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A custodial symmetry for $Zb\bar{b}$

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Abstract

We show that a subgroup of the custodial symmetry O(3) that protects $\Delta\rho$ from radiative corrections can also protect the $Zb\bar{b}$ coupling. This allows one to build models of electroweak symmetry breaking, such as higgsless, little Higgs or 5D composite Higgs models, that are safe from corrections to $Z \to b\bar{b}$. We show that when this symmetry protects $Zb\bar{b}$ it cannot simultaneously protect $Zt\bar{t}$ and $Wt\bar{b}$. Therefore one can expect to measure sizable deviations from the SM predictions of these couplings at future collider experiments. We also show under what circumstances $Zb_R\bar{b}_R$ can receive corrections in the right direction to explain the anomaly in the LEP/SLD forward–backward asymmetry $A^b_{\rm FB}$.
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1. Introduction

One of the most elegant solutions to the hierarchy problem is to consider that the Higgs boson, the scalar field responsible for electroweak symmetry breaking (EWSB), is not a fundamental particle. This approach is clearly inspired by QCD, where scalar and pseudoscalar states appear as composites of the strong dynamics. In recent years there has been a revival of interest in this approach. The important new ingredient has been calculability, achieved by using either the idea of "collective breaking" [1] or extra dimensions.

As in the old technicolor [2] or composite Higgs models [3], the main challenge of these new scenarios is to pass successfully all the electroweak precision tests (EWPT). This is a non-trivial task, since in these theories deviations from the Standard Model (SM) predictions usually arise at the tree level due to mixing effects between SM fields and the heavy states of the new sector. One of the main difficulties is to avoid large deviations in the $Zb_L\bar{b}_L$ coupling, whose measured value is in agreement with the SM prediction at the 0.25% level. This is difficult to overcome, since in these models the top, being heavy, couples strongly to the new sector. Since b_L is in the same weak doublet as t_L, it usually suffers from large modifications to its couplings.

In this Letter we will show that the custodial symmetry O(3), advocated long ago to protect $\Delta\rho$ [4], can also protect $Zb\bar{b}$. In particular we will see that the $Zb_L\bar{b}_L$ coupling can be safe from corrections and at the same time the $SU(2)_L$-related couplings $Zt_L\bar{t}_L$ and $Wt_L\bar{b}_L$ can receive sizable modifications. As an example, we will present the explicit calculations of these effects in a 5D scenario of EWSB. The custodial symmetry can also be used to protect the coupling of the b_R to the Z. However, the LEP and SLD experimental measurements of the forward–backward asymmetry $A^b_{\rm FB}$ suggest that the coupling $Zb_R\bar{b}_R$ might deviate from its SM value. We will then study the possibility of having large effects in $Zb_R\bar{b}_R$ of the right magnitude and sign as suggested by the experimental data.

Our analysis can be useful for any scenario of EWSB that contains a new sector beyond the SM (BSM) invariant under the global custodial symmetry. This sector is defined to include the Higgs field as well. Examples are the strongly interacting sector of technicolor models, the extra fields added in little Higgs theories to avoid quadratic divergences, or the bulk of a warped

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extra dimension present in some higgsless [5] and composite Higgs [6,7] models.

2. The coupling $Z\psi\bar{\psi}$ and the custodial symmetry

We will consider BSM sectors with the following global symmetry breaking pattern [4]:

$$O(4) \to O(3). \tag{1}$$

This breaking is equivalent to the more familiar custodial pattern $SU(2)_L \otimes SU(2)_R \to SU(2)_V$ together with a parity defined as the interchange $L \leftrightarrow R$ (P_{LR}). As we will see below, this discrete symmetry plays an important role to protect the coupling of the Z to fermions from non-zero corrections. The BSM sector also has to respect an $SU(3)_c \otimes U(1)_X$ symmetry corresponding to the SM color group and an extra U(1) needed to fit the hypercharges of the SM fields ($Y = T_R^3 + X$). As usual [4], we will parametrize the symmetry breaking in Eq. (1) by a 2×2 unitary matrix field U transforming as a $(\mathbf{2},\mathbf{2})_\mathbf{0}$ under $SU(2)_L \otimes SU(2)_R \otimes U(1)_X$, whose VEV is given by $\langle U \rangle = \mathbb{1}$.

