Development of scientific bases of the dynamics of machines as a section of applied mechanics

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Abstract

The characteristic of the inertia forces within the classification of forces during solving the problems of the mechanics of machines was given. This characteristic is based on the simultaneous and unconditional compliance with the laws of Newton and the law of the energy conservation. It is proved that every potential field of motion of a physical body corresponds to a potential velocity field and the poles of these fields are located in the same point in space. For an absolutely rigid body moving with a variable velocity the inertia force is the internal potential force and potential of the inertia force is the velocity of the physical body.

Keywords: mechanics, applied mechanics, the mechanics of machines, classification of forces, the force of inertia.

1. Introduction

All variety of problems solved in mechanics as a science “...the motion of the substance” [2] “... is based on a number of basic physical concepts such as time, space, simultaneity, mass, force” [2]. However, there are incorrect results from the point of view of classical mechanics in terms of non-compliance of the solution to the law of conservation of energy and one of Newton's laws when solving a number of problems of dynamics of machines by the methods of mathematical modeling [3-6]. This is primarily connected with the violation of the fundamental ideas during determining the various forces in mechanical systems taking into account the energy changes during mechanical work performing. Nowadays the question whether forces of inertia real or not is still actual [5-11].

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Without force of inertia no analysis of the dynamics of modern high-speed machines is made. Moreover, there are a clear definition of the inertial force\(^2\), methods of calculation and use for the determination of machines dynamics but the question whether forces of inertia real of not remains open [4-9,12].

2. The problem formulation

The paper proposes to consider a question about the forces of inertia in mechanics of machines, their definition and classification from the point of view of Applied Mechanics as a branch of the mechanics associated with the solution of engineering (practical) problems while moving and deforming the certain mechanical systems in a limited region of space for a certain period of time when using certain sources of mechanical energy with “non-living” source [13]. This point of view with the correct solution of problems of mechanics of machines (Applied Mechanics) [12] based on the fundamental principles of Classical Mechanics described in the three laws of Newton [3] concerning the law of conservation of energy [14,15] and based on the following conditions [13,16,17]:

- the force \(P_i\) (fig. 1) can be balanced by another force only;
- the force \(P\) can’t be applied to the body\(^4\), if the body does not resist to this force;
- the force \(P\) can perform the mechanical work \(\Delta P = PsP\) connected with displacement of the point of application \((s_p)\)if the source of energy of force \(E_P\) can change its own mechanical energy \(\Delta E_P\neq0\);
- change of the mechanical energy of the source of the force is the work produced by the force by moving the point of application of force respectively to that position which places its energy source;

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1 “Mechanics is an area of science about mechanical movement and mechanical interaction of physical bodies” [1].
2 “Force of inertia is a force that equals to the product of the mass of a particle and its acceleration. This force is directed opposite to the acceleration” [1].
4 Here and hereinafter the term “body” (“part of a body”) is a physical object, the volume of it \(V_m\) (\(\Delta V_m < V_m\)) is finite and can be less than any certain small amount of space \(\Delta V_a\) (\(\Delta V_m < \Delta V_a \to 0\)).
to the body moving with the certain velocity can’t be applied external driving force more than in the certain period of time provides the source of energy of that force during displacement of the point of application with velocity equal to the velocity of the movable body.

It is proposed [16,17] classification of forces (see fig. 1 and table 1) to solve the problems of mechanics of machines (Applied Mechanics). It is based on the normative terminology of [1,17] and the scientific literature [3-5, 13-15, 18-19].

