PERGAMON

Vision Research 42 (2002) 2235–2251

Vision Research

www.elsevier.com/locate/visres

Numerical modelling of the accommodating lens

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Received 10 July 2001; received in revised form 4 March 2002

Abstract

Data on geometric and material properties of the human lens derived from various published sources are used to construct axisymmetric, large displacement, finite element models of the accommodating lens of subjects aged 11, 29 and 45 years. The nucleus, cortex, capsule and zonule are modelled as linearly elastic materials. The numerical model of the 45-year lens is found to be significantly less effective in accommodating than the 29-year lens, suggesting that the modelling procedure is capable of capturing at least some of the features of presbyopia. The model of the 11-year lens shows some anomalous behaviour, and reasons for this are explored.

Keywords: Accommodation; Age dependence; Computational; Modelling; Presbyopia

1. Introduction

1.1. Accommodation and presbyopia

In the unaccommodated state, according to the classical theory (Helmholtz, 1909), the lens is held in a state of radial tension by the zonular fibres that connect the ciliary body to the lens equator. During accommodation, contraction of the ciliary muscle causes the tension in the zonular fibres to reduce and, as a consequence, the lens changes its shape. This change in shape causes the optical power of the lens to increase. It is well known that the amplitude of accommodation reduces steadily with age and that at an age of 40 to 50 years, the ability of the eye to accommodate generally becomes minimal. The Helmholtz mechanism of accommodation in the young eye is widely (although not universally e.g. Schachar, Huang, & Huang, 1993) accepted. The age-related changes that lead to presbyopia in middle age are, by comparison, rather poorly understood.

Current explanations of presbyopia may be classified into two broad groups: 'lenticular theories' which are based on the assumption that presbyopia is caused by age-related changes in the geometry and/or mechanical properties of the lens itself, and 'extra lenticular theories' in which presbyopia is attributed to mechanical and/or geometric changes within the zonule, ciliary body and other tissues surrounding the lens. It has also been suggested that presbyopia may be linked to combined effects from both groups; this leads to theories that are referred to as 'multifactorial'. Much has been written on these various hypotheses, and the basis for their development (e.g. Atchison, 1995; Glasser & Campbell, 1998).

Various experimental studies, both in vivo and in vitro, have been carried out on the accommodation process, the anatomy of the lens and the development of presbyopia (see below for further details). Although an extensive range of plausible hypotheses for the development of presbyopia exist (e.g. Atchison, 1995), it is a curious fact that the experimental studies that have been completed to date are generally regarded as being unable to offer decisive evidence for the correctness of any one of these views. These continuing uncertainties seem linked to the difficulty of carrying out experimental studies in this area, as outlined below.

1.2. In vivo and in vitro studies of accommodation and presbyopia

In vivo studies: In vivo studies suffer from the difficulty that the lens is normally partially obscured by the

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PII: S0042-6989(02)00094-9

iris and so direct measurements of changes in ciliary body and lens diameter during the accommodation process are difficult to make. In addition, optical measurements of the shape of the lens are subject to errors caused by refraction at the cornea and aqueous interfaces. These difficulties can be overcome by the use of alternative techniques, such as ultra-sound and magnetic resonance imaging (MRI), but these methods tends to produce data of an unsatisfactorily low resolution. Where geometric measurements of lenses are reported, they tend to be for particular features e.g. changes in lens radius during accommodation (Storey & Rabie, 1985; Strenk et al., 1999); lens curvatures and thickness (Brown, 1973, 1974; Dubbelman & Van der Heijde, 2001), or geometry of the zonular apparatus (Rohen, 1979; Ludwig, Wegscheider, Hoops, & Kampik, 1999). It appears that no one study has yet captured a complete set of correlated geometric and kinematic data for individual lenses.

The mechanical properties of the various materials that make up the lens and accommodation apparatus appear not to have been measured in vivo, and perhaps this is not a practical possibility. A knowledge of these properties, particularly those of the lens, is crucial, however, to assess lenticular theories of presbyopia. It would also not seem to be possible to measure directly the forces applied by the ciliary muscle to the lens (via the zonule) during the accommodation process. This is an unfortunate limitation, because it seems likely that these forces will depend on the growth and stiffness changes of the lens and zonule as the eye ages. A knowledge of how these forces vary with age would therefore be of considerable assistance in developing an improved understanding of presbyopia.

It has been suggested that the mechanism of presbyopia in non-human primates is similar to that in humans, and that as a consequence, studies of the mechanism of accommodation in monkeys may give useful insight into ageing of the human eye. A study of this sort is described by Croft et al. (1999). Although the use of animal models allows more extensive data to be obtained, in vivo studies of primates are subject to many of the shortcomings and difficulties that are experienced in human in vivo studies.

In vitro studies: In vitro studies provide the opportunity of making more detailed measurements of the way in which changes in the shape of the lens are related to the changes in lens equatorial radius that occur during accommodation. In principle, in vitro studies also allow the possibility of measurements of the variation of the forces applied to the lens during accommodation, although accurate measurements appear to be difficult to achieve. An ingenious attempt at this type of measurement is the spinning lens experiment described by Fisher (1971). It should be noted, however, that Fisher's experiment provided an estimate of the zonular forces based on an approximate analysis of the centripetal forces developed in a spinning lens; zonular forces were not measured directly.

More recently Pierscionek (1995) investigated the effect of equatorial stretching displacements on lens geometry, and Glasser and Campbell (1998) carried out stretching experiments on the lenses of cadaver eyes in which the variation of optical performance with equatorial stretch was measured directly. These latter experiments included measurements of the way in which accommodation amplitude and age influences the spherical aberration of the lens. Glasser and Campbell's experiments did not, however, include measurements of the stretching forces that were applied to the lens.

Although in vitro studies have certain advantages over in vivo techniques, they are subject to the important uncertainty that the conditions of the lens and surrounding tissues, and the way in which movement of the ciliary body is applied to the lens, may not be equivalent to conditions occurring in vivo.

1.3. Theoretical modelling of accommodation and presbyopia

Recently, attempts have been made to study accommodation, and the development of presbyopia, using theoretical methods to model the mechanical and optical performance of the lens. This approach benefits from the application of the methods of strength of materials and structural analysis that are well-established in the fields of mechanical and civil engineering. Although some work in this area has been carried out with very simple mechanical models (e.g. Weale, 2000; Wyatt, 1993), opportunities are now becoming available for more sophisticated analysis (e.g. Schachar et al., 1993) often using computer-aided design and finite element analysis software (e.g. Burd, Judge, & Flavell, 1999a; Burd, Judge, & Cross, 1999b; Schachar & Bax, 2001). Theoretical modelling relies heavily on geometric and material data from experimental studies and is therefore subject to any inherent experimental errors. If the experimental data used to specify a model are incomplete or in error, then the model may yield poor quality results.

Theoretical modelling does, however, provide important possibilities that are not available in experimental studies. The fact that a modelling exercise involves the synthesis of disparate bodies of data means that it can be used to test the consistency of sets of previous experimental measurements. Modelling can also suggest where improved experimental data are needed. If a model could be developed that reflected the main physiological features of presbyopia, then the relative importance of different ageing processes in the accommodation apparatus (changes in lens stiffness, differential growth of lens and ciliary muscle, changes in geometry of the lens, etc.) could be investigated by appropriate adjustments to the model. It is argued, then, that theoretical modelling (despite its limitations) is a useful supplement to experimental studies of the accommodation process.

1.4. Scope of the current paper

The purpose of the current paper is to propose theoretical procedures, based on finite element analysis, for modelling the human lens. These procedures are used to construct models of lenses of three different ages to investigate their mechanical and optical performance. The methods described in this paper have been developed from those outlined in a preliminary communication by Burd, Judge, and Cross (1999). They involve combined modelling of age-related changes occurring in the lens, zonule and ciliary body.

The procedures described in this paper are intended to establish appropriate methods of modelling the lens that are based on reasonable interpretations of the available published data. Unfortunately, relatively few relevant experimental data are available. This fact causes some difficulty in the development of the model because, in many cases, the reliability of individual sets of data cannot be assessed by comparison with comparable results from other sources. In other cases, reliable published data relating to some of the parameters required by the model do not appear to be available.

This modelling exercise has three broad purposes. Firstly, it is a means by which the consistency of the current archive of experimental data can be investigated. Secondly, it suggests where new or improved experimental data are required to achieve a better understanding of accommodation and presbyopia. Thirdly, it is an initial step towards developing a theoretical model that is capable of representing in detail the mechanisms that lead to the development of presbyopia.

The modelling procedure described in this paper followed the following steps. Firstly, an attempt was made to assemble the most credible set of geometric and mechanical data for lenses of ages 11, 29 and 45. (Lenses of these ages were selected because they were the ages of the subjects of the detailed geometric measurements reported by Brown (1973).) Reasoned assumptions were used to specify the model when the necessary experimental data appear to be unavailable. Secondly, the data were used to assemble a finite element model of the accommodating lens, and the behaviour predicted by the model was assessed. It is emphasised that the authors took as a priority the need to ensure that this modelling exercise represents a credible attempt to investigate the implications of drawing together experimental data from a variety of sources. The parameters and assumptions used to specify the model were based entirely on what the authors regard as sensible interpretations of the available data. No adjustments to the input parameters were permitted for the purpose of improving the fit of the model to the data. This avoids the kind of circular argument that arises when ad hoc adjustments to parameters are made (especially those that involve increasing the degrees of freedom).

Lenses of the same age will exhibit variations in their mechanical and geometric characteristics. Experimental measurements on the lens and surrounding tissues may also be subject to error and uncertainty. The effect of the variability of the input parameters on the predicted performance of the model is an important issue but, apart from a limited investigation of certain input parameters and assumptions, it is beyond the scope of the current study.