Since the BSM sector is invariant under O(4), we can rotate to a basis in which each BSM field (or operator), $\mathcal{O}_{\rm BSM}$, has a definite left and right isospin quantum number, $T_{L,R}$, and its 3rd component, $T^3_{L,R}$. We will assume that every SM field Φ is coupled to a single BSM field (or operator): $\mathcal{L}_{\rm int} = \Phi^\dagger \mathcal{O}_{\rm BSM} + {\rm h.c.}$ This assumption is always fulfilled in the BSM models that we are interested in. It guarantees that we can univocally assign to each SM field definite quantum numbers $T_{L,R}$, T^3_{LR}, corresponding to those of the operator $\mathcal{O}_{\rm BSM}$ to which it couples. Notice that this does not imply that the SM fields are in complete representations of $SU(2)_L \otimes SU(2)_R$, as it is known not to be the case.

Let us consider the implications of the custodial symmetry $O(3) = SU(2)_V \otimes P_{LR}$ on the coupling $Z\psi\bar{\psi}$, where ψ denotes a generic SM fermion. At zero momentum, this coupling is given by

$$\frac{g}{\cos\theta_W}\left[Q_L^3 - Q\sin^2\theta_W\right] Z^\mu \bar{\psi}\gamma_\mu\psi, \tag{2}$$

where Q_L^3 and Q are respectively the 3rd-component $SU(2)_L$ charge and the electric charge of ψ. Since the electric charge Q is conserved, possible modifications to the coupling $Z\psi\bar{\psi}$ can only arise from corrections to Q_L^3. Before EWSB we have $Q_L^3 = T_L^3$, but this is not guaranteed anymore after EWSB. We will be interested only in non-universal corrections induced by the BSM fields, and we will treat the SM W_L^3 field as an external classical source which probes the left charge Q_L^3. This is consistent since corrections induced through the renormalization of the Z kinetic term are universal.

We found two subgroups of the custodial symmetry $SU(2)_V \otimes P_{LR}$ that can protect Q_L^3. The first one is the subgroup $U(1)_L \otimes U(1)_R \otimes P_{LR}$ that it is broken by $\langle U \rangle$ down to $U(1)_V \otimes P_{LR}$. Although P_{LR} is a symmetry of the BSM sector, it is not, in general, respected by the coupling of ψ to the BSM sector. For P_{LR} to be a symmetry also of $\mathcal{L}_{\rm int} = \bar{\psi}\mathcal{O}_\psi + {\rm h.c.}$, we must demand that ψ is an eigenstate of P_{LR}. This implies

$$T_L = T_R, \qquad T_R^3 = T_L^3, \tag{3}$$

for the field ψ. If this is the case, the non-universal corrections to the charge Q_L^3 of ψ are zero. The proof goes as follows. By $U(1)_V$ invariance, we have that $Q_V^3 = Q_L^3 + Q_R^3$ is conserved, and therefore it cannot receive corrections:

$$\delta Q_V^3 = \delta Q_L^3 + \delta Q_R^3 = 0. \tag{4}$$

On the other hand, by P_{LR} invariance we have that the shift in Q_L^3 must be equal to the shift in Q_R^3:

$$\delta Q_L^3 = \delta Q_R^3. \tag{5}$$

Eqs. (4) and (5) imply that $\delta Q_L^3 = 0$. This proves that SM fermions that fulfill the condition (3) have their coupling to the Z protected by the subgroup $U(1)_V \otimes P_{LR}$ of the custodial symmetry.

