely covered but also deeply rooted in its domain of utilization. Application is ranging from ripple reduction of a very simple rectifier circuit to highly sophisticated application zones of biological and astrological signal analysis along with noise reduction of raw signal, video signal enhancement and graphic equalization in hi-fi systems. Basically, a filter is a frequency

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Peer review under responsibility of King Saud University.

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http://dx.doi.org/10.1016/j.jksues.2012.12.003

selective device which extracts the useful portion of input signal lying within its operating frequency range which could be contaminated with random noise due to unavoidable circumstances. On the basis of physical makeup and the way filtration is done, filters are broadly classified as analog and digital ones. Analog filters are made up with discrete components like resistor, capacitor, inductor and op-amp. Discrete component dependent design, prone to high component tolerance sensitivity, poor accuracy, highly susceptible to thermal drift and large physical size are the major retractions of analog filter implementation. On the contrary, digital filter performs mathematical operations on a sampled, discrete timed signal to achieve the desired features with the help of a specially designed digital signal processor (DSP) chip or a processor used in a general purpose computer.

Digital filters are broadly classified into two main categories namely; finite impulse response (FIR) filter and infinite impulse response (IIR) filter (Oppenheim et al., 1999; Proakis and Manolakis, 1996). The output of FIR filter depends on present and past values of input, so the name non-recursive is aptly suited to this filter. On the other hand, the output of IIR filter depends not only on previous inputs, but also on previous outputs with impulse responses continuing forever in time at least theoretically, so the name 'recursive' is aptly suited to this filter; anyway, a large memory is required to store the previous outputs for the recursive IIR filter. Hence, due to these aspects FIR filter realization is easier with the requirement of less memory space and design complexity. Ensured stability and linear phase response over a wide frequency range are the additional advantages. On the other hand, IIR filter distinctly meets the desired specifications of sharp transition width, lower pass band ripple and higher stop band attenuation with lower order as compared to the FIR filter. As a consequence, properly designed IIR filter can meet the magnitude response close to ideal and more finely as compared to FIR filter though stability is always to be ensured. Due to these challenging features with a wide field of applications, performances of IIR filters designed with various evolutionary optimization algorithms are compared to find out the comparative effectiveness of the algorithms and the best optimal IIR filters.

In the conventional approach, IIR filters of various types (Butterworth, Chebyshev and Elliptic, etc.) can be implemented with two methods. In the first case frequency sampling technique is adopted for Least Square Error (Lang, 2000) and Remez Exchange (Jackson and Lemay, 1990) process. In the second method, filter coefficients and minimum order are calculated for a prototype low pass filter in analog domain which is then transformed into digital domain with bilinear transformation. This frequency mapping works well at low frequency, but in high frequency domain this method is liable to frequency warping (Hussain et al., 2011).

IIR filter design is a challenging optimization problem. So far, gradient based classical algorithms such as steepest descent and quasi Newton algorithms have been used aptly for the design of IIR filters (Antoniou, 2005; Lu and Antoniou, 2000). In general, these algorithms are very fast and efficient to obtain the optimum solution of the objective function for a unimodal problem. But the error surface (typically the mean square error between the desired response and estimated filter output) of IIR filter is multimodal and hence superior evolutionary optimization techniques are required to find out better global solution.

The shortfalls of classical optimization techniques for handling any multimodal optimization problem are as follows: (i) Requirement of continuous and differentiable error fitness function (cost or objective function), (ii) Usually converges to the local optimum solution or revisits the same sub-optimal solution, (iii) Incapable to search the large problem space, (iv) Requirement of the piecewise linear cost approximation (linear programming) and (v) Highly sensitive to starting points when the number of solution variables is increased and as a result the solution space is also increased.

So, it can be concluded that classical search techniques are only suitable for handling differentiable unimodal objective function with a constricted search space. So, the various evolutionary heuristic search algorithms applied for filter optimization problems in recent times are as follows: Genetic Algorithm (GA) is inspired by the Darwin's "Survival of the Fittest" strategy (Karaboga and Cetinkaya, 2004; Tsai et al., 2006; Yu and Xinjie, 2007); Simulated Annealing (SA) is designed from the thermodynamic effects (Chen et al., 2001); Artificial Immune Systems (AIS) mimic the biological immune systems (Kalinli and Karaboga, 2005); Ant Colony Optimization (ACO) simulates the ants' food searching behavior (Karaboga et al., 2004); Bee Colony Optimization mimics the honey collecting behavior of the bee swarm (Karaboga and Cetinkaya, 2011); Cat Swarm Optimization (CSO) is based upon the behavior of cats for tracing and seeking of an object (Panda et al., 2011); and Particle Swarm Optimization (PSO) simulates the behavior of bird flocking or fish schooling (Pan and Chang, 2011; Das and Konar, 2007; Fang et al., 2009; Gao et al., 2008; Sun et al., 2010; Chen and Luk, 2010; Luitel and Venayagamoorthy, 2008a,b; Mandal et al., 2011, 2012; Wang et al., 2011); in Quantum behaved PSO (QPSO) quantum behavior of particles in a potential well is adopted in conventional PSO algorithm (Fang et al., 2009; Gao et al., 2008; Sun et al., 2010); to get rid of premature convergence and stagnation, chaotic perturbation is applied on the particles in Chaos PSO (CPSO) optimization technique (Gao et al., 2008); Differential Evolution PSO (DEPSO) is the hybridization of DE and PSO in which new offspring is created by the mutation of parent (Luitel and Venayagamoorthy, 2008a,b); in Craziness based PSO (CRPSO), sudden direction changing behavior of a particle in a swarm is mimicked in the conventional velocity equation of PSO with the incorporation of 'craziness factor' (Mandal et al., 2011, 2012).

In this paper, the capability of global searching and near optimum result finding features of RGA, PSO and GSA are individually investigated thoroughly for solving 8th order IIR filter design problems. GA is a probabilistic heuristic search optimization technique developed by Holland (Holland, 1975). The features such as multi-objective, coded variable and natural selection made this technique distinct and suitable for finding the near global solution of filter coefficients.

Particle Swarm Optimization (PSO) is swarm intelligence based algorithm developed by Kennedy and Eberhart (1995) and Eberhart and Shi (1998). Several attempts have been taken to design digital filter with basic PSO and its modified versions (Pan and Chang, 2011; Das and Konar, 2007; Fang et al., 2009; Gao et al., 2008; Sun et al., 2010; Chen and Luk, 2010; Luitel and Venayagamoorthy, 2008a,b; Mandal et al., 2011, 2012; Wang et al., 2011). The main attraction of PSO is its simplicity in computation and a few steps are required in the algorithm.

The limitations of the conventional PSO are premature convergence and stagnation problem (Ling et al., 2008; Biswal et al., 2009). To overcome these problems a recently proposed evolutionary optimization technique called gravitation search algorithm (GSA) is suggested by the authors for the design of 8th order LP, HP, BP and BS IIR filters.

The paper is organized as follows: Section 2 describes the IIR filter design problem. Different evolutionary algorithms namely, RGA, PSO and GSA along their comparative results are discussed in Section 3. Section 4 discusses the simulation results obtained for the designed IIR filter using different algorithms. Finally Section 5 concludes the paper.