2. Geometric model of the lens and zonule

2.1. Introduction

A first step in developing a finite element model of the lens is the establishment of a suitable geometric model. This should account for the following two factors:

Firstly, although the optical performance of the lens is determined by its local thickness and curvature on its polar axis (as well as its refractive index), the mechanical behaviour of the lens during accommodation depends (in addition to its physical properties) on the detailed geometry of the entire lens. A realistic geometric model is therefore required for the complete lens. Many of the previous biometric studies of the lens, however, have concentrated on particular features such as equatorial diameter or polar curvatures. Sets of correlated geometric data for individual lenses do not appear to be available. In the absence of complete geometric information about a particular lens, it is necessary to construct geometric models based on data from a variety of different sources. This process is far from straightforward because, whereas some aspects of the lens are extensively reported (such as polar thickness), other data (for example relating to the geometric arrangement of the zonular fibres) are relatively sparse. The current paucity of experimental data means that it is necessary to assemble data from a variety of sources to model a complete lens. Since lenses of the same age may be different in several important respects, these procedures lead to an idealised model; it is an open question whether the model is strictly representative of a lens at a particular age.

Secondly, a suitable reference configuration must be established to serve as a starting point for any analysis involving the model. This starting configuration should consist of a geometric arrangement of the lens in which the stresses are known. The most convenient reference configuration, from a computational point of view, would be one in which all stresses are zero. A simple approach (and the one adopted in this paper) is to

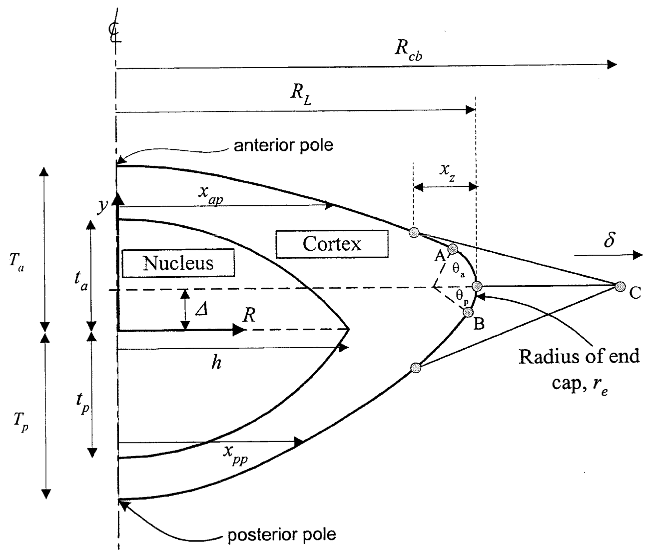

Fig. 1. Geometric parameters defining the reference configuration of lens.

assume that when the eye is in a fully accommodated state the lens, the capsule and the zonule are all stress-free. This reference state is illustrated diagrammatically in Fig. 1. Although this is the approach adopted in this paper, it should be emphasised that the behaviour of the lens may well depend critically on the assumptions made about the reference configuration. The force in the zonular fibres may not be close to zero in the fully accommodated state, for example. Also, Glasser and Campbell (1999) report that significant geometric changes occur within the isolated lens when the capsule is removed, particularly in young lenses. This suggests that that significant tensile pre-stresses may exist in the capsule (and consequential compressive stresses in the lens matrix) even when external forces applied to the lens are zero. This would mean that a stress-free configuration of the lens (that is, the configuration adopted in this model as a reference state) would not exist. The influence of pre-stress in the capsule on the performance of the lens would require additional data and modelling procedures, however, and this is beyond the scope of the paper.

2.2. Lens geometry

Few published data seem to exist that describe the geometry of the lens when in a fully accommodated state (i.e. corresponding to the assumed reference configuration). Brown (1973, 1974) does, however, describe in vivo measurements of the curvatures of the anterior and posterior surfaces of the lens using slit-lamp biomicroscopy. Brown (1973) presents detailed data on lens thicknesses and central and peripheral curvatures for lenses of different ages and under different accommodative conditions. This study is a unique source of information on changes of lens curvature during accommodation, although it must be noted that the data are only approximately corrected for the distortions caused by refraction by the cornea and lens itself. Although these data are incomplete (in the sense that further information about the lens equatorial radius and shape are required to construct a complete geometric model of the lens), the data are adopted here for this study. Brown (1973) gives data for subjects of ages 11, 29 and 45 and these are adopted in this study. (Brown also records data for a 19-year lens but this lens was not modelled in the present study because some of the required data are not recorded.) Brown (1973) records geometric information for accommodative demands of 10D for the 11 and 29 year lenses and 4D for the age 45 lens. With these accommodative demands, it is assumed (for the purpose of the modelling described in this paper) that the measured geometry corresponds to a fully accommodated state. (Note however that the 11-year lens should be capable of accommodation in excess of 10D; although it would be preferable to use a fully accommodated 11-year geometry, these data are not provided by Brown (1973).) These geometric data were used to calculate the coefficients of fifth order polynomials to define the shape of the anterior and posterior surfaces of the lens (see Fig. 1). The equatorial region of the lens is modelled using a circular cap. The constants in the polynomial expressions for the anterior and posterior surfaces are evaluated by matching the central and peripheral curvatures and also the lens thickness with values reported by Brown (1973). (Peripheral curvature was measured by Brown at distances x_{ap} and x_{pp} from the lens axis for the anterior and posterior surfaces respectively; values of x_{ap} and x_{pp} are given in Table 1.) The slope of the anterior and posterior surfaces on the lens axis is set to zero and the condition is imposed that the slope (but not necessarily the curvature) should be continuous at points A and B, where the polynomial surfaces join with the circular end cap; these points are at specified angular positions $\theta_a = 63°$ and $\theta_p = 37°$. The coefficients of the polynomials used to define the lens outline are given in Table 2. The radius of the end cap, r_e is assumed to be related to the lens equatorial radius R_L by $r_e/R_L = 0.1208$. This procedure to model the lens profile (including the particular choice of θ_a and θ_p, and the magnitude of the ratio $r_e/R_L = 0.1208$) follows closely the procedure adopted by Schachar et al. (1993). (Note, however, that Schachar et al. (1993) used the *unaccommodated* state as the reference configuration rather than the *accommodated* state that is adopted in this model.) Schachar et al. (1993), in turn, appear to have obtained these values of θ_a, θ_p and r_e/R_L by direct scaling of the lens cross-section given in Fig. 2 of Fincham (1937).

To complete the model of the external surface of the lens, values are required for the lens equatorial radius, R_L. Since, for a reference configuration, these values

Table 1
Geometric lens data

Dimension	11-year	29-year	45-year	Source
Lens radius, R_L	Values from Eq. (1)			Strenk et al. (1999)
Ciliary body radius, R_{cb}	Values from Eq. (2)			Strenk et al. (1999)
Anterior lens thickness, T_a (mm)	2.18	2.04	2.42	Brown (1973)
Posterior lens thickness, T_p (mm)	2.13	2.09	2.42	Brown (1973)
Radius of curvature of anterior pole, R_{ac} (mm)	6.8	6.5	7.6	Brown (1973)
Radius of curvature of posterior pole, R_{pc} (mm)	5.6	5.3	8.1	Brown (1973)
Anterior peripheral radius of curvature, R_{ap} (mm)	13.5	25.2	12.7	Brown (1973)
Radius of anterior peripheral curvature measurement, x_{ap} (mm)	2.8	2.8	2.8	Brown (1973)
Posterior peripheral radius of curvature, R_{pp} (mm)	8.1	12.2	8.0	Brown (1973)
Radius of posterior peripheral curvature measurement, x_{pp} (mm)	2.4	2.4	2.4	Brown (1973)
Anterior end cap angle, θ_a (°)	63	63	63	Scaled from Fig. 2 of Fincham (1937)
Posterior end cap angle, θ_p (°)	37	37	37	Scaled from Fig. 2 of Fincham (1937)
Variation of capsule thickness	Fifth order polynomial fit to published data			Fisher and Pettet (1972), see Fig. 3 and Table 3
Anterior thickness of nucleus, t_a (mm)	1.78	1.32	1.57	Brown (1973)
Posterior thickness of nucleus, t_p (mm)	1.68	1.36	1.64	Brown (1973)
Position of zonule attachments, x_z	Values from Eq. (3)			Farnsworth and Shyne (1979)
Amplitude of radial movement of ciliary body, δ	Values from Eq. (5)			Strenk et al. (1999)
Equatorial radius of nucleus, h (mm)	3.20	2.85	3.10	Brown (1973)
Nucleus equatorial offset, Δ (mm)	0.5853	0.5119	0.6292	Scaled from Fincham (1937) with additional data from Brown (1973)

Table 2
Coefficients of the polynomials defining the lens outline, where $y = aR^5 + bR^4 + cR^3 + dR^2 + f$

	a	b	c	d	f
Anterior (11-year)	−0.00048433393427	0.00528772036011	−0.01383693844808	−0.07352941176471	2.18
Posterior (11-year)	0.00300182571400	−0.02576464843559	0.06916082660799	0. 08928571428571	−2.13
Anterior (29-year)	−0.00153004454939	0.01191111565048	−0.02032562095557	−0.07692307692308	2.04
Posterior (29-year)	0.00375558685672	−0.03036516318799	0.06955483582257	0.09433962264151	−2.09
Anterior (45-year)	−0.00026524088453	0.00449862869630	−0.01657250977510	−0.06578947368421	2.42
Posterior (45-year)	0.00266482873720	−0.02666997217562	0.08467905191557	0. 06172839506173	−2.42

The symbols y and R are defined in Fig. 1. All dimensions are in millimetres. The large number of decimal places in coefficients a to d was used to minimise the effect of truncation errors in the definition of the lens geometry.

should correspond to a fully accommodated state, it may be the case that in vitro measurements of the radius of an isolated lens (such as those reported by Smith (1883) and referred to by others, including Fincham (1937)) might be appropriate. More recently, however, MRI studies by Strenk et al. (1999) have provided in vivo data on lens equatorial radius for subjects of a range of ages when subjected to accommodation demands of 0.1D and 8D. Although the data show substantial scatter, a linear fit though the Strenk et al. (1999) data for lens radius, for an accommodative demand of 8D, gives:

$$\text{Lens radius (mm)} = 4.07 + 0.0084 \times \text{age (years)} \quad (1)$$

Note that in computing this regression, data on lenses above the age of 70 years were discarded on the evidence of the study by Glasser and Campbell (1999) which suggested that above this age the growth of the lens slows significantly. It is interesting to note that this regression line agrees very closely with the measurements on isolated lenses reported by Smith (1883). Eq. (1) was therefore adopted to provide values of R_L for use in the geometric model.