The second example of a symmetry that can protect Q_L^3 is that of the discrete transformation $|T_L, T_R; T_L^3, T_R^3\rangle \to |T_L, T_R; -T_L^3, -T_R^3\rangle$, a subgroup of the custodial $SU(2)_V$. We will denote this symmetry by P_C. Its action on 2-component spinors is given by $P_C = i\sigma_1$, while SO(3) vectors transform with $P_C = {\rm Diag}(1,-1,-1)$. According to our rule then, the SM W_L^3 can be assigned an odd parity under $P_C : W_L^3 \to -W_L^3$. For ψ to be an eigenstate of this symmetry, it must have

$$T_L^3 = T_R^3 = 0. \tag{6}$$

If this is the case, we have that $\delta Q_L^3 = 0$ at any order. Indeed, if ψ is an eigenstate of P_C, then $\bar{\psi}\gamma^\mu\psi$ is even under P_C and it cannot couple to W_L^3 that is odd. Thus, the coupling of the Z to SM fermions that fulfill Eq. (6) is protected by the subgroup P_C of the custodial symmetry.

It is important to notice that the symmetries discussed above can only protect the coupling of the Z to fermions at zero momentum. However, momentum dependent corrections to $Z\psi\bar{\psi}$ are parametrically suppressed in strongly coupled BSM sectors. A naive estimate gives $\delta g/g \sim \lambda_t/g_{\rm BSM}\, q^2/\Lambda^2_{\rm BSM}$, where λ_t is the top Yukawa coupling and $g_{\rm BSM}$ is the coupling among the BSM particles. Therefore, $\delta g/g$ can be sufficiently small for $g_{\rm BSM} \gg \lambda_t$.

3. Corrections to $Zb_L\bar{b}_L$ in custodial invariant models

The symmetry argument given in the previous section shows how to build higgsless or composite Higgs models in which $Zb\bar{b}$ does not receive corrections from the BSM sector. Let us start with the $Zb_L\bar{b}_L$ coupling. In these models it has been commonly assumed that b_L transforms as a $(\mathbf{2},\mathbf{1})_{\mathbf{1/6}}$ representation of the $SU(2)_L \otimes SU(2)_R \otimes U(1)_X$ group. In that case, b_L has the quantum numbers $T_L = 1/2$, $T_R = 0$, $T_L^3 = -1/2$ and $T_R^3 = 0$, which fulfill neither the condition (3) nor (6). As a consequence, $Zb_L\bar{b}_L$ is not protected by the custodial symmetry. Condition (3), however, suggests us a better assignment for the b_L quantum numbers:

$$T_L = 1/2 = T_R, \quad \text{and} \quad T_L^3 = -1/2 = T_R^3. \tag{7}$$

This assignment guarantees that $Z\bar{b}_L b_L$ does not receive corrections from the BSM sector. Eq. (7) implies that t_L, being in the same $SU(2)_L$ doublet as b_L, has to have the following assignments: $T_L = T_R = 1/2$ and $T_L^3 = -T_R^3 = 1/2$. Therefore, condition (3) is not satisfied for t_L and there will be corrections to the $Zt_L\bar{t}_L$ coupling. Similarly, the custodial symmetry does not protect $Wt_L\bar{b}_L$ (see below), and one can have large modifications in this coupling as well, without affecting $Zb_L\bar{b}_L$. At present, the couplings of the top to the gauge bosons are not accurately measured. Future accelerators, however, will improve the measurements of these couplings and will be able to test this scenario.

3.1. Operator analysis

We give here an operator analysis for the couplings of $q_L = (t_L, b_L)$ to the Z and the W based on the custodial symmetry. For the assignment of Eq. (7), we must embed b_L in a $\mathbf{4_{2/3}}$ of $O(4) \otimes U(1)_X$, or, equivalently,

$$q_L \in (\mathbf{2}, \mathbf{2})_{\mathbf{2/3}} \equiv Q_L \tag{8}$$

under $SU(2)_L \otimes SU(2)_R \otimes U(1)_X$. In addition to the SM doublet, this representation contains an extra $SU(2)_L$ doublet q'_L that, not corresponding to any SM field, will play the role of a non-dynamical spectator. We find two single-trace dimension-4 operators that can contribute to the Z couplings:

$$\mathcal{L} = c_1 \operatorname{Tr}\big[\bar{Q}_L \gamma^\mu Q_L \hat{V}_\mu\big] + c_2 \operatorname{Tr}\big[\bar{Q}_L \gamma^\mu V_\mu Q_L\big], \tag{9}$$