### Table 1. Characteristics of forces of Applied Mechanics.

<table>
<thead>
<tr>
<th>Force (P) is vector quantity which is a measure of the mechanical action of one physical body on another.</th>
</tr>
</thead>
<tbody>
<tr>
<td>External force is a force acting on any physical body of mechanical system from the side which does not belong to this system. The source energy of external force located beyond the limits of the system of bodies.</td>
</tr>
<tr>
<td>Internal force is a force acting on any physical body (point of the body) of a mechanical system from the side of another physical bodies (body point) belonging to the considered mechanical system (the body). The source of energy of internal force is placed inside of this system of bodies.</td>
</tr>
<tr>
<td>Active forces perform mechanical work (Ap≠0) connected with displacement and (or) deformation of the body on which these forces act.</td>
</tr>
<tr>
<td>Driving force (P_G or F, G_F, ( \Phi_F, T_F )) is a force the point of application of which to the movable body is moving in the direction of the line of action of force on the body (Ap&gt;0).</td>
</tr>
<tr>
<td>Resistance force (P_Q or G_Q, ( \Phi_Q, T_Q )) is a force the point of application of which to the movable body is moving in the direction opposite to the direction of action of force on the body (Ap&lt;0).</td>
</tr>
<tr>
<td>Potential force - is a force with fixed point of application (s_F≠s≠0) respectively to the surface (volume) of this body (( \nu_F≠0 )) and the direction of action of this force does not change. Non-potential force (F(n), Q(n)) is a force the point of application of which to the movable body is moving respectively to the point of surface (volume) of this body (s_F≠s, s_Q≠s) and (or) the direction of action of this force changes its position in space (0&lt;A_p(n)&lt;A_p, 0&gt;A_Q(n)&lt;A_Q).</td>
</tr>
<tr>
<td>Passive forces acting on body do not perform mechanical work (Ap=0) that needs for displacement and (or) deformation of this body:</td>
</tr>
<tr>
<td>Dissipative force (P(D)) is a force acting on body in such a way that point of application of this force moves (( s_F≠s≠0 )) relatively to the surface (volume) of the fixed body on which it acts; reaction force (R) is a force acting on a fixed body and the point of application of which does not move (s_F=s=0) respectively to the point of surface (volume) of this body; normal force (N) is a force acting on body in the direction perpendicular to the direction of motion of this body.</td>
</tr>
</tbody>
</table>

### 3. Resolving the problem

The condition of linear motion with acceleration of the body \( m \) is defined by the Newton’s second law\(^5\)

\[
P(t) \sim \Delta(m(t)v(t))
\]

For the absolutely rigid body \( m(t)=\text{const} \) the equation (1) can be represented as

\(^5\) "Lex II: Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur”. This was translated quite closely in Motte’s 1729 translation as: “Law II: The alteration of motion is ever proportional to the motive force impress’d; and is made in the direction of the right line in which that force is impress’d”. The author highlighted in bold the word of Newton which indicates that Newton's second law defines only a quality relationship between the change of the linear momentum and applied driving force.
where \( k(t) \) is a coefficient of proportionality which equals \( k(t) = t \) [17-19] and represents the duration of action of the driving force in the direction of the line along which the linear momentum of the body \( m \) changes. If \( t \rightarrow dt \) then \( \Delta v(t) 
rightarrow dv \) and equation (2) will look like this

\[
P(t)k(t) = m\Delta v(t),
\]

\( (2) \)

Considering \( \dot{x}(t) = \phi(\dot{x}(t), t) \) from (3) must be

\[
P(t) = m(\ddot{x} + \ddot{x}).
\]

\( (4) \)

Analysis of (4) shows:

\[
P(t) = \begin{cases} m(\ddot{x} + \ddot{x}) = \text{const} \iff (\ddot{x} \neq 0); \\ m\ddot{x} = \text{const} \iff (\ddot{x} = 0). \end{cases}
\]

\( (5) \)

Non uniform linear motion of body \( m \) with constant acceleration \( (\ddot{x} > 0) \) or deceleration \( (\ddot{x} < 0) \) is possible at constant value and direction of the driving force \( P_F \) only. The equation (5) determined from the Newton’s second law does not give the characteristics of the driving force \( P \) and this requires additional analysis of the equation (5). In the work [18] at \( \ddot{x} = \text{const} \) received an equation which in the mentioned above notation will be represented in the form

\[
P = \pm m\ddot{x}.
\]