2. IIR filter design formulation

This section discusses the design strategy of IIR filters. The input–output relation is governed by the following difference equation (Proakis and Manolakis, 1996):

$$y(p) + \sum_{k=1}^{n} a_k y(p-k) = \sum_{k=0}^{m} b_k x(p-k) \quad (1)$$

where $x(p)$ and $y(p)$ are the filter's input and output, respectively and $n(\geqslant m)$ is the filter's order. With the assumption of coefficient $a_0 = 0$ the transfer function of the IIR filter is expressed as:

$$H(z) = \frac{\sum_{k=0}^{m} b_k z^{-k}}{1 + \sum_{k=1}^{n} a_k z^{-k}} \quad (2)$$

Let $z = e^{j\Omega}$. Then the frequency response of the IIR filter becomes

$$H(\Omega) = \frac{\sum_{k=0}^{m} b_k e^{-jk\Omega}}{1 + \sum_{k=1}^{n} a_k e^{-jk\Omega}} \quad (3)$$

or,

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{b_0 + b_1 e^{-j\Omega} + b_2 e^{-j2\Omega} + \ldots + b_m e^{-jm\Omega}}{1 + a_1 e^{-j\Omega} + a_2 e^{-j2\Omega} + \ldots + b_n e^{-jn\Omega}} \quad (4)$$

where $\Omega = 2\pi\left(\frac{f}{f_s}\right)$ in $[0, \pi]$ is the digital frequency; f is the analog frequency and f_s is the sampling frequency. Different fitness functions are used for IIR filter optimization problem (Karaboga and Cetinkaya, 2004; Luitel and Venayagamoorthy, 2008a,b). The commonly used approach to IIR filter design is to represent the problem as an optimization problem with the mean square error (MSE) as the error fitness function (Karaboga and Cetinkaya, 2004; Luitel and Venayagamoorthy, 2008a,b), expressed in (5).

$$J_1(\omega) = \frac{1}{N_s}[(d(p) - y(p))^2] \quad (5)$$

where N_s is the number of samples used for the computation of the error fitness function; $d(p)$ and $y(p)$ are the filter's desired and actual responses, respectively. The difference $e(p) = d(p) - y(p)$ is the error between the desired and the actual filter responses. The design goal is to minimize the MSE $J_1(\omega)$ with proper adjustment of coefficient vector ω represented as:

$$\omega = [a_0 a_1 \ldots a_n b_0 b_1 \ldots b_m]^T \quad (6)$$

In this paper, a novel error fitness function given in (7) is adopted in order to achieve higher stop band attenuation and to have better control on the transition width. Using (7), it is found that the filter design approach results in considerable improvement in stop band attenuation over other optimization techniques.

$$J_2(\omega) = \sum_{\omega} abs[abs(|H_d(\omega)| - 1) - \delta_p] + \sum_{\omega}[abs(|H_d(w)| - \delta_s)] \quad (7)$$

For the first term of (7), $\omega\epsilon$ pass band including a portion of the transition band and for the second term of (7), $\omega\epsilon$ stop band including the rest portion of the transition band. The portions of the transition band chosen depend on pass band edge and stop band edge frequencies.

The error fitness function given in (7) represents the generalized fitness function to be minimized using the evolutionary algorithms RGA, conventional PSO, and the proposed GSA individually. Each algorithm tries to minimize this error fitness $J_2(\omega)$ and thus optimizes the filter performance. Unlike other error fitness functions as given in Karaboga and Cetinkaya (2004) and Luitel and Venayagamoorthy, 2008a,b which consider only the maximum errors, $J_2(\omega)$ involves summation of all absolute errors for the whole frequency band, and minimization of $J_2(\omega)$ yields much higher stop band attenuation and lesser pass band and stop band ripples.

3. Evolutionary optimization algorithms employed

Evolutionary optimization algorithms stand upon the platform of heuristic search methods, which are characterized by features as stochastic, adaptive and learning in order to produce intelligent optimization schemes. Such schemes have the potential to adapt to their ever changing dynamic environment through the previously acquired knowledge. Few such efficient algorithms are discussed for the purpose of designing as well as comparison of performances for handling the optimization problem of design of IIR filters.

3.1. Real coded genetic algorithm (RGA)

Standard Genetic Algorithm (also known as real coded GA) is mainly a probabilistic search technique, based on the principles of natural selection and evolution built upon the Darwin's ''Survival of the Fittest'' strategy (Holland, 1975). Each encoded chromosome that constitutes the population is a solution to the filter designing optimization problem.

Steps of RGA as implemented for the optimization of coefficient vector ω are as follows (Mondal et al., 2010, 2011; Mandal et al., 2012):

Step 1: Initialize the real coded chromosome strings (ω) of n_p population, each consisting of equal number of numerator and denominator filter coefficients b_k and a_k, respectively; total coefficients = $(n + 1) \times 2$ for nth order filter to be designed; minimum and maximum values of the coef-

ficients are −2 and +2, respectively; number of samples = 128; $\delta_p = 0.01$, $\delta_s = 0.001$.
Step 2: Decoding of the strings and evaluation of error fitness values $J_2(\omega)$ according to (7).
Step 3: Selection of elite strings in order of increasing error fitness values from the minimum value.
Step 4: Copying the elite strings over the non-selected strings.
Step 5: Crossover and mutation generate offsprings.
Step 6: Genetic cycle updating.
Step 7: The iteration stops when maximum number of cycles is reached. The grand minimum error fitness and its corresponding chromosome string or the desired solution having $(n+1)\times 2$ number of filter coefficients are finally obtained.

3.2. Particle Swarm Optimization (PSO)

PSO is flexible, robust, population based stochastic search algorithm with attractive features of simplicity in implementation and ability to quickly converge to a reasonably good solution. Additionally, it has the capability to handle larger search space and non-differential objective function, unlike traditional optimization methods. Kennedy and Eberhart (1995) and Eberhart and Shi (1998) developed PSO algorithm to simulate random movements of bird flocking or fish schooling.

The algorithm starts with the random initialization of a swarm of individuals, which are known as particles within the multidimensional problem search space, in which each particle tries to move toward the optimum solution, where next movement is influenced by the previously acquired knowledge of particle best and global best positions once achieved by individual and the entire swarm, respectively.

To some extent, IIR filter design and other designs with PSO are already reported in Pan and Chang (2011), Das and Konar (2007), Fang et al. (2009), Gao et al. (2008), Sun et al. (2010), Chen and Luk (2010), Luitel and Venayagamoorthy (2008a,b) and Mandal et al. (2011, 2012).

The basic steps of the PSO algorithm are as follows (Mandal et al., 2011, 2012):

Step 1: Initialize the real coded particles (ω) of n_p population, each consists of equal number of numerator and denominator filter coefficients b_k and a_k, respectively; total coefficients $D = (n+1)\times 2$ for nth order filter to be designed; minimum and maximum values of the coefficients are −2 and +2, respectively; number of samples = 128; $\delta_p = 0.01$; $\delta_s = 0.001$; V_{max} and V_{min} values.
Step 2: Compute the error fitness value for the current position S_i of each particle.
Step 3:Each particle can remember its best position (*pbest*) which is known as cognitive information and that would be updated with each iteration.
Step 4: Each particle can also remember the best position the swarm has ever attained (*gbest*) and is called social information and would be updated in each iteration.
Step 5: Velocity and position of each particle are modified according to (8) and (10), respectively (Kennedy and Eberhart, 1995).

$$V_i^{(k+1)} = w * V_i^{(k)} + C_1 * rand_1 * \{pbest_i^{(k)} - S_i^{(k)}\} + C_2 * rand_2 * \{gbest_i^{(k)} - S_i^{(k)}\} \quad (8)$$

$$\text{where} \quad V_i = V_{max} \quad for \quad V_i > V_{max} = V_{min} \quad for \quad V_i < V_{min} \quad (9)$$

$$S_i^{(k+1)} = S_i^{(k)} + V_i^{(k+1)} \quad (10)$$

Step 6: The iteration stops when maximum number of cycles is reached. The grand minimum error fitness and its corresponding particle or the desired solution having $(n+1)\times 2$ number of filter coefficients are finally obtained.

3.3. Gravitation search algorithm (GSA)

To overcome the problems of parameter convergence and stagnation associated with PSO (Ling et al., 2008; Biswal et al., 2009), this paper adopts one recently proposed novel evolutionary optimization algorithm known as gravitational search algorithm (GSA) for the purpose of IIR digital filter design.

In GSA (Rashedi et al., 2009, 2011; Bahrololoum et al., 2012), agents/solution vectors are considered as objects and their performances are measured by their masses. All these objects attract each other by the gravity forces, and these forces produce a global movement of all objects towards the objects with heavier masses. Hence, masses cooperate using a direct form of communication through gravitational forces. The heavier masses (which correspond to better solutions) move more slowly than lighter ones. This guarantees the exploitation step of the algorithm.

Three kinds of masses are defined in theoretical physics:

(a) *Active gravitational mass* (M_a) is a measure of the strength of the gravitational field due to a particular object. Gravitational field of an object with small active gravitational mass is weaker than the object with more active gravitational mass.
(b) *Passive gravitational mass* (M_p) is a measure of the strength of an object's interaction with the gravitational field. Within the same gravitational field, an object with a smaller passive gravitational mass experiences a smaller force than an object with a larger passive gravitational mass.
(c) *Inertial mass* (M_i) is a measure of an object's resistance to changing its state of motion when a force is applied. An object with large inertial mass changes its motion more slowly, and an object with small inertial mass changes it rapidly.