The lens volumes, computed on the basis of this geometric model, are plotted in Fig. 2 and compared with data on the volume of excised human lenses reported by Bellows (1944). The data that Bellows report appear to have been based on measurements by

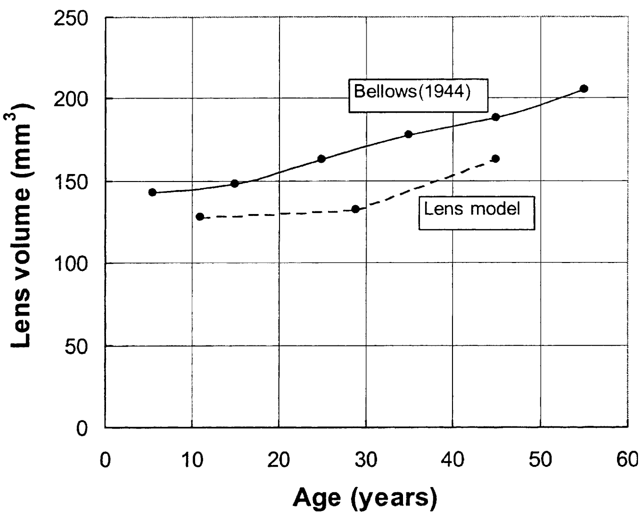

Fig. 2. Volume of lens model compared with Bellows (1944).

Scammon and Hesdorffer (1937). It is clear that the lens model shows a similar tendency to increase in volume with age as is evident in the experimental data, although the volume of the model lenses is consistently lower than the Bellows data. It seem likely, because of the axisymmetric nature of the model, that this apparent inconsistency may be related to assumptions about the geometry of the lens in the equatorial region.

2.3. Zonule and ciliary body

The lens is connected to the ciliary muscle by a series of zonular fibres that originate at the ora serrata and are anchored to the ciliary body before attaching to the lens capsule. Few detailed studies have been carried out of the detailed geometry of the zonule, and attachments to the ciliary body. The studies that have been reported (e.g. Farnsworth & Shyne, 1979; Ludwig et al., 1999; Canals, Costa-Vila, Potau, Merindano, & Ruano, 1996; Rohen, 1979) indicate that the geometry of the zonule is highly complex and that it varies significantly with age. These studies also suggest that the zonule consists of three separate groups; an anterior group, a central (or equatorial) group and a posterior group; these three groups of zonular fibres are idealised in the geometric model shown in Fig. 1. The three sets of zonular fibres are assumed in this model to connect with the ciliary body at a single point (point C in Fig. 1).

Recent MRI studies by Strenk et al. (1999) show that the ciliary body radius, in the accommodated state, tends to reduce slightly with age. A linear regression fit to Strenk's ciliary body radius data for an 8D accommodation gives:

$$\text{Ciliary body radius (mm)} = 6.735 - 0.009 \times \text{age (years)} \qquad (2)$$

This equation is used to define the reference value of R_{cb} in the numerical model.

Farnsworth and Shyne (1979) show that the projected distance between the anterior zonule attachments and the lens radius (dimension x_Z on Fig. 1) increases with age. It is far from clear, however, whether the posterior and equatorial attachments are subject to similar movements. For simplicity, the attachment of the equatorial zonule in this model is assumed to remain stationary on the lens equator, and the posterior attachments are assumed to be located at the same projected distance (dimension x_Z) from the lens equator as the anterior attachments. The Farnsworth and Shyne (1979) data may be used to obtain the following equation for x_Z (the distance between the anterior zonule attachment and the lens radius, see Fig. 1) by regression:

$$x_Z(\text{mm}) = 0.0311 + 0.0124 \times \text{age (years)} \qquad (3)$$

This assumed relationship, in conjunction with Eqs. (1) and (2) and the assumed geometry of the lens outline, implies that the length of the anterior and posterior zonule in the accommodated state remains roughly constant with age. This is consistent with observations reported by Farnsworth and Shyne (1979). In the accommodated state, the equatorial zonule, however, is shorter in the older lenses (as a result of age-related increases in lens radius and reduction in ciliary body radius). This seems physically unrealistic (because there is no obvious mechanism by which the equatorial zonule reduces in length with age) but for purposes of this model this lack of realism is neglected.

The zonule consists of a large number of individual fibres arranged around the circumference of the lens. To model these fibres using an axisymmetric finite element model, each set of zonular fibres is represented by a continuous annular sheet. Selection of the appropriate thickness for these sheets is a difficult matter because few data are available on the size and number of the zonule. It is noted, however, that studies of the zonule (e.g. Farnsworth & Shyne, 1979) suggest that the anterior zonule is more numerous than the posterior zonule and that the number of central fibres is relatively few. For the purposes of this study it is assumed that the number of anterior, posterior and central zonular fibres are in the ratio 6:3:1; appropriate stiffnesses of the annular sheets used to model the zonule are therefore specified to be in this ratio.

2.4. Lens nucleus

Following Fisher (1971) the lens is modelled by assuming that it consists of two separate linearly elastic regions; nucleus and cortex. Geometric data for the nucleus given by Brown (1973) are sufficient for the anterior and posterior boundaries of the nucleus to be defined by circular arcs (see Fig. 1). It should be noted that the data used to define the geometry of the nucleus and the shape of the lens surface were always taken from the same lens.

2.5. Capsule thickness

The capsule thickness is known to vary with radial position and also with age (e.g. Fincham, 1937; Fisher & Pettet, 1972). It appears that in much of this previous experimental work, measurements have been made either of the spatial variation in capsule thickness for a particular age, or the variation of capsule thickness with age for a particular position. The study by Fisher and Pettet (1972), however, provides complete data on the spatial variation of capsule thickness for a set of lenses of different ages; these data are used to deduce suitable spatial variations for use in the numerical model, as described below.

Fisher and Pettet (1972) provide capsule thickness data for lenses of age 22, 37 and 47 years, as well as data for two older lenses. Best-fit fifth order polynomials for the variation of capsule thickness with position were obtained for each of these lenses (Fig. 3, Table 3). The youngest lens reported by Fisher and Pettet was 22 years and, in the absence of data for a younger lens, the polynomial fit to this particular lens was used directly in the 11-year numerical model. Similarly, the best-fit polynomial constructed from Fisher and Pettet's measurements for the 47-year lens was used directly in the 45-year numerical model. To obtain data for the 29-year lens, however, raw data from Fisher and Pettet's graphs for 22- and 37-year lenses were linearly interpolated. These interpolated data were then used to obtain a best-fit fifth order polynomial for use in the numerical model.

It should be noted that Fisher and Pettet's data (and the corresponding best-fit polynomials) are presented in terms of distance measured along the lens surface from the lens equator, rather than in terms of a radial dimension. This fact was carefully accounted for when using the best-fit polynomials in the numerical model.

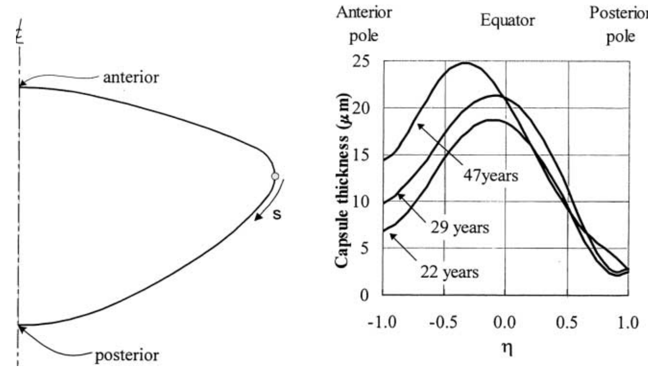

Fig. 3. Assumed variation of capsule thickness with position.

3. Mechanical properties of the lens and zonule

The lens model consists of three distinct materials; the nucleus, the cortex and the lens capsule. For the purpose of this model each material is assumed to be isotropic and linearly elastic. Although these materials may behave in a non-linear way (e.g. Krag, Olsen, & Andreassen, 1997) and their microstructure suggests that their mechanical properties may be anisotropic, isotropic linear elasticity is adopted on the basis that insufficient published data are available to calibrate more complex constitutive models. The capsule is a thin membrane and is represented by axisymmetric membrane finite elements; these do not possess any bending stiffness. The lens cortex and nucleus are modelled by axisymmetric elastic continuum elements.

3.1. Capsule

A limited amount of data are available relating to the stiffness of the lens capsule. Fisher (1969), for example, describes a series of measurements based on an experiment in which a portion of the anterior capsule was inflated by the application of an external pressure. Fisher concluded that the Young's modulus of the capsule decreased with age from about 6 $\mathrm{N\,mm^{-2}}$ at birth to about 1.5 $\mathrm{N\,mm^{-2}}$ in extreme old age. Fisher (1969) suggests that a Poisson's ratio of 0.47 would be appropriate for the capsule; this value of Poisson's ratio

Table 3
Assumed variation of capsule thickness with position (see Table 3)

Age	a_c	b_c	c_c	d_c	e_c	f_c
22 years (used in the 11-year model)	0.8352	16.0266	2.9249	−29.7854	−5.8871	18.4139
29 years (obtained by interpolation from Fisher & Pettet, 1972 data for 22- and 37-year lenses)	4.7464	16.8928	−2.7198	−31.6306	−5.4702	21.0881
47 years (used in the 45-year model)	−16.2276	7.6695	31.7967	−20.0132	−21.3694	20.8891

Capsule thickness (μm) $= a_c\eta^5 + b_c\eta^4 + c_c\eta^3 + d_c\eta^2 + e_c\eta + f_c$. For the anterior surface $\eta = -s/s_a$ where s is distance measured along the lens outline from the lens equator and s_a is the value of s at the anterior pole. For the posterior surface, $\eta = s/s_p$ where s is distance measured along the lens outline and s_p is the value of s at the posterior pole.

is therefore incorporated in the numerical model described in this paper.