\( (6) \)

With the correct solution of problems of Applied Mechanics in the right side of equation (6) can be the force balancing the external force \( P \) only. If such an external force is absent then to balance the external force \( P \) in the equation (6) can be internal force of the body \( m \) only. In conditions of non uniform motion according to [1] such a force is the force of inertia

\[
\pm m\ddot{x} = \mp \Phi.
\]

\( (7) \)

Let us investigate (fig. 2) the process of linear motion of body with mass \( m = \rho V_m = \rho lbh \) (\( \rho \) - density, \( lbh \) - sizes of the body \( m \)) with width \( b = l \) on the horizontal surface “without friction” (with rollers \( r \)) with acceleration \( a_m = \ddot{x} > 0 \) (fig. 2, a) while this body is driving by the external force \( P_F = F \) applied to the body \( m \) at the point \( A \) on the surface of the body \( (F_A = F_A^{(e)}) \). Another case: the motion of the body with deceleration \( a_m < \ddot{x} < 0 \) (fig. 2, b) while this body is driving by the external resistance force \( P_0 = Q^{(e)} \) applied to the body \( m \) at the point \( B \) \( (Q^{(e)} = Q_B^{(e)}) \). To characterize the parameters of the forces of the body \( m \) under the influence of external forces we will use a fixed coordinate system \( XOY \) which is connected directly \( (x_{EF}) \) or indirectly with fixed source of energy of external driving force \( E_F^{(e)} \) (fig. 2, a) or resistance force \(-E_Q^{(e)} \) \((x_{EQ} \) on the fig. 2, b). To characterize the internal forces acting in the body \( m \) with length \( l \) we will use moveable coordinate system \( X'OY' \) with axis \( O'Y' \) that goes through the center mass of body \( m \) (in the middle of length \( l \) with \( m(x) = m(x) = \text{const} \) parallel to the axis \( OY \) and axis \( O'X' \) in the direction of

\[\text{In Newton’s second law is not indicated the characteristics of the driving force which can be either an external force or internal. The change of linear momentum of the body in the direction of line of action of the driving force can be either with a plus sign (movement with an acceleration) and with a minus sign (movement with a deceleration).}\]
velocity \( v_m \) of motion of body \( m \). The relationship between the coordinate systems is determined by the equation

\[
x = x_0 + \frac{x}{l},
\]

where \( x_0 \) – coordinate of centre of mass of the body \( m \) in the coordinate system \( XOY \); \( x' = x_0' \) – coordinate of material point of the body (element of the body \( dm = (m/l)dx \)). This coordinate defines the position of \( dm \) in the coordinate system \( X'O'Y' \) (fig. 2, a, b). It does not matter for the kinematics of the body (element of the body) whether it is an external force or a component of the internal force acting on the body (element of the body). If the vectors of these forces at any given period of time are equal (Newton’s second law) from (8) and fig. 2, a must be

\[
\vec{F}^{(c)} = \vec{F}_A^{(c)} = \vec{F}^{(c)}(x_A) = \vec{F}^{(i)}(x_A'),
\]

In its turn the influence of the internal driving force \( F^{(i)}(x_A') \) in accordance with Newton’s third law7 will be balanced by the internal resistance force \( \phi^{(i)}(x_A') \) [9]

\[
\vec{F}^{(i)}(x_A') = -\phi^{(i)}(x_A') = -\phi^{(i)}(x_A).
\]

To determine the law of distribution of the internal forces in the body \( m \) let us select (fig. 2, a) at a distance \( x' \) from center of mass of this body element \( dm \). The forces \(-\Phi^{(i)}(x') = F^{(i)}(x') = F^{(i)}(x') \) and \( F^{(i)}(x+dx) = F^{(i)}(x'+dx') = -\phi^{(i)} \).

