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$\mathcal{N} = 2$ dilaton-Weyl multiplets in 5D and Nishino-Rajpoot supergravity off-shell

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Abstract: We describe in detail the derivation of a superconformal off-shell formulation of the alternative $\mathcal{N} = 2$, $d = 5$ ungauged supergravity of Nishino and Rajpoot, coupled to n Abelian vector multiplets, using a general dilaton-Weyl multiplet. We generalize the vector multiplet coupling available in the literature and show under which assumptions that the scalar manifold reduces to the known case of $\mathrm{SO}(1,1) \times \mathrm{SO}(1,n)/\mathrm{SO}(n)$. As an application of the formalism we propose generalized vector multiplet coupled higher curvature terms, whose construction we sketch briefly.

Keywords: Supergravity Models, Supersymmetry and Duality

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Contents

1 Introduction

A conventional on-shell formulation of $\mathcal{N}=2$ supergravity in five dimensions was initially given in [1, 2] and the U(1) gauged case was first described in [3]. In [4–7] on-shell methods were used to treat the case of this supergravity coupled to vector multiplets. Hypermultiplet couplings and gaugings were considered in [8–10] and tensor multiplet matter in [11, 12] along with gaugings of isometries of a subgroup of the isometry group of the scalar manifold. The theory can also be obtained from compactification of M-theory on a Calabi-Yau threefold CY_3 [13, 14]. The resulting Lagrangian depends on topological data of the compactification manifold, namely the Calabi-Yau intersection numbers.

This formulation of supergravity doesn't include the N-S two-form $B_{\mu\nu}$ and dilaton explicitly and in order to investigate effective descriptions of string theory it became important to include the dilaton and antisymmetric fields, so off-shell formulations [15–20] were explored to facilitate the construction of matter coupled supergravities, although these theories lack a manifest σ-model structure for the scalars before eliminating the auxiliary fields [21]. In [22], Nishino and Rajpoot proposed an alternative on-shell formulation of $\mathcal{N}=2$ $d=5$ supergravity starting from a supergravity multiplet with a larger field content which contains the N-S antisymmetric field $B_{\mu\nu}$ and a dilaton σ. This multiplet's vielbein $e_\mu{}^m$, gravitini $\psi_\mu{}^{\mathbf{i}}$ and graviphoton A_μ coincide with the conventional fields and in addition to the two-form and dilaton, there is a dilatino $\chi^{\mathbf{i}}$, giving rise to $12+12$ on-shell degrees of freedom. Vector and hypermultiplets [23] have been coupled to this supergravity theory, with a structure of the couplings similar to that of $\mathcal{N}=1$ $d=9$ supergravity [24]. A priori, both formulations are rather similar if one dualizes the antisymmetric tensor $B_{\mu\nu}$ into a vector field B_μ. However, after coupling to vector multiplets, the resulting σ-model structure is different. In fact, it was shown in [25] that the dilaton-Weyl multiplet can be obtained by coupling the standard multiplet to an improved vector multiplet.

The matter couplings of $\mathcal{N} = 2$ d=5 supergravity were studied extensively in [26] from a superspace perspective, and further work using the superconformal formulation [25, 27] allowed the construction of superconformal multiplets and their corresponding actions [19, 20, 25, 27, 28], leading to quite general $d = 5$ matter couplings in the superconformal formulation [29]. The resulting theories preserve eight supersymmetries[1] [30] and can be studied at depth with the tools of special geometry [31–34], the condition for which arises in the off-shell theory as a constraint coming from a scalar Lagrange multiplier auxiliary field of the standard-Weyl multiplet. The advantage of the off-shell formulation is that we may find higher derivative densities, which are important from a string theory perspective, without changing the supersymmetry transformations, and therefore inducing corrections to our original action, an iterative process that may never terminate. The higher derivative densities that are supersymmetric completions of the square of the Ricci scalar and the square of the Weyl tensor have been produced in the background of the standard-Weyl superconformal gravitational multiplet in [35, 36].

In [25] dilaton-Weyl multiplets were introduced including the two form, the dilaton and the dilatino, whilst in [27] dilaton-Weyl multiplets incorporating more than one vector multiplet were introduced. In [37–42] an off-shell superspace formulation of the superconformal theory has been developed, which should lead to the most general couplings, and indeed the dilaton-Weyl multiplet was considered in these works. We find it useful to add to the literature an explicit derivation of the N-R supergravity from the off-shell formulation by means of gauge fixing and field redefinitions, complimenting the work done in [27]. We shall discuss in detail the vector multiplet couplings of this theory. We shall also discuss simple generalizations of two of the higher derivative densities [35, 36] found in the literature.

This paper is organized as follows. In section 2 we discuss the derivation of the minimal N-R supergravity and in section 3 we couple to Abelian vector multiplets and relegate to appendix B the explicit constant field redefinitions needed to arrive at the conventions of [22, 23]. In section 4 we generalize the known higher derivative densities to the extended dilaton-Weyl multiplets that we describe in appendix A, in which we make use of a composition of a vector multiplet in terms of a linear multiplet [43] that we give in appendix C. We conclude in section 5.

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[1] We use the terminology $\mathcal{N} = 2$ due to the fact we use symplectic Majorana spinors, which are an SU(2) doublet of complex spinors obeying the symplectic Majorana condition, $\psi^{\mathbf{i}} = \epsilon^{\mathbf{ij}}(\psi^{\mathbf{j}})^c$ where c denotes charge conjugation. In the literature sometimes the notation $\mathcal{N} = 1$ is used in the case that the theory is presented in terms of Dirac spinors. Of course these two descriptions both have 8 real components of the supercharges.

2 Pure N-R supergravity from the off-shell superconformal formalism

In this section we give the details of the construction of the N-R supergravity [22, 23] from the off-shell formalism based on the superconformal dilaton-Weyl multiplet described in [43]. We also describe an alternative procedure put forward in [27]. To couple the theory of [43] to vector multiplets one may use the results of [36], however following the procedure of [27] we will be led to introduce a larger generalized dilaton-Weyl multiplet, which includes an arbitrary number of vector multiplets. It is instructive to consider the case of the pure N-R supergravity first, and then the coupling to vector multiplets separately.

The pure N-R supergravity can be constructed straightforwardly using exactly the results of [43], whose conventions we will follow, which are described in detail in [25]. However we shall construct it in a slightly different way that was suggested in [27], as we will emphasize below. The two derivative theory is constructed by combining a vector multiplet action and a compensating linear multiplet action, obtained in the background of a Weyl multiplet. We suppress the spinor indices in bilinears using the NW-SE convention and we raise and lower the SU(2) indices using the totally antisymmetric tensor $\epsilon_{\mathbf{ij}}$ where $\epsilon_{\mathbf{12}} = \epsilon^{\mathbf{12}} = 1$, e.g. $\bar{\psi}_\mu \psi_\nu = \bar{\psi}^{\mathbf{i}}_\mu \psi_{\mathbf{i}\nu} = \bar{\psi}^{\mathbf{i}}_\mu \psi^{\mathbf{j}}_\nu \epsilon_{\mathbf{ji}}$. We will frequently use the notation that for two p-forms α, β, we define $\alpha \cdot \beta = \alpha_{\mu_1 \cdots \mu_p} \beta^{\mu_1 \cdots \mu_p}$, and $\alpha^2 = \alpha \cdot \alpha$.

There are two types of Weyl multiplet, the so called standard-Weyl multiplet and the dilaton-Weyl multiplet. The standard-Weyl multiplet consists of the vielbien e_μ^m, gravitino $\psi^{\mathbf{i}}_\mu$, an auxiliary two form T_{mn}, an auxiliary scalar D, an auxiliary fermion $\chi^{\mathbf{i}}$, an auxiliary SU(2) triplet of vectors $V^{\mathbf{ij}}_\mu$ with $V^{\mathbf{ij}}_\mu = V^{\mathbf{ji}}_\mu$ and a gauge field for local dilatations, b_μ. These transform under supersymmetry with parameter $\epsilon^{\mathbf{i}}$ and special supersymmetry with parameter $\eta^{\mathbf{i}}$ as

$$
\begin{aligned}
\delta e_\mu^m &= \frac{1}{2}\bar{\epsilon}\gamma^m \psi_\mu \,, \\
\delta \psi^{\mathbf{i}}_\mu &= \left(\nabla_\mu + \frac{1}{2} b_\mu\right)\epsilon^{\mathbf{i}} - V^{\mathbf{ij}}_\mu \epsilon_{\mathbf{j}} + i\gamma_{mn}T^{mn}\gamma_\mu \epsilon^{\mathbf{i}} - i\gamma_\mu \eta^{\mathbf{i}} \,, \\
\delta V^{\mathbf{ij}}_\mu &= -\frac{3i}{2}\bar{\epsilon}^{(\mathbf{i}}\underline{\phi}^{\mathbf{j})}_\mu + 4\bar{\epsilon}^{(\mathbf{i}}\gamma_\mu \chi^{\mathbf{j})} + i\bar{\epsilon}^{(\mathbf{i}}\gamma_{mn}T^{mn}\psi^{\mathbf{j})}_\mu + \frac{3i}{2}\bar{\eta}^{(\mathbf{i}}\psi^{\mathbf{j})}_\mu \,, \\
\delta T_{mn} &= \frac{i}{2}\bar{\epsilon}\gamma_{mn}\chi - \frac{3i}{32}\bar{\epsilon}\underline{\hat{R}}_{mn}(Q) \,, \\
\delta \chi^{\mathbf{i}} &= \frac{1}{4}D\epsilon^{\mathbf{i}} - \frac{1}{64}\gamma^{mn}\underline{\hat{R}}^{\mathbf{ij}}_{mn}(V)\epsilon_{\mathbf{j}} + \frac{i}{8}\gamma^{mn}\gamma_p \mathcal{D}^p T_{mn}\epsilon^{\mathbf{i}} \\
&\quad - \frac{i}{8}\gamma^m \mathcal{D}^n T_{mn}\epsilon^{\mathbf{i}} - \frac{1}{4}\gamma^{mnpq}T_{mn}T_{pq}\epsilon^{\mathbf{i}} + \frac{1}{6}T^2\epsilon^{\mathbf{i}} + \frac{1}{4}\gamma_{mn}T^{mn}\eta^{\mathbf{i}} \,, \\
\delta D &= \bar{\epsilon}\gamma^m \mathcal{D}_m \chi - \frac{5i}{3}\bar{\epsilon}\gamma_{mn}T^{mn}\chi - i\bar{\eta}\chi \,, \\
\delta b_\mu &= \frac{i}{2}\bar{\epsilon}\underline{\phi}_\mu - 2\bar{\epsilon}\gamma_\mu \chi + \frac{i}{2}\bar{\eta}\psi_\mu \,,
\end{aligned}
\tag{2.1}
$$

where the the spin covariant derivative is defined by

$$
\nabla_\mu \epsilon^{\mathbf{i}} = \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{mn}\gamma_{mn}\right)\epsilon^{\mathbf{i}} \,, \tag{2.2}
$$

and we have underlined the composite fields apart from the spin connection. Explicit expressions for the composite fields are

$$
\begin{aligned}
\omega_\mu^{mn} &= 2e^{\nu[m}\partial_{[\mu}e_{\nu]}^{n]} - e^{\nu[m}e^{n]\sigma}e_{\mu p}\partial_\nu e_\sigma^p + 2e_\mu^{[m}b^{n]} - \frac{1}{2}\bar\psi^{[n}\gamma^{m]}\psi_\mu - \frac{1}{4}\bar\psi^n\gamma_\mu\psi^m\,,\\
\underline{\phi}_\mu^{\mathbf{i}} &= \frac{i}{3}\gamma^m\underline{\hat{R}'}_{\mu m}{}^{\mathbf{i}}(Q) - \frac{i}{24}\gamma_\mu\gamma^{mn}\underline{\hat{R}'}_{mn}{}^{\mathbf{i}}(Q)\,,\\
\underline{\hat{R}'}_{\mu\nu}{}^{\mathbf{i}}(Q) &= 2\nabla_{[\mu}\psi_{\nu]}^{\mathbf{i}} + b_{[\mu}\psi_{\nu]}^{\mathbf{i}} - 2V_{[\mu}^{\mathbf{ij}}\psi_{\nu]\mathbf{j}} + 2i\gamma_{mn}T^{mn}\gamma_{[\mu}\psi_{\nu]}^{\mathbf{i}}\,,\\
\underline{\hat{R}}_{\mu\nu}^{\mathbf{i}}(Q) &= \underline{\hat{R}'}_{\mu\nu}{}^{\mathbf{i}}(Q) - 2i\gamma_{[\mu}\underline{\phi}_{\nu]}^{\mathbf{i}}\;,\\
\underline{\hat{R}}_{\mu\nu}^{\mathbf{ij}}(V) &= 2\partial_{[\mu}V_{\nu]}^{\mathbf{ij}} - 2V_{[\mu}^{\mathbf{k}(\mathbf{i}}V_{\nu]\mathbf{k}}^{\mathbf{j})} - 3i\bar{\underline{\phi}}_{[\mu}^{(\mathbf{i}}\psi_{\nu]}^{\mathbf{j})} - 8\bar\psi_{[\mu}^{(\mathbf{i}}\gamma_{\nu]}\chi^{\mathbf{j})} - i\bar\psi_{[\mu}^{(\mathbf{i}}(\gamma\cdot T)\psi_{\nu]}^{\mathbf{j})}\,,\\
\underline{R'}(M)_{\mu\nu}{}^{mn} &= 2\partial_{[\mu}\omega_{\nu]}{}^{mn} + 2\omega_{[\mu}{}^{mp}\omega_{\nu]p}{}^n + i\bar\psi_{[\mu}\gamma^{mn}\psi_{\nu]} + i\bar\psi_{[\mu}\gamma^{[m}(\gamma\cdot T)\gamma^{n]}\psi_{\nu]}\\
&\quad + \bar\psi_{[\mu}\gamma^{[m}\underline{R}_{\nu]}{}^{n]}(Q) + \frac{1}{2}\bar\psi_{[\mu}\gamma_{\nu]}\underline{R}^{mn}(Q) - 8\bar\psi_{[\mu}e_{\nu]}{}^{[m}\gamma^{n]}\chi + i\bar{\underline{\phi}}_{[\mu}\gamma^{mn}\psi_{\nu]}\,,\\
\underline{f}_m^m &= -\frac{1}{16}\underline{\mathcal{R}}\,,\qquad \mathcal{R} = \underline{R'}(M)_{\mu\nu}{}^{\mu\nu}\,,
\end{aligned}
\tag{2.3}
$$

where the relevant superconformal derivatives are given by

$$
\begin{aligned}
\mathcal{D}_\mu\chi^{\mathbf{i}} = \left(\nabla_\mu - \frac{3}{2}b_\mu\right)\chi^{\mathbf{i}} - V_\mu^{\mathbf{ij}}\chi_{\mathbf{j}} - \frac{1}{4}D\psi_\mu^{\mathbf{i}} + \frac{1}{64}\gamma^{mn}\hat{R}_{mn}^{\mathbf{ij}}(V)\psi_{\mu\mathbf{j}} - \frac{i}{8}\gamma^{mn}\gamma^p(\mathcal{D}_pT_{mn})\psi_\mu^{\mathbf{i}}\\
+ \frac{i}{8}\gamma^m(\mathcal{D}^nT_{mn})\psi_\mu^{\mathbf{i}} + \frac{1}{4}\gamma^{mnpq}T_{mn}T_{pq}\psi_\mu^{\mathbf{i}} - \frac{1}{6}T^2\psi_\mu^{\mathbf{i}} - \frac{1}{4}\gamma_{mn}T^{mn}\underline{\phi}_\mu^{\mathbf{i}}\,,
\end{aligned}
\tag{2.4}
$$

and

$$
\mathcal{D}_\mu T_{mn} = (\nabla_\mu - b_\mu)T_{mn} - \frac{i}{2}\bar\psi_\mu\gamma_{mn}\chi + \frac{3i}{32}\bar\psi_\mu\hat{R}_{mn}(Q)\,. \tag{2.5}
$$

The superconformal linear multiplet is formed from an SU(2) triplet $L^{\mathbf{ij}} = L^{\mathbf{ji}}$, a constrained vector E_m, a scalar N and a fermion $\varphi^{\mathbf{i}}$ which transform, in the background of the standard-Weyl multiplet, as

$$
\begin{aligned}
\delta L^{\mathbf{ij}} &= i\bar\epsilon^{(\mathbf{i}}\varphi^{\mathbf{j})}\;,\\
\delta\varphi^{\mathbf{i}} &= -\frac{i}{2}\gamma^m\mathcal{D}_mL^{\mathbf{ij}}\epsilon_{\mathbf{j}} - \frac{i}{2}\gamma^mE_m\epsilon^{\mathbf{i}} + \frac{N}{2}\epsilon^i - \gamma_{mn}T^{mn}L^{\mathbf{ij}}\epsilon_{\mathbf{j}} + 3L^{\mathbf{ij}}\eta_{\mathbf{j}}\,,\\
\delta E_m &= -\frac{i}{2}\bar\epsilon\gamma_{mn}\mathcal{D}^n\varphi - 2\bar\epsilon\gamma^n\varphi T_{nm} - 2\bar\eta\gamma_m\varphi\,,\\
\delta N &= \frac{1}{2}\bar\epsilon\gamma^m\mathcal{D}_m\varphi + \frac{3i}{2}\bar\epsilon\gamma_{mn}T^{mn}\varphi + 4i\bar\epsilon^{\mathbf{i}}\chi^{\mathbf{i}}L_{\mathbf{ij}} + \frac{3i}{2}\bar\eta\varphi\,,
\end{aligned}
\tag{2.6}
$$

where

$$
\begin{aligned}
\mathcal{D}_\mu L^{\mathbf{ij}} &= (\partial_\mu - 3b_\mu)L^{\mathbf{ij}} + 2V_\mu^{(\mathbf{i}}{}_{\mathbf{k}}L^{\mathbf{j})\mathbf{k}} - i\bar\psi_\mu^{(\mathbf{i}}\varphi^{\mathbf{j})}\,,\\
\mathcal{D}_\mu\varphi^{\mathbf{i}} &= \left(\nabla_\mu - \frac{7}{2}b_\mu\right)\varphi^{\mathbf{i}} - V_\mu^{\mathbf{ij}}\varphi_{\mathbf{j}} - \frac{i}{2}\gamma^m\mathcal{D}_mL^{\mathbf{ij}}\psi_{\mu\mathbf{j}} + \frac{i}{2}\gamma^mE_m\psi_\mu^{\mathbf{i}} - \frac{N}{2}\psi_\mu^{\mathbf{i}}\\
&\quad + \gamma_{mn}T^{mn}L^{\mathbf{ij}}\psi_{\mu\mathbf{j}} - 3L^{\mathbf{ij}}\phi_{\mu\mathbf{j}}\,,\\
\mathcal{D}_\mu E_m &= (\nabla_\mu - 4b_\mu)E_m + \frac{i}{2}\bar\psi_\mu\gamma_{mn}\mathcal{D}^n\varphi + 2\bar\psi_\mu\gamma^n\varphi T_{nm} + 2\bar{\underline{\phi}}_\mu\gamma_m\varphi\,.
\end{aligned}
\tag{2.7}
$$

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The constraint on the vector E^m, which reads $\mathcal{D}^m E_m = 0$ can be solved by the introduction of a three form $E_{\mu\nu\rho}$ such that

$$E^m = -\frac{1}{12} e_\mu^m e^{-1} \epsilon^{\mu\nu\rho\sigma\lambda} \mathcal{D}_\nu E_{\rho\sigma\lambda}\,, \tag{2.8}$$

and it is useful to define the two form $E_{\mu\nu\rho} = e\epsilon_{\mu\nu\rho\sigma\lambda} E^{\sigma\lambda}$, so that we have $E^m = e_\mu^m \mathcal{D}_\nu E^{\mu\nu}$.