In GSA, each mass (agent) has four specifications: position, inertial mass, active gravitational mass, and passive gravitational mass. The position of the mass corresponds to the solution of the problem, and its gravitational and inertial masses are determined using a fitness function. In other words, each mass presents a solution, the algorithm is navigated by properly adjusting the gravitational, and inertial masses. By lapse of iteration cycles, it is expected that masses be attracted by

Table 1 Design specifications of IIR LP, HP, BP and BS filters.

Type of Filter	Pass band ripple (δ_p)	Stop band ripple (δ_s)	Pass band normalized edge frequencies (ω_p)	Stop band normalized edge frequencies (ω_s)
LP	0.01	0.001	0.45	0.50
HP	0.01	0.001	0.35	0.30
BP	0.01	0.001	0.35 and 0.65	0.3 and 0.7
BS	0.01	0.001	0.25 and 0.75	0.3 and 0.7

Table 2 Control parameters of RGA, PSO and GSA.

Parameter	RGA	PSO	GSA
Population size	120	25	25
Iteration cycles	500	500	500
Crossover rate	1.0	–	–
Crossover	Single Point Crossover	–	–
Mutation rate	0.01	–	–
Mutation	Gaussian Mutation	–	–
Selection	Roulette	–	–
Selection probability	1/3	–	–
C_1	–	2.05	–
C_2	–	2.05	–
v_i^{min}	–	0.01	–
0_i^{max}	–	1.0	–
w_{max}	–	1.0	–
w_{min}	–	0.4	–
α	–	–	20
G_0	–	–	1000
rNORM	–	–	2
rPower	–	–	1
ε	–	–	0.0001

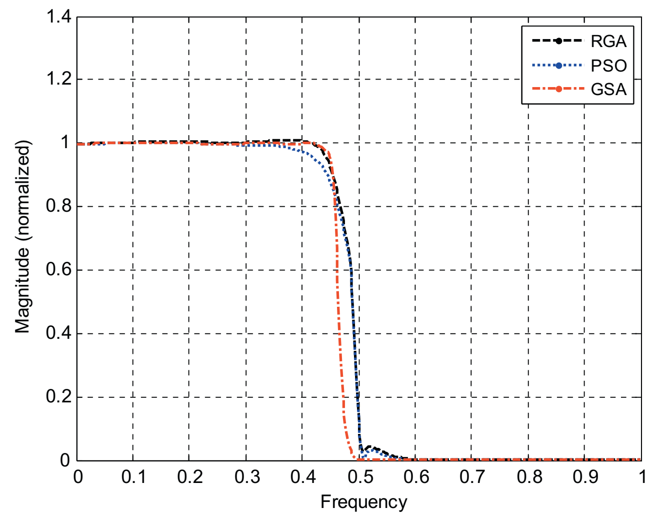

Figure 2 Normalized gain plots for 8th order IIR LP filter using RGA, PSO and GSA.

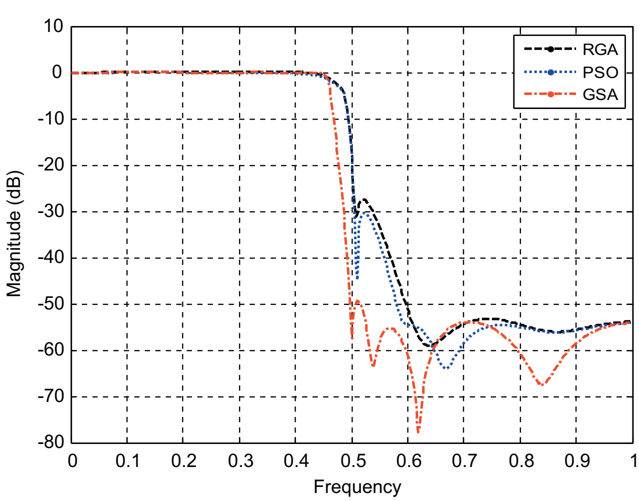

Figure 1 Gain plots in dB for 8th order IIR LP filter using RGA, PSO and GSA.

the heaviest mass. This heaviest mass will present an optimum solution in the search space.

The GSA could be considered as an isolated system of masses. It is like a small artificial world of masses obeying the Newtonian laws of gravitation and motion. More precisely, masses obey the following two laws.

i. *Law of gravity:* Each particle attracts every other particle and the gravitational force between two particles is directly proportional to the product of their masses and inversely proportional to the square of the distance (R) between them. R is used as R^{rPower} ($rPower = 1$) because R offered better results than R^2 in all the experimental cases with benchmark functions.
ii. *Law of motion:* The current velocity of any mass is equal to the sum of the fraction of its previous velocity and the variation in the velocity. Variation in the velocity or acceleration of any mass is equal to the force acted on the system divided by the mass of inertia.

Now, let us consider a system with N agents (masses). The position of the ith agent is defined by

$$X_i = (x_i^1, \ldots\ldots x_i^d, \ldots\ldots x_i^n) \quad \text{for} \quad i = 1, 2, \ldots\ldots N \tag{11}$$

where x_i^d presents the position of ith agent in the dth dimension.

At a specific iteration cycle t, the force acting on ith mass from jth mass is defined as in the following equation

$$F_{ij}^d(t) = G(t)\frac{M_{pi}(t) \times M_{aj}(t)}{R_{ij}(t) + \varepsilon}\left(X_j^d(t) - X_i^d(t)\right) \tag{12}$$

where $M_{aj}(t)$ is the active gravitational mass related to the jth agent at iteration cycle t; $M_{pi}(t)$ is the passive gravitational mass related to the jth agent at iteration cycle t; $G(t)$ is gravitational constant at iteration cycle t; ε is a small constant, and

Table 3 Optimized coefficients and performance comparison of concerned algorithms for 8th order IIR LP filter.

Algorithm	Numerator Coefficients (b_k)	Denominator Coefficients (a_k)	Maximum Stop band attenuation (dB)
RGA	0.0415 0.1234 0.2676 0.3806 0.4206 0.3484 0.2164 0.0925 0.0233	0.9994 −1.1555 2.7421 −2.3022 2.4552 −1.4037 0.7776 −0.2480 0.0524	27.5145
PSO	0.0413 0.1241 0.2668 0.3791 0.4202 0.3478 0.2165 0.0936 0.0235	1.0001 −1.1546 2.7413 −2.3016 2.4547 −1.4044 0.7781 −0.2483 0.0519	30.3635
GSA	0.0298 0.0778 0.1680 0.2378 0.2717 0.2340 0.1596 0.0739 0.0261	1.0001 −1.6888 3.3754 −3.4260 3.2805 −2.0249 1.0290 −0.3239 0.0594	49.3552

$R_{ij}(t)$ is the Euclidian distance between the two agents i and j given by (13).

$$R_{ij}(t) = \|X_i(t), X_j(t)\|_{rNorm} \quad rNorm \text{is usually 2} \tag{13}$$

To give a stochastic characteristic to the algorithm, it is expected that the total force that acts on ith agent in dth dimension be a randomly weighted sum of dth components of the forces exerted from other agents given by (14).

$$F_i^d(t) = \sum_{j=1, j\neq i}^{N} rand_j F_{ij}^d(t) \tag{14}$$

where $rand_j$ is a random number in the interval [0,1].

Hence, by the law of motion, the acceleration of the ith agent at iteration cycle t, and in dth dimension, $a_i^d(t)$ is given by (15).

$$a_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)} \tag{15}$$

where $M_{ii}(t)$ is the inertial mass of the ith agent.

Furthermore, the next velocity of an agent is considered as a fraction of its current velocity added to its acceleration. Therefore, its position and its velocity can be calculated by employing (16) and (17), respectively.

$$v_i^d(t+1) = rand_i \times v_i^d(t+1) + a_i^d(t) \tag{16}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \tag{17}$$

In (16), $rand_i$ is a uniform random variable in [0, 1]. This random number is utilized to give a randomized characteristic to the search. The gravitational constant (G) is initialized at the beginning and will be reduced with iteration cycle to control the search accuracy. In other words, G as a function of the initial value (G_0) and iteration cycle (t) is expressed as in (18).

Table 6 Radii of zeros for 8th order IIR LP filter.