A more recent study, by Krag et al. (1997), involved uni-axial testing of a ring of material removed from the anterior capsule of 67 eyes. The data plotted by Krag et al. (1997) relate to large strain tests (strain up to 80%) and little detail is given about the behaviour of the material at the much lower strains (estimated, on the basis of calculations described later in this paper, to be in the region of 5%) that the capsule is subjected to during the accommodation process. A regression analysis of the measured Young's modulus at a strain of 10% has been provided by Krag, Andreassen, and Olsen (1996), however, and this gives:

$$E_c = a(A - 35) + 1.45 \tag{4}$$

where A is age in years, E_c is the Young's modulus in units of $\mathrm{N\,mm^{-2}}$, $a = 0.03\ \mathrm{N\,mm^{-2}\,years^{-1}}$ for $A < 35$ years and $a = 0$ for $A \geqslant 35$ years.

Values of Young's modulus derived from Eq. (4) are lower than the values reported by Fisher (1969). Fisher's results relate to the stiffness at strains that are considerably larger than 10%, however, and these values would be expected to be significantly larger than the stiffness at lower strains because of the marked material non-linearity of the capsule material. Eq. (4) is broadly consistent with the measured data on capsule modulus reported by van Alphen and Graebel (1991). It should be noted, however, that the tests employed by van Alphen and Graebel would have generated highly complex strain fields within the lens; this makes their experimental results difficult to interpret.

Information about Young's modulus relevant to the relatively small strain levels that are expected to develop within the capsule during the accommodation process is required for the purpose of this model. The authors are not aware of any published data corresponding to low levels of strain and so the analyses described in this paper have been based on Eq. (4). It is noted from the data given by Krag et al. (1996, 1997), however, that the Young's modulus at, say, 5% strain may be somewhat less than the 10% values given by from Eq. (4).

3.2. Cortex and nucleus

An ingeneous experiment to measure the stiffness of the cortex and nucleus was described by Fisher (1971). Fisher spun a lens on its polar axis and then photographed the shape of the lens as the rotational speed was varied. On the assumption that the lens surface was ellipsoid, that the nucleus was spherical and of diameter equal to the polar depth of the lens, and that the cortex and nucleus were both linear isotropic elastic materials, Fisher deduced values of Young's modulus for the cortex and nucleus for lenses of different ages. Fisher then proposed a cubic variation with age for the Young's modulus of the nucleus and a quadratic variation with age for the Young's modulus of the cortex to provide a best fit with these data. These functions are adopted here to deduce values of nucleus and cortex Young's moduli for use in this numerical model. (Note that an error appears to exist in one of the constants reported by Fisher (1971) in the polynomial used to describe the nucleus Young's modulus. For the correlation to be consistent with Fisher's Fig. 13, then the value of the constant c_n given in Fisher's paper should be multiplied by -1. The analyses in this paper are therefore conducted with $c_n = -4.286 \times 10^{-4}$).

It appears that Fisher conducted his experiments on lenses for which the capsule was intact. In the analysis of his data, however, he adopts an analytical approach in which the structural action of the capsule is neglected. Fisher justifies this assumption on the basis of his observation that the presence of the capsule appeared to have minimal influence on the deformations of the lens when it is spun. However, this assumption that the stiffness of the capsule is insufficient to influence the deformations in the spinning lens seems open to question.

3.3. Zonule

Few data appear to be available in the literature on the mechanical properties of the zonule. This paucity of data reflects the difficulties in making force and displacement measurements on such a small and delicate physiological structure. Fisher (1986) describes lens-stretching experiments which provided estimates of the zonular forces acting during the accommodation process. These forces, combined with assumptions about the geometry changes that occur during the accommodation process, led Fisher to suggest an age-independent value of $0.35\ \mathrm{N\,mm^{-2}}$ for the Young's modulus of the zonule. In separate tests, van Alphen and Graebel (1991) proposed a value of 1.5 $\mathrm{N\,mm^{-2}}$ for the zonule Young's modulus based on tests involving uniaxial loading of the lens.

A difficulty exists in specifying the Young's modulus of the zonular fibres since the combined stiffness of the zonular fibres depends on their number and size as well as the value of Young's modulus for the material. Since few data are available on the cross-sectional area and number of the zonular fibres, an alternative approach was used in this study to determine appropriate parameters for the zonule. This is described below.

Farnsworth and Shyne (1979) suggest that three groups of zonular fibres exist: anterior, posterior and central. Farnsworth and Shyne suggest that the anterior zonular fibres are more numerous than the posterior and that the number of equatorial zonular fibres is relatively few. For the purposes of this numerical model, therefore, the numbers of zonular fibres in each of these three groups are assumed to be in the ratio 6:3:1. To model this in the finite element analysis, three separate thin sheets of material are used to model each set of zonular fibres. The axial stiffnesses of these sheets (corresponding to the product of Young's modulus and thickness of the annular sheet) are specified to be in the ratio 6:3:1 to model the assumed distribution of the zonular fibres. To ensure that the annular sheets represented individual fibres, a model was adopted in which the circumferential stiffness was zero. Fisher (1986) suggests that the zonule stiffness does not change appreciably with age; it was therefore assumed in the development of this numerical model that one value of zonule stiffness would be appropriate for lenses of all ages. To deduce this zonule stiffness, finite element calculations were conducted for the 29-year eye, Mesh B (see Fig. 4b), and the zonule stiffness was adjusted, manually, to ensure that the lens equator movements computed using the model was consistent with the MRI data given by Strenk et al. (1999).

Unfortunately, the Strenk data for ciliary body and lens equator movement are subject to a substantial amount of scatter. However, regression lines fit to the Strenk data suggest that at age 29 years a typical value of ciliary body radius change during full accommodation would be 0.3607 mm (Eq. (5)) and the corresponding change in lens radius would be 0.2903 mm. The values of zonule stiffness (corresponding to the product of Young's modulus and thickness of the annular sheet representing the zonule) derived using this approach are given in Table 4.

4. Finite element model of the ageing lens

The numerical analyses described in this paper are based on the use of the commercial finite element program ABAQUS (ABAQUS—Hibbitt, Karlsson and Sorensen, UK, Ltd.). Care needs to be exercised in the choice of the particular finite element formulation for use in the analysis to ensure that the performance of the

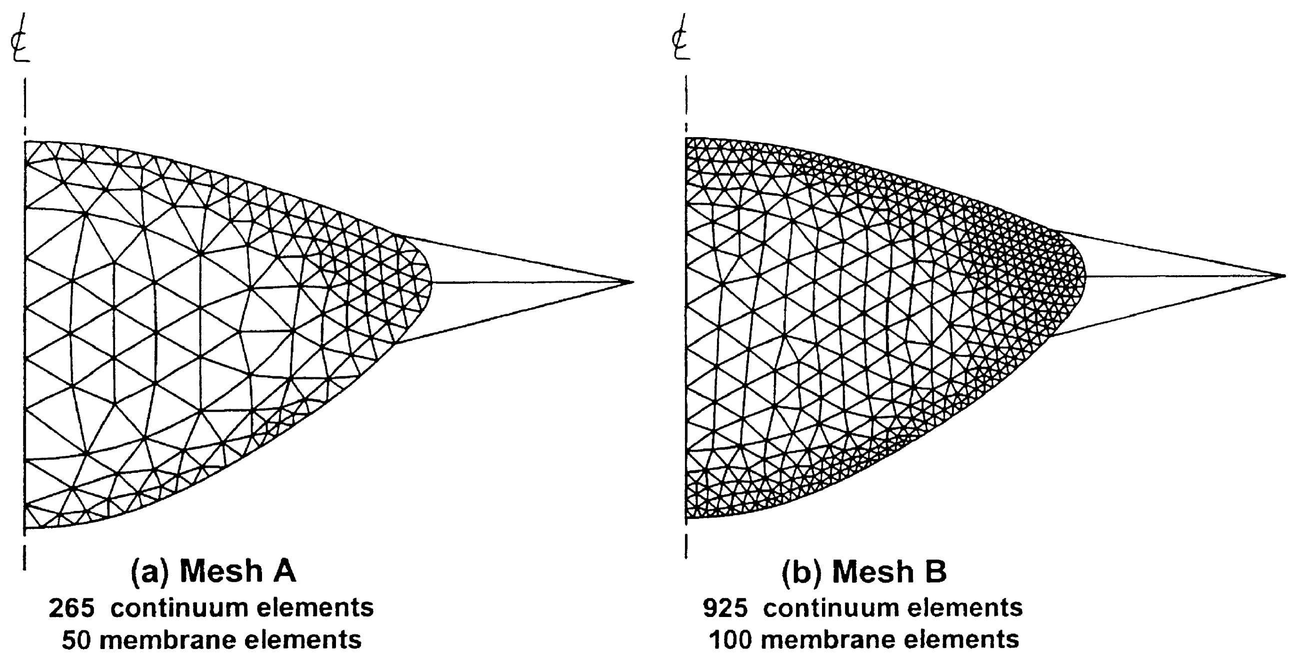

Fig. 4. Finite element meshes for 29-year lens. (a) 265 continuum elements and 50 membrane elements modelling the capsule. (b) 925 continuum elements 100 membrane elements modelling the capsule.