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7 “Lex III: Actioni contrariam semper et æqualem esse reactionem: sive corporum duorum actiones in se mutuo semper esse æquales et in partes contrarias dirigis”. This was translated quite closely in Motte’s 1729 translation as: “Law III: The alteration of motion is ever proportional to the motive force impress’d; and is made in the direction of the right line in which that force is impress’d”.

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Fig. 2. (a): the scheme of action of forces at non uniform linear motion of the body \( m \) with constant acceleration; (b): the scheme of action of forces at non uniform linear motion of the body \( m \) constant deceleration
\( \Phi_0^{(i)}(x^t + dx^t) \) (on fig. 2, a these forces are not shown) will act on the side surfaces of this element in points b and a accordingly to the equation (10). Consequently the condition of motion of element \( dm \) with uniform acceleration (fig. 2, a):

\[
\begin{align*}
    d\tilde{F}^{(t)}(x^t) &= d\tilde{F}^{(e)} = const > 0; \\
    d\Phi_0^{(i)}(x^t) &= -d\tilde{F}^{(i)}(x^t) = const < 0.
\end{align*}
\] (11)

From (11) taking into account the boundary conditions (9) we obtain (diagram of forces on the fig. 2, a)

\[
\begin{align*}
    \tilde{F}^{(t)}(x^t) &= \frac{\tilde{F}^{(e)}}{l}(x^t + x^t) > 0; \\
    \Phi_0^{(i)}(x^t) &= -\frac{\tilde{F}^{(i)}}{l}x^t = -\tilde{F}^{(i)}(x^t) < 0,
\end{align*}
\] (12)

where \( \tilde{F}^{(i)}_0 = -\tilde{a}_m \int \frac{m}{l} dx^t = -\tilde{a}_m m \) force of inertia, internal resistance force of absolutely rigid body \( m \) while moving with constant acceleration respectively to the center of mass of this body in the coordinate system \( X'O'Y' \):

\[
\int_{-0.5l}^{0.5l} \Phi_0^{(i)}(x^t)|x^t| dx^t = 0 \text{ (fig. 2, a).}
\]

The work of driving internal force during \( t = \Delta t \) with the displacement \( x_{q'}(t) = \frac{x(\Delta t)}{2} \) of body \( m \) equals

\[
A_{q'}^{(i)}(\Delta t) = \int_0^{0.5l} \int_{-0.5l}^{0.5l} F^{(i)}(x^t)|x^t| dx^t dx = F^{(e)}x_{q'}(t) = A_{q'}^{(e)}(\Delta t)
\] (13)

and it is provided by the change of work of the source of energy \( E_{q'}^{(e)} \) [2]

\[
A_{q'}^{(e)}(\Delta t) = -\Delta E_{q'}^{(e)}.
\] (14)

In its turn the work of internal resistance force \( \Phi_0^{(i)} \) equals

\[
A_{q'}^{(i)}(\Delta t) = \int_0^{0.5l} \int_{-0.5l}^{0.5l} F^{(i)}(x^t)|x^t| dx^t dx = -\frac{m(\Delta \nu_m)^2}{2} = -A_{q'}^{(e)}(\Delta t)
\] (15)

and provides the change of the internal mechanical energy \( E_{m}^{(i)} \) of the body \( m \)

\[
A_{q'}^{(i)}(\Delta t) = -\Delta E_{m}^{(i)} = \frac{m(\Delta \nu_m)^2}{2}.
\] (16)

This also based on the law of conservation of energy in the process of motion of the body \( m \) with constant acceleration.
\[ \Delta E_F^{(e)} + \Delta E_m^{(i)} = 0. \]  

(17)