Algorithm	Zeros			
	Z1	Z2	Z3	Z4
GSA	0.937409	1.009397	0.983378	1.006001

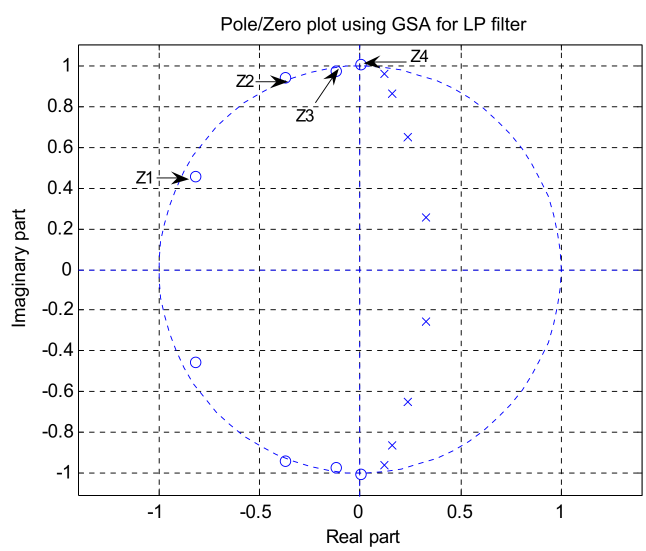

Figure 3 Pole-zero plot of 8th order IIR LP filter using GSA.

Table 4 Statistical data for stop band attenuation (dB) for 8th order IIR LP filter.

Algorithm	Maximum	Mean	Variance	Standard deviation
RGA	27.5145	40.3870	165.7013	12.8725
PSO	30.3635	46.5478	130.9666	11.4441
GSA	49.3552	53.1923	5.1670	2.2731

Table 5 Qualitatively analyzed data for 8th order IIR LP filter.

Algorithm	Maximum pass band ripple (normalized)	Stop band ripple (normalized)			Transition width
		Maximum	Minimum	Average	
RGA	0.0095	4.2100×10^{-2}	15.7130×10^{-4}	2.1836×10^{-2}	0.0297
PSO	0.0021	3.0300×10^{-2}	6.2811×10^{-4}	1.5464×10^{-2}	0.0338
GSA	0.0028	0.3406×10^{-2}	1.2959×10^{-4}	0.1768×10^{-2}	0.0400

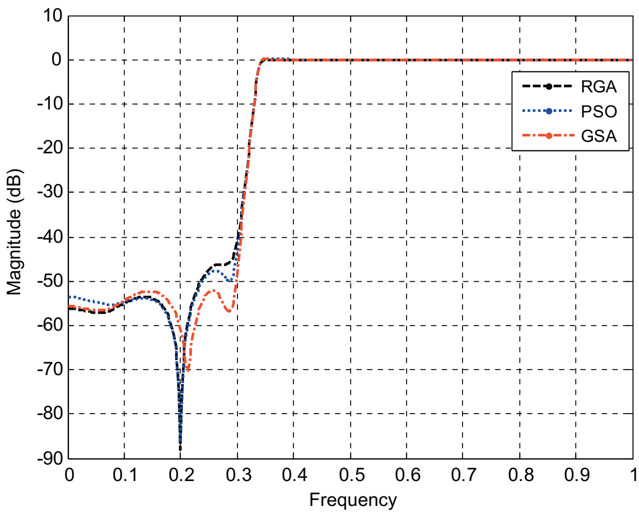

Figure 4 Gain plots in dB for 8th order IIR HP filter using RGA, PSO and GSA.

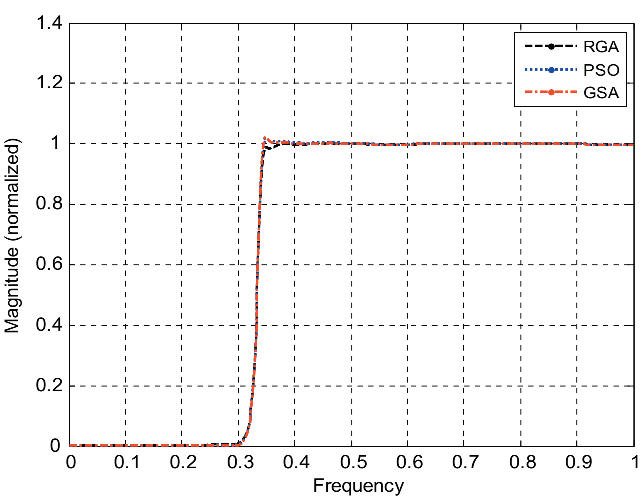

Figure 5 Normalized gain plots for 8th order IIR HP filter using RGA, PSO and GSA.

Table 8 Statistical data for stop band attenuation (dB) for 8th order IIR HP filter.

Algorithm	Maximum	Mean	Variance	Standard deviation
RGA	46.2199	49.8589	13.2467	2.6391
PSO	47.7018	50.7807	9.4796	3.0789
GSA	52.1714	53.3914	2.7025	1.6439

$$G = G_0 \exp\left(-\alpha * \left(\frac{t}{\text{maxcycles}}\right)\right) \tag{18}$$

Gravitational and inertia masses are simply calculated by the error fitness evaluation as defined by (7). A heavier mass means a more efficient agent. This means that better agents have higher attractions and walk more slowly. Assuming the equality of the gravitational and the inertia mass, the values of masses are calculated using the map of error fitness values. Gravitational and inertial masses are updated by the following equations:

$$M_{ai} = M_{pi} = M_{ii} \quad \text{for} \quad i = 1, 2, \ldots\ldots\ldots N \tag{19}$$

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \tag{20}$$

$$M_i(t) = \frac{m_i(t)}{\sum_{1}^{N} m_i(t)} \tag{21}$$

where $fit_i(t)$ represents the error fitness value of the ith agent at iteration cycle t, and $worst(t)$ and $best(t)$ are defined in (22) and (23), respectively, for the minimization problem as considered in this work.

$$best(t) = \min_{j \in \{1,\ldots N\}} fit_j(t) \tag{22}$$

$$worst(t) = \max_{j \in \{1,\ldots N\}} fit_j(t) \tag{23}$$

One way to perform a good compromise between exploration and exploitation is to reduce the number of agents with lapse of time in (14). Hence, it is supposed that a set of agents with bigger masses apply their forces to the other. However, this policy is to be adopted carefully because it may reduce the exploration power and increase the exploitation capability.

Table 7 Optimized coefficients and performance comparison of concerned algorithms for 8th order IIR HP filter.

Algorithm	Numerator coefficients (b_k)	Denominator coefficients (a_k)	Maximum stop band attenuation (dB)
RGA	0.1250 −0.7092 1.9588 −3.3672 3.9090 −3.1264 1.6821 −0.5585 0.0881	0.9999 −2.1875 3.8221 −3.6220 2.9095 −1.3332 0.5678 −0.0861 0.0285	46.2199
PSO	0.1252 −0.7091 1.9587 −3.3671 3.9091 −3.1263 1.6821 −0.5584 0.0881	1.0001 −2.1874 3.8222 −3.6220 2.9096 −1.3333 0.5678 −0.0861 0.0285	47.7018
GSA	0.1252 −0.7092 1.9587 −3.3672 3.9090 −3.1264 1.6820 −0.5585 0.0882	0.9999 −2.1875 3.8222 −3.6220 2.9095 −1.3333 0.5679 −0.0861 0.0285	52.1714

Table 9 Qualitatively analyzed data for 8th Order IIR HP filter.