where $Q_L = \sigma^\alpha Q_L^\alpha$ is a 2×2 matrix field,[1] $V_\mu = (iD_\mu U)U^\dagger$, $\hat{V}_\mu = (iD_\mu U)^\dagger U$, and the covariant derivative is defined as $D_\mu U = \partial_\mu U + ig\sigma_a W_\mu^a U/2 - ig' B_\mu U \sigma_3/2$. By imposing P_{LR}, under which $U \to U^\dagger$, $V_\mu \leftrightarrow \hat{V}_\mu$ and $Q_L \to \sigma^{\alpha\,\dagger} Q_L^\alpha$, we obtain $c_1 = c_2$. There is also a double-trace operator that can contribute to the Z coupling to q_L:

$$\mathcal{L} = c_3 \operatorname{Tr}\big[\bar{Q}_L \gamma^\mu i D_\mu U\big] \operatorname{Tr}\big[U^\dagger Q_L\big] + \text{h.c.} \tag{10}$$

To obtain the contributions to $Zb_L\bar{b}_L$, $Zt_L\bar{t}_L$ and $Wt_L\bar{b}_L$ we plug

$$Q_L = \sigma_- b_L + \sigma_0 t_L + \cdots, \qquad U = \mathbb{1},$$
$$D_\mu U = \frac{ig\sigma_3}{2\cos\theta_W} Z_\mu + \frac{ig\sigma_+}{\sqrt{2}} W_\mu^+ + \cdots, \tag{11}$$

into Eqs. (9) and (10), where $\sigma_\pm = (\sigma_1 \pm i\sigma_2)/2$ and $\sigma_0 = (\mathbb{1} + \sigma_3)/2$. This gives

$$\frac{g}{\cos\theta_W}\left[\frac{c_2 - c_1}{2}\, \bar{b}_L \gamma^\mu b_L - \frac{c_1 + c_2 + 2c_3}{2} \bar{t}_L \gamma^\mu t_L\right] Z_\mu$$
$$- \frac{g}{\sqrt{2}}(c_2 + c_3)\bar{t}_L \gamma^\mu b_L W_\mu^+ + \text{h.c.} \tag{12}$$

As expected from the symmetry argument, the contributions to $Zb_L\bar{b}_L$ vanish after imposing invariance under P_{LR} ($c_1 = c_2$), while the contributions to the couplings of the top quark are different from zero.

The embedding of t_R in a multiplet of $SU(2)_L \otimes SU(2)_R \otimes U(1)_X$ is determined by the top mass operator $\bar{q}_L U t_R$. There are two possible invariant operators:

$$\text{(a)} \quad \overline{(\mathbf{2}, \mathbf{2})}_{\mathbf{2/3}} (\mathbf{2}, \mathbf{2})_{\mathbf{0}} (\mathbf{1}, \mathbf{1})_{\mathbf{2/3}}, \quad \text{or}$$
$$\text{(b)} \quad \overline{(\mathbf{2}, \mathbf{2})}_{\mathbf{2/3}} (\mathbf{2}, \mathbf{2})_{\mathbf{0}} (\mathbf{1}, \mathbf{3})_{\mathbf{2/3}}, \tag{13}$$

implying respectively the two following embeddings for t_R:

$$\text{(a)} \quad t_R \in (\mathbf{1}, \mathbf{1})_{\mathbf{2/3}}, \quad \text{or}$$
$$\text{(b)} \quad t_R \in (\mathbf{1}, \mathbf{3})_{\mathbf{2/3}} \oplus (\mathbf{3}, \mathbf{1})_{\mathbf{2/3}}, \tag{14}$$