It is easy to get an equation for the motion of element \(dm\) with constant deceleration (fig. 2, b):

\[
\begin{align*}
\frac{d\bar{Q}^{(i)}}{dt}(x^i) &= d\bar{Q}^{(e)} = \text{const} < 0; \\
\frac{d\bar{Q}^{(i)}}{dt}(x^i) &= -d\bar{Q}^{(e)}(x^i) = \text{const} > 0,
\end{align*}
\]

(18)

These equations (18) define the characteristics (diagram) of distribution of internal forces during motion of the body \(m\) with constant deceleration (fig. 2, b)

\[
\begin{align*}
\bar{\Phi}^{(i)}(x^i) &= \bar{\Phi}^{(e)} \left( \frac{1}{2} + \frac{x^i}{2} \right) > 0,
\\
\bar{Q}^{(i)}(x^i) &= -\bar{\Phi}^{(i)} < 0
\end{align*}
\]

(19)

and make it possible to get equations

\[
\begin{align*}
A^{(i)}_{\Phi}(\Delta r) &= \frac{m(\Delta v_m)^2}{2} > 0; \\
A^{(i)}_{Q}(\Delta r) &= -A^{(i)}_{\Phi}(\Delta r) = -A^{(e)}_{Q} < 0,
\end{align*}
\]

(20)

which provide the law of conservation of energy during that kind of motion

\[ A^{(i)}_{\Phi}(\Delta r) = -\Delta E^{(i)}_m = \Delta E^{(e)}_m = -A^{(e)}_{Q}. \]  

(21)

In accordance with [2] for potential force “the equation for elementary work must be the total differential of some scalar function…”. Taking into account that the kinetic energy of the body \(m\) is a scalar function we will receive:

\[
\begin{align*}
d\left(\frac{mr^2}{2}\right) = m\ddot{r}dr = m\ddot{r}dr &= \left\{ \begin{array}{l}
\Phi^{(i)} dr = dA^{(i)}_{\Phi} > 0 \text{ npu } \bar{r} < 0 \text{ npu } dE^{(i)}_m < 0; \\
-Q^{(i)} dr = -dA^{(i)}_{Q} < 0 \text{ npu } \bar{r} > 0 \text{ npu } dE^{(i)}_m > 0,
\end{array} \right.
\end{align*}
\]

(22)

where \(r=r(x,y,z)\) - the radius vector defines the position of the body \(m\) (centre of gravity of the body) in the coordinate system\(XOYZ\); \(dA^{(i)}_{\Phi}\) and \(dA^{(i)}_{Q}\) - elementary works of the force of inertia during motion with constant deceleration and constant acceleration of the body \(m\) respectively; \(dE^{(i)}_m\) - elementary change of internal (kinetic) energy of the body \(m\) in the process of motion with variable velocity.

From the equations (10)-(22) and table 1 it is received that the force of inertia of a rigid body is an internal potential force that resists the external force. This external force provides motion of the inertial mass body with variable velocity\(^8\). The source of energy of the force of inertia is kinetic energy. This kinetic energy is an internal energy of the moveable physical body and the potential of the force of inertia is a velocity\(^9\) of this body.

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\(^8\) It should be noted that the force of inertia for plastically deformable body is not a potential force because the condition (22) is not fulfilled [20].

\(^9\) “Depending on velocity potential…was primarily used (and erroneously) by Weber W. in classical electrodynamics where postulated forces depending on velocity. German mathematician Schering E. was probably the first who seriously tried to use such a force in mechanics (see GÖtt. Abh. 18, 3,1873). For example, in the first edition Whittaker E.”Analytical dynamics” published in
To confirm this let us prove the Theorem.
If in any space there is the potential function \( U(x,y,z) \) then in this space there is the potential function \( V(\mathbf{r},\mathbf{r},\mathbf{I}) \) and the pole of the force field \( U(x,y,z) \) is a pole of the force field \( V(\mathbf{r},\mathbf{r},\mathbf{I}) \).