Algorithm	Maximum pass band ripple (normalized)	Stop band ripple (normalized)			Transition width
		Maximum	Minimum	Average	
RGA	0.0146	0.48863×10^{-2}	0.39587×10^{-4}	0.24629×10^{-2}	0.0598
PSO	0.0186	0.41201×10^{-2}	0.47667×10^{-4}	0.20839×10^{-2}	0.0500
GSA	0.0207	0.24628×10^{-2}	3.11350×10^{-4}	0.13871×10^{-2}	0.0518

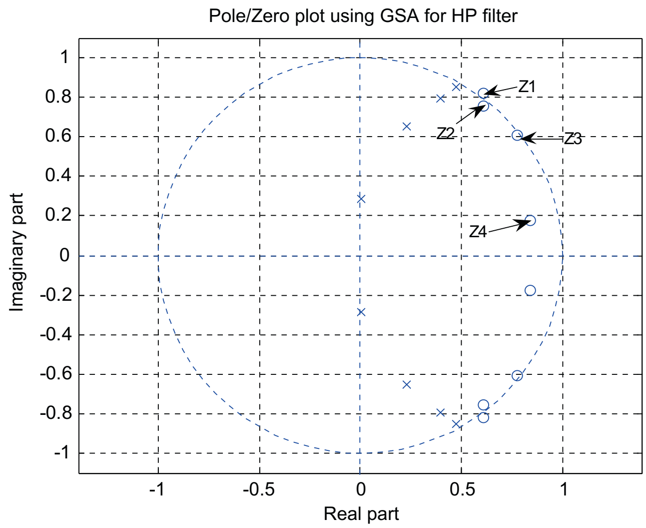

Figure 6 Pole-zero plot of 8th order IIR HP filter using GSA.

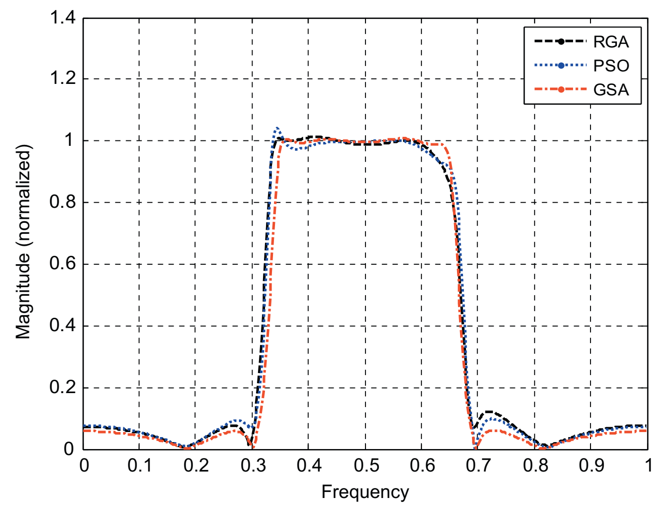

Figure 8 Normalized gain plots for 8th order IIR BP filter using RGA, PSO and GSA.

Table 10 Radii of zeros for 8th Order IIR HP filter.

Algorithm	Zeros			
	Z1	Z2	Z3	Z4
GSA	1.024214	0.967517	0.986437	0.858579

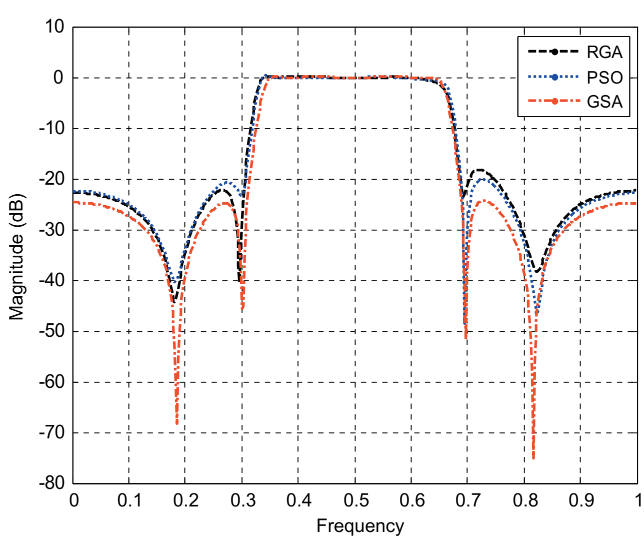

Figure 7 Gain plots in dB for 8th order IIR BP filter using RGA, PSO and GSA.

In order to avoid trapping into a local optimum, the algorithm must use the exploration at the beginning. By lapse of iterations, exploration must fade out and exploitation must fade in. To improve the performance of GSA by controlling exploration and exploitation only the *Kbest* agents will attract the others. *Kbest* is a function of iteration cycle with the initial value K_0 and it decreases with iteration cycle. In such a way, all agents apply the force at the beginning, and as iteration cycle progresses, *Kbest* is decreased linearly. At the end, there will be just one agent applying force to the others. Therefore, (14) could be modified as (24).

$$F_i^d(t) = \sum_{j \in Kbest,\ j \neq i} rand_j F_{ij}^d(t) \tag{24}$$

In (24), *Kbest* is the set of first *K* agents with the minimum error fitness values and the biggest masses.

3.3.1. Implementation of the GSA for the IIR filter design problem

The steps of the GSA, as implemented for the solution of IIR filter design carried out in this work, are shown below:

Step 1: Initialization: population (swarm size) of agent vectors, $n_p = 25$; maximum iteration cycles = 500; for equal number of numerator and denominator coefficients b_k and a_k, respectively; total coefficients = $(n + 1) \times 2$ for *n*th order filter to be designed; minimum and maximum values of the coefficients are -2 and $+2$, respectively; number

Table 11 Optimized coefficients and performance comparison of concerned algorithms for 8th order IIR BP filter.

Algorithm	Numerator coefficients (b_k)	Denominator coefficients (a_k)	Maximum stop band attenuation (dB)
RGA	0.1369 −0.0069 −0.0200 −0.0043 0.1897 0.0069 −0.0338 −0.0056 0.1253	0.9971 −0.0075 1.5866 −0.0094 1.7020 0.0000 0.8246 −0.0025 0.2247	18.2445
PSO	0.1274 0.0071 −0.0209 0.008 0.1857 0.0001 −0.0292 −0.0052 0.1299	0.9927 −0.002 1.5940 0.0029 1.6978 −0.0002 0.8079 −0.0034 0.2058	20.1389
GSA	0.1040 −0.0003 −0.0158 0.0006 0.1543 0.0005 −0.0162 −0.0003 0.1043	1.0005 −0.0000 1.7574 0.0003 1.8299 0.0004 0.8934 −0.0008 0.2168	24.3104

Table 12 Statistical data for stop band attenuation (dB) for 8th order IIR BP filter.

Algorithm	Maximum	Mean	Variance	Standard deviation
RGA	18.2445	20.3032	4.2382	2.0587
PSO	20.1389	21.4826	1.8054	1.3437
GSA	24.3104	24.5265	0.0467	0.2161

Table 14 Radii of zeros for 8th order IIR BP filter.

Algorithm	Zeros			
	Z1	Z2	Z3	Z4
GSA	0.999874	1.000812	1.001116	0.999676

of samples = 128; δ_p = 0.01, δ_s = 0.001; α = 20; G_0 = 1000; rNorm = 2; rPower = 1; initial velocities = zeros (n_p, $(n + 1) \times 2$); ε = 0.0001.

Step 2: Generate initial agent vectors n_p having $(n + 1) \times 2$ number of filter coefficients $h(n)$ randomly within limits.

Step 3: Computation of error fitness values of the total population, n_p, as defined by (7).

Step 4: Computation of the population based best solution (h_{gbest}) vector.

Step 5: Update $G(t)$, $best(t)$, $worst(t)$ and $M_i(t)$ for i = 1, 2, ..., n_p; t is current iteration cycle.

Step 6: Calculation of the total forces in different directions.

Step 7: Calculation of accelerations and velocities of agents.

Step 8: Updating agents' positions.

Step 9: Repeat Steps 3–8 until the stopping criterion (either maximum iteration cycles or near global optimal solution or agent, h_{gbest}) is met.

Finally, h_{gbest} is the vector of optimal filter coefficients $(n + 1) \times 2$. Extensive simulation study has been individually performed for comparison of optimization performances of three algorithms RGA, PSO and GSA, respectively, for the 8th order LP, HP, BP and BS IIR filter optimization problems. The design specifications followed for all algorithms are given in Table 1.