which correspond respectively to a $\mathbf{1_{2/3}}$ and a $\mathbf{6_{2/3}}$ multiplet of $O(4) \otimes U(1)_X$. In both cases t_R has $T_L^3 = T_R^3 = 0$, fulfilling the condition (6). Therefore, its coupling to the Z is protected by the P_C symmetry.[2] We can also perform an operator analysis for the Z coupling to t_R. For the case (a), no invariant operator can be written since $\operatorname{Tr}[V_\mu] = \operatorname{Tr}[\hat{V}_\mu] = 0$. For the case (b), we have that t_R corresponds to the $T_L^3 = T_R^3 = 0$ component of $(\mathbf{1}, \mathbf{3})_{\mathbf{2/3}} \equiv U_R$. There are two dimension-4 operators that can contribute to the Z coupling to t_R:

$$\mathcal{L} = c_4 \operatorname{Tr}\big[\bar{U}_R \gamma^\mu U_R \hat{V}_\mu\big] + c_5 \operatorname{Tr}\big[\bar{U}_R \gamma^\mu \hat{V}_\mu U_R\big]. \tag{15}$$

Using $U_R = \sigma_3 t_R + \cdots$ we find that, as expected, the contribution to $Zt_R\bar{t}_R$ vanishes.

In theories in which the Higgs arises as a pseudo-Goldstone boson (PGB) from the symmetry breaking $SO(5) \to O(4)$, one has to embed the fermion multiplets into $SO(5)$ representations. We find two very simple options. For the case (a) we can use a $\mathbf{5_{2/3}}$ of $SO(5) \otimes U(1)_X$, that decomposes as

$$\mathbf{5_{2/3}} = (\mathbf{2}, \mathbf{2})_{\mathbf{2/3}} \oplus (\mathbf{1}, \mathbf{1})_{\mathbf{2/3}} \tag{16}$$

under $SU(2)_L \otimes SU(2)_R \otimes U(1)_X$, and contains the multiplets of Eqs. (8) and (14). For the case (b) we can embed the top in a $\mathbf{10_{2/3}}$:

$$\mathbf{10_{2/3}} = (\mathbf{2}, \mathbf{2})_{\mathbf{2/3}} \oplus (\mathbf{1}, \mathbf{3})_{\mathbf{2/3}} \oplus (\mathbf{3}, \mathbf{1})_{\mathbf{2/3}}. \tag{17}$$

In the composite Higgs model of Ref. [7] the SM fermions were embedded in spinorial representations of $SO(5)$ (**4**'s of $SO(5)$), and the shift in the $Zb_L\bar{b}_L$ coupling implied severe bounds on the masses of the new particles [8]. By simply embedding the SM fields in either of the representations (16), (17), one can avoid large corrections to $Zb_L\bar{b}_L$ and build successful composite Higgs models with a much lighter spectrum of new particles [9].

3.2. Explicit calculations in 5D models of EWSB

In this section we focus on 5D composite Higgs models realized in AdS_5 space–time [6,7], and compute the correction to $Z\psi\bar{\psi}$ induced by the first Kaluza–Klein (KK) mode. In these

[1] We use the basis $\sigma^\alpha = (\mathbb{1}, i\sigma_1, i\sigma_2, i\sigma_3)$ where σ_a, $a = 1, 2, 3$, are the Pauli matrices.

[2] For the case (a) it is interesting to notice that t_R is a singlet of the custodial symmetry and therefore loop effects involving this field will not generate corrections to $\Delta\rho$.

theories the EWSB scale is given by $v = \epsilon f_\pi$, where f_π is the analog of the pion decay constant and ϵ is a model-dependent parameter bounded to be $0 < \epsilon \leqslant 1$. The experimental constraint from the Peskin–Takeuchi S parameter generically requires $\epsilon \lesssim 0.5$. Our result for $Z\psi\bar{\psi}$ will also apply to the class of higgsless models in AdS_5 [5] after setting $\epsilon = 1$.