The vector multiplet is formed from an SU(2) triplet of scalars $Y^{\mathbf{ij}}$, the gauge field A_μ, a gaugino $\lambda^{\mathbf{i}}$ and a scalar ρ. These transform under the supersymmetries in the background of the standard-Weyl multiplet as

$$\begin{aligned}
\delta A_\mu &= -\frac{i}{2}\rho\bar{\epsilon}\psi_\mu + \frac{1}{2}\bar{\epsilon}\gamma_\mu\lambda\,,\\
\delta Y^{\mathbf{ij}} &= -\frac{1}{2}\bar{\epsilon}^{(\mathbf{i}}\gamma^m \mathcal{D}_m \lambda^{\mathbf{j})} + \frac{i}{2}\bar{\epsilon}^{(\mathbf{i}}(\gamma\cdot T)\lambda^{\mathbf{j})} - 4i\rho\bar{\epsilon}^{(\mathbf{i}}\chi^{\mathbf{j})} + \frac{i}{2}\bar{\eta}^{(\mathbf{i}}\lambda^{\mathbf{j})}\,,\\
\delta\lambda^{\mathbf{i}} &= -\frac{1}{4}\gamma_{mn}\hat{F}^{mn}\epsilon^{\mathbf{i}} - \frac{i}{2}\gamma^m(\mathcal{D}_m\rho)\epsilon^{\mathbf{i}} + \rho\gamma_{mn}T^{mn}\epsilon^{\mathbf{i}} - Y^{\mathbf{ij}}\epsilon_{\mathbf{j}} + \rho\eta^{\mathbf{i}}\,,\\
\delta\rho &= \frac{i}{2}\bar{\epsilon}\lambda\,,
\end{aligned} \tag{2.9}$$

where

$$\begin{aligned}
\mathcal{D}_\mu\rho &= (\partial_\mu - b_\mu)\rho - \frac{i}{2}\bar{\psi}_\mu\lambda\,,\\
\mathcal{D}_\mu\lambda^{\mathbf{i}} &= \left(\nabla_\mu - \frac{3}{2}b_\mu\right)\lambda^{\mathbf{i}} - V_\mu^{\mathbf{ij}}\lambda_{\mathbf{j}} + \frac{1}{4}\gamma_{mn}\hat{F}^{mn}\psi_\mu^{\mathbf{i}} + \frac{i}{2}\gamma^m(\mathcal{D}_m\rho)\psi_\mu^{\mathbf{i}}\\
&\quad + Y^{\mathbf{ij}}\psi_{\mu\mathbf{j}} - \rho\gamma_{mn}T^{mn}\psi_\mu^{\mathbf{i}} - \rho\underline{\phi}_\mu^{\mathbf{i}}\,,\\
\hat{F}_{\mu\nu} &= F_{\mu\nu} - \bar{\psi}_{[\mu}\gamma_{\nu]}\lambda + \frac{i}{2}\rho\bar{\psi}_{[\mu}\psi_{\nu]}\,,
\end{aligned} \tag{2.10}$$

and where $F = dA$.

A superconformally invariant density formula constructed from a vector multiplet and a linear multiplet is given by

$$\begin{aligned}
e^{-1}\mathcal{L}_{VL} &= Y^{\mathbf{ij}}L_{\mathbf{ij}} + i\bar{\lambda}\varphi - \frac{1}{2}\bar{\psi}_m^{\mathbf{i}}\gamma^m\lambda^{\mathbf{j}}L_{\mathbf{ij}} + C_m P^m\\
&\quad + \rho\left(N + \frac{1}{2}\bar{\psi}_m\gamma^m\varphi + \frac{i}{4}\bar{\psi}_m^{\mathbf{i}}\gamma^{mn}\psi_n^{\mathbf{j}}L_{\mathbf{ij}}\right)\,,
\end{aligned} \tag{2.11}$$

where P^m is the bosonic part of the supercovariant E^m

$$P^m = E^m + \frac{i}{2}\bar{\psi}_n\gamma^{nm}\varphi + \frac{1}{4}\bar{\psi}_n^{\mathbf{i}}\gamma^{mnp}\psi_p^{\mathbf{j}}L_{\mathbf{ij}}\,. \tag{2.12}$$

In order to describe vector-vector couplings one can compose the linear multiplet appearing in the above action from a vector multiplet and to describe linear-linear couplings one can compose the vector multiplet appearing in the action from a linear multiplet. The composition of the vector multiplet from the linear multiplet is given in detail in [36, 43] and we list the bosonic parts in appendix C.1. As noted in [27], where only the scalar composition was given, this embedding leads to fairly long expressions when including the fermions. We will be interested in the bosonic part of the resulting action which reads

$$\begin{aligned} e^{-1}\mathcal{L}_L = {} & L^{-1}L_{\mathbf{ij}}\Box L^{\mathbf{ij}} - L^{\mathbf{ij}}\mathcal{D}_\mu L_{\mathbf{k(i}}\mathcal{D}^\mu L_{\mathbf{j)m}}L^{\mathbf{km}}L^{-3} - N^2L^{-1} \\ & - P_\mu P^\mu L^{-1} + \frac{8}{3}LT^2 + 4DL - \frac{1}{2}L^{-3}P^{\mu\nu}L^{\mathbf{l}}_{\mathbf{k}}\partial_\mu L^{\mathbf{kp}}\partial_\nu L_{\mathbf{pl}} \\ & + 2P^{\mu\nu}\partial_\mu\left(L^{-1}P_\nu + V_\nu^{\mathbf{ij}}L_{\mathbf{ij}}L^{-1}\right), \end{aligned} \tag{2.13}$$

where $L^2 = L_{\mathbf{ij}}L^{\mathbf{ij}}$, $P^{\mu\nu}$ is the bosonic part of $E^{\mu\nu}$ and

$$\begin{aligned} L_{\mathbf{ij}}\Box L^{\mathbf{ij}} = {} & L_{\mathbf{ij}}\left(\partial^m - 4b^m + \omega_n{}^{nm}\right)\mathcal{D}_m L^{\mathbf{ij}} + 2L_{\mathbf{ij}}V_n^{\mathbf{i}}{}_{\mathbf{k}}\mathcal{D}^nL^{\mathbf{jk}} \\ & + 6L^2\underline{f}_m^m - iL_{\mathbf{ij}}\bar\psi^{m\mathbf{i}}\mathcal{D}_m\varphi^{\mathbf{j}} - 6L^2\bar\psi^m\gamma_m\chi \\ & - L_{\mathbf{ij}}\bar\varphi^{\mathbf{i}}\gamma_{mn}T^{mn}\gamma^p\psi_p^{\mathbf{j}} + L_{\mathbf{ij}}\bar\varphi^{\mathbf{i}}\gamma^m\underline{\phi}_m^{\mathbf{j}}. \end{aligned} \tag{2.14}$$

The composition of the linear multiplet in terms of a single vector multiplet is well known [20, 25, 27], which we take from (A.1) of [36],[2] and reads

$$\begin{aligned} L_{\mathbf{ij}}(\mathbf{V}) &= 2\rho Y_{\mathbf{ij}} - \frac{i}{2}\bar\lambda_{\mathbf{i}}\lambda_{\mathbf{j}}, \\ \varphi_{\mathbf{i}}(\mathbf{V}) &= i\rho\gamma^m\mathcal{D}_m\lambda_{\mathbf{i}} + 2\rho\gamma_{mn}T^{mn}\lambda_{\mathbf{i}} - 8\rho^2\chi_{\mathbf{i}} - \frac{1}{4}\gamma^{mn}\hat F_{mn}\lambda_{\mathbf{i}} + \frac{i}{2}\gamma^m(\mathcal{D}_m\rho)\lambda_{\mathbf{i}} - Y_{\mathbf{ij}}\lambda^{\mathbf{j}}, \\ E^m(\mathbf{V}) &= \mathcal{D}_n\left(-\rho\hat F^{mn} + 8\rho^2T^{mn} - \frac{i}{4}\bar\lambda\gamma_{mn}\lambda\right) - \frac{1}{8}\epsilon^{mnpqr}\hat F_{np}\hat F_{qr}, \\ N(\mathbf{V}) &= \rho\Box\rho + \frac{1}{2}(\mathcal{D}_m\rho)(\mathcal{D}^m\rho) - \frac{1}{4}\hat F_{mn}\hat F^{mn} + Y^{\mathbf{ij}}Y_{\mathbf{ij}} + 8\rho\hat F_{mn}T^{mn} \\ &\quad - 4\rho^2\left(D + \frac{26}{3}T^2\right) - \frac{1}{2}\bar\lambda\gamma^m\mathcal{D}_m\lambda + i\bar\lambda\gamma_{mn}T^{mn}\lambda + 16i\rho\bar\chi\lambda. \end{aligned} \tag{2.15}$$

With this at hand we can now write down an action by taking the Lagrangian $\mathcal{L}_L - 3\mathcal{L}_{\mathbf{V}}$, where $\mathcal{L}_{\mathbf{V}}$ can be formed in two ways: by taking another copy of the same vector multiplet $\mathbf{V} = (\rho, A_\mu, \lambda^{\mathbf{i}}, Y^{\mathbf{ij}})$, or by considering a second vector multiplet. Let us first consider using the same vector multiplet that we have embedded in the linear multiplet as done in [43]. We obtain for the bosonic part of the vector multiplet density

$$\begin{aligned} e^{-1}\mathcal{L}_V = {} & -\frac{1}{4}\rho F^2 + \frac{1}{3}\rho^2\Box\rho + \frac{\rho}{6}(\mathcal{D}\rho)^2 + \rho Y^{\mathbf{ij}}Y_{\mathbf{ij}} \\ & - \frac{4}{3}\rho^3\left(D + \frac{26}{3}T^2\right) + 4\rho^2F_{\mu\nu}T^{\mu\nu} - \frac{e^{-1}}{24}\epsilon^{\mu\nu\rho\sigma\lambda}A_\mu F_{\nu\rho}F_{\sigma\lambda}, \end{aligned} \tag{2.16}$$

where

$$\begin{aligned} \Box\rho = {} & (\nabla^m - 2b^m)\mathcal{D}_m\rho - \frac{i}{2}\bar\psi_m\mathcal{D}^m\lambda - 2\rho\bar\psi_m\gamma^m\underline{\chi} \\ & + \frac{1}{2}\bar\psi_m\gamma^m\gamma_{np}\underline{T}^{np}\lambda + \frac{1}{2}\underline{\bar\phi}^m\gamma_m\lambda + 2\rho\underline{f}_m^m. \end{aligned}$$

[2] We have corrected a typo of a missing factor of ρ in the last term of the first line of the expression for $N(\mathbf{V})$ and a missing factor of i in the penultimate term in the expression for $\varphi^{\mathbf{i}}(\mathbf{V})$.

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It turns out that the equations of motion for the vector multiplet fields imply[3]

$$\begin{aligned}
\underline{T}^{mn} &= \frac{\rho^{-2}}{8}\left(\rho\hat{F}^{mn} + \frac{1}{6}\epsilon^{mnpqr}\hat{H}_{pqr} + \frac{i}{4}\bar{\lambda}\gamma^{mn}\lambda\right)\,, \\
\underline{\chi}^{\mathbf{i}} &= \frac{i}{8}\rho^{-1}\gamma^m\mathcal{D}_m\lambda^{\mathbf{i}} + \frac{i}{16}\rho^{-2}\gamma^m(\mathcal{D}_m\rho)\lambda^{\mathbf{i}} - \frac{\rho^{-2}}{32}\gamma_{mn}\hat{F}^{mn}\lambda^{\mathbf{i}} \\
&\quad + \frac{\rho^{-1}}{4}\gamma_{mn}\underline{T}^{mn}\lambda^{\mathbf{i}} + \frac{i\rho^{-1}}{32}\lambda_{\mathbf{j}}\bar{\lambda}^{\mathbf{i}}\lambda^{\mathbf{j}}\,, \\
\underline{D} &= \frac{\rho^{-1}}{4}\Box\rho + \frac{\rho^{-2}}{8}(\mathcal{D}\rho)^2 - \frac{\rho^{-2}}{16}\hat{F}^2 - \frac{\rho^{-2}}{8}\bar{\lambda}\gamma^m\mathcal{D}_m\lambda - \frac{\rho^{-4}}{64}\bar{\lambda}^{\mathbf{i}}\lambda^{\mathbf{j}}\bar{\lambda}_{\mathbf{i}}\lambda_{\mathbf{j}} - 4i\rho^{-1}\lambda\underline{\chi} \\
&\quad + \left(2\rho^{-1}\hat{F}_{mn} - \frac{26}{3}\underline{T}_{mn} + \frac{i\rho^{-2}}{4}\bar{\lambda}\gamma_{mn}\lambda\right)\underline{T}^{mn}\,, \\
\underline{Y}^{\mathbf{ij}} &= \frac{i}{4}\rho^{-1}\bar{\lambda}^{\mathbf{i}}\lambda^{\mathbf{j}}\,,
\end{aligned} \tag{2.17}$$

where

$$\begin{aligned}
\hat{H}_{\mu\nu\rho} &= H_{\mu\nu\rho} - \frac{3}{4}\rho^2\bar{\psi}_{[\mu}\gamma_\nu\psi_{\rho]} - \frac{3i}{2}\rho\bar{\psi}_{[\mu}\gamma_{\nu\rho]}\lambda\,, \\
H_{\mu\nu\rho} &= 3\partial_{[\mu}B_{\nu\rho]} + \frac{3}{2}A_{[\mu}F_{\nu\rho]}\,,
\end{aligned} \tag{2.18}$$

and for H to be gauge invariant we need that B transforms under gauge transformations as

$$\delta B_{\mu\nu} = 2\partial_{[\mu}\Lambda_{\nu]} - \frac{1}{2}\Lambda F_{\mu\nu}\,. \tag{2.19}$$

Now we note that the equation of motion for D is given by

$$L = \rho^3\,. \tag{2.20}$$

This must be implemented as a constraint if one is to use the above solutions of the equations of motion in the action, and obtain an equivalent theory. However the gauge fixing performed in [36, 43] demands that L be constant,

$$L_{\mathbf{ij}} = \pm\frac{1}{\sqrt{2}}\delta_{\mathbf{ij}} \qquad b_\mu = 0 \qquad \lambda = 0\,. \tag{2.21}$$

So the action given in [43] should be supplemented by the contraint arising from the equations of motion of the standard Weyl fields we have eliminated. This is compatible, for example, with the ρ equation of motion however when we come to consider higher derivative theories the form of this contraint will change.[4] Alternatively one could impose the gauge fixing conditions $L_{\mathbf{ij}} = \pm\frac{L'}{\sqrt{2}}\delta_{\mathbf{ij}}$ where L' is a non-constant scalar field, and the normalization is chosen such that $L^2 = L_{ij}L^{ij} = L'^2$, however in such a case the local SU(2) symmetry of the superconformal gravity will only have been fixed down to local U(1). Furthermore the necessary compensating special supersymmetry transformation to maintain this gauge will become dependant on dL. This may be an interesting theory, but it is somewhat different from the ungauged N-R supergravity we wish to construct here, and we hope to return to this in future work.

[3] For the details see [43] where the relevant fermionic terms in the action are given.

[4] We shall discuss how we can avoid this in the remainder of this section, which is particularly useful when considering higher derivative theories.