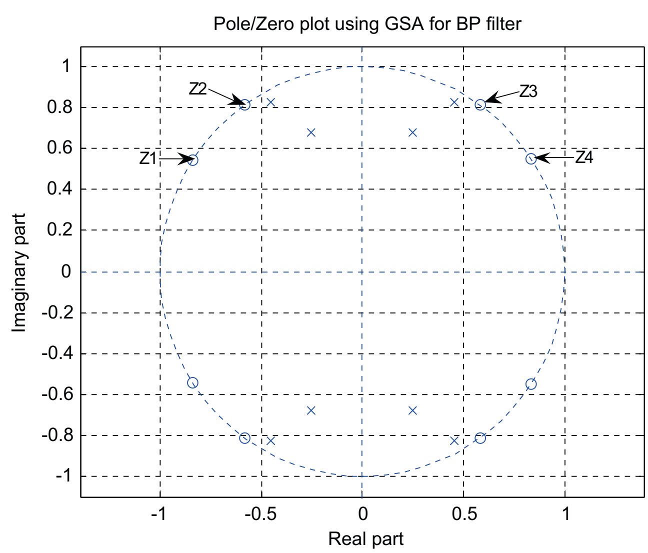

Figure 9 Pole-zero plot of 8th order IIR BP filter using GSA.

4. Results and discussions

The values of the control parameters of RGA, PSO and GSA are given in Table 2. Each algorithm is run for 30 times to get the best solution and the best results are reported in this paper.

Table 13 Qualitatively analyzed data for 8th order IIR BP filter.

Algorithm	Maximum pass band ripple (normalized)	Stop band ripple (normalized)			Transition width
		Maximum	Minimum	Average	
RGA	0.0134	12.24×10^{-2}	12.0000×10^{-3}	6.7200×10^{-2}	0.0311
PSO	0.0399	9.84×10^{-2}	3.7771×10^{-3}	5.1089×10^{-2}	0.0277
GSA	0.0130	6.09×10^{-2}	0.1756×10^{-3}	3.0538×10^{-2}	0.0366

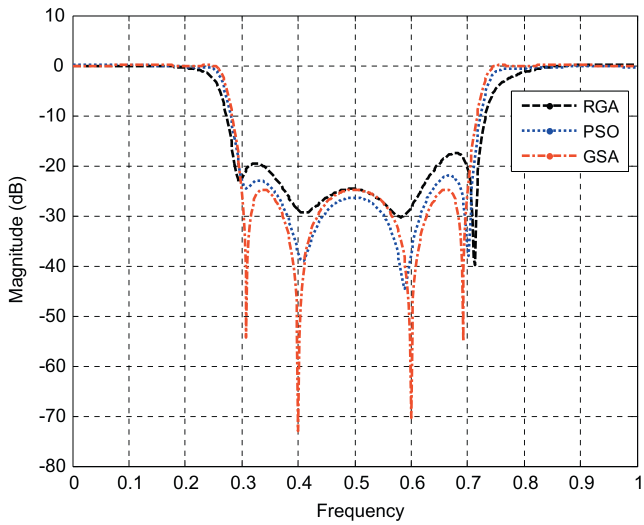

Figure 10 Gain plots in dB for 8th order IIR BS filter using RGA, PSO and GSA.

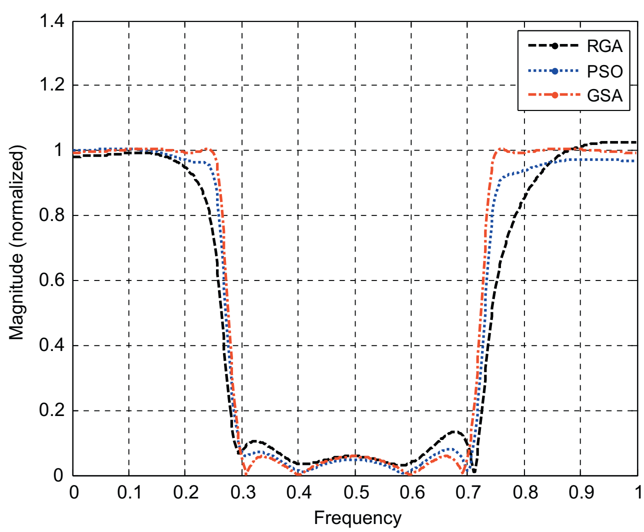

Figure 11 Normalized gain plots for 8th order IIR BS filter using RGA, PSO and GSA.

All optimization programs are run in MATLAB 7.5 version on core (TM) 2 duo processor, 3.00 GHz with 2 GB RAM.

Table 16 Statistical data for stop band attenuation (dB) for 8th order IIR BS filter.

Algorithm	Maximum	Mean	Variance	Standard deviation
RGA	17.4734	21.0867	13.0559	3.6133
PSO	21.9740	24.1658	4.8038	2.1918
GSA	24.7606	24.7761	0.0001	0.0116

Three aspects of the algorithms are investigated in this work namely, their accuracy, speed of convergence and stability. Fig. 1 shows the comparative gain plots in dB for the designed 8th order IIR LP filter obtained for different algorithms. Fig. 2 represents the comparative normalized gain plots for 8th order IIR LP filter. The best optimized numerator coefficients (b_k) and denominator coefficients (a_k) obtained are reported in Table 3. It is observed that maximum stop band attenuations 27.5145 dB, 30.3635 dB and 49.3552 dB are obtained for RGA, PSO and GSA algorithms, respectively. Gain plots and Tables 4 and 5 also explore that the proposed 8th order IIR filter design using GSA attains the highest stop band attenuation, the lowest stop band ripple, variance and standard deviation with appreciably small transition width and pass band ripple against the results produced by the rest algorithms (See Table 5). Table 6 shows the radii of zeros for the designed 8th order IIR LP filter.

Fig. 3 shows the pole-zero plot for 8th order IIR LP filter designed with the GSA algorithm. This figure demonstrates the existence of poles within the unit circle which ensures the bounded input bounded output (BIBO) stability condition. Fig. 4 shows the comparative gain plot in dB for the 8th order IIR HP filter with the individual application of RGA, PSO and GSA optimization techniques, respectively. Fig. 5 represents the comparative normalized gain plots for 8th order IIR HP filter. The best optimized numerator coefficients (b_k) and denominator coefficients (a_k) obtained are reported in Table 7. It has been observed that maximum stop band attenuations 46.2199 dB, 47.7018 dB and 52.1714 dB are obtained for RGA, PSO and GSA algorithms, respectively. Gain plots and Tables 8 and 9 also prove that the proposed optimization technique, GSA attains the highest stop band attenuation, lowest stop band ripple along with the smallest variance and standard deviation compared to the results produced by others.

Fig. 6 shows the pole-zero plot of 8th order IIR HP filter designed with the GSA optimization technique. It is noticed that all poles are within the unit circle which ensures the stability condition of designed filter. Radii of zeros located above the real part of z plane are shown in Table 10.

Table 15 Optimized coefficients and performance comparison of concerned algorithms for 8th order IIR BS filter.

Algorithm	Numerator coefficients (b_k)	Denominator coefficients (a_k)	Maximum stop band attenuation (dB)
RGA	0.2269 −0.0189 0.5039 0.0170 0.6409 −0.0136 0.4866 0.0093 0.2189	1.0190 −0.0067 0.0968 0.0109 0.8671 0.0180 −0.0322 0.0177 0.1182	17.4734
PSO	0.2142 −0.0058 0.4833 −0.0008 0.6503 0.0097 0.4976 0.0041 0.2091	1.0073 −0.0069 0.0980 −0.0077 0.8902 −0.0073 −0.0198 −0.0048 0.1089	21.9740
GSA	0.2215 0.0000 0.5175 0.0001 0.6995 −0.0000 0.5172 0.0001 0.2211	1.0000 −0.0001 0.1572 0.0000 0.9085 0.0000 0.0055 −0.0001 0.1181	24.7606

Table 17 Qualitatively analyzed data for 8th order IIR BS filter.

Algorithm	Maximum Pass band ripple (normalized)	Stop band ripple (normalized)			Transition width
		Maximum	Minimum	Average	
RGA	0.0268	13.38×10^{-2}	30.6000×10^{-3}	8.2200×10^{-2}	0.0535
PSO	0.0303	7.97×10^{-2}	5.8373×10^{-3}	4.2769×10^{-2}	0.0377
GSA	0.0063	5.78×10^{-2}	0.2207×10^{-3}	2.9010×10^{-2}	0.0395

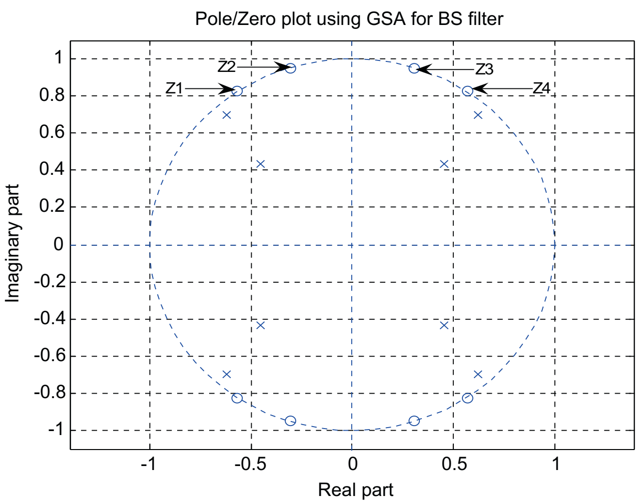

Figure 12 Pole-zero plot of 8th order IIR BS filter using GSA.