Let us denote with c the fermion 5D bulk mass in units of the AdS curvature. We will assume $-1/2 < c < 1/2$, since for $|c| > 1/2$ the fermion zero modes are quite decoupled from the 5D bulk and non-universal corrections to $Z\psi\bar{\psi}$ from the exchange of KK modes are exponentially suppressed (this is the case for the first and second generation fermions). There are two types of diagrams contributing to $Z\psi\bar{\psi}$, one involving the exchange of gauge KKs, the other involving fermionic KKs. The contribution from the tower of $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ gauge KKs is, at the tree level and at zero momentum:

$$\delta g \simeq \left(T_R^3 - T_L^3\right)\frac{1-2c}{2\sqrt{2}(3-2c)}\epsilon^2, \tag{18}$$

where $\delta g (g/\cos\theta_W)\bar{\psi}\gamma^\mu\psi Z_\mu$ gives the non-universal correction to the SM vertex.[3] Effects from the fermion KKs are of the form

$$\delta g = \sum_{\mathrm{KK}} \sin^2\theta_{\mathrm{KK}}\left(T_L^{3\mathrm{KK}} - T_L^3\right), \tag{19}$$

where θ_{KK} is the mixing angle between the KK and ψ. This mixing occurs after EWSB and it is of order $\sin\theta_{\mathrm{KK}} \sim \epsilon\sqrt{1/2 - c}$.[4] Although the sum in Eq. (19) is over all the KK tower, a good approximation is obtained by considering only the lowest mode.

In the case in which q_L belongs to a $(\mathbf{2},\mathbf{2})_{\mathbf{2/3}}$ of $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_X$, only fermionic KKs in the representations $(\mathbf{1},\mathbf{1})_{\mathbf{2/3}}$, $(\mathbf{1},\mathbf{3})_{\mathbf{2/3}} \oplus (\mathbf{3},\mathbf{1})_{\mathbf{2/3}}$ and $(\mathbf{3},\mathbf{3})_{\mathbf{2/3}}$ can mix with b_L or t_L at order ϵ. The coefficients of the operators in Eqs. (9) and (10) then read

$$\begin{aligned} c_1 = c_2 &\simeq \frac{1-2c_q}{2\sqrt{2}(3-2c_q)}\epsilon^2 + \frac{1}{2}\sin^2\theta_{\mathrm{KK}}^{(1,1)} + \frac{1}{2}\sin^2\theta_{\mathrm{KK}}^{(3,1)} \\ &\quad - \frac{3}{4}\sin^2\theta_{\mathrm{KK}}^{(3,3)}, \\ c_3 &= 0. \end{aligned} \tag{20}$$

Here $\theta_{\mathrm{KK}}^{(1,1)}$ is the mixing angle between t_L and the KK in the $(\mathbf{1},\mathbf{1})_{\mathbf{2/3}}$ representation, and $\theta_{\mathrm{KK}}^{(3,1)}$ ($\theta_{\mathrm{KK}}^{(3,3)}$) is the mixing angle between b_L and the KK in the $(\mathbf{3},\mathbf{1})_{\mathbf{2/3}}$ ($(\mathbf{3},\mathbf{3})_{\mathbf{2/3}}$) representation. In the case of a composite Higgs model where q_L is embedded in a $\mathbf{5}_{\mathbf{2/3}}$ of SO(5), the result is that of Eq. (20) with only the gauge and $(\mathbf{1},\mathbf{1})_{\mathbf{2/3}}$ fermionic contributions turned on. Eq. (12) together with Eq. (20) give us the tree-level correction to the couplings of the Z and the W to the SM fermions b_L, t_L. Corrections of order $\sim \epsilon^2 \sim$ 10–20% are thus possible if q_L is strongly coupled to the 5D bulk dynamics (i.e., for $-1/2 < c \lesssim 0$), and they could be observed in future experiments that probe the couplings of the top quark.

4. The coupling $Zb_R\bar{b}_R$

The small ratio m_b/m_t can be naturally explained in the class of models under consideration by assuming that the SM b_R couples weakly to the BSM sector. The shift in the coupling of b_R to the Z due to the BSM sector, δg_{Rb}, will then be small. This is the case, for example, when $q_L \in (\mathbf{2},\mathbf{2})_{\mathbf{2/3}}$ and both b_R and t_R couple to the same BSM operator transforming as a $(\mathbf{1},\mathbf{3})_{\mathbf{2/3}} \oplus (\mathbf{3},\mathbf{1})_{\mathbf{2/3}}$, case (b) of Eq. (13).