Table 4
Data on material properites of lens

	11-year	29-year	45-year	Reference
Capsule Young's modulus ($N\,mm^{-2}$)	Values from Eq. (4)			Krag et al. (1996)
Cortex Young's modulus ($N\,mm^{-2}$)	1.875×10^{-3}	3.417×10^{-3}	3.980×10^{-3}	Fisher (1971)
Nucleus Young's modulus ($N\,mm^{-2}$)	0.5695×10^{-3}	0.5474×10^{-3}	0.9966×10^{-3}	Fisher (1971)
Anterior zonule stiffness ($N\,mm^{-1}$)	66×10^{-3}	66×10^{-3}	66×10^{-3}	
Central zonule stiffness ($N\,mm^{-1}$)	11×10^{-3}	11×10^{-3}	11×10^{-3}	
Posterior zonule stiffness ($N\,mm^{-1}$)	33×10^{-3}	33×10^{-3}	33×10^{-3}	
Capsule Poisson's ratio	0.47	0.47	0.47	Fisher (1969)
Cortex and nucleus Poisson's ratio	0.49	0.49	0.49	

model is satisfactory. An axisymmetric analysis procedure is adopted, on the assumption that the lens may be regarded in geometric terms as being a body of revolution. (The use of an axisymmetric analysis allows the three-dimensional performance of the lens to be modelled using a two-dimensional mesh of the lens cross-section.) An important feature to consider is that the cortex and nucleus are assumed to be nearly incompressible (a Poisson's ratio of 0.49 is assumed in the calculations except where noted otherwise) and it is known that finite element analysis of nearly incompressible materials can be subject to significant errors unless suitable elements are adopted (Sloan & Randolph, 1982). This issue is particularly important for axisymmetric calculations of the sort described in this paper. In these calculations, six-noded hybrid elements were used to model the lens cortex and nucleus; these elements have been specially formulated to avoid errors associated with modelling incompressible materials.

The capsule was modelled using three-noded membrane elements. This choice was determined by the need to ensure that the elements conformed with the six-noded elements used to model the lens cortex. These membrane elements possess axial stiffness but bending stiffness is zero. The axial stiffness is determined by the product of the Young's modulus and the specified geometric thickness. This axial stiffness will clearly vary with position because of the assumed variation in capsule thickness. In these calculations the capsule was modelled as being fully bonded with the cortex. Although a sliding interface between the membrane and cortex would be an alternative assumption, Bassnett, Missey, and Vucemilo (1999) suggest that the lens epithelial cell attachments to the capsule are 'strong enough to resist the forces generated during lens accommodation'. On the basis of this suggestion, the assumption of a fully bonded interface is thought to be reasonable.

The zonule was modelled using two-noded membrane elements. These were based on an approach in which circumferential stresses were maintained at zero to represent the behaviour of separate radial fibres.

The lens capsule acts as a structural membrane and this suggests (and was confirmed by Burd, Judge, and Flavell (1999)) that geometric non-linear theory may be required to develop a realistic model and to avoid misleading results. The fact that the capsule acts principally as a membrane also suggests that care needs to exercised in the development of suitable mathematical functions to describe the shape of the lens to ensure that the curvature varies in a reasonably well-ordered way. It is also necessary to ensure that slope discontinuities do not occur in the geometry used to model the reference configuration of the lens.

The calculations were conducted by applying a displacement, δ, to the ciliary body (point C in Fig. 1) that would correspond to the expected amplitude of movement. Data on the amplitude of ciliary body displacement are given by Strenk et al. (1999). Although the data show considerable scatter, a linear regression analysis was used to give:

$$\delta(\text{mm}) = 0.5129 - 0.00525 \times \text{age (years)} \tag{5}$$

4.1. Studies of 29-year lens

Two preliminary calculations were carried out using the axisymmetric meshes shown in Fig. 4 to assess the influence of mesh density on the results. In each case, a displacement, δ, was applied in increments to the point representing the ciliary body and the resulting geometry of the lens was computed using the non-linear solution algorithms available in ABAQUS. Solutions for the lens geometry were available at the end of each calculation increment; these solutions corresponded to particular values of ciliary body displacement as it was increased during the analysis. At the end of each calculation increment, the geometry of the portion of the anterior and posterior capsule within a circular aperture of radius 0.8 mm was used to determine the optical power of the lens on the basis of simple paraxial theory. Best fit spheres for the anterior and posterior surfaces were found; the computed radii of curvature, r_a and r_p for the anterior and posterior surfaces respectively, with the computed polar lens thickness, t, were then used to calculate the optical power using the conventional thick lens formula

$$\text{Optical power} = \frac{n_l - n_a}{r_a} + \frac{n_l - n_a}{r_p} - \frac{t(n_l - n_a)^2}{r_a r_p n_l} \tag{6}$$

where n_l, the refractive index of the lens is assumed to be 1.42 and n_a, the refractive index of the aqueous and vitreous to be 1.336. Note that this process follows closely that described by Burd, Judge, and Flavell (1999). It should be noted, however, that there is some evidence to suggest that the refractive index of the lens may reduce significantly with age. Dubbelman and Van der Heijde (2001), for example, suggest that n_l reduces from about 1.436 at age 11 to about 1.425 at age 45. Although using these age-related values of refractive index would have a significant influence on the computed performance of the lens, details of possible age-related variations of lens refractive index are regarded as unproven and so they are not adopted in this study.

The deformed mesh for Mesh B is shown in Fig. 5 for a ciliary body displacement corresponding to the unaccommodated state, together with the lens surfaces in the fully accommodated state. The paraxial anterior surface is flatter in the unaccommodated state than in the accommodated state; this is consistent with the Helmholtz view of accommodation. The computed results indicate that the paraxial anterior curvature reduces by 49% as the lens is stretched from the accommodated to the

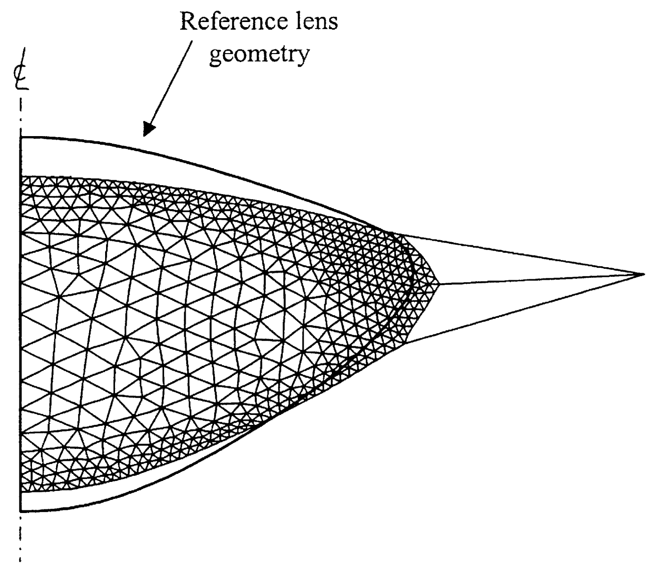

Fig. 5. Deformed mesh for 29-year lens; Mesh B.

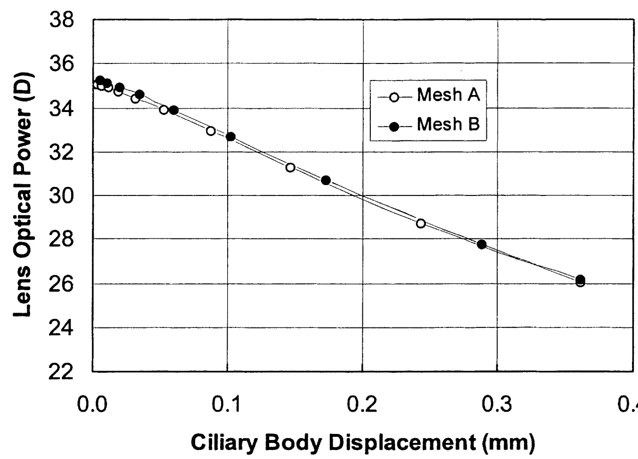

Fig. 6. Variation of optical power with ciliary body movement, 29-year lens.

unaccommodated state. It is less clear from the figure whether on the posterior surface the paraxial curvature is altered, though the computed results indicate a 13% reduction in paraxial posterior curvature as the lens is stretched from the accommodated to the unaccommodated state. The conventional understanding of the accommodation process (Fincham, 1937), emphasises the small change in curvature of the posterior surface, but Garner and Yap (1997) point out that this view relies heavily on studies of two subjects by Fincham. Garner and Yap present new measurements in 11 subjects showing that about a third of the change in lens power with accommodation of 8D is caused by changes in posterior lens curvature. Although Koretz, Handelman, and Brown (1984) state that the posterior curvatures change little with accommodation, their Fig. 4 suggests that there were clear changes in the posterior lens surface curvature in accommodation. [1] In our model, with accommodation, the pole of the anterior surface moves forward about twice as much as the pole of the posterior surface moves backwards, which agrees qualitatively with other measurements (e.g. Drexler, Baumgartner, Findl, Hitzenberger, & Fercher, 1997), although the model's polar movements are about twice as great as those observed by Drexler et al. In Fig. 5, the deformed mesh indicates sharp changes in slope at points where the zonular fibres join the lens; this is a consequence of the tensile stresses developed within the zonule in the unaccommodated state.

The resulting variation of optical power with ciliary body movement is shown in Fig. 6. It is clear that for both calculations, the optical power reduces with radial movement of the ciliary body, as expected. Although the mesh density has a small effect on the results the results are regarded as being sufficiently close to each other to suggest that further refinement (to test whether a converged solution has been obtained) is unnecessary. The element density employed in the more refined mesh (Mesh B) is therefore regarded as being acceptable. Accordingly a similar mesh density to that adopted in Mesh B is used for the study, described later, of the 11- and 45-year lenses, (see Fig. 7).

4.2. Study of 11-, 29- and 45-year lenses

Computations have been carried out for 11- and 45-year lenses using the meshes shown in Fig. 7, where the displacement applied to the ciliary body corresponds to the amplitude of ciliary body displacement given by Eq. (5). The computed variation of optical power with ciliary body displacement for each lens is shown in Fig. 8. The 29-year lens shows a strong reduction in optical power with increasing ciliary body displacement, as has already been noted in Fig. 6. In contrast, the 45-year lens exhibits a much flatter response, particularly for small values of ciliary body displacement. This, of course, is what would be expected of a presbyopic lens. The performance of the 11-year lens is anomalous, however; it exhibits an accommodation amplitude that is unexpectedly less than that observed in the 29-year lens. Discussion of this anomalous behaviour is given later in the paper.