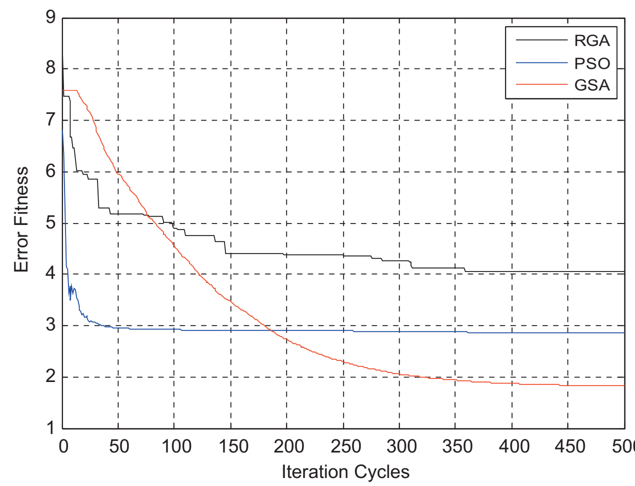

Figure 13 Convergence profiles for RGA, PSO and GSA for 8th order IIR LP filter.

Table 18 Radii of zeros for 8th order IIR BS filter.

Algorithm	Zeros			
	Z1	Z2	Z3	Z4
GSA	1.000006	0.999196	0.999862	0.999923

Comparative gain plots in dB are demonstrated in Fig. 7. Fig. 8 also represents the comparative normalized gain plots for 8th order IIR BP filter designed by the optimization techniques. The best optimized numerator coefficients (b_k) and denominator coefficients (a_k) obtained are reported in Table 11. It is observed that the maximum stop band attenuations 18.2445 dB, 20.1389 dB and 24.3104 dB are obtained for RGA, PSO and GSA optimization techniques, respectively. Gain plots and Tables 12 and 13 also indicate that the proposed 8th order IIR filter design using GSA attains the highest stop band attenuation, lowest pass band and stop band ripples, variance and standard deviation with significantly small transition width as compared to the results produced by others.

Fig. 9 shows the pole-zero plot of 8th order IIR BP filter designed with GSA optimization technique. The designed filter is stable due to the location of poles within the unit circle. Radii of zeros located above the real part of z plane are reported in Table 14.

Fig. 10 shows the comparative gain plot in dB for the 8th order IIR BS filter with the application of RGA, PSO and

Table 19 Comparison of performance criteria attained by other reported works.

Reference	Algorithm considered	Filter type	Order	Stop band attenuation (dB)	Pass band ripple	Stop band ripple	Transition width
Karaboga and Cetinkaya (2004)	GA	LP	10th	14	–	–	–
Gao et al. (2008)	CPSO	LP	8th	34	–	–	–
Luitel and Venayagamoorthy (2008a,b)	PSO, DE-PSO	LP	9th	25, 22	–	–	
Luitel and Venayagamoorthy (2008a,b)	PSO, PSO-QI	LP	9th	22, 27	–	–	–
Wang et al. (2011)	LS-MOEA	LP, HP, BP, BS	11th	–	–	0.12, 0.16, 0.15, 0.05	–
Present paper	GSA	LP, HP, BP, BS	8th	49.3552, 52.1714, 24.3104, 24.7606	0.0028, 0.0207, 0.0130, 0.0063	0.3406×10^{-2}, 0.2462×10^{-2}, 6.09×10^{-2}, 5.78×10^{-2}	0.04, 0.0518, 0.0366, 0.0395

Table 20 Convergence profile results for RGA, PSO and GSA for 8th order low pass IIR filter.

Algorithm	Minimum error value	Iteration cycles	Convergence speed (per cycle)	Execution time for 100 cycles (s)
RGA	4.054	500	8.608×10^{-3}	7.795833
PSO	2.850	500	7.918×10^{-3}	5.693405
GSA	1.825	500	11.530×10^{-3}	4.037103

GSA optimization techniques, respectively. Fig. 11 represents the comparative normalized gain plots for 8th order IIR BS filter. The best optimized numerator coefficients (b_k) and denominator coefficients (a_k) obtained after extensive simulation study are reported in Table 15. It has been observed that maximum stop band attenuations 17.4734 dB, 21.9740 dB and 24.7606 dB are obtained for RGA, PSO and GSA algorithms, respectively. Gain plots and Tables 16 and 17 also explore that the proposed optimization technique, GSA attains the highest stop band attenuation, smallest variance and standard deviation with lowest pass band and stop band ripples and appreciably small transition width compared to the results produced by others.

Fig. 12 shows the pole-zero plot of 8th order IIR BS filter designed with GSA optimization technique. The designed filter is stable due to the location of poles within the unit circle. Radii of zeros located above the real part of z plane are reported in Table 18. Table 18 dictates the same inference of non minimum phase characteristic of the filter.

It is observed from Table 4 that the maximum stop band attenuations 27.5145 dB, 30.3635 dB, and 49.3552 dB are obtained for RGA, PSO, and GSA algorithms, respectively, for 8th order IIR LP filter design. In Gao et al. (2008) applied CPSO technique for designing 8th order IIR LP filter and reported maximum stop band attenuation of approximately 34 dB; in this work the proposed algorithm GSA shows much more stop band attenuation. Luitel et al. reported the design of 9th order IIR LP filter using PSO and PSO-QI and approximate attenuations of 22 dB and 27 dB, respectively, have been achieved in Luitel and Venayagamoorthy, 2008a,b. Luitel et al. in (Luitel and Venayagamoorthy, 2008a,b) reported for 9th order IIR LP filter using PSO and DEPSO in which maximum attenuations approximately of 25 dB and 22 dB, respectively, have been reported. In this paper, maximum attenuation obtained for PSO is higher even though it is designed with lower order. In Karaboga and Cetinkaya (2004) have reported for 10th order minimum phase IIR LP filter of maximum attenuation approximately of 14 dB when GA is employed. In this work the maximum attenuation of 27.5145 dB for RGA with lower order is achieved. In (Wang et al., 2011) Wang et al. have reported for the maximum stop band ripple of approximately 0.12, 0.16, 0.15 and 0.05 for 11th order IIR LP, HP, BP and BS filters when LS-MOEA technique is adopted. In this study with RGA improved maximum results are obtained. The GSA yields the improved stop band ripples even with lower order IIR LP, HP, BP and BS filters, respectively, as 0.3406×10^{-2}, 0.24628×10^{-2}, 6.09×10^{-2} and 5.78×10^{-2}. The aforementioned results can be verified from Table 19.

4.1. Comparative effectiveness and convergence profiles of RGA, PSO and GSA

In order to compare the algorithms in terms of the error fitness values, Fig. 13 depicts the comparative convergences of error fitness values obtained by RGA, PSO and the GSA for the 8th order IIR LP filter. Similar plots are also obtained for the rest filters, which are not shown.

As shown in Fig. 13, RGA converges to the minimum error fitness value of 4.054 in 38.9791 s; PSO converges to the minimum error fitness value of 2.850 in 28.4670 s; whereas, GSA converges to the minimum error fitness value of 1.825 in 20.185515 s. The above-mentioned execution times may be verified from Table 20. Similar observations can be made for the rest filters, which are not shown. Table 20 summarizes the convergence profile results for RGA, PSO and GSA applied for the design of IIR LP filter.