It is however interesting to consider the possibility that b_R couples more strongly to the BSM sector, since a positive shift $\delta g_{Rb} \sim +0.02$ would explain the 3σ anomaly in the forward–backward asymmetry A_{FB}^b measured by the LEP and SLD experiments (see [10]).[5]

If, for example, b_R and t_R couple to two different BSM operators, possibly with the same $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_X$ quantum numbers, then $m_b \ll m_t$ could follow from hierarchies in the couplings of the BSM sector. In the case of the 5D models of Section 3.2 one can use Eqs. (18) and (19) to calculate δg_{Rb}. For $b_R \in (\mathbf{1},\mathbf{3})_{\mathbf{2/3}}$, only KK fermions in a $(\mathbf{2},\mathbf{2})_{\mathbf{2/3}}$ and $(\mathbf{2},\mathbf{4})_{\mathbf{2/3}}$ can mix with b_R at order ϵ. This gives, for $|c_b| < 1/2$,

$$\delta g_{Rb} \simeq -\frac{1-2c_b}{2\sqrt{2}(3-2c_b)}\epsilon^2 - \frac{1}{2}\sin^2\theta_{\mathrm{KK}}^{(2,2)} + \sin^2\theta_{\mathrm{KK}}^{(2,4)}. \tag{21}$$

Here and in the following, $\theta_{\mathrm{KK}}^{(r,s)}$ denotes the mixing angle between b_R and the KK state with electric charge $-1/3$ in a $(\mathbf{r},\mathbf{s})$ representation of $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ (if the representation $(\mathbf{r},\mathbf{s})$ contains more than one state with electric charge $-1/3$, then $\theta_{\mathrm{KK}}^{(r,s)}$ will refer to the KK with $T_L^3 = -1/2$). Thus, one can obtain a positive δg_{Rb} from the mixing of b_R with the KKs in the $(\mathbf{2},\mathbf{4})_{\mathbf{2/3}}$, as needed to explain the A_{FB}^b anomaly.

A different possibility is that the SM q_L itself couples to two different BSM operators: the first responsible for generating the top mass, the second for generating the bottom mass.[6] The coupling to this latter operator will in general violate the custodial symmetry subgroup protecting g_{Lb}, but it is natural to assume that its coefficient is small, in order to reproduce the small ratio m_b/m_t. The resulting δg_{Lb} will also be small, allowing at the same time a large coupling of b_R to the BSM sector. There are many choices for embedding b_R in $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_X$, giving δg_{Rb} of either sign. The simplest choice is

$$b_R \in (\mathbf{1},\mathbf{1})_{-\mathbf{1/3}}, \tag{22}$$

which can be embedded in a $\mathbf{5}$ of SO(5). In this case the BSM operator coupled to q_L responsible for the bottom mass has to

[3] Eq. (18) is valid for $-1/2 \leqslant c < 1/2$. In the limit $c \to 1/2$ the same formula applies with $(1-2c) \to 1/(\pi k R)$, where πR is the proper length of the extra dimension and k is the curvature of AdS_5.

[4] This holds if all the KKs have similar masses of order Λ_{BSM}. If the KK state mixing with ψ has a smaller mass $m \ll \Lambda_{\mathrm{BSM}}$, then $\sin\theta_{\mathrm{KK}}$ is larger by a factor $(\Lambda_{\mathrm{BSM}}/m)$.

[5] A larger and negative shift, $\delta g_{Rb} \sim -0.17$ would also explain the data [11], but to obtain such a large shift would require a very light spectrum of new particles. We do not consider here this possibility.

[6] An explicit realization of this scenario in the context of a 5D composite Higgs model will be given in [9].