Following [43] we then find the action and supersymmetry transformations given below in (2.25), (2.29) under the gauge fixings given in (2.27). We can also obtain this theory in a different way which was suggested in [27], which will be useful to generalise the coupling to vector multiplets and higher derivative theories in the next sections. We introduce an additional vector multiplet $\mathbf{V}_\flat = (\rho^\flat, A^\flat_\mu, \lambda^{\flat\mathbf{i}}, Y^{\flat\mathbf{ij}})$. Combining this with a linear multiplet composed of a vector multiplet that we shall denote $\mathbf{V}_D = (\sigma, C_\mu, \psi^{\mathbf{i}}, Y^{\mathbf{ij}})$ in the density formula (2.11) we obtain a suitable Lagrangian density which we denote $\mathcal{L}_{\mathbf{V}'}$, and we will take the Lagrangian to be

$$\mathcal{L} = \mathcal{L}_L + \mathcal{L}_{\mathbf{V}'}\,. \tag{2.22}$$

Examining the equations of motion for the vector multiplet $\mathbf{V}_\flat = (\rho^\flat, A^\flat_\mu, \lambda^{\flat\mathbf{i}}, Y^{\flat\mathbf{ij}})$ directly in the action formula (2.11), since the composite linear multiplet does not now depend on these fields, we see that the fields $\mathbf{V}_\flat$ act as Lagrange multipliers, whose equations of motion set the fields of the composite linear multiplet to zero[5] and one obtains expressions for the standard-Weyl multiplet matter fields in terms of $\mathbf{V}_D$,

$$\begin{aligned}
\underline{T}^{mn} &= \frac{\sigma^{-2}}{8}\left(\sigma\hat{G}^{mn} + \frac{1}{6}\epsilon^{mnpqr}\hat{H}_{pqr} + \frac{i}{4}\bar{\psi}\gamma^{mn}\psi\right),\\
\underline{\chi}^{\mathbf{i}} &= \frac{i}{8}\sigma^{-1}\gamma^m\mathcal{D}_m\psi^{\mathbf{i}} + \frac{i}{16}\sigma^{-2}\gamma^m(\mathcal{D}_m\sigma)\psi^{\mathbf{i}} - \frac{\sigma^{-2}}{32}\gamma_{mn}\hat{G}^{mn}\psi^{\mathbf{i}}\\
&\quad + \frac{\sigma^{-1}}{4}\gamma_{mn}\underline{T}^{mn}\psi^{\mathbf{i}} + \frac{\sigma^{-2}}{8}\underline{Y}^{\mathbf{ij}}\psi_{\mathbf{j}}\,,\\
\underline{D} &= \frac{1}{4\sigma}\hat{\Box}\sigma + \frac{1}{8\sigma^2}(\mathcal{D}\sigma)^2 - \frac{1}{16\sigma^2}\hat{G}^2 + \frac{1}{2}\underline{f}_m^{\ m}\\
&\quad + \left(2\sigma^{-1}\hat{G}_{mn} - \frac{26}{3}\underline{T}_{mn} + \frac{i\sigma^{-2}}{4}\bar{\psi}\gamma_{mn}\psi\right)\underline{T}^{mn} + \frac{1}{2}\bar{\psi}_m\gamma^m\gamma_{np}\underline{T}^{np}\psi\\
&\quad + \frac{1}{2}\underline{\bar{\phi}}^m\gamma_m\psi - \frac{\sigma^{-2}}{8}\bar{\psi}\gamma^m\mathcal{D}_m\psi - \frac{\sigma^{-4}}{64}\bar{\psi}^{\mathbf{i}}\psi^{\mathbf{j}}\bar{\psi}_{\mathbf{i}}\psi_{\mathbf{j}} - 4i\sigma^{-1}\psi\underline{\chi}\,,\\
\underline{Y}^{\mathbf{ij}} &= \frac{i}{4}\sigma^{-1}\bar{\psi}^{\mathbf{i}}\psi^{\mathbf{j}}\,,
\end{aligned} \tag{2.23}$$

where

$$\begin{aligned}
\hat{G}_{\mu\nu} &= G_{\mu\nu} - \bar{\psi}_{[\mu}\gamma_{\nu]}\psi + \frac{i}{2}\sigma\bar{\psi}_{[\mu}\psi_{\nu]}\,,\\
\hat{H}_{\mu\nu\rho} &= H_{\mu\nu\rho} - \frac{3}{4}\sigma^2\bar{\psi}_{[\mu}\gamma_\nu\psi_{\rho]} - \frac{3i}{2}\sigma\bar{\psi}_{[\mu}\gamma_{\nu\rho]}\psi\,,\\
H_{\mu\nu\rho} &= 3\partial_{[\mu}B_{\nu\rho]} + \frac{3}{2}C_{[\mu}G_{\nu\rho]}\,,\\
\hat{\Box}\sigma &= (\nabla^m - 2b^m)\mathcal{D}_m\sigma - \frac{i}{2}\bar{\psi}_m\mathcal{D}^m\psi - 2\sigma\bar{\psi}_m\gamma^m\underline{\chi}\,,
\end{aligned} \tag{2.24}$$

and $G = dC$. The equation of motion for D now implies $L = \sigma^2\rho^\flat$, so the gauge fixing conditions (2.21) can be implemented, as the constraint arising from the D equation of motion can be solved in terms of $\rho^\flat$ which is a Lagrange multiplier and the other fields of $\mathbf{V}_\flat$ can be similarly used to solve the $T_{mn}, \chi^{\mathbf{i}}$ equations of motion. As above we use

[5] Note that this also clearly satisfies the constraint $\mathcal{D}^m E_m = 0$.

the expressions (2.23) to define a new gravitational multiplet, and will take them to be identities, so that the term involving the Lagrange multipliers can be neglected in the action, since the composite linear multiplet is now identically vanishing. In particular we can always solve the contraints coming from the standard weyl fields we have eliminated using the Lagrange multipliers.

Note that in this case the contribution to the superconformal action from the vector multiplets is completely contained in the expressions for the previously independent standard-Weyl multiplet matter fields, which are now composite. If we take the most general contribution from the vector multiplet $\mathbf{V}_D$ that still allows for the $\mathbf{V}_\flat$ vector multiplet to be a Lagrange multiplier, i.e. we add the Lagrangian density $\mathcal{L}_{\mathbf{V}_D}$ formed from the three copies of $\mathbf{V}_D$ we find, using the expressions (2.23), that $\mathcal{L}_{\mathbf{V}_D} = 0$. Indeed this must be the case as there are no terms in the Lagrangian density (2.11) that do not involve the composite linear multiplet, which, as we have seen above, vanishes.

Let us now summarize the details of the dilaton-Weyl multiplet, which is made up of the vielbien e_μ^m, gravitino $\psi_\mu^{\mathbf{i}}$, graviphoton gauge field C_μ, a two-form gauge field $B_{\mu\nu}$, the dilaton σ, the dilatino $\psi^{\mathbf{i}}$ and an auxiliary SU(2) triplet of vectors $V_\mu^{\mathbf{ij}}$ with $V_\mu^{\mathbf{ij}} = V_\mu^{\mathbf{ji}}$ and a gauge field for local dilatations b_μ. These transform under supersymmetry with parameter $\epsilon^{\mathbf{i}}$ and special supersymmetry with parameter $\eta^{\mathbf{i}}$ as

$$\begin{aligned}
\delta e_\mu^m &= \frac{1}{2}\bar{\epsilon}\gamma^m\psi_\mu\,,\\
\delta\psi_\mu^{\mathbf{i}} &= \left(\nabla_\mu + \frac{1}{2}b_\mu\right)\epsilon^{\mathbf{i}} - V_\mu^{\mathbf{ij}}\epsilon_{\mathbf{j}} + i\gamma_{mn}\underline{T}^{mn}\gamma_\mu\epsilon^{\mathbf{i}} - i\gamma_\mu\eta^{\mathbf{i}}\,,\\
\delta V_\mu^{\mathbf{ij}} &= -\frac{3i}{2}\bar{\epsilon}^{(\mathbf{i}}\underline{\phi}_\mu^{\mathbf{j})} + 4\bar{\epsilon}^{(\mathbf{i}}\gamma_\mu\underline{\chi}^{\mathbf{j})} + i\bar{\epsilon}^{(\mathbf{i}}\gamma_{mn}\underline{T}^{mn}\psi_\mu^{\mathbf{j})} + \frac{3i}{2}\bar{\eta}^{(\mathbf{i}}\psi_\mu^{\mathbf{j})}\,,\\
\delta C_\mu &= -\frac{i}{2}\sigma\bar{\epsilon}\psi_\mu + \frac{1}{2}\bar{\epsilon}\gamma_\mu\psi\,,\\
\delta B_{\mu\nu} &= \frac{1}{2}\sigma^2\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]} + \frac{i}{2}\sigma\bar{\epsilon}\gamma_{\mu\nu}\psi + C_{[\mu}\delta(\epsilon)C_{\nu]}\,,\\
\delta\psi^{\mathbf{i}} &= -\frac{1}{4}\gamma_{mn}\hat{G}^{mn}\epsilon^{\mathbf{i}} - \frac{i}{2}\gamma^m(\mathcal{D}_m\sigma)\epsilon^{\mathbf{i}} + \sigma\gamma_{mn}\underline{T}^{mn}\epsilon^{\mathbf{i}} - \frac{i}{4}\sigma^{-1}\epsilon_{\mathbf{j}}\bar{\psi}^{\mathbf{i}}\psi^{\mathbf{j}} + \sigma\eta^{\mathbf{i}}\,,\\
\delta\sigma &= \frac{i}{2}\bar{\epsilon}\psi\,,\\
\delta b_\mu &= \frac{i}{2}\bar{\epsilon}\underline{\phi}_\mu - 2\bar{\epsilon}\gamma_\mu\underline{\chi} + \frac{i}{2}\bar{\eta}\psi_\mu\,,
\end{aligned} \tag{2.25}$$

where we have underlined composite fields the expressions for which are listed in (2.3) but now additionally $\underline{T}_{mn}$,$\underline{D}$ and $\underline{\chi}^{\mathbf{i}}$ are given by their expressions in (2.23). The supercovariant field strength $\hat{H}$ defined in (2.24) obeys the generalized Bianchi identity

$$\mathcal{D}_{[\mu}\hat{H}_{\nu\rho\sigma]} = \frac{3}{4}\hat{G}_{[\mu\nu}\hat{G}_{\rho\sigma]}\,, \tag{2.26}$$

where $G = dC$.

Armed with the superconformal theory we now wish to gauge fix down to the N-R supergravity. First we choose

$$b_\mu = 0\,, \qquad L_{\mathbf{ij}} = \frac{L}{\sqrt{2}}\delta_{\mathbf{ij}}\,, \qquad \varphi^{\mathbf{i}} = 0\,. \tag{2.27}$$

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The first condition breaks local dilatational invariance and fixes the form of the necessary compensating special conformal boosts, the second breaks local SU(2) down to $\mathrm{U}(1)_R$, where L is constant, and the third fixes special supersymmetry. Choosing the value of L is a choice of dilatation. In order to maintain this gauge we must set

$$\eta_{\mathbf{k}} = \frac{1}{3}(\gamma \cdot T)\epsilon_{\mathbf{k}} - \frac{i}{2}(\gamma^m E_m)\delta_{\mathbf{ik}}\epsilon^{\mathbf{i}} \;, \tag{2.28}$$

where in order to avoid confusion we point out that E_m is the vector of the compensating linear multiplet, not the composite one.

Under these gauge fixing conditions we obtain for the bosonic part of the action[6]

$$\begin{aligned} e^{-1}L^{-1}\mathcal{L}_L = &-\frac{1}{2}R + \frac{1}{4}\sigma^{-2}G^2 + \frac{1}{6}\sigma^{-4}H^2 + \frac{3}{2}\sigma^{-2}(d\sigma)^2 \\ &- V'^{\mathbf{ij}}_{\mu}V'^{\mu}_{\mathbf{ij}} - N^2 + L^{-2}P_\mu P^\mu + \sqrt{2}L^{-2}P^\mu V_\mu \,, \end{aligned} \tag{2.29}$$

where we have decomposed $V_\mu^{\mathbf{ij}}$ into its traceful and traceless parts [43]

$$V_\mu^{\mathbf{ij}} = V'^{\mathbf{ij}} + \frac{1}{2}\delta^{\mathbf{ij}}V_\mu \;, \qquad V'^{\mathbf{ij}}\delta_{\mathbf{ij}} = 0 \,, \tag{2.30}$$

and P^μ denotes the bosonic part of E^μ. Finally we set $L = 1$. The action (2.29) is invariant under the supersymmetry transformations (2.25), with the special supersymmetry parameter $\eta^{\mathbf{i}}$ replaced by its expression (2.28). To arrive at the on-shell formulation we may next eliminate auxiliary fields P^μ, N and $V_\mu^{\mathbf{ij}}$ by their equations of motion which imply these fields vanish, and the supersymmetry transformations become:[7]

$$\begin{aligned} \delta e_\mu^m &= \frac{1}{2}\bar{\epsilon}\gamma^m\psi_\mu \,, \\ \delta\psi_\mu^{\mathbf{i}} &= \nabla_\mu\epsilon^{\mathbf{i}} + i\underline{T}^{mn}\left(\gamma_{mn}\gamma_\mu - \frac{1}{3}\gamma_\mu\gamma_{mn}\right)\epsilon^{\mathbf{i}} \,, \\ \delta C_\mu &= -\frac{i}{2}\sigma\bar{\epsilon}\psi_\mu + \frac{1}{2}\bar{\epsilon}\gamma_\mu\psi \,, \\ \delta B_{\mu\nu} &= \frac{1}{2}\sigma^2\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]} + \frac{i}{2}\sigma\bar{\epsilon}\gamma_{\mu\nu}\psi + C_{[\mu}\delta(\epsilon)C_{\nu]} \,, \\ \delta\psi^{\mathbf{i}} &= -\frac{1}{4}\gamma_{mn}G^{mn}\epsilon^{\mathbf{i}} - \frac{i}{2}\gamma^m(\partial_m\sigma)\epsilon^{\mathbf{i}} + \frac{4}{3}\sigma\gamma_{mn}\underline{T}^{mn}\epsilon^{\mathbf{i}} \,, \\ \delta\sigma &= \frac{i}{2}\bar{\epsilon}\psi \;. \end{aligned} \tag{2.31}$$

We must now perform some field and parameter redefinitions to bring the supersymmetry transformations to a same form as those in [22, 23]. We will take

$$\epsilon^{\mathbf{i}} = -\sqrt{2}\epsilon'^{\mathbf{i}} \,, \quad \psi_\mu^{\mathbf{i}} = -\sqrt{2}\psi'^{\mathbf{i}}_\mu \,, \quad \sigma = e^{\sigma'} \,, \quad \psi^{\mathbf{i}} = -\frac{\sqrt{2}}{\sqrt{3}}e^{\sigma'}\chi'^{\mathbf{i}} \,, \quad C_\mu = \sqrt{2}A'_\mu \,, \tag{2.32}$$

noting that the definition of the three form field strength has therefore changed to

$$G'_{\mu\nu\rho} = H'_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]} + 3A'_{[\mu}F'_{\nu\rho]} \,, \tag{2.33}$$

[6] We have written the action in this way to emphasize the fact that the relative signs of the terms appearing are not dependent on the gauge fixing choice $L = \pm 1$. Rather this choice only gives an overall sign to this contribution to the action.

[7] Note that due to the equation of motion for P^μ the special supersymmetry parameter now reads $\eta^{\mathbf{i}} = \frac{1}{3}\gamma_{mn}T^{mn}\epsilon^{\mathbf{i}}$ if we ignore terms quadratic in the fermionic fields.

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where $F' = dA' = \sqrt{2}G$, and so from (2.26) the Bianchi identity for G now reads

$$\partial_{[\mu}G'_{\nu\rho\sigma]} = \frac{3}{2}F'_{[\mu\nu}F'_{\rho\sigma]}\,. \tag{2.34}$$

Dropping the primes we find the following supersymmetry transformations

$$\begin{aligned}
\delta e_\mu{}^m &= \bar{\epsilon}\gamma^m\psi_\mu\,,\\
\delta\sigma &= \frac{i}{\sqrt{3}}\bar{\epsilon}\chi\,,\\
\delta\psi_\mu{}^{\mathbf{i}} &= \nabla_\mu\epsilon^{\mathbf{i}} + \frac{i}{6\sqrt{2}}e^{-\sigma}\Big(\gamma_\mu{}^{\rho\sigma} - 4\delta_\mu{}^\rho\gamma^\sigma\Big)\epsilon^{\mathbf{i}}F_{\rho\sigma} + \frac{1}{18}e^{-2\sigma}\Big(\gamma_\mu{}^{\rho\sigma\tau} - \frac{3}{2}\delta_\mu{}^\rho\gamma^{\sigma\tau}\Big)\epsilon^{\mathbf{i}}G_{\rho\sigma\tau}\,,\\
\delta A_\mu &= -\frac{i}{\sqrt{2}}e^\sigma\bar{\epsilon}\psi_\mu + \frac{1}{\sqrt{6}}e^\sigma\bar{\epsilon}\gamma_\mu\chi\,,\\
\delta B_{\mu\nu} &= e^{2\sigma}\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]} + \frac{i}{\sqrt{3}}e^{2\sigma}\bar{\epsilon}\gamma_{\mu\nu}\chi + 2A_{[\mu|}\delta A_{|\nu]}\,,\\
\delta\chi^{\mathbf{i}} &= -\frac{1}{2\sqrt{6}}e^{-\sigma}\gamma^{\mu\nu}\epsilon^{\mathbf{i}}F_{\mu\nu} + \frac{i}{6\sqrt{3}}e^{-2\sigma}\gamma^{\mu\nu\rho}\epsilon^{\mathbf{i}}G_{\mu\nu\rho} - \frac{\sqrt{3}i}{2}\gamma^\mu\epsilon^{\mathbf{i}}\partial_\mu\sigma\,,
\end{aligned}$$

under which the Lagrangian with bosonic part

$$-e^{-1}\frac{1}{2}\mathcal{L}_L = \frac{1}{4}R - \frac{3}{4}(d\sigma)^2 - \frac{1}{4}e^{-2\sigma}G^2 - \frac{1}{12}e^{-4\sigma}H^2\,,$$

is invariant. Taking account of the different curvature conventions between [22, 23] and [25, 36, 43] by changing the sign of the Ricci scalar, we have thus arrived at the pure N-R formulation $\mathcal{N} = 2$, $d = 5$ supergravity. The fermionic terms up to quadratic level are given in [22, 23] and may also be cross checked using the results of [43].

3 Coupling to Abelian vector multiplets

The superconformal vector multiplets, labelled by capital Latin indices $I, J, K, \ldots$ are each formed from an SU(2) triplet field $Y^{\mathbf{ij}}$, the gauge field A_μ, a gaugino $\lambda^{\mathbf{i}}$ and a scalar ρ. These transform under the supersymmetries as

$$\begin{aligned}
\delta A_\mu^I &= -\frac{i}{2}\rho^I\bar{\epsilon}\psi_\mu + \frac{1}{2}\bar{\epsilon}\gamma_\mu\lambda^I\,,\\
\delta Y^{I\mathbf{ij}} &= -\frac{1}{2}\bar{\epsilon}^{(\mathbf{i}}\gamma^m\mathcal{D}_m\lambda^{I\mathbf{j})} + \frac{i}{2}\bar{\epsilon}^{(\mathbf{i}}\gamma_{mn}T^{|mn|}\lambda^{\mathbf{j})I} - 4i\rho^I\bar{\epsilon}^{(\mathbf{i}}\chi^{\mathbf{j})} + \frac{i}{2}\bar{\eta}^{(\mathbf{i}}\lambda^{\mathbf{j})I}\,,\\
\delta\lambda^{\mathbf{i}I} &= -\frac{1}{4}\gamma_{mn}\hat{F}^{Imn}\epsilon^{\mathbf{i}} - \frac{i}{2}\gamma^m\mathcal{D}_m\rho^I\epsilon^{\mathbf{i}} + \rho^I\gamma_{mn}T^{mn}\epsilon^{\mathbf{i}} - Y^{I\mathbf{ij}}\epsilon_{\mathbf{j}} + \rho^I\eta^{\mathbf{i}}\,,\\
\delta\rho^I &= \frac{i}{2}\bar{\epsilon}\lambda^I\,,
\end{aligned} \tag{3.1}$$

where

$$\begin{aligned}
\mathcal{D}_\mu\rho^I &= (\partial_\mu - b_\mu)\rho^I - \frac{i}{2}\bar{\psi}_\mu\lambda^I\,,\\
\mathcal{D}_\mu\lambda^{I\mathbf{i}} &= \left(\nabla_\mu - \frac{3}{2}b_\mu\right)\lambda^{I\mathbf{i}} - V_\mu^{\mathbf{ij}}\lambda_{\mathbf{j}}^I + \frac{1}{4}\gamma_{mn}\hat{F}^{Imn}\psi_\mu^{\mathbf{i}} + \frac{i}{2}\gamma^m\left(\mathcal{D}_m\rho^I\right)\psi_\mu^{\mathbf{i}}\\
&\quad + Y^{I\mathbf{ij}}\psi_{\mu\mathbf{j}} - \rho^I\gamma_{mn}\underline{T}^{mn}\psi_\mu^{\mathbf{i}} - \rho^I\underline{\phi}_\mu^{\mathbf{i}}\,,\\
\hat{F}_{\mu\nu}^I &= F_{\mu\nu}^I - \bar{\psi}_{[\mu}\gamma_{\nu]}\lambda^I + \frac{i}{2}\rho^I\bar{\psi}_{[\mu}\psi_{\nu]}\,,
\end{aligned} \tag{3.2}$$

and where $F^I = dA^I$.