From Fig. 13 it can be concluded that the proposed algorithm GSA obtains the minimum error fitness value as compared to PSO and RGA. It is also noticed that the proposed algorithm, GSA has the faster rate of convergence in terms of sharp reduction in error fitness value shown in Fig. 13, compared to the rest error fitness curves obtained by RGA and PSO algorithms for obtaining the optimum results. With a view to the above fact, it may finally be inferred that the performance of the GSA is the best among all the mentioned algorithms. All optimization programs are run in MATLAB 7.5 version on core (TM) 2 duo processor, 3.00 GHz with 2 GB RAM.

5. Conclusion

In this paper, a recently developed evolutionary optimization algorithm, GSA, based on interaction of masses and guided by the law of gravity has been applied to the optimal designing of 8th order low pass, high pass, band pass and band stop IIR digital filters. The optimal filters thus obtained meet the stability criterion and show the best attenuation characteristics with reasonably good transition widths. The GSA converges very fast to the best quality optimal solution and reaches the lowest minimum error fitness value in a moderately low execution time. Statistically improved results obtained for the GSA also justify the potential of the proposed algorithm for the realization of digital IIR filters.

References

Antoniou, A., 2005. Digital Signal Processing: Signals, Systems and Filters. McGraw Hill.

Bahrololoum, A., Nezamabadi-pour, H., Bahrololoum, H., Saeed, M., 2012. A prototype classifier based on gravitational search algorithm. Appl. Soft Comput. 12 (2), 819–825.

Biswal, B., Dash, P.K., Panigrahi, B.K., 2009. Power quality disturbance classification using fuzzy c-means algorithm and adaptive particle swarm optimization. IEEE Trans. Ind. Electron. 56 (1), 212–220.

Chen, S., Luk, B.L., 2010. Digital IIR filter design using particle swarm optimization. Int. J. Model. Ident. Control 9 (4), 327–335.

Chen, S., Istepanian, R., Luk, B.L., 2001. Digital IIR filter design using adaptive simulated annealing. Digit. Signal Process. 11 (3), 241–251.

Das, S., Konar, A., 2007. A swarm intelligence approach to the synthesis of two-dimensional IIR filters. Eng. Appl. Artif. Intel. 20 (8), 1086–1096.

Eberhart, R., Shi, Y.,1998. Comparison between genetic algorithm and particle swarm optimization. In: Evolutionary Programming VII, Springer, pp. 611-616.

Fang, W., Sun, J., Xu, W., 2009. A new mutated quantum-behaved particle swarm optimization for digital IIR filter design. EURASIP J. Adv. Signal Process., 1–7.

Gao, Y., Li, Y., Qian, H., 2008. The design of IIR digital filter based on chaos particle swarm optimization algorithm. IEEE Int. Conf. Genet. Evol. Comput., 303–306.

Holland, J.H., 1975. Adaptation in Natural and Artificial Systems. University of Michigan Press, Ann Arbor.

Hussain, Z.M., Sadik, A.Z., O'Shea, P., 2011. Digital Signal Processing – An Introduction with MATLAB Applications. Springer-Verlag.

Jackson, L.B., Lemay, G.J., 1990. A simple Remez exchange algorithm to design IIR filters with zeros on the unit circle. IEEE Int. Conf. Acoust. Speech Signal Process. USA 3, 1333–1336.

Kalinli, A., Karaboga, N., 2005. Artificial immune algorithm for IIR filter design. Eng. Appl. Artif. Intel. 18 (8), 919–929.

Karaboga, N., Cetinkaya, B., 2004. Design of minimum phase digital IIR filters by using genetic algorithm. In: Proc. IEEE 6th Nordic Signal Processing Symposium, June 2004, Finland, pp. 29–32.

Karaboga, N., Cetinkaya, M.H., 2011. A novel and efficient algorithm for adaptive filtering: artificial bee colony algorithm. Turk. J. Elec. Eng. Comp. Sci. 19 (1), 175–190.

Karaboga, N., Kalinli, A., Karaboga, D., 2004. Designing digital IIR filters using ant colony optimization algorithm. Eng. Appl. Artif. Intel. 17 (3), 301–309.

Kennedy, J., Eberhart, R., 1995. Particle swarm optimization. Proc. IEEE Int. Conf. on Neural Network 4, 1942–1948.

Lang, M.C., 2000. Least-squares design of IIR filters with prescribed magnitude and phase responses and pole radius constraint. IEEE Trans. Signal Process. 48 (11), 3109–3121.

Ling, S.H., Iu, H.H.C., Leung, F.H.F., Chan, K.Y., 2008. Improved hybrid particle swarm optimized wavelet neural network for modelling the development of fluid dispensing for electronic packaging. IEEE Trans. Ind. Electron. 55 (9), 3447–3460.

Lu, W.-S., Antoniou, A., 2000. Design of digital filters and filter banks by optimization: a state of the art review. In: Proc. European Signal Processing Conf., vol. 1, September 2000, Finland, pp. 351–354.

Luitel, B., Venayagamoorthy, G.K., 2008. Particle swarm optimization with quantum infusion for the design of digital filters. In: IEEE Swarm Intelligence Symposium, September 2008, St. Louis, MO USA, pp. 1–8.

Luitel, B., Venayagamoorthy, G.K., 2008a. Differential evolution particle swarm optimization for digital filter design. IEEE Congr. Evol. Comput., 3954–3961.

Mandal, S., Ghoshal, S.P., Kar, R., Mandal, D., 2011. Optimal linear phase FIR band pass filter design using craziness based particle swarm optimization algorithm. J. Shanghai Jiaotong Univ. (Science), Springer 16 (6), 696–703.

Mandal, S., Ghoshal, S.P., Kar, R., Mandal, D., 2012. Design of optimal linear phase FIR high pass filter using craziness based particle swarm optimization technique. J. King Saud Univ. Comput. Inf. Sci. (Elsevier) 24, 83–92.

Mandal, S., Mandal, D., Kar, R., Ghoshal, S.P., 2012. Non-recursive FIR band pass filter optimization by improved particle swarm optimization. In: Int. Conf. on Information Systems Design and Intelligent Applications-2012, (Springer-Verlag), AISC 132, India, pp. 405–412.

Mondal, D., Ghosal, S.P., Bhattacharya, A.K., 2011. Application of evolutionary optimization techniques for finding the optimal set of concentric circular antenna array. Expert Syst. Appl. (Elsevier) 38 (4), 2942–2950.

Mondal, D., Ghosal, S.P., Bhattacharya, A.K., 2010. Radiation pattern optimization for concentric circular antenna array with central element feeding using craziness based particle swarm optimization. International Journal of RF and Microwave Computer-Aided Engineering (Wiley) 20 (5), 577–586.

Oppenheim, A.V., Schafer, R.W., Buck, J.R., 1999. Discrete-Time Signal Processing. Prentice-Hall, NJ.

Pan, S.T., Chang, C.Y., 2011. Particle swarm optimization on D-stable IIR filter design. In: IEEE World Congress on Intelligent Control Automation, WCICA '11, Taipei, pp. 621–626.

Panda, G., Pradhan, P.M., Majhi, B., 2011. IIR system identification using cat swarm optimization. Expert Syst. Appl. 38 (10), 12671–12683.

Proakis, J.G., Manolakis, D.G., 1996. Digital Signal Processing. Prentice-Hall, NJ.

Rashedi, E., Nezamabadi-pour, H., Saryazdi, S., 2009. GSA: a gravitational search algorithm. Inf. Sci. (Elsevier) 179 (13), 2232–2248.

Rashedi, E., Nezamabadi-Pour, H., Saryazdi, S., 2011. Filter modelling using gravitational search algorithm. Eng. Appl. Artif. Intell. (Elsevier) 24 (1), 117–122.

Sun, J., Fang, W., Xu, W., 2010. A quantum-behaved particle swarm optimization with diversity-guided mutation for the design of two-dimensional IIR digital filters. IEEE Trans. Circuits and Systems-II 57 (2), 141–145.

Tsai, J.T., Chou, J.H., Liu, T.K., 2006. Optimal design of digital IIR filters by using hybrid Taguchi genetic algorithm. IEEE Trans. Ind. Electron. 53 (3), 867–879.

Wang, Y., Li, B., Chen, Y., 2011. Digital IIR filter design using multi-objective optimization evolutionary algorithm. Appl. Soft Comput. (Elsevier) 11, 1851–1857.

Yu, Y., Xinjie, Y., 2007. Cooperative co-evolutionary genetic algorithm for digital IIR filter design. IEEE Trans. Ind. Electron. 54 (3), 1311–1318.