Note that, for the three lens models, the computed optical power is seen to reduce steadily with ciliary body displacement. This is in contrast to the observations

[1] Koretz et al. fit parabolas $y = a + cx^2$ to the lens layers seen in Brown's slit lamp photographs, and presented their findings in terms of plots of the co-variation between values of a and c. Higher c-values imply a greater curvature. In these plots, the posterior surface is represented by the points with the most negative a-values. As the points with the most negative a-values have significantly higher c-values in the accommodated state, posterior curvature must have increased with accommodation.

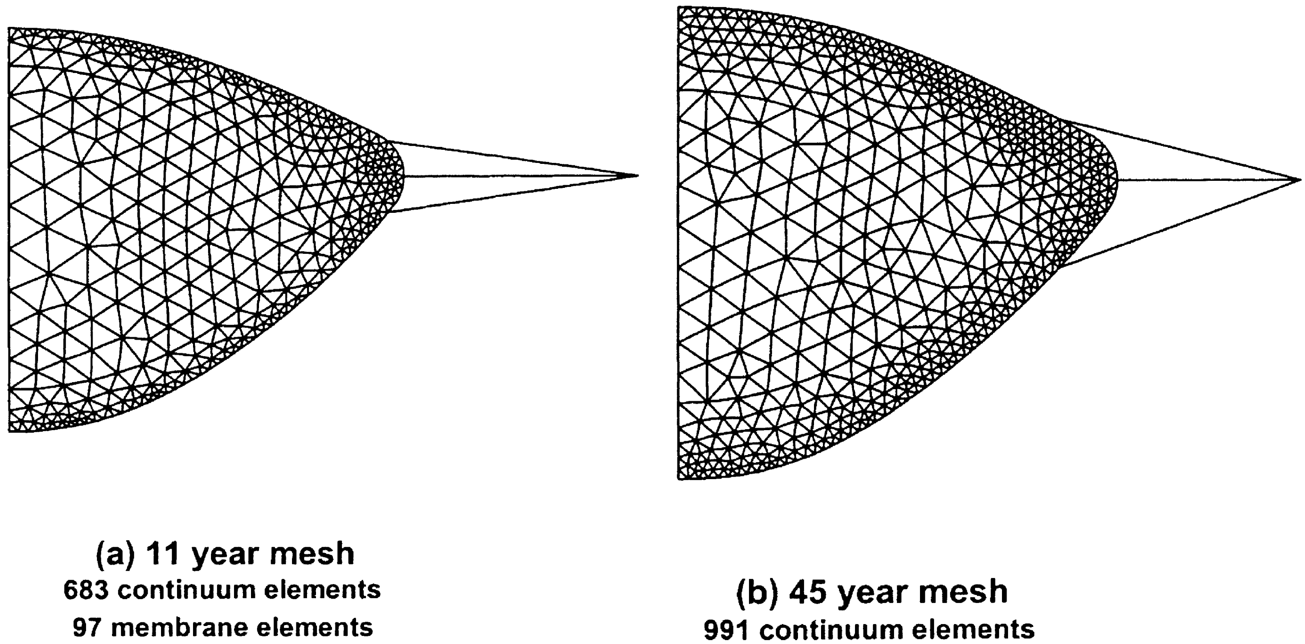

Fig. 7. Finite element meshes of 11- and 45-year lenses. (a) 683 continuum elements and 97 membrane elements modelling the capsule. (b) 991 continuum elements and 106 membrane elements modelling the capsule.

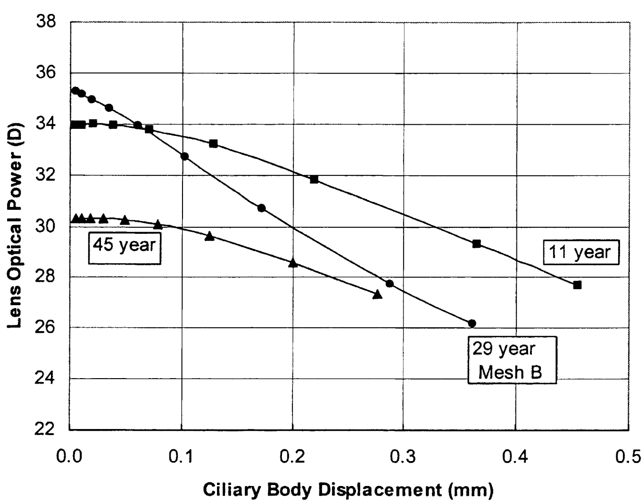

Fig. 8. Variation of optical power with ciliary body displacement. 11-, 29- and 45-year lenses.

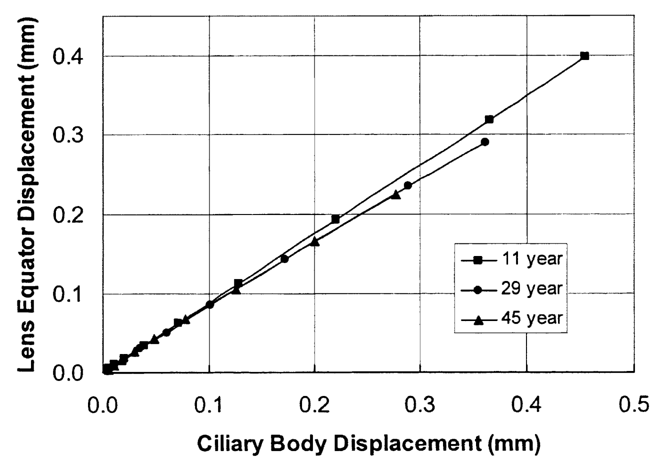

Fig. 9. Relationship between ciliary body displacement and lens equator displacement.

reported by Fisher (1977) which show that, in cadaver eyes subject to radial stretch, a point is reached when further increases in ciliary body radius have little effect on the lens power. (Note also that Fisher's stretching experiments applied a considerably larger ciliary body displacement than is simulated in these numerical calculations.) Krag et al. (1997) observed that the stiffness of the capsule increases significantly with increased strain level; it is possible that this material non-linearity could lead to significant increases in the stiffness of the lens as the ciliary body is stretched, and this would be consistent with Fisher's observations. It would be possible to use a non-linear model for the capsule in a finite element analysis of the lens but this is beyond the scope of this paper.

Fig. 9 shows the relationship between lens displacement and ciliary body displacement for each of the lenses. For a given value of ciliary body displacement, the 11-year lens has a slightly larger equatorial displacement than the older lenses. This feature of the lens would be expected, on the basis that the younger lens is more flexible than the older ones. It is clear, however, that stiffness differences between the three lenses have only a minor effect on the amplitude of lens equator movement. On the basis of the material stiffness data

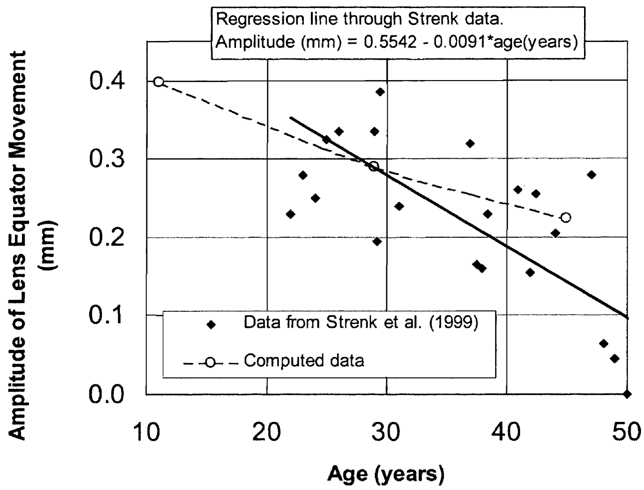

Fig. 10. Computed lens equator amplitude compared with data from Strenk et al. (1999).

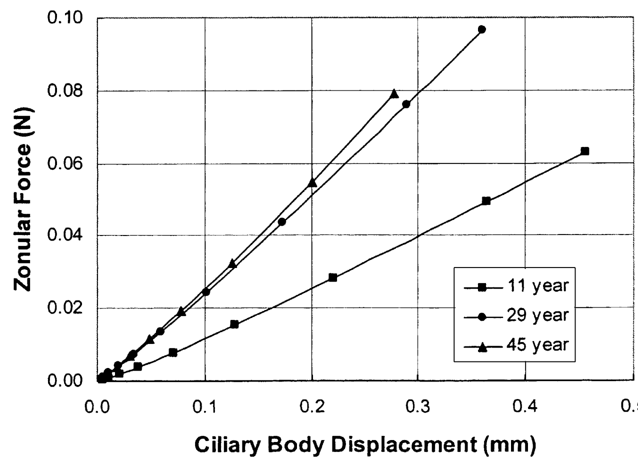

Fig. 11. Zonular force as a function of ciliary body movement.

used to construct this model, therefore, increases in the stiffness of the lens with age do not appear to play a significant role in the development of presbyopia.

A plot of the amplitude of the radial movement of the lens equator, for maximum amplitude of accommodation, from the MRI studies by Strenk et al. (1999) is shown in Fig. 10. Also plotted for comparison are the corresponding data for the three numerical lens models. The 29-year results fit exactly to the regression line through the Strenk et al. (1999) data; this is to be expected because the stiffness of the zonule was specially selected to ensure that this would be the case. The MRI data do not extend to ages below 20 years and so direct comparison with the results of the 11-year model is difficult. It is clear, however, that an extension of the regression line through the Strenk data to 11 years would provide a reasonable fit to the numerical data. The amplitude of lens equator movement for the 45-year lens falls within the general region of the Strenk data although it is somewhat greater than the value that would be obtained from the regression line. Note that an improved fit to Strenk's data for equatorial movement could have been obtained by appropriate adjustment of the zonule stiffness. However, this approach has not been adopted on the basis that no experimental data are available to suggest the zonule stiffness changes appreciably with age.