These are embedded into a linear multiplet

$$
\begin{aligned}
L(\mathbf{V})_{\mathbf{ij}} &= a_{IJ}\left(2\rho^I Y^J_{\mathbf{ij}} - \frac{i}{2}\bar{\lambda}^I_{\mathbf{i}}\lambda^J_{\mathbf{j}}\right) , \\
\varphi_{\mathbf{i}}(\mathbf{V}) &= a_{IJ}\Big(i\rho^I\gamma^m\mathcal{D}_m\lambda^J_{\mathbf{i}} + 2\rho^I\gamma_{mn}T^{mn}\lambda^J_{\mathbf{i}} - 8\rho^I\rho^J\chi_{\mathbf{i}} \\
&\qquad -\frac{1}{4}\gamma^{mn}\hat{F}^I_{mn}\lambda^J_{\mathbf{i}} + \frac{i}{2}\gamma^m\left(\mathcal{D}_m\rho^I\right)\lambda^J_{\mathbf{i}} - Y^I_{\mathbf{ij}}\lambda^{J\mathbf{j}}\Big) , \\
E^m(\mathbf{V}) &= a_{IJ}\left(\mathcal{D}_n\left(-\rho^I\hat{F}^{Jmn} + 8\rho^I\rho^J T^{mn} - \frac{i}{4}\bar{\lambda}^I\gamma^{mn}\lambda^J\right) - \frac{1}{8}\epsilon^{mnpqr}F^I_{np}F^J_{qr}\right) , \\
N(\mathbf{V}) &= a_{IJ}\Big(\rho^I\Box\rho^J + \frac{1}{2}\left(\mathcal{D}_m\rho^I\right)\left(\mathcal{D}^m\rho^J\right) - \frac{1}{4}\hat{F}^I_{mn}\hat{F}^{Jmn} + Y^{I\mathbf{ij}}Y^J_{\mathbf{ij}} + 8\rho^I\hat{F}^J_{mn}T^{mn} \\
&\qquad -4\rho^I\rho^J\left(D + \frac{26}{3}T^2\right) - \frac{1}{2}\bar{\lambda}^I\gamma^m\mathcal{D}_m\lambda^J + i\bar{\lambda}^I\gamma_{mn}T^{mn}\lambda^J + 16i\rho^I\bar{\chi}\lambda^J\Big) , \qquad (3.3)
\end{aligned}
$$

where a_{IJ} is a symmetric constant matrix. We then compose a density from these with the Lagrange multiplier vector multiplet, $\mathbf{V}_\flat$, and solve the equations of motion of the fields of $\mathbf{V}_\flat$ in terms of the standard-Weyl fields, i.e we get the equations $L(\mathbf{V}) = 0, E^a(\mathbf{V}) = 0, \varphi^{\mathbf{i}}(\mathbf{V}) = 0$ and $N(\mathbf{V}) = 0$. Similarly to the above we can implement the D equation as a constraint by defining a new extended dilaton-Weyl multiplet containing the vector fields and solving the constraints for the Lagrange multipliers.

Note we can diagonalize a_{IJ} using a constant $\mathrm{GL}(n,\mathbb{R})$ transformation, which is just a constant linear field redefinition of the vector multiplets. Furthermore we can set the diagonal entries to be ± 1.[8] We shall take a Lorentzian signature, $\eta_{IJ} = \mathrm{diag}(-1, 1, \ldots, 1)$, so that we arrive at the N-R formulation, which is presumably needed to ensure the absence of ghosts.[9]

Defining $\mathcal{A} = \eta_{IJ}\rho^I\rho^J$, $\mathcal{A}_I = \eta_{IJ}\rho^J$ and solving the equations of motion for the Lagrange multiplier vector multiplet we obtain

$$
\begin{aligned}
\mathcal{A}\underline{T}^{mn} &= -\frac{1}{8}\left(\frac{1}{6}\epsilon^{mnpqr}\hat{H}_{pqr} - \mathcal{A}_I\hat{F}^{Imn} - \eta_{IJ}\frac{i}{4}\bar{\lambda}^I\gamma^{mn}\lambda^J\right) , \\
\mathcal{A}\underline{\chi}^{\mathbf{i}} &= \eta_{IJ}\Big(\frac{i}{8}\rho^I\gamma^m\mathcal{D}_m\lambda^{J\mathbf{i}} + \frac{i}{16}\gamma^m\left(\mathcal{D}_m\rho^I\right)\lambda^{J\mathbf{i}} \\
&\qquad -\frac{1}{32}\gamma_{mn}\hat{F}^{Imn}\lambda^{J\mathbf{i}} + \frac{1}{4}\rho^I\gamma_{mn}\underline{T}^{mn}\lambda^{J\mathbf{i}} - \frac{1}{8}\underline{Y}^I_{\mathbf{ij}}\lambda^{J\mathbf{j}}\Big) , \\
\mathcal{A}\underline{D} &= -\frac{26}{3}\mathcal{A}\underline{T}^2 + \eta_{IJ}\Big(\frac{1}{4}\rho^I\Box\rho^J + \frac{1}{8}\left(\mathcal{D}\rho^I\right)\left(\mathcal{D}\rho^J\right) - \frac{1}{16}\hat{F}^I_{mn}\hat{F}^{Jmn} - \frac{1}{8}\bar{\lambda}^I\gamma^m\mathcal{D}_m\lambda^J \\
&\qquad +\frac{1}{4}\underline{Y}^I_{\mathbf{ij}}\underline{Y}^{J\mathbf{ij}} - 4i\rho^I\lambda^J\underline{\chi} + \left(2\rho^I\hat{F}^J_{mn} + \frac{i}{4}\bar{\lambda}^I\gamma_{mn}\lambda^J\right)\underline{T}^{mn}\Big) , \\
\mathcal{A}_I\underline{Y}^{\mathbf{ij}I} &= \frac{i}{4}\eta_{IJ}\bar{\lambda}^{I\mathbf{i}}\lambda^{J\mathbf{j}} , \qquad (3.4)
\end{aligned}
$$

[8] We shall assume for the time being that $\det a \neq 0$, however it is clear that as we may still diagonalize a_{IJ} the vector multiplet directions for which a_{IJ} has zero eigenvalues will not contribute to this action.

[9] Note that we have not analysed fully whether there is any way to avoid the introduction of ghosts in different signatures for the matrix a_{IJ}, but we expect that the Lorentzian signature is necessary.

where

$$\Box\rho^I = (\nabla^m - 2b^m)\mathcal{D}_m\rho^I - \frac{i}{2}\bar{\psi}_m\mathcal{D}^m\lambda^I - 2\rho^I\bar{\psi}_m\gamma^m\underline{\chi} + \frac{1}{2}\bar{\psi}_m\gamma^m\gamma_{np}\underline{T}^{np}\lambda^I$$
$$+ \frac{1}{2}\underline{\bar{\phi}}^m\gamma_m\lambda^I + 2\underline{f}_m{}^m\rho^I\,. \tag{3.5}$$

We will interpret the last equation in (3.4) as a definition for $\underline{Y}^0_{\mathbf{ij}}$ in terms of the fields of the dilaton-Weyl multiplet and the remaining vector multiplets, which is why we have underlined $Y^I_{\mathbf{ij}}$ in the above expressions. We have introduced the three form H

$$\begin{aligned} H_{\mu\nu\rho} &= 3\partial_{[\mu}B_{\nu\rho]} - \frac{3}{2}\eta_{IJ}A^I_{[\mu}F^J_{\nu\rho]}\,,\\ \hat{H}_{\mu\nu\rho} &= H_{\mu\nu\rho} + \frac{3}{4}\mathcal{A}\bar{\psi}_{[\mu}\gamma_\nu\psi_{\rho]} + \frac{3i}{2}\mathcal{A}_I\bar{\psi}_{[\mu}\gamma_{\nu\rho]}\lambda^I\,, \end{aligned} \tag{3.6}$$

with modified Bianchi identity

$$\nabla_{[m}\hat{H}_{npq]} = -\frac{3}{4}\eta_{IJ}\hat{F}^I_{[mn}\hat{F}^J_{pq]}\,, \tag{3.7}$$

in order to solve the composite linear multiplet vector equation, $E^m = 0$. The two form gauge field $B_{\mu\nu}$ transforms under supersymmetry as

$$\delta B_{\mu\nu} = -\frac{1}{2}\mathcal{A}\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]} - \frac{i}{2}\mathcal{A}_I\bar{\epsilon}\gamma_{\mu\nu}\lambda^I - \eta_{IJ}A^I_{[\mu}\delta A^J_{\nu]}\,, \tag{3.8}$$

and the gauge invariance of H implies a suitable gauge transformation of B is

$$\delta B_{\mu\nu} = 2\partial_{[\mu}\Lambda_{\nu]} + \frac{1}{2}\eta_{IJ}\Lambda^I F^J_{\mu\nu}\,. \tag{3.9}$$

We summarize the general dilaton-Weyl multiplets we have constructed following [27], in which this enlarged algebra was shown to close off-shell, in the conventions of [36, 43] in appendix A.

Inserting these expressions into (2.13) and performing the gauge fixing (2.21), and setting $L = 1$ we obtain for the bosonic part of the off-shell Lagrangian[10]

$$\begin{aligned} e^{-1}\mathcal{L}_L = &-\frac{1}{2}R - V'^{\mathbf{ij}}_\mu V'^\mu_{\mathbf{ij}} - N^2 + P_\mu P^\mu + \sqrt{2}P^\mu V_\mu\\ &-\frac{1}{4}\mathcal{A}^{-1}a_{IJ}F^I\cdot F^J + \frac{1}{2}\mathcal{A}^{-2}\mathcal{A}_I\mathcal{A}_J F^I\cdot F^J + \mathcal{A}^{-1}\eta_{IJ}\underline{Y}^{I\mathbf{ij}}\underline{Y}^J_{\mathbf{ij}}\\ &-\frac{1}{2}\mathcal{A}^{-1}a_{IJ}\left(d\rho^I\right)\cdot\left(d\rho^J\right) + \mathcal{A}^{-2}\mathcal{A}_I\mathcal{A}_J\left(d\rho^I\right)\cdot\left(d\rho^J\right) - \frac{1}{6}\mathcal{A}^{-2}H^2\,, \end{aligned} \tag{3.10}$$

where we have yet to implement the identity involving $Y^{I\mathbf{ij}}$ coming from the last equation in (3.4) in this Lagrangian.

Next we make a non-constant redefinition of the scalar fields

$$\sigma' = \frac{1}{2}\ln(-\mathcal{A})\,, \qquad \rho'^i = (-\mathcal{A})^{-\frac{1}{2}}\rho^i\,, \tag{3.11}$$

[10] Note that the only terms that will change with respect to the Lagrangian of the pure case are those involving D and T.

so inverting this we get

$$\mathcal{A} = -e^{2\sigma'}\,, \quad \rho^i = e^{\sigma'}\rho'^i \qquad \Longrightarrow \qquad \rho^0 = e^{\sigma'}\sqrt{1+\delta_{ij}\rho'^i\rho'^j} := e^{\sigma'}L^0\,. \tag{3.12}$$

Note that as $\mathcal{A}$ is just a quadratic polynomial of the fields ρ^I it is continuous. We shall assume it never vanishes, otherwise our definitions for the standard-Weyl multiplet fields we have eliminated become singular. Thus we shall take the case $\mathcal{A} < 0$ in what follows, and the positive case could be treated identically changing the sign of $\mathcal{A}$ in the transformation, although this appears to change the signature of the scalar manifold and would therefore introduce ghosts. Note that this transformation is a well defined coordinate transformation for the subspace $\mathcal{A} < 0$. Similarly we transform the gauginos such that

$$\lambda^{0\mathbf{i}} = e^{\sigma'}L^0\lambda'^{0\mathbf{i}} + e^{\sigma'}\left(L^0\right)^{-1}\rho'^i\delta_{ij}\lambda'^{j\mathbf{i}}\,, \qquad \lambda^{i\mathbf{i}} = e^{\sigma'}\left(\lambda'^{i\mathbf{i}} + \rho'^i\lambda'^{0\mathbf{i}}\right), \tag{3.13}$$

and the inverse of this transformation is

$$\begin{aligned}\lambda'^{0\mathbf{i}} &= -\mathcal{A}^{-1}\mathcal{A}_I\lambda^{I\mathbf{i}}\,,\\ \lambda'^{i\mathbf{i}} &= \frac{1}{\sqrt{-\mathcal{A}}}\left(\lambda^{i\mathbf{i}} + \rho^i\mathcal{A}^{-1}\mathcal{A}_I\lambda^{I\mathbf{i}}\right)\,.\end{aligned} \tag{3.14}$$

We leave all other fields fixed. Note that after this transformation the condition on the auxiliary fields $Y^{I\mathbf{ij}}$ from (3.4) becomes

$$Y^{0\mathbf{ij}} = (L^0)^{-1}\delta_{ij}\rho'^iY^{j\mathbf{ij}}\,. \tag{3.15}$$

So dropping the primes, the bosonic part of the resulting off-shell Lagrangian is

$$\begin{aligned}e^{-1}\mathcal{L}_L = &-\frac{1}{2}R - V'^{\mathbf{ij}}_{\mu}V'^{\mu}_{\mathbf{ij}} - N^2 + P_\mu P^\mu + \sqrt{2}P^\mu V_\mu\\ &+\frac{1}{4}e^{-2\sigma}\left(\eta_{IJ}+2L_IL_J\right)F^I\cdot F^J + e^{-2\sigma}\eta_{IJ}L^I_\alpha L^J_\beta\delta^\alpha_i\delta^\beta_jY^{i\mathbf{ij}}Y^j_{\mathbf{ij}}\\ &+\frac{3}{2}(d\sigma)\cdot(d\sigma) + \frac{1}{2}\eta_{IJ}L^I_\alpha L^J_\beta\delta^\alpha_i\delta^\beta_j\left(d\rho^i\right)\cdot\left(d\rho^j\right) + \frac{1}{6}e^{-4\sigma}H^2\,,\end{aligned} \tag{3.16}$$

where we have defined

$$L^I = \left(L^0,\rho^i\right), \qquad L^0_\alpha = \left(L^0\right)^{-1}\delta_{\alpha i}\rho^i\,, \qquad L^i_\alpha = \delta^i_\alpha\,, \tag{3.17}$$

and

$$L_I = \left(L^0, -\delta_{ij}\rho^j\right), \qquad L^\alpha_0 = -\left(L^0\right)\delta^\alpha_i\rho^i\,, \qquad L^\alpha_i = \delta^\alpha_i + \delta_{ij}\delta^\alpha_k\rho^j\rho^k\,, \tag{3.18}$$

and where L^0 is defined by (3.12). One may check that after introducing indices $A = (0,a)$ which are raised and lowered with the metric $\eta_{AB} = \mathrm{diag}(-,+,\cdots,+)$ and identifying[11] $L^I_0 = L^I$ and $L^0_I = L_I$ we have

$$L^I_AL^A_J := L^IL_J + L^I_aL^a_J = \delta^I_J\,, \qquad L^A_IL^I_B = \delta^A_B\,, \tag{3.19}$$

[11] Note that with this definition $\eta_{IJ}L^IL^J = -L_IL^I = -1$.

independently of the frame, as long as the vielbein are invertible. Of course we also need these vielbein in order to define the fermions locally on the scalar manifold. Explicitly the vielbein V^a_α is

$$V^a_\alpha = \delta^a_\alpha - \frac{1}{L^0(L^0+1)}\delta^a_i \delta_{\alpha j}\rho^i\rho^j \,, \tag{3.20}$$

with inverse

$$V^\alpha_a = \delta^\alpha_a + \frac{1}{(L^0+1)}\delta^\alpha_i \delta_{a j}\rho^i\rho^j \;. \tag{3.21}$$

We note that this means that the transformations (3.13), (3.14) may be written

$$\lambda^{I\mathbf{i}} = e^{\sigma'}\left(L^I\lambda'^{0\mathbf{i}} + L^I_\alpha \delta^\alpha_i \lambda'^{i\mathbf{i}}\right) = e^{\sigma'}L^I_A\lambda'^{A\mathbf{i}} \,, \qquad \lambda'^{A\mathbf{i}} = \frac{1}{\sqrt{-\mathcal{A}}}L^A_I\lambda^I \,, \tag{3.22}$$

where we defined $\lambda'^A = \delta^A_J\lambda'^J$ and that the condition (3.15) implies

$$Y^{I\mathbf{ij}} = L^I_a V^a_\alpha \delta^\alpha_i Y^{i\mathbf{ij}} \,. \tag{3.23}$$