The proof of the Theorem.
If potential forces are acting on moveable body then according to the law of conservation of mechanical energy

\[
E = P + T = \text{const},
\]

where \( P = -U(x,y,z) = -U(r) \) - potential energy \( (P) \) of force field \( U(x,y,z) = U(r) \) interacting with the body with center of mass \( m \) located in the point of space \( U(x,y,z) \) defined by the radius vector \( r = \sqrt{x^2 + y^2 + z^2} \gg l \) \((l-\text{maximum linear size of the body})\) in the coordinate system \( XOYZ \) with the centre \( O \) located in the pole of the force field \( U(x,y,z) \):

\[
T = \frac{mv^2}{2} = -V(m,\dot{x}(x),\dot{y}(y),\dot{z}(z)) = -V(m,\dot{r}(r)) \quad \text{- kinetic energy of the body \( m \) in the coordinate system} \ XOYZ:
\]

The equation (23) can be represented:

\[
E(x, y, z, \dot{x}, \dot{y}, \dot{z}) = U(x, y, z) + V(\mathbf{r}, \mathbf{r}, \mathbf{I}) = \text{const}.
\]

Consequently

\[
dE = \Delta U + \Delta V = 0.
\]

From (16,17), (20–22), (25) we will receive

\[
dE = (P_{x}^{(e)} + \Phi_{x} \, dx) + (P_{y}^{(e)} + \Phi_{y} \, dy) + (P_{z}^{(e)} + \Phi_{z} \, dz) = 0,
\]

where \( P_{x}^{(e)} = \frac{\partial U}{\partial x}, \quad P_{y}^{(e)} = \frac{\partial U}{\partial y}, \quad P_{z}^{(e)} = \frac{\partial U}{\partial z} \) - graphical projections onto a coordinate axes \( XOYZ \) of the potential force \( P^{(e)}(m,r) = \text{grad}U \); \( \Phi_{x} = \frac{\partial V}{\partial x}, \quad \Phi_{y} = \frac{\partial V}{\partial y}, \quad \Phi_{z} = \frac{\partial V}{\partial z} \) - graphical projections onto a coordinate axes \( XOYZ \) of the force of inertia \( \Phi = \Phi(m,\dot{r}(r)) = \text{grad}V \).

The equation (26) with \( dx\neq0, \, dy\neq0 \) and \( dz\neq0 \) has a solution if \( \text{grad}U = -\text{grad}V \).

Consequently fields \( U(x,y,z) \) and \( V(\mathbf{r},\mathbf{r},\mathbf{I}) \) have the same pole. End of proof.

The Theorem gives us the idea that potential of the force \( P^{(e)}(m) \) is the position of the point of application of this force respectively to the pole of the force field \( U(x,y,z) \) \((\text{source of energy of the force} \ P^{(e)})\) and the potential of the force of inertia \( \Phi^{(1)} \) is a velocity of the body \( m \) respectively to the source of energy of external force acting on this body.

Let us consider an example. The body \( m \) in the form of a particle having an electric charge moves in an electrostatic field of the body \( M >> m \) (fig. 3).

1904 year there is a reference to the potential in the sense of "potential function of Schering." This term does not come into use because in the following publications it was deleted" [4]. This statement of the author [4] underlined by me because it is not undisputable.
Without forces of resistance to motion of the body \( m \) in the uniform potential field of the body \( M \) this body can perform two kinds of movement (fig. 3, \( a \) and fig. 3, \( b \)).