Fig. 11 shows the total zonular force (i.e. the zonular force applied over the complete circumference) applied by the ciliary body plotted as a function of the ciliary body radial displacement. This suggests that in a state of rest, when the eye is unaccommodated, the ciliary body applies a radial force of between 0.08 and 0.1 N to the lens via the zonule, in the adult eye. The computed resting force is substantially lower in the 11-year eye and this is presumably due to the fact that the capsule and lens are significantly less stiff than in the older lenses. It should be noted that the predicted amplitude of the tensile force applied to the zonule by the ciliary body (between 0.08 and 0.1 N in the adult eye) is significantly greater than the value of about 0.015 N that would be suggested by Fisher's spinning lens experiment (see Fig. 8 of Fisher (1977)). The causes of this discrepancy are not clear.

Fig. 12 shows the accommodation amplitude curves given by Duane (1922), for the complete optical system of the eye. The data from the numerical model that has been described so far relates to the optical power of the isolated lens. To estimate how the accommodation amplitude of the complete eye might relate to the range of optical power of the lens, it is noted that in the schematic eye of Bennett and Rabbetts (1998), a change of lens power of 13.15D is required to change the optical power of the eye by 10D. To estimate the accommodation amplitude of the complete eye, corresponding to the numerical model of the isolated lens, the computed lens power amplitude is simply divided by 1.315. Three data points, corresponding to the three lens models developed in this study, corrected in this way to give esti-

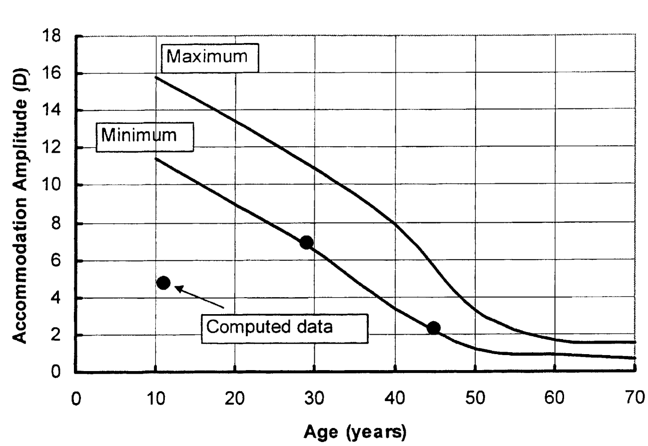

Fig. 12. Amplitude of accommodation compared with Duane (1922).

mated accommodation amplitude for the complete eye, are plotted in Fig. 12. For the two older lenses, the numerical model lies on the lower range of Duane's data; the 11-year eye (as has already been noted) exhibits an amplitude of accommodation that is anomalous. It should be noted that the amplitude of ciliary body displacement used in the finite element analysis was derived from in vivo data (Strenk et al., 1999) based on an accommodation demand of 8D. It seems possible that Strenk's data on ciliary body movement do not represent the full range of available movement for young eyes (which would be capable of an accommodation range in excess of 8D). Since the numerical model is based on Strenk's data, there is likely to be a tendency for the analysis to under-predict the range of accommodation in young eyes.

4.3. Anomalous behaviour of the 11-year lens

In broad terms, the performance of the 29- and 45-year models are found to conform closely to previous expectations. The predicted accommodation amplitude of the 11-year model, however, is unexpectedly low. The performance of the model depends critically on the input parameters and assumptions used in its development. Although it is not currently clear which aspects of the model are responsible, a discussion of the likely sources of this unexpected behaviour is given below.

Experience from previous work on the development of numerical models of the accommodating lens (Burd, Judge, & Flavell, 1999; Burd, Judge, & Cross, 1999) indicates that the results of the modelling are strongly conditioned by the geometric parameters and procedures that are used to specify the initial lens geometry. To illustrate this point, a further analysis has been carried out (which is referred to below as Analysis 5). This analysis was similar to the 29-year run employing Mesh B, with the exception that the shape and size of the nucleus, the lens thickness and the lens curvatures were taken to be equal to the data from Brown (1973) for the unaccommodated 11-year (rather than the 29-year) lens. This analysis was intended to investigate the sensitivity of the 29-year model to changes in its specified geometry. The computed amplitude of accommodation (based on Eq. (6)) for the original 29-year eye model was 9.07D, whereas the amplitude for Analysis 5 was 6.02D, i.e. a reduction of 34%. While Analysis 5 is not intended to represent any real case, it does indicate that the geometric parameters used to define the lens are crucial to its computed performance. It appears, in broad terms, that Brown's data for the 11-year lens lead to a model that is significantly less effective in accommodation than a lens based on Brown's 29-year data.

Since it is suspected that the computed performance of the lens may be strongly dependent on the quality of the data used to define its geometry, an investigation

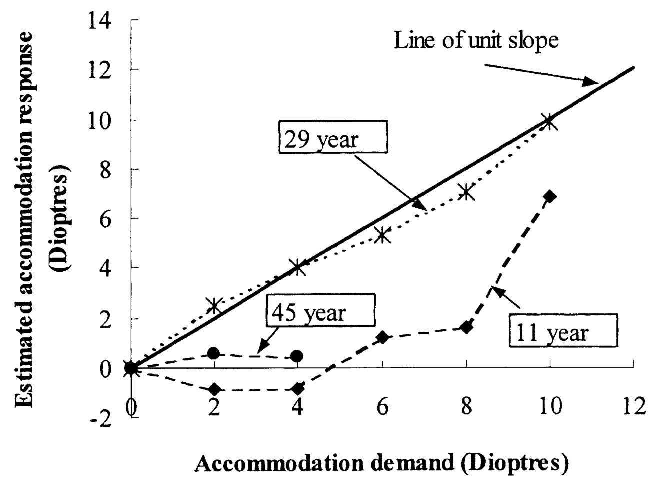

Fig. 13. Analysis of the data of Brown (1973).

was conducted of the reliability of the data given by Brown (1973) in his Tables 1 and 3. Brown's geometric data were used directly, in conjunction with Eq. (6), to estimate the accommodation response of the lens, taking the unaccommodated state to be 0D. This response was then divided by 1.315 to estimate the accommodation response of the complete eye. The results, plotted against accommodation demand in Fig. 13, show that the estimated accommodation response of the 29-year eye matches closely the accommodation demand. As expected, the 45-year eye shows very little accommodation response. The 11-year data appear anomalous, however, in that the estimated accommodation response is less than 20% of the accommodation demand, other than with 10D demand, where it is about 65% of demand. This leads us to doubt whether Brown's data for the 11-year old eye can be accepted as a valid estimate of 11-year old lens behaviour. Also, the 10D accommodative demand adopted by Brown (1973) will have been insufficient to induce a fully accommodated state in the 11-year lens. This will mean that the reference configuration adopted in the 11-year lens may be significantly different from the fully accommodated state.

Other factors may be relevant in developing an understanding of the computed behaviour of the 11-year model. Dubbelman and Van der Heijde (2001) for example suggest that the lens refractive index reduces with age; their data suggest a lens refractive index of 1.436 at age 11. If this value of refractive index had been used in the computation of the accommodation amplitude of the 11-year lens (rather than the value of 1.42 that was adopted), then this would have increased the predicted amplitude of accommodation of the lens by 18%. Note, however, that Glasser and Campbell (1999) concluded on the basis of their experimental studies that the equivalent refractive index of the eye does not vary significantly with age. Also, the 8D accommodation demand used in the experiments reported by Strenk et al. (1999) would not have been sufficient to induce maximum accommodation in a young eye. Noting that these data were used to determine the ciliary body displacement for use in the model, it seems possible that the ciliary body displacement applied to the 11-year eye may have been too low perhaps, therefore, contributing to its anomalous behaviour. The variation of capsule thickness adopted for the 11-year model was based on data, from Fisher and Pettet (1972), for a lens of age 22 years. It is not thought, however, that this will have had a major effect on the performance of the 11-year model.

4.4. Sensitivity studies

Although a detailed study of the effects of variations in the input parameters on the predictions made by the model is beyond the scope of the paper, a limited sensitivity exercise has been attempted as described below.

The calculations described in this paper were based on the use of a Poisson's ratio of 0.49 for the nucleus and cortex. This value was adopted on the basis that it corresponds closely to the value of 0.5 that would specify an incompressible material. Although numerical difficulties occur when a Poisson's ratio of exactly 0.5 is used, an analysis of the 29-year lens has been carried out in which the Poisson's ratio of the nucleus and cortex was set to 0.4999999; this analysis is referred to here as Analysis 6. The results of Analysis 6 indicated an accommodation amplitude that was less than that computed for the 29-year run, with Mesh B, by about 3%. This difference in amplitude is not regarded as being significant.

In the development of the numerical models, the stiffness of the zonule was apportioned in the ratio 6:3:1 between the anterior, posterior and equatorial zonular fibres respectively. Two further runs, based on the 29-year model and Mesh B, were conducted to investigate the sensitivity of this particular assumption on the performance of the model. In Analysis 7, the stiffness of all three sets of zonular fibres was set to be equal. In Analysis 8, the stiffness of the anterior and posterior zonular fibres was set to zero with the consequence that only the equatorial zonule was active. In both of these additional analyses, the same approach was used to select the zonule stiffness so that it matched the data published by Strenk et al. (1999) (i.e. by ensuring that the radial movement of the lens equator was 0.2903 mm for a ciliary body movement of 0.3607 mm). The results of these analyses are shown in Fig. 14. It is clear that details of the way in which the zonule stiffness is apportioned between the three zonule groups have a significant effect on the computed response. Analysis 8, in which only the central zonule is active, exhibits an amplitude of accommodation that is rather less that is evident in the other two models. This suggests that the equatorial zonule is less effective in contributing to the

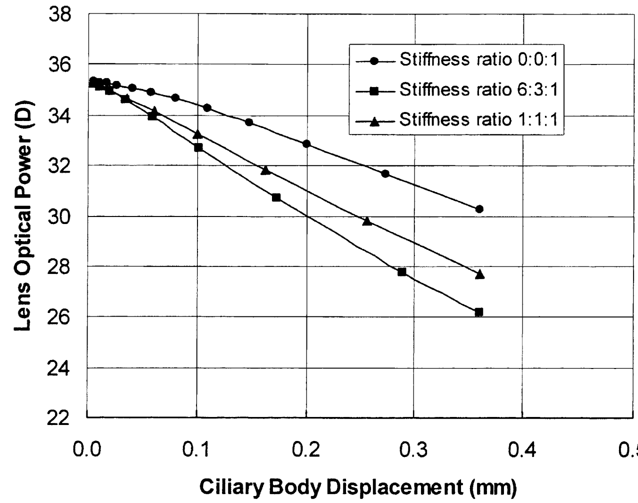

Fig. 14. Effect of different zonule configurations on the performance of the 29-year lens.

process of accommodation than the anterior and posterior zonular fibres. Although the behaviour of the model is seen to depend significantly on detailed assumptions regarding the zonule stiffness, it is thought that a stiffness apportionment in the ratio 6:3:1 remains an appropriate assumption, based on the available experimental data.