To maintain our gauge fixing recall the special supersymmetry parameter must be set to

$$\eta_{\mathbf{k}} = \frac{1}{3}(\gamma\cdot T)\epsilon_{\mathbf{k}} - \frac{i}{2}(\gamma^m P_m)\delta_{\mathbf{ik}}\epsilon^{\mathbf{i}} + \dots \;, \tag{3.24}$$

where ... signifies terms higher order in the fermions. The supersymmetry transformations become

$$\begin{aligned}
\delta e^m_\mu &= \frac{1}{2}\bar\epsilon\gamma^m\psi_\mu \,, \\
\delta\psi^{\mathbf{i}}_\mu &= \nabla_\mu\epsilon^{\mathbf{i}} - V^{\mathbf{ij}}_\mu\epsilon_{\mathbf{j}} + i\left(\gamma_{mn}\gamma_\mu - \frac{1}{3}\gamma_\mu\gamma_{mn}\right)\underline{T}^{mn}\epsilon^{\mathbf{i}} - \frac{1}{2}\gamma_\mu\gamma_m P^m\epsilon^{\mathbf{ij}}\delta_{\mathbf{jk}}\epsilon^{\mathbf{k}} \,, \\
\delta V^{\mathbf{ij}}_\mu &= -\frac{3i}{2}\bar\epsilon^{(\mathbf{i}}\underline{\phi}^{\mathbf{j})}_\mu + 4\bar\epsilon^{(\mathbf{i}}\gamma_\mu\underline{\chi}^{\mathbf{j})} + i\bar\epsilon^{(\mathbf{i}}\gamma_{mn}\underline{T}^{mn}\psi^{\mathbf{j})}_\mu + \frac{3i}{2}\underline{\bar\eta}^{(\mathbf{i}}\psi^{\mathbf{j})}_\mu \,, \\
\delta A^I_\mu &= -\frac{i}{2}e^\sigma L^I\bar\epsilon\psi_\mu + \frac{1}{2}e^\sigma\bar\epsilon\gamma_\mu L^I\lambda^0 + \frac{1}{2}e^\sigma\bar\epsilon\gamma_\mu L^I_a\lambda^a \,, \\
\delta B_{\mu\nu} &= \frac{1}{2}e^{2\sigma}\bar\epsilon\gamma_{[\mu}\psi_{\nu]} + \frac{i}{2}e^{2\sigma}\bar\epsilon\gamma_{\mu\nu}\chi - \eta_{IJ}A^I_{[\mu}\delta(\epsilon)A^J_{\nu]} \,, \\
\delta Y^{i\mathbf{ij}} &= -\frac{1}{2}\bar\epsilon^{(\mathbf{i}}\gamma^m\mathcal{D}_m\left(e^\sigma L^i_A\lambda^{\mathbf{j})A}\right) + \frac{i}{2}e^\sigma L^i_A\bar\epsilon^{(\mathbf{i}}\gamma_{mn}\underline{T}^{|mn|}\lambda^{\mathbf{j})A} - 4i\rho^i\bar\epsilon^{(\mathbf{i}}\underline{\chi}^{\mathbf{j})} + \frac{i}{2}e^\sigma L^i_A\underline{\bar\eta}^{(\mathbf{i}}\lambda^{\mathbf{j})A} \,, \\
\delta\lambda^{\mathbf{i}0} &= -\frac{1}{4}e^{-\sigma}L_I F^{Imn}\gamma_{mn}\epsilon^{\mathbf{i}} - \frac{i}{2}\gamma^m(\partial_m\sigma)\epsilon^{\mathbf{i}} + \frac{4}{3}\gamma_{mn}\underline{T}^{mn}\epsilon^{\mathbf{i}} \;, \\
\delta\lambda^{\mathbf{i}a} &= -\frac{1}{4}e^{-\sigma}L^a_I F^{Imn}\gamma_{mn}\epsilon^{\mathbf{i}} - \frac{i}{2}\gamma^m V^a_\alpha(\partial_m\varphi^\alpha)\epsilon^{\mathbf{i}} - e^{-\sigma}V^a_\alpha\delta^\alpha_i Y^{i\mathbf{ij}}\epsilon_{\mathbf{j}} \;, \\
\delta\sigma &= \frac{i}{2}\bar\epsilon\lambda^0 \,, \\
\delta\rho^i &= \frac{i}{2}\bar\epsilon\delta^i_\alpha V^\alpha_a\lambda^a \,,
\end{aligned} \tag{3.25}$$

where we have underlined composite fields, explicit expressions for which are given in (A.4), (A.5), (3.24).

The action (3.16) with supersymmetry transformations (3.25) is an off-shell version of the N-R supergravity presented in [22, 23] which was first described in [27].[12]

[12] We add to that work here by describing fully the scalar manifold, identifying the dilaton from regularity of the supersymmetry transformations and giving an explicit map to the conventions of [22, 23].

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Next we relabel the scalars $\varphi^\alpha = \rho^i\delta_i^\alpha$, and introducing the associated $(n-1)$-dimensional Riemannian metric $g_{\alpha\beta}$ we find

$$g_{\alpha\beta} = \delta_{\alpha\beta} - \frac{1}{(L^0)^2}\delta_{\alpha\delta}\delta_{\beta\gamma}\varphi^\delta\varphi^\gamma\,, \tag{3.26}$$

with inverse

$$g^{\alpha\beta} = \delta^{\alpha\beta} + \varphi^\alpha\varphi^\beta\,, \tag{3.27}$$

with $L^0 = \sqrt{1+\delta_{\alpha\beta}\varphi^\alpha\varphi^\beta}$. Next considering the tensor

$$L_{IJ} = -L_I L_J + L_I^\alpha L_J^\beta g_{\alpha\beta} = \eta_{AB}L_I^A L_J^B = -L_I L_I + L_I^a L_J^b \delta_{ab} \tag{3.28}$$

we see that

$$L_{IJ} = \eta_{IJ} = \mathrm{diag}(-,+,\cdots,+). \tag{3.29}$$

We find that in our frame

$$\begin{aligned} L^I &= \left(L^0, \delta^i_\alpha\varphi^\alpha\right), \qquad L^0_a = \delta_{a\alpha}\varphi^\alpha\,, \quad L^i_a = \delta^i_a + \frac{1}{(L^0+1)}\delta_{a\alpha}\delta^i_\beta\varphi^\alpha\varphi^\beta\,, \\ L_I &= \left(L^0, -\delta_{i\alpha}\varphi^\alpha\right), \quad L^a_0 = -\delta^a_\alpha\varphi^\alpha\,, \quad L^a_i = \delta^a_i + \frac{1}{(L^0+1)}\delta^a_\alpha\delta^i_\beta\varphi^\alpha\varphi^\beta\,. \end{aligned} \tag{3.30}$$

Now using the Cartan equation

$$dV^a + A^a{}_b V^b = 0\,, \tag{3.31}$$

we may read off the connection 1-form $A^a{}_b$ and verify the differential identities

$$L_A{}^I\partial_\alpha L_I{}^B = \frac{1}{2}A_\alpha{}^{ab}(H_{ab})_A{}^B + V_\alpha{}^a(K_a)_A{}^B\,, \tag{3.32}$$

where

$$(H_{ab})_c{}^d = \delta_{ac}\delta_b{}^d - \delta_{bc}\delta_a{}^d \qquad (K_a)_{0b} = \frac{1}{\xi}\delta_{ab}\,, \tag{3.33}$$

and with our current conventions we find $\xi = -1$. Following [22, 23] we then find

$$D_\alpha L_I = \partial_\alpha L_I = \frac{1}{\xi}L_I{}^a V_{\alpha a}\,, \qquad D_\alpha L_I{}^a = \frac{1}{\xi}L_I V_\alpha{}^a\,. \tag{3.34}$$

Moreover, given that

$$L_{IJ}L^J = -L_I\,, \qquad L_{IJ}L_a{}^J = L_{Ia}\,, \tag{3.35}$$

we can evaluate the commutator

$$[D_\alpha, D_\beta]L_I{}^a = -\frac{1}{\xi^2}\left(V_\alpha{}^a V_\beta{}^b - V_\beta{}^a V_\alpha{}^b\right)L_{Ib}\,, \tag{3.36}$$

so we can read off the curvature tensor of H^n which is

$$R_{\alpha\beta}{}^{ab} = -\frac{1}{\xi^2}\left(V_\alpha{}^a V_\beta{}^b - V_\beta{}^a V_\alpha{}^b\right), \tag{3.37}$$

and the Ricci scalar is negative and given by $R = -n(n-1)/\xi^2$.

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In particular we have the coset algebra for the coset generators K_a and SO(n) generators H_{ab},

$$\Big[H_{ab}, H_{cd}\Big] = \delta_{bc}H_{ad} - \delta_{ac}H_{bd} + \delta_{ad}H_{bc} - \delta_{bd}H_{ac} \; ,$$
$$\Big[H_{ab}, K_c\Big] = \delta_{bc}K_a - \delta_{ac}K_b \, , \qquad [K_a, K_b] = \frac{1}{\xi^2}H_{ab} \, . \tag{3.38}$$

So the scalar manifold is simply the coset space SO(1,n)/SO(n).

Note that the conditions (3.19) are identities and not constraints, which is somewhat different to the case of vector multiplets in the background of the standard-Weyl multiplet, where the scalar field D acts as a Lagrange multiplier to implement the very special geometry constraint in two derivative theories.[13] Here though the constraint coming from the D equation of motion is avoided by moving to the dilaton-Weyl multiplet and solving for the Lagrange multipliers which no longer occur in the action.

Integrating out $V_\mu^{\mathbf{ij}}, V_\mu, P_\mu, N$ and $Y^{i\mathbf{j}}$ the action for the linear multiplet becomes

$$\begin{aligned} -e^{-1}\mathcal{L}_L = &+\frac{1}{2}R - \frac{1}{4}e^{-2\sigma}(L_I L_J + L_I^a L_{Ja})F^I \cdot F^J - \frac{1}{6}e^{-4\sigma}H^2 \\ &- \frac{3}{2}(d\sigma)^2 - \frac{1}{2}g_{\alpha\beta}(d\varphi^\alpha)\cdot(d\varphi^\beta) \; , \end{aligned} \tag{3.39}$$

and the supersymmetry variations are now

$$\begin{aligned} \delta e_\mu^m &= \frac{1}{2}\bar{\epsilon}\gamma^m\psi_\mu \, , \\ \delta\psi_\mu^{\mathbf{i}} &= \nabla_\mu\epsilon^{\mathbf{i}} + i\gamma_{\nu_1\nu_2}\underline{T}^{\nu_1\nu_2}\gamma_\mu\epsilon^{\mathbf{i}} - \frac{i}{3}\gamma_\mu\gamma_{\nu_1\nu_2}\underline{T}^{\nu_1\nu_2}\epsilon^{\mathbf{i}} \, , \\ \delta A_\mu^I &= -\frac{i}{2}e^\sigma L^I\bar{\epsilon}\psi_\mu + \frac{1}{2}e^\sigma L^I\bar{\epsilon}\gamma_\mu\chi + \frac{1}{2}e^\sigma\bar{\epsilon}\gamma_\mu L_a^I\lambda^a \, , \\ \delta B_{\mu\nu} &= \frac{1}{2}e^{2\sigma}\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]} + \frac{i}{2}e^{2\sigma}\bar{\epsilon}\gamma_{\mu\nu}\chi - \eta_{IJ}A_{[\mu}^I\delta(\epsilon)A_{\nu]}^J \, , \\ \delta\chi^{\mathbf{i}} &= -\frac{1}{12}\gamma_{mn}e^{-\sigma}L_I F^{Imn}\epsilon^{\mathbf{i}} - \frac{i}{2}\gamma^\mu(d\sigma)_\mu\epsilon^{\mathbf{i}} + \frac{i}{18}e^{-2\sigma}H^{mnp}\gamma_{mnp}\epsilon^{\mathbf{i}} \, , \\ \delta\lambda^{\mathbf{i}a} &= -\frac{1}{4}e^{-\sigma}L_I^a F^{Imn}\gamma_{mn}\epsilon^{\mathbf{i}} - \frac{i}{2}\gamma^m V_\alpha^a(\partial_m\varphi^\alpha)\epsilon^{\mathbf{i}} \, , \\ \delta\sigma &= \frac{i}{2}\bar{\epsilon}\chi \, , \\ \delta\varphi^\alpha &= \frac{i}{2}\bar{\epsilon}V_a^\alpha\lambda^a \; . \end{aligned} \tag{3.40}$$

Note that there are still some differences between this formulation and the N-R supergravity presented in [22, 23], in particular here the parameter $\xi = -1$, whereas in [22, 23] $\xi = -\frac{1}{\sqrt{2}}$. However the differences are merely due to conventions, and the explicit (constant) field redefinition is given in appendix B. We find it useful to keep these conventions, as we will be interested in adding high derivative terms which are simple generalizations of those presented in [36].

[13] One can see that we do have one field that can act in this way, which is the scalar field of the compensating linear multiplet. It is possible that the scalar manifolds for physical tensor multiplet scalars in the background of the dilaton-Weyl gravitational multiplet will be similar to those of the vector multiplets in the background of the standard-Weyl multiplet.

Note that the on-shell theory with action (3.39) is invariant under the scaling symmetry

$$\sigma \to \sigma + c \qquad B_{\mu\nu} \to e^{2c} B_{\mu\nu} \qquad A^I_\mu \to e^c A^I_\mu \qquad G_{\mu\nu\rho} \to e^{2c} G_{\mu\nu\rho}\,, \tag{3.41}$$

and the off-shell theory (3.16) with supersymmetry transformations (3.25) maintains this symmetry if we also scale

$$Y^{I\mathbf{ij}} \to e^c Y^{I\mathbf{ij}}\,. \tag{3.42}$$

Before we turn to higher derivative terms we wish to consider whether the vector multiplet coupling of this theory can be generalized from that presented in [22, 23]. To this end we may also add the most general vector multiplet coupling that is compatible with the Lagrange multiplier vector multiplet continuing to function as such. This reads

$$\begin{aligned} e^{-1}\mathcal{L}_V = C_{IJK}\Big(&-\frac{1}{4}\rho^I F^J\cdot F^K + \frac{1}{3}\rho^I\rho^J\Box\rho^K + \frac{1}{6}\rho^I\left(\mathcal{D}\rho^J\right)\cdot\left(\mathcal{D}\rho^K\right) + \rho^I Y^{J\mathbf{ij}}Y^K_{\mathbf{ij}} \\ &-\frac{4}{3}\rho^I\rho^J\rho^K\left(D + \frac{26}{3}T^2\right) + 4\rho^I\rho^J F^K_{\mu\nu}T^{\mu\nu} - \frac{e^{-1}}{24}\epsilon^{\mu\nu\rho\sigma\lambda}A^I_\mu F^J_{\nu\rho}F^K_{\sigma\lambda}\Big)\,, \end{aligned} \tag{3.43}$$

which is completely independent of the Lagrange multiplier vector multiplet.

There are two special cases where the density (3.43) vanishes $\mathcal{L}_V = 0$, where either $C_{IJK} = 0$ or less trivially when $C_{IJK} = d_{(I}a_{JK)}$. To see that the density vanishes in the later the case note that it is formed from the combination of the vanishing composite linear multiplet and another set of vector multiplets, and each term in the density contains an element of the linear composite multiplet. One can also verify this by direct computation of course. Another way to see this is by considering the original cubic prepotential involving the Lagrange multiplier vector multiplet, $\rho^\flat\mathcal{A}$. Indeed making a field redefinition of the Lagrange multiplier vector multiplet of the form

$$\rho^\flat = \rho'^\flat + d_I\rho^I \tag{3.44}$$

will not change the theory and simply generates the vanishing term considered above.

For general C_{IJK} we define

$$\mathcal{C} = C_{IJK}\rho^I\rho^J\rho^K\,, \qquad \mathcal{C}_I = C_{IJK}\rho^J\rho^K\,, \qquad \mathcal{C}_{IJ} = C_{IJK}\rho^K\,, \tag{3.45}$$

and the density (3.43) becomes

$$\begin{aligned} e^{-1}\mathcal{L}_V = &-\frac{1}{4}\left(\mathcal{C}_{IJ} - \frac{\mathcal{C}}{3}\mathcal{A}^{-1}a_{IJ} - 2\mathcal{A}^{-1}\mathcal{A}_I\mathcal{C}_J + \frac{4\mathcal{C}}{3}\mathcal{A}^{-2}\mathcal{A}_I\mathcal{A}_J\right)F^I\cdot F^J \\ &+\left(\mathcal{C}_{IJ} - \frac{\mathcal{C}}{3}\mathcal{A}^{-1}a_{IJ}\right)\underline{Y}^{I\mathbf{ij}}\underline{Y}^J_{\mathbf{ij}} - \frac{1}{24}C_{IJK}\epsilon^{mnpqr}A^I_m F^J_{np}F^K_{qr} \\ &-\frac{1}{2}\left(\mathcal{C}_{IJ} - 2\mathcal{A}^{-1}\mathcal{A}_I\mathcal{C}_J - \frac{\mathcal{C}}{3}\mathcal{A}^{-1}a_{IJ} + \frac{4\mathcal{C}}{3}\mathcal{A}^{-2}\mathcal{A}_I\mathcal{A}_J\right)\left(d\rho^I\right)\cdot\left(d\rho^J\right) \\ &-\frac{1}{12}\left(\mathcal{A}^{-1}\mathcal{C}_I - \frac{2\mathcal{C}}{3}\mathcal{A}^{-2}\mathcal{A}_I\right)\epsilon^{mnpqr}F^I_{mn}H_{pqr}\,. \end{aligned} \tag{3.46}$$

Note however that this density contains terms not present in the original formulation, and as such this represents a generalization of the vector multiplet couplings, and furthermore the dilatonic couplings break the symmetry (3.41). Also note that the two Ricci

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scalar contributions to this density coming from the superconformal d'Alembertion have cancelled. Applying the transformations (3.11), (3.14) we obtain

$$
\begin{aligned}
e^{-1}\mathcal{L}_V = &+e^{\sigma}\left(\tilde{\mathcal{C}}_{IJ}+\frac{\tilde{\mathcal{C}}}{3}\eta_{IJ}\right)\left(L^I_\alpha L^J_\beta \delta^\alpha_i\delta^\beta_j\right)Y^{i\mathbf{ij}}Y^j_{\mathbf{ij}}+\frac{1}{12}\left(\tilde{\mathcal{C}}_I-\frac{2\tilde{\mathcal{C}}}{3}L_I\right)\epsilon^{mnpqr}F^I_{mn}H_{pqr}\\
&-\frac{1}{4}e^{\sigma}\left(\tilde{\mathcal{C}}_{IJ}+\frac{\tilde{\mathcal{C}}}{3}\eta_{IJ}-2L_I\tilde{\mathcal{C}}_J+\frac{4\tilde{\mathcal{C}}}{3}L_IL_J\right)F^I\cdot F^J-\frac{1}{24}C_{IJK}\epsilon^{mnpqr}A^I_mF^J_{np}F^K_{qr}\\
&-\frac{1}{2}e^{3\sigma}\left(\tilde{\mathcal{C}}_{IJ}+\frac{\tilde{\mathcal{C}}}{3}\eta_{IJ}\right)L^I_\alpha L^J_\beta\delta^\alpha_i\delta^\beta_j\left(d\rho^i\right)\cdot\left(d\rho^j\right). \qquad (3.47)
\end{aligned}
$$

The explicit Chern-Simons term and the term involving both the 2- and 3-form field strengths F^I and H do not occur in the N-R formulation. If we demand their absence we find the condition

$$
C_{IJK}=\left(2\tilde{\mathcal{C}}L_{(I}-3\tilde{\mathcal{C}}_{(I}\right)\eta_{JK)}\,, \qquad (3.48)
$$

which implies

$$
\tilde{\mathcal{C}}_{IJ}=-\frac{\tilde{\mathcal{C}}}{3}\eta_{IJ}+2L_I\tilde{\mathcal{C}}_J-\frac{4\tilde{\mathcal{C}}}{3}L_IL_J\,, \qquad (3.49)
$$

but this implies that the entire density vanishes and we are left with the N-R supergravity coming from the linear multiplet density only. Note that the Chern-Simons term clearly breaks the symmetry (3.41). Demanding (3.48) is the only way to restore it, apart from the exceptional case when we have only one vector multiplet in which case (3.48) is automatic, but the density in that case again vanishes as discussed in the previous section.