- The linear motion of the body \( m \) with acceleration. This motion directed along the normal \( O n \) to the center of mass of the body \( M \) located in the pole \( p_s \) of the linear force field at the intersection of the axes \( OX \) and \( OY \) (fig. 3, \( a \)). The internal force \( P_{M-m}^{(i)} \) acts on these bodies. The components of this force are forces \( P_{M-m}^{(i)} \) and \( P_{m-M}^{(i)} = -P_{M-m}^{(i)} \) acting on the body \( M \) from the side of the body \( m \) and on the body \( m \) from the side of the body \( M \) respectively. It can be written \( P_{M-m}^{(i)} = \vec{P}_M \) and \( P_{m-M}^{(i)} = \vec{P}_m = \vec{F}_m \) for each of these bodies accordingly to the mentioned above classification. In this case accelerations of the bodies \( M \) and \( m \): \( \ddot{a}_M = \frac{\vec{B}_M}{M} = 0 \); \( \vec{P}_M = \vec{R}_M \); \( \ddot{a}_m = \frac{\vec{F}_m}{m} ; \Phi^{(i)} = -\dot{\Phi}_m m = -\vec{F}_m \). The force \( F_m \) performs the work \( A_F = F_m(r_1 - r_2 ) > 0 \) during motion of the body \( m \) from the point \( A(r_1) \) to the point \( B(r_2 < r_1) \). This work changes the potential energy of the body \( m \). The change of this energy: \( \Delta\Pi = F_m(r_2 - r_1) = -A_F \). At the same time at the motion of the body \( m \) with constant acceleration from the point \( A(r_1) \) to the point \( B(r_2 < r_1) \) the velocity of this body increases: \( \Delta v_m = \sqrt{2a_m(r_1 - r_2)} \) and change of the kinetic energy of the body \( m \): \( \Delta \Gamma = \frac{m (\Delta v_m)^2}{2} = F_m(r_1 - r_2) > 0 \). The characteristics of motion of the body \( m \) in the potential force field are provided by the following condition

\[
\Delta E_F = F_m(r_2 - r_1) = A_F .
\]  

and up to the moment of the contact of the bodies \( M \) and \( m \) this condition corresponds with the law of conservation of energy (23). The mechanical energy or its part transforms from one form to another at the moment of interaction between bodies \( M \) and \( m \) but this question in the present article is not considered.

- Let us consider the motion of the body \( m \) (fig. 3, \( b \)) along the circumference of a circle which is the level of the potential field \( U(r) \) with pole \( p_s \) and radius \( r_y \). This motion at the same time is a motion with constant velocity
$v_m$ along the circumference of a circle which is the level of the potential field $V(r)$ with pole $p_v$. The force of inertia $\Phi_m = m \frac{v_m^2}{r_y}$ acts on the body $m$. The angle between the force vector and the vector of velocity of the body is $\varphi = 0.5\pi$. The work of the force of inertia during period of time $t : A_\varphi = \Phi_m v_m t \cos \varphi = 0$. The same way the work of the force $P_m^{(i)} = P_N$ balancing the force of inertia equals zero. In this case $\Delta E_I = 0$ and $\Delta T = 0$. Consequently $\Delta E = 0$ and the condition (27) is met irrespectively to the duration of motion of the body $m$ in the potential field of the body $M$. In its turn the action of force $P_m^{(i)} = -\Phi_m^{(i)}$ defines the value of the force $P_m^{(i)} = -P_N$. It is necessary to balance the force $P_m^{(i)} = R^M_M$ by the force $R^{(i)}_M = -R^M_M$ to provide a constant position of the center of mass of the body $M$ in the coordinate system XOY. This constant position is achieved by the rotation of the body $m_i = m$ along the circumference of a circle with radius $r_y$ with constant velocity $v_m = v_m$ at diametrically opposite locations of these bodies.

**Conclusion**

- The force of inertia of absolutely rigid body relatively to this body is an internal potential force acting during change of kinetic energy of the body and balancing the resultant force of the external forces (external force) applied to the rigid body at the process of its motion with acceleration.
- Kinetic energy is an internal mechanical energy of the moveable body. This energy provides the work of forces of inertia while acting unbalanced external forces (external force) on this body. The potential of the force of inertia is the velocity of motion of the body at the current time.
- Each of the potential field of displacement of the physical body corresponds to the field of velocities. The poles of these fields are located in the same point of space and force functions of the fields have opposite signs.

**References**