5. Conclusions

A review of the literature on measurements of the mechanical properties of the tissues making up the lens and accommodation apparatus has been used to construct numerical models of lenses of different ages. The behaviour of the numerical models show good, or reasonable, agreement with a range of published data, although the computed behaviour of the youngest lens is anomalous. Initial studies led to the hypothesis that the unexpected performance of the 11-year lens may be linked to details of the assumed geometry. Further data on the geometry of young lenses will be needed to test this hypothesis and, currently, these data do not seem to be available.

Numerical modelling requires access to high quality data on geometric and material properties of the lens. The current archive is such that a numerical model is reliant on data from a small number of sources. If numerical modelling is to be developed as a successful approach in the study of accommodation and presbyopia, then this will rely on new, high quality, experimental data becoming available in the future. Of critical importance is the need for new measurements of the geometry of the lens, in eyes of different ages, for different states of accommodation. It will also be necessary for future experimental work to address the issue of the variability in mechanical and geometric properties of lenses of similar ages. A knowledge of the likely variation of these parameters will be of fundamental importance for future sensitivity studies.

The outcome of this study is broadly consistent with the numerical results obtained by Schachar and Bax (2001), when their reference geometry was based on the accommodated state. Schachar and Bax (2001) described an analysis of a 29-year eye that, in common with the current study, was based on data obtained by Brown (1973), although some of the detailed modelling procedures and parameters used by Schachar and Bax differ from those adopted in the current study.

The results of the models described in this paper are broadly in agreement with those given in a preliminary communication (Burd, Judge, & Cross, 1999). Although these numerical models have been shown, generally, to accord with expectation, the calculations do not, in themselves, serve to settle the question of which, if any, of the proposed hypotheses of presbyopia are correct. If the results for the 11-year lens are, for the moment, discounted, then it does appear that the numerical model has captured the essential features of presbyopia. Further work is needed (which will be aided by access to high quality test data) to confirm this. If this confirmation is forthcoming then it will be possible to investigate independently each one of the age-related features of the model to assess their relative importance in presbyopia.

References

van Alphen, G. W. H. M., & Graebel, W. P. (1991). Elasticity of tissues involved in accommodation. *Vision Research, 31*(7/8), 1417–1438.

Atchison, D. A. (1995). Accommodation and presbyopia. *Ophthalmic and Physiological Optics, 15*, 255–277.

Bassnett, S., Missey, H., & Vucemilo, I. (1999). Molecular architecture of the lens fiber cell basal membrane complex. *Journal of Cell Science, 112*, 2155–2165.

Bellows, J. G. (1944). *Cataract and anomalies of the lens* (p. 67) London: Henry Kimpton.

Bennett, A. G., & Rabbetts, R. B. (1998). Clinical Visual Optics. Butterworth–Heinemann, 3rd ed.

Brown, N. (1973). The change in shape and internal form of the lens of the eye on accommodation. *Experimental Eye Research, 15*, 441–459.

Brown, N. (1974). The change in lens curvature with age. *Experimental Eye Research, 19*, 175–183.

Burd, H. J., Judge, S. J., & Flavell, M. J. (1999a). Mechanics of accommodation of the human eye. *Vision Research, 39*, 1591–1595.

Burd, H. J, Judge, S. J., & Cross, J. A. (1999b). Numerical modelling of ocular accommodation (ARVO Abstract). *Investigative Opthamology and Visual Science, 40*(4), Abstract no. 4678.

Canals, M., Costa-Vila, J., Potau, J. M., Merindano, M. D., & Ruano, D. (1996). Scanning electron microscopy of the human zonule of the lens (zonula ciliaris). *Acta Anatomica, 157*, 309–314.

Croft, M. A., Glasser, A., Gabelt, B., Ebbert, T., Heatley, G. A., & Kaufman, P. L. (1999). Lens and ciliary muscle function in young and old rhesus monkeys. (ARVO Abstract). *Investigative Ophthalmology and Visual Science, 40*(4), Abstract no. 1918.

Drexler, W., Baumgartner, A., Findl, O., Hitzenberger, C. K., & Fercher, A. F. (1997). Biometric investigation of changes in the anterior eye segment during accommodation. *Vision Research, 37*, 2789–2800.

Duane, A. (1922). Studies in monocular and binocular accommodation with their clinical applications. *American Journal of Ophthalmology, 5*, 865–877.

Dubbelman, M., & Van der Heijde, G. L. (2001). The shape of the aging human lens: curvature, equivalent refractive index and the lens paradox. *Vision Research, 41*, 1867–1877.

Farnsworth, P. N., & Shyne, S. E. (1979). Anterior zonular shifts with age. *Experimental Eye Research, 28*, 291–297.

Fincham, E. F. (1937). The mechanism of accommodation. *British Journal of Ophthalmology* (Suppl. 8), 5–80.

Fisher, R. F. (1969). The elastic constants of the human lens capsule. *Journal of Physiology, 201*, 1–19.

Fisher, R. F. (1971). The elastic constants of the human lens capsule. *Journal of Physiology, 212*, 147–180.

Fisher, R. F., & Pettet, B. E. (1972). The postnatal growth of the capsule of the human crystalline lens. *Journal of Anatomy, 112*(2), 207–214.

Fisher, R. F. (1977). The force of contraction of the human ciliary muscle during accommodation. *Journal of Physiology, 270*, 51–74.

Fisher, R. F. (1986). The ciliary body in accommodation. *Transactions of the Ophthalmological Societies of the United Kingdom, 105*, 208–219.

Garner, L. F., & Yap, M. K. H. (1997). Changes in ocular dimensions and refraction with accommodation. *Ophthalmic Physiological Optics, 17*, 12–17.

Glasser, A., & Campbell, M. C. W. (1998). Presbyopia and the optical changes in the human crystalline lens with age. *Vision Research, 38*, 209–229.

Glasser, A., & Campbell, M. C. W. (1999). Biometric, optical and physical changes in the isolated human crystalline lens with age in relation to presbyopia. *Vision Research, 39*(11), 1991–2015.

Helmholtz, H. von. (1909). Physiological optics, vol. 1, New York: Dover, 1962, pp. 143–172; English translation by JPC Southall for the Optical Society of America (1924) from the 3rd German edition of Handbuch der Physiologischen Optik. Hamburg: Voss.

Koretz, J. F., Handelman, G. H., & Brown, N. P. (1984). Analysis of human crystalline lens curvature as a function of accommodative state and age. *Vision Research, 24*, 1141–1151.

Krag, S., Andreassen, T. T., & Olsen, T. (1996). Elastic properties of the lens capsule in relation to accommodation and presbyopia. *Investigative Ophthalmology and Visual Science, 37*(3), Abstract no. 774.

Krag, S., Olsen, T., & Andreassen, T. T. (1997). Biomechanical characteristics of the human anterior lens capsule in relation to age. *Investigative Ophthalmology and Visual Science, 38*, 357–363.

Ludwig, K., Wegscheider, E., Hoops, J. P., & Kampik, A. (1999). In vivo imaging of the human zonular aparatus with high-resolution ultrasound biomicroscopy. *Graefe's Archive for Clinical and Experimental Ophthalmology, 237*(5), 361–371.

Pierscionek, B. K. (1995). Age-related response of human lenses to stretching forces. *Experimental Eye Research, 60*(3), 325–332.

Rohen, J. W. (1979). Scanning electron microscopic studies of the zonular apparatus in human and monkey eyes. *Investigative Ophthalmology and Visual Science, 18*, 133–144.

Scammon, R. E., & Hesdorffer, M. B. (1937). Growth in mass and volume of the human lens in postnatal life. *Archives of Ophthamology Chicago, 17*, 104–112.

Schachar, R. A., Huang, T., & Huang, X. (1993). Mathematic proof of Schachar's hypothesis of accommodation. *Annals of Ophthalmology, 25*(1), 5–9.

Schachar, R. A., & Bax, A. J. (2001). Mechanism of accommodation as analyzed by nonlinear finite element analysis. *Annals of Ophthalmology Clinics, 33*(2), 103–112.

Sloan, S. W., & Randolph, M. F. (1982). Numerical prediction of collapse loads using finite element methods. *International Journal of Numerical and Analytical Methods in Geomechanics, 6*, 47–76.

Smith, P. (1883). Diseases of crystalline lens and capsule. *Transactions of the Ophthalmological Society UK, 3*, 79–99.

Storey, J. K., & Rabie, E. P. (1985). Ultrasonic measurement of transverse lens diameter during accommodation. *Ophthalmic and Physiological Optics, 5*, 145–148.

Strenk, S. A., Semmlow, J. L., Strenk, L. M., Munoz, P., Gronlund-Jacob, J., & DeMarco, J. K. (1999). Age-related changes in human ciliary muscle and lens: a magnetic resonance imaging study. *Investigative Ophthalmology and Visual Science, 40*(6), 1162–1169.

Weale, R. A. (2000). Why we need reading-glasses before a zimmer-frame. *Vision Research, 40*, 2233–2240.

Wyatt, H. J. (1993). Application of a simple mechanical model of accommodation to the aging eye. *Vision Research, 33*(5/6), 731–738.