Now we turn to the case in which $\det a=0$. In this case we can still diagonalize the rank r tensor a_{IJ} with a constant $\mathrm{GL}(r,\mathbb{R})$ transformation. Putting a tilde on the indices in (3.43) and then splitting indices into $\tilde{I}=(I,\hat{I})$ with $I=(0,\cdots,r-1)$ and $\hat{I}=(r,\cdots n)$. We will refer to the r I directions as internal vector multiplets as they occur in the gravitational multiplet, and the remaining $\hat{I}$ directions as external vector multiplets. As the contribution to the density formed from the Lagrange multiplier vector multiplet and the composite linear multiplet (3.3) vanishes for the external vector multiplets, we only have the contribution to the density (3.43). Substituting the expressions for the composite standard-Weyl multiplet fields this reads

$$
\begin{aligned}
e^{-1}\mathcal{L}_V = &-\frac{1}{4}\left(\mathcal{C}_{IJ}-\frac{\mathcal{C}}{3}\mathcal{A}^{-1}a_{IJ}-2\mathcal{A}^{-1}\mathcal{A}_I\mathcal{C}_J+\frac{4\mathcal{C}}{3}\mathcal{A}^{-2}\mathcal{A}_I\mathcal{A}_J\right)F^I\cdot F^J\\
&-\frac{1}{4}\mathcal{C}_{\hat{I}\hat{J}}F^{\hat{I}}\cdot F^{\hat{J}}-\frac{1}{4}\left(2\mathcal{C}_{I\hat{J}}-2\mathcal{A}^{-1}\mathcal{A}_I\mathcal{C}_{\hat{J}}\right)F^I\cdot F^{\hat{J}}\\
&+\left(\mathcal{C}_{IJ}-\frac{\mathcal{C}}{3}\mathcal{A}^{-1}a_{IJ}\right)\underline{Y}^{I\mathbf{ij}}\underline{Y}^J_{\mathbf{ij}}+2\mathcal{C}_{I\hat{J}}\underline{Y}^{I\mathbf{ij}}Y^{\hat{J}}_{\mathbf{ij}}+\mathcal{C}_{\hat{I}\hat{J}}Y^{\hat{I}\mathbf{ij}}Y^{\hat{J}}_{\mathbf{ij}}\\
&-\frac{1}{2}\left(\mathcal{C}_{IJ}-2\mathcal{A}^{-1}\mathcal{A}_I\mathcal{C}_J-\frac{\mathcal{C}}{3}\mathcal{A}^{-1}a_{IJ}+\frac{4\mathcal{C}}{3}\mathcal{A}^{-2}\mathcal{A}_I\mathcal{A}_J\right)\left(d\rho^I\right)\cdot\left(d\rho^J\right)\\
&-\frac{1}{2}\left(2\mathcal{C}_{I\hat{J}}-2\mathcal{A}^{-1}\mathcal{A}_I\mathcal{C}_{\hat{J}}\right)\left(d\rho^I\right)\cdot\left(d\rho^{\hat{J}}\right)-\frac{1}{2}\mathcal{C}_{\hat{I}\hat{J}}\left(d\rho^{\hat{I}}\right)\cdot\left(d\rho^{\hat{J}}\right)\\
&-\frac{1}{12}\left(\mathcal{A}^{-1}\mathcal{C}_I-\frac{2\mathcal{C}}{3}\mathcal{A}^{-2}\mathcal{A}_I\right)\epsilon^{mnpqr}F^I_{mn}H_{pqr}-\frac{1}{12}\mathcal{A}^{-1}\mathcal{C}_{\hat{I}}\epsilon^{mnpqr}F^{\hat{I}}_{mn}H_{pqr}\\
&-\frac{1}{24}C_{\tilde{I}\tilde{J}\tilde{K}}\epsilon^{mnpqr}A^{\tilde{I}}_mF^{\tilde{J}}_{np}F^{\tilde{K}}_{qr}\,, \qquad (3.50)
\end{aligned}
$$

where

$$\mathcal{C} = C_{\tilde{I}\tilde{J}\tilde{K}}\rho^{\tilde{I}}\rho^{\tilde{J}}\rho^{\tilde{K}}\,, \qquad \mathcal{C}_{\tilde{I}} = C_{\tilde{I}\tilde{J}\tilde{K}}\rho^{\tilde{J}}\rho^{\tilde{K}}\,, \qquad \mathcal{C}_{\tilde{I}\tilde{J}} = C_{\tilde{I}\tilde{J}\tilde{K}}\rho^{\tilde{K}}\,. \tag{3.51}$$

As discussed above the explicit Chern-Simons term breaks the symmetry (3.41), so if we wish to maintain it extended to the external vector multiplets we need that the last two lines of the above density cancel up to a surface term. In this case we immediately obtain

$$C_{\hat{I}\hat{J}\hat{K}} = C_{\hat{I}\hat{J}K} = 0, \quad C_{IJK} = 3\mathcal{A}^{-1}\mathcal{C}_{(I}\eta_{JK)} - 2\mathcal{C}\mathcal{A}^{-2}\mathcal{A}_{(I}\eta_{JK)}, \quad C_{\hat{I}JK} = \mathcal{A}^{-1}\mathcal{C}_{\hat{I}}\eta_{JK}, \tag{3.52}$$

but again we find that in this case the density vanishes as these imply that

$$\mathcal{C}_{IJ} = \frac{\mathcal{C}}{3}\mathcal{A}^{-1}\eta_{IJ} + 2\mathcal{A}^{-1}\mathcal{A}_I\mathcal{C}_J - \frac{4\mathcal{C}}{3}\mathcal{A}^{-2}\mathcal{A}_I\mathcal{A}_J\,. \tag{3.53}$$

Again the vanishing of the denisty in this case can be seen from a redefinition of the Lagrange multiplier of the form

$$\rho^\flat = \rho'^\flat + d_{\tilde{I}}\rho^{\tilde{I}}\,, \tag{3.54}$$

which generates the terms

$$C_{IJK} = d_{(I}a_{JK)} \qquad C_{\hat{I}JK} = \frac{1}{3}d_{\hat{I}}a_{JK} \tag{3.55}$$

which are equivalent to (3.52) for some constants $d_{\tilde{I}}$. The density (3.50) is the most general vector multiplet coupling we can add, and we have shown that it generically breaks the symmetry (3.41). Indeed the original prepotential $\rho^\flat\mathcal{A} + \mathcal{C}$ exhibits the symmetry

$$\rho^\flat \to e^{-2c}\,, \qquad \rho^I \to e^c\rho^I\,, \qquad \rho^{\hat{I}} \to e^c\rho^{\hat{I}}\,, \tag{3.56}$$

only in the case $\mathcal{C} = 0$, up to the terms (3.55) which can be generated by the redefinition of the Lagrange multiplier.

If however we require only that the internal vector multiplets have the symmetry (3.41) whilst the external vector multiplets are inert under this transformation, it is clear that we may add couplings between external vector multiplets $\hat{I}$ whilst preserving (3.41), i.e we take $C_{IJK} = 3\mathcal{A}^{-1}\mathcal{C}_{(I}\eta_{JK)} - 2\mathcal{C}\mathcal{A}^{-2}\mathcal{A}_{(I}\eta_{JK)}$ and $C_{\hat{I}JK} = \mathcal{A}^{-1}\mathcal{C}_{\hat{I}}\eta_{JK}$ but now we allow $C_{\hat{I}\hat{J}\hat{K}}$ to be arbitrary,[14] so that we maintain the symmetry

$$\sigma \to \sigma + c \qquad A^I_\mu \to e^c A^I_\mu \qquad B_{\mu\nu} \to e^{2c}B_{\mu\nu} \qquad A^{\hat{I}}_\mu \to A^{\hat{I}}_\mu\,. \tag{3.57}$$

The density in this case reads

$$e^{-1}\mathcal{L}_V = -\frac{1}{4}\mathcal{C}_{\hat{I}\hat{J}}F^{\hat{I}}\cdot F^{\hat{J}} + \mathcal{C}_{\hat{I}\hat{J}}Y^{\hat{I}\mathbf{ij}}Y^{\hat{J}}_{\mathbf{ij}} - \frac{1}{2}\mathcal{C}_{\hat{I}\hat{J}}\left(d\rho^{\hat{I}}\right)\cdot\left(d\rho^{\hat{J}}\right) - \frac{1}{24}C_{\hat{I}\hat{J}\hat{K}}\epsilon^{mnpqr}A^{\hat{I}}_m F^{\hat{J}}_{np}F^{\hat{K}}_{qr}. \tag{3.58}$$

Note that we must therefore not transform the external scalars with our coordinate transformation (3.11), so the supersymmetry transformations of the external vector multiplets

[14] We cannot have $C_{I\hat{J}\hat{K}}$ different from zero and maintain the symmetry as the corresponding Chern Simons term explicitly breaks it and there is no candidate cancellation term coming from the $*(F\wedge H)$ terms.

are given by (3.1). If we allow for different scaling behaviour of the external multiplets, we may construct densities which respect the symmetry

$$\sigma \to \sigma + c \qquad A^I_\mu \to e^c A^I_\mu \qquad B_{\mu\nu} \to e^{2c} B_{\mu\nu} \qquad A^{\hat{I}}_\mu \to e^{-kc} A^{\hat{I}}_\mu \,, \tag{3.59}$$

by transforming the external scalars such that

$$\rho'^{\hat{I}} = e^{k\sigma} \rho^{\hat{I}} \,, \tag{3.60}$$

and the gauginos by

$$\lambda'^{\hat{I}} = k e^{k\sigma} \lambda^0 + e^{k\sigma} \lambda^{\hat{I}} \tag{3.61}$$

in the following cases. We have discussed the case $k = 0$ above, which corresponds to allowing us to take $C_{\hat{I}\hat{J}\hat{K}}$ non-zero. It is clear we can never take C_{IJK} different from its expression above. In the case $k = \frac{1}{2}$ we may take $C_{IJ\hat{K}} \neq 0$ but then we need $C_{\hat{I}\hat{J}\hat{K}} = 0$ and $C_{IJ\hat{K}}$ must be equal to its expression above. Finally in the case $k = 1$ we may allow $C_{\hat{I}JK}$ to differ from its expression above, but need $C_{\hat{I}\hat{J}\hat{K}} = C_{\hat{I}\hat{J}K} = 0$. This can also be seen easily by inspection of the original cubic prepotential.

The case of one internal multiplet is exceptional as we shall now discuss. Recall that the vector density formed from the internal vector multiplet vanishes identically. Indeed it is also the case that a density formed from two internal multiplets and arbitrarily many external multiplets must vanish. This means that we may take arbitrary $C_{000}, C_{00\hat{I}}$, however terms involving these quantities will not appear in the action, and will therefore not break the symmetry (3.41). Indeed we may read off the most general contribution to the density from (3.50).

$$\begin{aligned}
&e^{-1}\mathcal{L}_V = \\
&\quad -\frac{1}{4}\left(e^{-\sigma}\mathcal{D} + e^{-2\sigma}\hat{\mathcal{C}}\right)\left(F^0\right)^2 + \frac{1}{2}\left(\hat{\mathcal{C}}_{\hat{I}\hat{J}} + e^{-\sigma}\mathcal{D}_{\hat{I}\hat{J}}\right) F^{\hat{I}} \cdot F^{\hat{J}} + \frac{1}{2}\left(\mathcal{D}_{\hat{I}} + e^{-\sigma}\hat{\mathcal{C}}_{\hat{I}}\right) F^0 \cdot F^{\hat{I}} \\
&\quad -\frac{1}{2}\left(\hat{\mathcal{C}} + e^{\sigma}\mathcal{D}\right)(d\sigma)^2 + \left(\hat{\mathcal{C}}_{\hat{I}} + e^{\sigma}\mathcal{D}_{\hat{I}}\right)(d\sigma)\cdot\left(d\rho^{\hat{I}}\right) - \frac{1}{2}\left(\hat{\mathcal{C}}_{\hat{I}\hat{J}} + e^{\sigma} D_{\hat{I}\hat{J}}\right)\left(d\rho^{\hat{I}}\right)\cdot\left(d\rho^{\hat{J}}\right) \\
&\quad -\frac{1}{12}\left(e^{-2\sigma}\mathcal{D} - e^{-3\sigma}\frac{2}{3}\hat{\mathcal{C}}\right)\epsilon^{mnpqr} F^0_{mn} H_{pqr} + \frac{1}{12}\left(e^{-2\sigma}\mathcal{C}_{\hat{I}} + 2e^{-\sigma}\mathcal{D}_{\hat{I}}\right)\epsilon^{mnpqr} F^{\hat{I}}_{mn} H_{pqr} \\
&\quad -\frac{1}{24} C_{\hat{I}\hat{J}\hat{K}}\epsilon^{mnpqr} A^{\hat{I}}_m F^{\hat{J}}_{np} F^{\hat{K}}_{qr} - \frac{1}{8} D_{\hat{I}\hat{J}}\epsilon^{mnpqr} A^0_m F^{\hat{I}}_{np} F^{\hat{J}}_{qr} + \left(\hat{\mathcal{C}}_{\hat{I}\hat{J}} + e^{\sigma} D_{\hat{I}\hat{J}}\right) Y^{\hat{I}\mathbf{ij}} Y^{\hat{J}}_{\mathbf{ij}} \,,
\end{aligned} \tag{3.62}$$

where we defined

$$\hat{\mathcal{C}} = C_{\hat{I}\hat{J}\hat{K}}\rho^{\hat{I}}\rho^{\hat{J}}\rho^{\hat{K}} \,, \qquad \hat{\mathcal{C}}_{\hat{I}} = C_{\hat{I}\hat{J}\hat{K}}\rho^{\hat{J}}\rho^{\hat{K}} \,, \qquad \hat{\mathcal{C}}_{\hat{I}\hat{J}} = C_{\hat{I}\hat{J}\hat{K}}\rho^{\hat{K}} \,, \tag{3.63}$$

and

$$\mathcal{D} = C_{0\hat{I}\hat{J}}\rho^{\hat{I}}\rho^{\hat{J}} \,, \qquad \mathcal{D}_{\hat{I}} = C_{0\hat{I}\hat{J}}\rho^{\hat{J}} \,, \qquad D_{\hat{I}\hat{J}} = C_{0\hat{I}\hat{J}} \,. \tag{3.64}$$

Similarly to the above cases we may preserve the symmetry (3.57) only if $D_{\hat{I}\hat{J}} = 0$, but the theory exhibits a symmetry of the form (3.59) after a suitable scalar and gaugino redefinition when taking only one of $D_{\hat{I}\hat{J}}$ or $C_{\hat{I}\hat{J}\hat{K}}$ non-vanishing.

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To summarize if we demand that the symmetry (3.41) is extended to the external vector multiplets we may only add vector multiplet couplings of the form

$$C_{IJK} = 3\mathcal{A}^{-1}\mathcal{C}_{(I}\eta_{JK)} - 2\mathcal{C}\mathcal{A}^{-2}\mathcal{A}_{(I}\eta_{JK)}\,, \qquad C_{\hat{I}JK} = \mathcal{A}^{-1}\mathcal{C}_{\hat{I}}\eta_{JK}\,, \tag{3.65}$$

with all other components zero, but the density (3.50) vanishes, and the scalar manifold is simply $\mathrm{SO}(1,1)\times\mathrm{SO}(1,n)/\mathrm{SO}(n)$. On the other hand if we demand that the external vector multiplets are inert under this transformation (3.57), then we must take the expressions (3.65) with $C_{\hat{I}\hat{J}K} = 0$, but with arbitrary $C_{\hat{I}\hat{J}\hat{K}}$ and (3.58) is the corresponding density which allows for the preservation of the symmetry (3.41). The scalar manifold is then a product of $\mathrm{SO}(1,1)\times\mathrm{SO}(1,n)/\mathrm{SO}(n)\times\mathcal{M}$, with $\mathcal{M}$ some $m = n - r$ dimensional manifold, which seems only to be restricted by demanding the absence of ghosts in the theory. Also an explicit Chern-Simons term appears. On the other hand, if we relax the assumption that our theory should preserve the symmetry (3.41) then we may add the general vector multiplet couplings and obtain the density (3.50). In this case the entire scalar manifold is dependent on the form of $C_{\tilde{I}\tilde{J}\tilde{K}}$. In particular a Lagrange multiplier forcing a restriction of the scalar manifold, for example the very special geometry condition, is absent. If we view the theory as being defined by the $C_{\tilde{I}\tilde{J}\tilde{K}}$ from compactification then the symmetry (3.41) or even (3.59) is generically broken.

4 Higher derivative densities

In this section we shall describe how to simply generalize the known Ricci squared [36] and Weyl squared [35] invariants to an arbitrary number of internal and external vector multiplets. In [43] an off-shell superconformal Riemann squared invariant was derived in the $r = 1$ dilaton-Weyl multiplet that we used here to construct the pure N-R supergravity, but we leave the generalization of the Riemann squared invariant for future work.[15]

4.1 Ricci squared invariant

In [36] a Ricci squared invariant coupled to vector multiplets in the r=1 dilaton-Weyl multiplet was constructed in a particular basis of the superconformal fields. This basis is equivalent to a reversible gauge fixing of the theory by breaking the SU(2) down to U(1), and breaking the local dilatonic symmetry and special supersymmetry. We shall give the details of the construction without going to this basis, by using the construction of the Ricci squared invariant in the standard-Weyl multiplet, which was also given in [36]. The essential observation is that the Ricci scalar appears in the composite expression for the field $Y^{\mathbf{ij}}$ in terms of a linear multiplet, and that this is not cancelled by the contribution coming from the expression for $\underline{D}$ when moving to the general dilaton-Weyl multiplet. Thus in the standard-Weyl multiplet we may form the Ricci squared invariant by considering a composite linear multiplet, which is formed from two copies of a composite vector

[15]Deriving this invariant is equivalent to deriving the Riemann squared invariant in the standard-Weyl multiplet, which has yet to be given in components, but was recently analysed in superspace in [44].

multiplet, each of which is formed from our compensating linear multiplet. Schematically the density is

$$e^{-1}\mathcal{L} = \mathbf{V}^{\tilde{I}} \cdot \mathbf{L}\left(\mathbf{V}^{\#}, \mathbf{V}^{\#}\right) \tag{4.1}$$

where $\mathbf{V}^{\#} = \mathbf{V}(\mathbf{L}_0)$. Clearly as the density (3.43) was formed from composing the linear multiplet from two sets of vector multiplets, we may construct a density from (3.43) by setting $C_{\tilde{I}\#\#} = e_{\tilde{I}}$, where the vector multiplet $\mathbf{V}^{\#}$ is composite and is formed from our compensating linear multiplet. After the gauge fixing (2.27) and setting $L = 1$, the bosonic parts of the vector multiplet composed of our compensating linear multiplet, which we obtain from gauge fixing (C.1), are simply

$$\begin{aligned}
\underline{\rho}^{\#} &= 2N\,, \\
\underline{Y}^{\mathbf{ij}}_{\#} &= \frac{1}{\sqrt{2}}\delta^{\mathbf{ij}}\left(-\frac{3}{8}R - N^2 - P^2 + \frac{8}{3}T^2 + 4D - V'^{\mathbf{kl}}_{a}V'^{a}_{\mathbf{kl}}\right) \\
&\quad + 2P^a V'^{\mathbf{ij}}_{a} + \sqrt{2}\nabla^a V'^{(\mathbf{i}}_{a\mathbf{m}}\delta^{\mathbf{j})\mathbf{m}}, \\
F^{\#}_{\mu\nu} &= 4\partial_{[\mu}P_{\nu]} + 2\sqrt{2}\partial_{[\mu}V_{\nu]}\,,
\end{aligned} \tag{4.2}$$

where we have split $V^{\mathbf{ij}}$ into its traceful and traceless parts as in (2.30).