Table 1
Several possible embeddings of b_R in $SU(2)_L \otimes SU(2)_R \otimes U(1)_X$ and corresponding contributions to δg_{Rb} from the first KK modes in 5D models of EWSB: gauge contribution (size and sign as given by the b_R axial charge $Q_A = T^3_L - T^3_R$, where we have defined $\Delta_g = \frac{1-2c_b}{2\sqrt{2}(3-2c_b)}\epsilon^2$), and fermionic contribution

b_R	$\delta g_{Rb}\vert_{\text{gauge}}/\Delta_g = -Q_A$	$\delta g_{Rb}\vert_{\text{fermionic}}$
$(\mathbf{1},\mathbf{3})_{\mathbf{2/3}}$	-1	$-\frac{1}{2}\sin^2\theta^{(2,2)}_{\text{KK}} + \sin^2\theta^{(2,4)}_{\text{KK}}$
$(\mathbf{1},\mathbf{1})_{\mathbf{-1/3}}$	0	0
$(\mathbf{1},\mathbf{3})_{\mathbf{-1/3}}$	0	0
$(\mathbf{1},\mathbf{2})_{\mathbf{1/6}}$	$-1/2$	$-\frac{1}{2}\sin^2\theta^{(2,1)}_{\text{KK}} + \frac{1}{2}\sin^2\theta^{(2,3)}_{\text{KK}}$
$(\mathbf{1},\mathbf{2})_{\mathbf{-5/6}}$	$+1/2$	$\frac{1}{2}\sin^2\theta^{(2,1)}_{\text{KK}} - \frac{1}{4}\sin^2\theta^{(2,3)}_{\text{KK}}$
$(\mathbf{1},\mathbf{3})_{\mathbf{-4/3}}$	$+1$	$\frac{1}{2}\sin^2\theta^{(2,2)}_{\text{KK}} - \frac{1}{3}\sin^2\theta^{(2,4)}_{\text{KK}}$

transform as a $(\mathbf{2},\mathbf{2})_{\mathbf{-1/3}}$. Since however $T^3_{L,R} = 0$ for b_R, the P_C symmetry argument of Section 2 implies $\delta g_{Rb} = 0$ for both gauge and fermionic contributions. Another possible choice is

$$b_R \in (\mathbf{1},\mathbf{2})_{\mathbf{1/6}}, \tag{23}$$

which can be embedded into a **4** of SO(5). In this case the BSM operator coupled to q_L can transform as either a $(\mathbf{2},\mathbf{1})_{\mathbf{1/6}}$ or a $(\mathbf{2},\mathbf{3})_{\mathbf{1/6}}$. At order ϵ, b_R can mix with KKs in $(\mathbf{2},\mathbf{1})_{\mathbf{1/6}}$ and $(\mathbf{2},\mathbf{3})_{\mathbf{1/6}}$. We find

$$\delta g_{Rb} \simeq -\frac{1-2c_b}{4\sqrt{2}(3-2c_b)}\epsilon^2 - \frac{1}{2}\sin^2\theta^{(2,1)}_{\text{KK}} + \frac{1}{2}\sin^2\theta^{(2,3)}_{\text{KK}}. \tag{24}$$

Thus, one has $\delta g_{Rb} > 0$ from mixing with KKs in the $(\mathbf{2},\mathbf{3})_{\mathbf{1/6}}$, as needed to explain the A^b_{FB} anomaly. A few other examples with 1 Higgs insertion are indicated in Table 1.

5. Conclusions

In models where the electroweak symmetry breaking is induced by a new (strongly interacting) sector coupled to the SM fields, it is crucial for the new sector to respect a custodial symmetry in order to prevent large corrections to $\Delta\rho$. We have shown that the custodial symmetry O(3) can also protect the $Zb_L\bar{b}_L$ coupling from corrections. This suggests that the custodial invariance might be a key ingredient to build natural models of electroweak symmetry breaking with a relatively light spectrum of new fermions, as required by naturalness arguments. A way to test this scenario is to look for modifications in the couplings $Zt\bar{t}$, $Wt\bar{b}$, which cannot be protected at the same time by the custodial symmetry and can receive potentially large shifts. Finally, we investigated the possibility of a modification of the $Zb_R\bar{b}_R$ coupling, showing that a positive shift, as required to explain the anomaly in the LEP/SLD forward–backward asymmetry A^b_{FB}, is possible for certain choices of the b_R custodial quantum numbers.

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