We obtain the density

$$\begin{aligned}
&e^{-1}\mathcal{L}_{R^2} = \\
&\quad \mathcal{E}\left(\frac{3}{8}R - 4\left(\underline{D} + \frac{26}{3}\underline{T}^2\right) + 32\underline{T}^2 + N^2 + P^2 + V'^2\right)^2 - 16\mathcal{E}N^2\left(\underline{D} + \frac{26}{3}\underline{T}^2\right) \\
&\quad + 2\mathcal{E}\left(\sqrt{2}P^a V'^{\mathbf{ij}}_{a} + \nabla^a V'^{\mathbf{ij}}_{a}\right)\left(\sqrt{2}P^a V'_{a\mathbf{ij}} + \nabla^a V'_{a\mathbf{ij}}\right) + 16\mathcal{E}N\left(\sqrt{2}dV + 2dP\right)\cdot\underline{T} \\
&\quad - \frac{1}{2}\mathcal{E}(dV)^2 - \sqrt{2}\mathcal{E}(dV)\cdot(dP) - \mathcal{E}(dP)^2 - 2\mathcal{E}(dN)^2 - 4Ne_{\tilde{I}}\left(d\rho^{\tilde{I}}\right)\cdot(dN) \\
&\quad - \sqrt{2}e_{\tilde{I}}N\left(F^{\tilde{I}}\cdot dV\right) - 2e_{\tilde{I}}N\left(F^{\tilde{I}}\cdot dP\right) + 16e_{\tilde{I}}N^2\left(F^{\tilde{I}}\cdot\underline{T}\right) - \frac{1}{3}\mathcal{E}N^2R \\
&\quad - 2\sqrt{2}e_{\tilde{I}}\underline{Y}^{\tilde{I}\mathbf{ij}}\delta_{\mathbf{ij}}\left(\frac{3}{8}RN - 4\underline{D}N - \frac{8}{3}N\underline{T}^2 + N^3 + NP^2 + NV'^2\right) \\
&\quad - e_{\tilde{I}}\epsilon^{\mu\nu\rho\sigma\tau}A^{\tilde{I}}_{\mu}\partial_\nu V_\rho\partial_\sigma V_\tau - 2\sqrt{2}e_{\tilde{I}}\epsilon^{\mu\nu\rho\sigma\tau}A^{\tilde{I}}_{\mu}\partial_\nu P_\rho\partial_\sigma V_\tau - 2e_{\tilde{I}}\epsilon^{\mu\nu\rho\sigma\tau}A^{\tilde{I}}_{\mu}\partial_\nu P_\rho\partial_\sigma P_\tau \\
&\quad + 8e_{\tilde{I}}\underline{Y}^{\tilde{I}}_{\mathbf{ij}}V'^{\mathbf{ij}}_{a}NP^a - 4\sqrt{2}e_{\tilde{I}}\underline{Y}^{\tilde{I}}_{\mathbf{ij}}N\nabla^a V'^{\mathbf{m}(\mathbf{i}}_{a}\delta^{\mathbf{j})}_{\mathbf{m}}\,,
\end{aligned} \tag{4.3}$$

where $\mathcal{E} = e_{\tilde{I}}\rho^{\tilde{I}}$. Substituting the expressions for the composite standard-Weyl fields from (3.4) we obtain a supersymmetric Ricci squared invariant coupled to internal and external vector multiplets, whose leading term is $\frac{1}{4}\mathcal{E}R^2$. If we apply the map (3.11) to the internal multiplets we may add this to the two derivative actions derived in the previous section and the leading term becomes $e_I e^{\sigma}L^I R^2 + e_{\hat{I}}\rho^{\hat{I}}R^2$, so the symmetry (3.57) is maintained only in the case that we couple exclusively to external multiplets, i.e. $e_I = 0$. If we take the point of view that this correction is perturbative, and since at leading order the fields $V^{\mathbf{ij}}_{\mu}, Y^{\tilde{I}\mathbf{ij}}, N, P$ vanish the relevant contribution is

$$\mathcal{L}_{R^2} = \mathcal{E}\left(\frac{3}{8}R - 4\left(\underline{D} + \frac{26}{3}\underline{T}^2\right) + 32\underline{T}^2\right)^2 + \cdots\,. \tag{4.4}$$

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4.2 Weyl squared invariant

In [35] a supersymmetric invariant including a Weyl tensor squared term was constructed in the standard-Weyl multiplet and coupled to Abelian vector multiplets. This is given in the conventions we use in [36], which we will repeat below. We will consider the same construction as before, namely that we have a Lagrange multiplier vector multiplet coupled only to the other vectors in such a way as to implement the vanishing of the composite linear multiplet, providing expressions for the standard-Weyl fields $\underline{D}, \underline{T}_{\mu\nu}$ and $\underline{\chi}^{\mathbf{i}}$. In particular we will not couple the Lagrange multiplier vector multiplet to the higher derivative terms, and so do not induce higher derivative expressions in the definitions of these fields. The contribution to the bosonic action of the Weyl-squared term is given in [36] and reads

$$
\begin{aligned}
& e^{-1}\mathcal{L}_{C^2+\frac{1}{6}R^2} = \\
& \beta_{\tilde I}\Big(\frac{1}{8}\rho^{\tilde I} C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} + \frac{64}{3}\rho^{\tilde I}\underline{D}^2 + \frac{1024}{9}\rho^{\tilde I}\underline{T}^2\underline{D} - \frac{32}{3}\underline{D}\,\underline{T}_{\mu\nu}F^{\mu\nu\,\tilde I} \\
& - \frac{16}{3}\rho^{\tilde I}C_{\mu\nu\rho\sigma}\underline{T}^{\mu\nu}\,\underline{T}^{\rho\sigma} + 2C_{\mu\nu\rho\sigma}\underline{T}^{\mu\nu}F^{\rho\sigma\,\tilde I} + \frac{1}{16}\epsilon^{\mu\nu\rho\sigma\lambda}A^{\tilde I}_{\mu}C_{\nu\rho\tau\delta}C_{\sigma\lambda}{}^{\tau\delta} \\
& - \frac{1}{12}\epsilon^{\mu\nu\rho\sigma\lambda}A^{\tilde I}_{\mu}V_{\nu\rho}{}^{ij}V_{\sigma\lambda\,ij} + \frac{16}{3}Y^{\tilde I}_{ij}V_{\mu\nu}{}^{ij}\underline{T}^{\mu\nu} - \frac{1}{3}\rho^{\tilde I}V_{\mu\nu}{}^{ij}V^{\mu\nu}{}_{ij} \\
& + \frac{64}{3}\rho^{\tilde I}\nabla_{\nu}\underline{T}_{\mu\rho}\nabla^{\mu}\underline{T}^{\nu\rho} - \frac{128}{3}\rho^{\tilde I}\underline{T}_{\mu\nu}\nabla^{\nu}\nabla_{\rho}\underline{T}^{\mu\rho} - \frac{256}{9}\rho^{\tilde I}R^{\nu\rho}\underline{T}_{\mu\nu}\underline{T}^{\mu}{}_{\rho} \\
& + \frac{32}{9}\rho^{\tilde I}R\underline{T}^2 - \frac{64}{3}\rho^{\tilde I}\nabla_{\mu}\underline{T}_{\nu\rho}\nabla^{\mu}\underline{T}^{\nu\rho} + 1024\rho^{\tilde I}\,\underline{T}_{\mu\nu}\underline{T}^{\nu\rho}\underline{T}_{\rho\sigma}\underline{T}^{\sigma\mu} - \frac{2816}{27}\rho^{\tilde I}\left(\underline{T}^2\right)^2 \\
& - \frac{64}{9}\underline{T}_{\mu\nu}F^{\mu\nu\,\tilde I}\underline{T}^2 - \frac{256}{3}\underline{T}_{\mu\rho}\underline{T}^{\rho\lambda}\underline{T}_{\nu\lambda}F^{\mu\nu\,\tilde I} - \frac{32}{3}\epsilon_{\mu\nu\rho\sigma\lambda}\underline{T}^{\rho\tau}\nabla_{\tau}\underline{T}^{\sigma\lambda}F^{\mu\nu\,\tilde I} \\
& - 16\epsilon_{\mu\nu\rho\sigma\lambda}\underline{T}^{\rho}{}_{\tau}\nabla^{\sigma}\underline{T}^{\lambda\tau}F^{\mu\nu\,\tilde I} - \frac{128}{3}\rho^{\tilde I}\epsilon_{\mu\nu\rho\sigma\lambda}\underline{T}^{\mu\nu}\underline{T}^{\rho\sigma}\nabla_{\tau}\underline{T}^{\lambda\tau}\Big),
\end{aligned}
\tag{4.5}
$$

where $\beta_{\tilde I}$ are constants, $V^{\mathbf{ij}}_{\mu\nu} = 2\partial_{[\mu}V^{\mathbf{ij}}_{\nu]} - 2V^{\mathbf{ik}}_{[\mu}V_{\nu]\mathbf{k}}{}^{\mathbf{j}}$ and $C_{\mu\nu\rho\sigma}$ is the Weyl tensor. Note that the D^2 term contains a factor of the Ricci scalar squared, which is why we have labelled the invariant $C^2 + \frac{1}{6}R^2$. This fact is what allows one to combine it with the Riemann squared invariant to form the Gauss-Bonnet combination [45] in the $r = 1$ dilaton-Weyl multiplet, which is the only case that at present the Riemann squared invariant is known. Inserting the expressions for the composite fields $\underline{T}$, $\underline{D}$ and $\underline{Y}^{0\mathbf{ij}}$ given in (A.5) we obtain a supersymmetric invariant for arbitrary numbers of internal and external multiplets. We may then make the transformations (3.11), (3.13) in order to identify the dilaton. We note that the symmetry (3.57) is broken unless we couple exclusively to external multiplets, i.e. $\beta_I = 0$ and that only the third line of this invariant may be neglected in a perturbative treatment, due to the vanishing of the fields $V^{\mathbf{ij}}_{\mu}$ and $Y^{\tilde I\mathbf{ij}}$ at the two derivative level.

5 Conclusions

In this work we described in detail the construction of the $\mathcal{N} = 2$ $d = 5$ supergravity of Nishino and Rajpoot [22, 23] from the superconformal formulation [19, 20, 25, 27]. The construction of the minimal N-R model proceeded straightforwardly. In the case of the N-R model coupled to vector multiplets we paid particular attention to the identification of the dilaton amongst the scalars, and the resulting scalar manifolds. We found that in

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order for the supersymmetry transformations to be non-singular we must require that the homogeneous quadratic $\mathcal{A} = a_{IJ}\rho^I\rho^J$ must never vanish. Making the coordinate transformation (3.11) we then found it easy to identify the scalar manifold in the case that the only contribution from the vector multiplet coupling came from a quadratic coupling between them which in turn is coupled to a Lagrange multiplier vector multiplet, which gave rise to the original N-R formulation. It is well know that the general (two derivative) vector multiplet coupling is defined by a symmetric tensor C_{IJK} which can be viewed as the triple intersection of a Calabi-Yau manifold in the compactification of M-theory [13]. From this point of view, the coupling that results in the N-R formulation is schematically

$$C_{IJK}\mathbf{V}^I \cdot L\left(\mathbf{V}^J, \mathbf{V}^K\right) = \mathbf{V}_\flat \cdot L\left(a_{IJ}\mathbf{V}^I\mathbf{V}^J\right) \tag{5.1}$$

where a_{IJ} has Lorentzian signature and may be diagonalized so that in the new basis

$$a'_{IJ} = \eta_{IJ} = \mathrm{diag}(-1, 1\cdots, 1)\,. \tag{5.2}$$

As a shorthand for this we will use the notation

$$C_{\flat IJ} = a_{IJ} \tag{5.3}$$

indicating that only this component is non-zero. This can be plugged into the vector multiplet density (3.43), and we found that the scalar manifold is $\mathrm{SO}(1,1)\times\mathrm{SO}(1,n)/\mathrm{SO}(n)$ as described in [22, 23].

We generalized the vector multiplet matter coupling available in the literature, but this came at the price of breaking the global scaling symmetry of the action that is present in the N-R formulation. We always consider densities that preserve the function of $\mathbf{V}_\flat$ as Lagrange multipliers. In particular first we took

$$C_{\flat IJ} = a_{IJ}\,, \qquad C'_{IJK}\,, \tag{5.4}$$

non-zero and derived the density (3.47). This generically breaks the shift symmetry (3.41), and only respects it when the C'_{IJK} contribution to the density vanishes, the conditions for which are given in (3.48). We called the vector multiplets $\mathbf{V}^I$ in the above internal vector multiplets, as they appear in the gravitational multiplet. We can extend the coupling to external vector multiplets which do not appear in the gravitational multiplet by considering

$$C_{\flat IJ} = a_{IJ}, \qquad C'_{\tilde{I}\tilde{J}\tilde{K}}\,, \tag{5.5}$$

where $\tilde{I} = \left(I, \hat{I}\right)$ and in particular does not include the $\flat$ direction. The form of the coefficients (5.4) arises from a compactification of the low energy limit of M-theory on a Calabi-Yau which is a K3 fibration [14] where it is assumed that the rank of a is maximal. Taking (5.5) results in the most general vector multiplet coupling that allows for the $\mathbf{V}_\flat$ to function as a Lagrange multiplier, and we gave the density in (3.50). Not surprisingly this density generically breaks the symmetry (3.41), but we found that if we allow the external vector multiplets to be inert under these transformations we could preserve the symmetry (3.57) in the particular case that we take the vector density (3.58), so that the scalar manifold is now a product $\mathrm{SO}(1,1)\times\mathrm{SO}(1,n)/\mathrm{SO}(n)\times\mathcal{M}$. We then turned to higher derivative corrections and generalized the known Ricci squared and Weyl squared densities to include more than one internal multiplet. Again these break the symmetry (3.41), but if we take them to be coupled to only external multiplets we may maintain the symmetry (3.57).

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It would be interesting to explicitly consider the appropriate compactifications of the heterotic theory on suitable five manifolds and to understand better the relation of that theory to the off-shell theory presented here, and the duality to M-theory on a Calabi-Yau 3-fold. In [14] such a computation was carried out using the very special geometry condition $\mathcal{C} = C_{\tilde{I}\tilde{J}\tilde{K}}\rho^{\tilde{I}}\rho^{\tilde{J}}\rho^{\tilde{K}} = 1$ to produce a Lagangian for the effective heterotic theory by removing one of the scalars from the action in the case of two internal vector multiplets, which is equivalent to fixing the Lagrange multiplier scalar $\rho^\flat$ using the D equation of motion in the off-shell formulation, at least at the two derivative level. In the heterotic superstring picture the presence of the additional vector couplings C_{IJK} were related to 1-loop corrections, whilst the original N-R formulation is the tree level contribution. In the off-shell formulation in the standard-Weyl multiplet the very special geometry condition arises at the two derivative level by integrating out a Lagrange multiplier, the standard-Weyl field D. After the dualization we have no such constraint in the vector multiplet sector, as it can be solved using the Lagrange multiplier vector multiplet. These two approaches are equivalent at the two derivative level, but it seems that we ought to include the higher derivative corrections to the very special geometry constraint, or in our picture to include couplings between the higher derivative terms and the Lagrange multiplier vector multiplet, introducing higher derivative terms in the expression for the composite standard-Weyl fields. For the case of only one internal vector multiplet the heterotic result implies the absence of a one loop term corresponding to the vanishing vector density of section 2, which was straightforward to show in our set-up. It would be interesting to see how our external vector multiplets fit into this picture, and particularly how the higher derivative corrections in the standard- and dilaton-Weyl multiplets may be related by the heterotic/M-theory duality.

It would be highly desirable to derive a Riemann squared or Ricci tensor squared supersymmetric invariant in the standard-Weyl multiplet in order to construct arbitrary quadratic curvature supergravities. This would be of interest when considering higher order string theory corrections, but also within the framework of supersymmetric Lovelock theory or Chern-Simons supergravity [46], although this has been investigated in a rather different approach to that we have taken here. For generic higher order theories the auxiliary fields of the off-shell formulation become dynamical, and in order to avoid this one must take a perturbative approach to integrating out these fields, as done in [35, 47]. In [45] it was shown that for the supersymmetrization of the Gauss-Bonnet term, in the background of a dilaton-Weyl multiplet containing only one internal vector multiplet, that the kinetic terms for the auxiliary fields exactly cancel, meaning that they can be integrated out exactly. It would be interesting to see if this also happens in the background of the standard-Weyl multiplet, and to understand the compactifications of string and M-theory to this theory. Interestingly the coefficients of the Chern-Simons terms which along with supersymmetry specify the vector multiplet couplings completely, at both the two and four derivative level, have been investigated recently in [48–50] from a 6D and M-theoretic perspective. Whilst this article was in preparation the interesting article [44] appeared which addresses many of these issues from a superspace perspective.

We may also straightforwardly add on-shell hypermultiplet couplings to this theory which is desirable due to the presence of the universal hyper-multiplet in compactifications.

This was recently discussed in [51] in addition to higher derivative couplings. In the superconformal tensor calculus it is not known how to put general hypermultiplets off-shell, however in the superspace formulation this has been discussed in [37–42] and whilst this article was in preparation the interesting paper [52] also appeared. Since the field N appears in the linear multiplet sector which, on coupling to additional tensor multiplets, may provide a factor in the scalar manifold closer to the very special geometry of the standard formulation it would be interesting to include general linear multiplet couplings. It would be also be particularly interesting to gauge the models presented here, using the methods of [43], in particular for applications to four dimensional field theories via the AdS/CFT correspondence. It should also be possible to extend the internal gauging procedure of that work from gauging the internal U(1) gauge field of the dilaton-Weyl multiplet to gauging the full SU(2) R-symmetry using these methods to produce a Weyl multiplet with an internal Yang-Mills multiplet and find a suitable gauge fixing of the superconformal fields.

A Generalized dilaton-Weyl superconformal multiplets

A general dilaton-Weyl multiplet[16] is made up of the vielbien e_μ^a, gravitino $\psi_\mu^{\mathbf{i}}$, m gauge fields A_μ^I, a two-form gauge field $B_{\mu\nu}$, m scalars ρ^I, m gauginos $\psi^{I\mathbf{i}}$, an auxiliary SU(2) triplet of vectors $V_\mu^{\mathbf{ij}}$ with $V_\mu^{\mathbf{ij}} = V_\mu^{\mathbf{ji}}$, $(m-1)$ SU(2) triplets of scalars, $Y^{i\mathbf{ij}}$ and a gauge field for local dilatations b_μ. Using vector multiplet indices $I = (0, i)$ these transform under supersymmetry with parameter $\epsilon^{\mathbf{i}}$ and special supersymmetry with parameter $\eta^{\mathbf{i}}$ as

$$
\begin{aligned}
\delta e_\mu^a &= \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu\,,\\
\delta\psi_\mu^{\mathbf{i}} &= \left(\nabla_\mu + \frac{1}{2}b_\mu\right)\epsilon^{\mathbf{i}} - V_\mu^{\mathbf{ij}}\epsilon_{\mathbf{j}} + i\gamma_{mn}\underline{T}^{mn}\gamma_\mu\epsilon^{\mathbf{i}} - i\gamma_\mu\eta^{\mathbf{i}}\,,\\
\delta V_\mu^{\mathbf{ij}} &= -\frac{3i}{2}\bar{\epsilon}^{(\mathbf{i}}\underline{\phi}_\mu^{\mathbf{j})} + 4\bar{\epsilon}^{(\mathbf{i}}\gamma_\mu\underline{\chi}^{\mathbf{j})} + i\bar{\epsilon}^{(\mathbf{i}}\gamma_{mn}\underline{T}^{mn}\psi_\mu^{\mathbf{j})} + \frac{3i}{2}\bar{\eta}^{(\mathbf{i}}\psi_\mu^{\mathbf{j})}\,,\\
\delta b_\mu &= \frac{i}{2}\bar{\epsilon}\underline{\phi}_\mu - 2\bar{\epsilon}\gamma_\mu\underline{\chi} + \frac{i}{2}\bar{\eta}\psi_\mu\,,\\
\delta A_\mu^I &= -\frac{i}{2}\rho^I\bar{\epsilon}\psi_\mu + \frac{1}{2}\bar{\epsilon}\gamma_\mu\lambda^I\,,\\
\delta B_{\mu\nu} &= -\frac{1}{2}\mathcal{A}\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]} - \frac{i}{2}\mathcal{A}_I\bar{\epsilon}\gamma_{\mu\nu}\lambda^I - \eta_{IJ}A^I_{[\mu}\delta(\epsilon)A^J_{\nu]}\,,\\
\delta\lambda^{I\mathbf{i}} &= -\frac{1}{4}\gamma_{mn}\hat{F}^{Imn}\epsilon^{\mathbf{i}} - \frac{i}{2}\gamma^a\left(\mathcal{D}_a\rho^I\right)\epsilon^{\mathbf{i}} + \rho^I\gamma_{mn}\underline{T}^{mn}\epsilon^{\mathbf{i}} - \underline{Y}^{I\mathbf{ij}}\epsilon_{\mathbf{j}} + \rho^I\eta^{\mathbf{i}}\,,\\
\delta Y^{i\mathbf{ij}} &= -\frac{1}{2}\bar{\epsilon}^{(\mathbf{i}}\gamma^m\mathcal{D}_m\lambda^{\mathbf{j})i} + \frac{i}{2}\bar{\epsilon}^{(\mathbf{i}}\gamma_{mn}\underline{T}^{|mn|}\lambda^{\mathbf{j})i} - 4i\rho^i\bar{\epsilon}^{(\mathbf{i}}\underline{\chi}^{\mathbf{j})} + \frac{i}{2}\bar{\eta}^{(\mathbf{i}}\lambda^{\mathbf{j})i}\,,\\
\delta\rho^I &= \frac{i}{2}\bar{\epsilon}\lambda^I\,,
\end{aligned}
\tag{A.1}
$$

where the spin covariant derivative is defined by

$$
\nabla_\mu\epsilon^{\mathbf{i}} = \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{mn}\gamma_{mn}\right)\epsilon^{\mathbf{i}}\,,
\tag{A.2}
$$

[16] For $m \geq 1$.

where

$$\omega_\mu^{mn} = 2e^{\nu[m}\partial_{[\mu}e_{\nu]}^{n]} - e^{\nu[m}e^{n]}\sigma e_{\mu p}\partial_\nu e_\sigma^p + 2e_\mu^{[m}b^{n]} - \frac{1}{2}\bar{\psi}^{[n}\gamma^{m]}\psi_\mu - \frac{1}{4}\bar{\psi}^n\gamma_\mu\psi^m\,, \tag{A.3}$$

and we have underlined composite fields, expressions for which are given by

$$\begin{aligned}
\underline{\phi}_\mu^{\mathbf{i}} &= \frac{i}{3}\gamma^m\underline{\hat{R}'}^{\mathbf{i}}_{\mu m}(Q) - \frac{i}{24}\gamma_\mu\gamma^{mn}\underline{\hat{R}'}^{\mathbf{i}}_{mn}(Q)\,,\\
\underline{\hat{R}'}^{\mathbf{i}}_{\mu\nu}(Q) &= 2\nabla_{[\mu}\psi^{\mathbf{i}}_{\nu]} + b_{[\mu}\psi^{\mathbf{i}}_{\nu]} - 2V^{\mathbf{ij}}_{[\mu}\psi_{\nu]\mathbf{j}} + 2i\gamma_{mn}\underline{T}^{mn}\gamma_{[\mu}\psi^{\mathbf{i}}_{\nu]}\,,\\
\underline{\hat{R}}^{\mathbf{i}}_{\mu\nu}(Q) &= \underline{\hat{R}'}^{\mathbf{i}}_{\mu\nu}(Q) - 2i\gamma_{[\mu}\underline{\phi}^{\mathbf{i}}_{\nu]}\,,
\end{aligned} \tag{A.4}$$

as in the standard-Weyl multiplet but now we also have

$$\begin{aligned}
\underline{T}^{mn} &= -\frac{1}{8\mathcal{A}}\left(\frac{1}{6}\epsilon^{mnpqr}\hat{H}_{pqr} - \mathcal{A}_I\hat{F}^{Imn} - \eta_{IJ}\frac{i}{4}\bar{\lambda}^I\gamma^{mn}\lambda^J\right)\,,\\
\underline{\chi}^{\mathbf{i}} &= \eta_{IJ}\mathcal{A}^{-1}\left(\frac{i}{8}\rho^I\gamma^m\mathcal{D}_m\lambda^{J\mathbf{i}} + \frac{i}{16}\gamma^m\left(\mathcal{D}_m\rho^I\right)\lambda^{J\mathbf{i}}\right.\\
&\quad\left.-\frac{1}{32}\gamma_{mn}\hat{F}^{Imn}\lambda^{J\mathbf{i}} + \frac{1}{4}\rho^I\gamma_{mn}\underline{T}^{mn}\lambda^{J\mathbf{i}} - \frac{1}{8}\underline{Y}^I_{\mathbf{ij}}\lambda^{J\mathbf{j}}\right)\,,\\
\underline{D} &= -\frac{26}{3}\underline{T}^2 + \eta_{IJ}\mathcal{A}^{-1}\left(\frac{1}{4}\rho^I\Box\rho^J + \frac{1}{8}\left(\mathcal{D}\rho^I\right)\left(\mathcal{D}\rho^J\right) - \frac{1}{16}\hat{F}^I_{mn}\hat{F}^{Jmn} - \frac{1}{8}\bar{\lambda}^I\gamma^m\mathcal{D}_m\lambda^J\right.\\
&\quad\left.+\frac{1}{4}\underline{Y}^I_{\mathbf{ij}}\underline{Y}^{J\mathbf{ij}} - 4i\rho^I\lambda^J\underline{\chi} + \left(2\rho^I\hat{F}^J_{mn} + \frac{i}{4}\bar{\lambda}^I\gamma_{mn}\lambda^J\right)\underline{T}^{mn}\right)\,,\\
\underline{Y}^{\mathbf{ij}0} &= \left(\rho^0\right)^{-1}\mathcal{A}_i Y^{\mathbf{ij}i} - \frac{i}{4}\eta_{IJ}\left(\rho^0\right)^{-1}\bar{\lambda}^{I\mathbf{i}}\lambda^{J\mathbf{j}}\,,
\end{aligned} \tag{A.5}$$

where

$$\mathcal{A} = \eta_{IJ}\rho^I\rho^J\,,\qquad \mathcal{A}_I = \eta_{IJ}\rho^J\,,\qquad \eta_{IJ} = \mathrm{diag}(-,+,\cdots,+)\,. \tag{A.6}$$

As discussed at length in the main body of the text the dilaton of the N-R formulation is to be identitified as $\sigma = \frac{1}{2}\ln(-\mathcal{A})$ and we need $\mathcal{A} \neq 0$ for the expressions (A.5) to be non-singular.

B Explicit field redefinition

Here we give the explicit field redefinitions needed to arrive at the N-R formulation in the notation of [22, 23]. Starting from the on-shell theory with Lagrangian (3.39), which is invariant under supersymmetry transformations (3.40) we need to make the following field redefinitions

$$\begin{aligned}
&\epsilon^{\mathbf{i}} = -\sqrt{2}\epsilon'^{\mathbf{i}}\,, && \psi^{\mathbf{i}}_\mu = -\sqrt{2}\psi'^{\mathbf{i}}_\mu\,, && A^I_\mu = \sqrt{2}A'^I_\mu\,, && V^\alpha_a = \frac{1}{\sqrt{2}}V'^\alpha_a\,,\\
&\chi^{\mathbf{i}} = -\frac{\sqrt{2}}{\sqrt{3}}\chi'^{\mathbf{i}}\,, && \lambda^{a\mathbf{i}} = -\sqrt{2}\lambda'^{a\mathbf{i}}\,, && && V^a_\alpha = \sqrt{2}V'^a_\alpha\,,
\end{aligned} \tag{B.1}$$

and redefine

$$L^I_\alpha = \sqrt{2}L'^I_\alpha\,,\qquad L^\alpha_I = \frac{1}{\sqrt{2}}L'^\alpha_I\,. \tag{B.2}$$

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The definition of the three form field strength, which be now call G, has therefore changed to

$$G'_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]} - 3\eta_{IJ}A'^{I}_{[\mu}F'^{J}_{\nu\rho]}\,, \tag{B.3}$$

where $F'^I = dA'^I = \sqrt{2}F^I$ and the metric is rescaled to

$$g'_{\alpha\beta} = \frac{1}{2}\left(\delta_{\alpha\beta} - \frac{1}{(L^0)^2}\delta_{\alpha\gamma}\delta_{\beta\delta}\varphi^\gamma\varphi^\delta\right) = \frac{1}{2}g_{\alpha\beta}\,. \tag{B.4}$$

Note that L^I_A and the SO(n) connection 1-form A remains unchanged, however the parameter ξ has now become $\xi = -\frac{1}{\sqrt{2}}$, due to the appearance of the vielbein in (3.32). It is not difficult to see that on can further rescale the vielbein $V^a_\alpha = kV'^a_\alpha$, $V^a_\alpha = \frac{1}{k}V'^a_\alpha$ leaving L^I_A fixed and redefining the scalars $\varphi = \frac{1}{k}\varphi'$ whilst leaving all other fields fixed. The Lagrangian and supersymmetry transformations are invariant under this map, however the explicit expressions for the L^I_A in terms of the scalars will change. This is equivalent to scaling the spacelike directions in our coordinate transformations (3.11) and (3.13) and so the choice of the parameter ξ is, in this way, arbitrary. For each fixed value of the dilaton the physical scalar manifold with metric $g_{\alpha\beta}$ is a cone. The full scalar manifold including the dilaton is clearly the solid cone, and what we have described is a foliation by the dilaton of the full scalar manifold, whose leaves are hyperboloids of equal constant Ricci curvature. A different choice of the value of the Ricci scalar is then just an alternative foliation. The bosonic part of the action is now given by

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}R - \frac{1}{4}e^{-2\sigma}(L_I{}^aL_{Ja} + L_IL_J)F^I_{\mu\nu}F^{\mu\nu J} - \frac{1}{12}e^{-4\sigma}G^2\\ &-\frac{1}{2}g_{\alpha\beta}(d\varphi^\alpha)\cdot(d\varphi^\beta) - \frac{3}{4}(d\sigma)^2\end{aligned} \tag{B.5}$$

and its fermionic completion up the quadratic order in the fermions is given in [22, 23]. The supersymmetry transformations, up to quadratic order in fermions read:

$$\begin{aligned}
&\delta e_\mu{}^m = \bar\epsilon\gamma^m\psi_\mu\,, \qquad\qquad \delta\sigma = \frac{i}{\sqrt{3}}\bar\epsilon\chi\,,\\
&\delta\psi_\mu{}^{\mathbf{i}} = D_\mu\epsilon^{\mathbf{i}} + \frac{i}{6\sqrt{2}}e^{-\sigma}\left(\gamma_\mu{}^{\rho\sigma} - 4\delta_\mu{}^\rho\gamma^\sigma\right)\epsilon^{\mathbf{i}}L_IF_{\rho\sigma}{}^I + \frac{1}{18}e^{-2\sigma}\left(\gamma_\mu{}^{\rho\sigma\tau} - \frac{3}{2}\delta_\mu{}^\rho\gamma^{\sigma\tau}\right)\epsilon^{\mathbf{i}}G_{\rho\sigma\tau}\,,\\
&\delta A_\mu{}^I = -\frac{i}{\sqrt{2}}e^\sigma L^I\bar\epsilon\psi_\mu + \frac{1}{\sqrt{6}}e^\sigma L^I\bar\epsilon\gamma_\mu\chi + \frac{1}{\sqrt{2}}e^\sigma\bar\epsilon\gamma_\mu\lambda^aL_a{}^I\,,\\
&\delta B_{\mu\nu} = e^{2\sigma}\bar\epsilon\gamma_{[\mu}\psi_{\nu]} + \frac{i}{\sqrt{3}}e^{2\sigma}\bar\epsilon\gamma_{\mu\nu}\chi - 2L_{IJ}A_{[\mu|}{}^I\delta_QA_{|\nu]}{}^J\,,\\
&\delta\chi^{\mathbf{i}} = -\frac{1}{2\sqrt{6}}e^{-\sigma}\gamma^{\mu\nu}\epsilon^{\mathbf{i}}L_IF_{\mu\nu}{}^I + \frac{i}{6\sqrt{3}}e^{-2\sigma}\gamma^{\mu\nu\rho}\epsilon^{\mathbf{i}}G_{\mu\nu\rho} - \frac{\sqrt{3}i}{2}\gamma^\mu\epsilon^{\mathbf{i}}\partial_\mu\sigma\,,\\
&\delta\varphi^\alpha = \frac{i}{\sqrt{2}}V_a{}^\alpha\bar\epsilon\lambda^a\,, \qquad \delta_Q\lambda^{a\mathbf{i}} = -\frac{1}{2\sqrt{2}}e^{-\sigma}\gamma^{\mu\nu}\epsilon^{\mathbf{i}}L_I{}^aF_{\mu\nu}{}^I - \frac{i}{\sqrt{2}}\gamma^\mu\epsilon^{\mathbf{i}}V_\alpha{}^a\partial_\mu\varphi^\alpha\,.
\end{aligned} \tag{B.6}$$

C Vector multiplet composed of a linear multiplet

One can also construct the elements of vector multiplet in terms of the elements of a linear multiplet and a Weyl multiplet [36, 43, 45]. Here we just list the bosonic parts

$$\begin{aligned}\rho &= 2L^{-1}N\,,\\ Y_{\mathbf{ij}} &= L^{-1}\Box^{C}L_{\mathbf{ij}} - \mathcal{D}_a L_{\mathbf{k}(\mathbf{i}}\mathcal{D}^a L_{\mathbf{j})\mathbf{m}}L^{\mathbf{km}}L^{-3} - N^2 L_{\mathbf{ij}}L^{-3} - P_\mu P^\mu L_{\mathbf{ij}}L^{-3}\\ &\quad + \frac{8}{3}L^{-1}T^2 L_{\mathbf{ij}} + 4L^{-1}DL_{\mathbf{ij}} + 2P_\mu L_{\mathbf{k}(\mathbf{i}}\mathcal{D}^\mu L_{\mathbf{j})}{}^{\mathbf{k}}L^{-3},\\ F_{\mu\nu} &= 4\mathcal{D}_{[\mu}(L^{-1}P_{\nu]}) + 2L^{-1}R_{\mu\nu}{}^{\mathbf{ij}}(V)L_{\mathbf{ij}} - 2L^{-3}L^{\mathbf{l}}_{\mathbf{k}}\mathcal{D}_{[\mu}L^{\mathbf{kp}}\mathcal{D}_{\nu]}L_{\mathbf{lp}}\,. \end{aligned} \tag{C.1}$$

where the bosonic parts of the relevant covariant derivatives, d'Alembertion and the curvatures are given by

$$\begin{aligned}\mathcal{D}_\mu L^{\mathbf{ij}} &= (\nabla_\mu - 3b_\mu)L^{\mathbf{ij}} + 2V_\mu{}^{(\mathbf{i}}{}_{\mathbf{k}}L^{\mathbf{j})\mathbf{k}}\\ \mathcal{D}_\mu P_\nu &= (\nabla_\mu - 4b_\mu)P_\nu\\ \Box^C L^{\mathbf{ij}} &= (\nabla^a - 4b^a)\mathcal{D}_a L^{\mathbf{ij}} + 2V_a{}^{(\mathbf{i}}{}_{\mathbf{k}}\mathcal{D}^a L^{\mathbf{j})\mathbf{k}} + 6L^{\mathbf{ij}}f_a^a\\ R(V)^{\mathbf{ij}}_{\mu\nu} :&= V^{\mathbf{ij}}_{\mu\nu} = 2\partial_{[\mu}V^{\mathbf{ij}}_{\nu]} - 2V^{\mathbf{k}(\mathbf{i}}_{[\mu}V^{\mathbf{j})}_{\nu]\mathbf{k}} \end{aligned} \tag{C.2}$$

and for closure of the algebra the constraint $\mathcal{D}^a P_a = 0$ is needed, and f_a^a is given in (2.3).

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