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Conformal invariant equations for nucleus–nucleus scattering in perturbative QCD with $N_c \to \infty$

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Abstract

In the perturbative QCD with $N_c \to \infty$ the amplitude for the collision of two heavy nuclei is expressed via dipole densities in the nuclei. Coupled equations for these densities are derived in the configuration space. The equations are conformal invariant in absence of external sources. Passing to conformal basis and its possible truncation are discussed.

1. Introduction

In the perturbative QCD with $N_c \to \infty$, high-energy scattering of a pointlike projectile on a large nucleus is described by a sum of fan diagrams constructed from BFKL pomerons and vertexes for their splitting in two. Summation of these diagrams leads to the well-known BK equation [1–3], now well studied both theoretically and numerically. Generalization of these results to nucleus–nucleus scattering requires a symmetric treatment of projectile and target and obviously involves not only the vertex for the splitting of a pomeron in two but also fusing of two pomerons in one. Such a program was realized in our papers [4]. There we limited ourselves to the case when momenta transferred to both nuclei are negligible as compared to gluon momenta inside pomerons, which physically corresponds to the limit of very heavy nuclei and a finite pomeron slope. As a result all pomerons were propagating in the forward direction. This simplified the final equations considerably but their basic conformal invariance property remained hidden. Also written in the momentum space the equations are difficult to compare with the results following from the dipole picture and so-called JIMWLK approach, in which the problem of symmetric treatment of projectile and target (and also of inclusion of pomeron loops) is lately being studied very actively (see, e.g., [5] and references therein). For these reasons in this Letter we rederive equations describing nucleus–nucleus scattering for a general case, when the pomerons are allowed to change their momenta in their interaction with the nuclei and between themselves. Our final equations are in the transverse coordinate space, so that comparison with the dipole approach will be facilitated.

It is important to stress the approximations used in the derivation. We rely on the perturbative QCD in the limit of large number of colours, $N_c \to \infty$, and assume both nuclei to be large, with their atomic numbers $A, B \gg 1$. This allows to take into account only tree diagrams constructed of pomerons and their interaction vertexes (see Fig. 1 for the simplest examples). Both pomeron loops and contributions which cannot be described in terms of pomerons (e.g., gluonic interaction between two pomerons) are neglected, since they are damped by higher powers of $1/N_c$ and/or $1/A$, $1/B$. Obviously this approximation cannot work at superlarge energies when the pomeron can propagate to transverse distances larger than the nuclei dimensions, which occurs at rapidities Y such that $\alpha' Y > R_{A,B}^2$, where α' is the pomeron slope. All experimental data give $\alpha < 0.01$ fm^2, so that these rapidities seem to be well beyond our present and predictable possibilities. The approximation of large nuclei also allows to neglect correlations between

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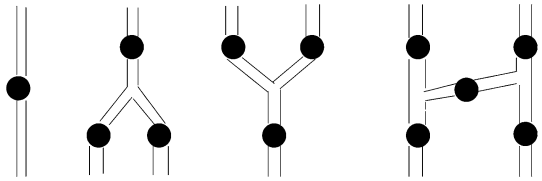

Fig. 1. Simplest tree diagrams for nucleus–nucleus scattering.

colour distributions of different nucleons and excludes diagrams in which a pomeron interacts simultaneously with two nucleons of the target or projectile.

In our formalism the nucleus–nucleus scattering amplitude is trivially expressed via a pair of non-local pomeron fields for the colliding nuclei. Our final result is a pair of equations for these pomeron fields, which possess full conformal invariance in absence of external sources. The equations include terms with the interaction between the two fields. If one neglects this interaction the equations decouple into a pair of BK equations for the projectile and target. The equations are in fact very complicated and not well suited for numerical studies, which are difficult already for the forward propagation case considered in [4]. Some simplification seems to be possible by passing to the conformal basis and restricting to lowest conformal states, which is also discussed in the Letter.

2. Pomeron diagrams and effective field theory

At fixed overall impact parameter b the AB amplitude $\mathcal{A}(Y,b)$ can be presented as an exponential of its connected part:

$$\mathcal{A}(Y,b) = 2is\big(1-e^{-T(Y,b)}\big). \tag{1}$$

The dimensionless T is an integral over two impact parameters b_A and b_B of the collision point relative to the centers of the nuclei A and B:

$$T(Y,b) = \int d^2b_A\, d^2b_B\, \delta^2(b-b_A+b_B)T(Y,b_A,b_B). \tag{2}$$

As mentioned in the introduction, in the perturbative QCD with $N_c\to\infty$ the amplitude $-T(Y,b_A,b_B)$ is given by a sum of all connected tree diagrams constructed of BFKL pomerons and the triple pomeron vertex. More concretely, in these diagrams a line ("propagator") describing propagation of a pair of gluons from rapidity y and points r_1 and r_2 to rapidity y' and points r_1' and r_2' corresponds to the BFKL Green function $G_{y-y'}(r_1,r_2|r_1',r_2')$ [6]:

$$G_{y-y'}(r_1,r_2|r_1',r_2') = \theta(y-y')\sum_\mu e^{\omega_\mu(y-y')}\lambda_\mu E_\mu(r_1,r_2)E_\mu^*(r_1',r_2'), \tag{3}$$

where $\mu=\{n,\nu,r_0\}\equiv\{h,r_0\}$, summation in (3) includes summation over n and integrations over ν and transverse vector r_0 with the weight $(\nu^2+n^2/4)/\pi^4$; functions E_μ form the conformal basis. In the complex notation $r=x+iy$, $r^*=x-iy$

$$E_\mu(r_1,r_2) = \left(\frac{r_{12}}{r_{10}r_{20}}\right)^h\left(\frac{r_{12}^*}{r_{10}^*r_{20}^*}\right)^{\bar h}, \tag{4}$$

where $r_{12}=r_1-r_2$, etc.; $h=(1-n)/2+i\nu$ and $\bar h=1-h^*$ are conformal weights. Function ω_μ is the BFKL eigenvalue

$$\omega_\mu=\omega_h=\bar\alpha\left[\psi(1)-\operatorname{Re}\psi\left(\frac{|n|+1}{2}+i\nu\right)\right], \tag{5}$$

where standardly $\bar\alpha=\alpha_s N_c/\pi$. Finally

$$\lambda_\mu=\lambda_h=\frac{1}{[(n+1)^2+4\nu^2][(n-1)^2+4\nu^2]}. \tag{6}$$

The interaction between pomerons is realized via the triple pomeron vertex. It is non-local and not symmetric in the incoming and outgoing pomerons. For arbitrary values of N_c the vertex for splitting of a pomeron in two was found in [7]. In the limit $N_c\to\infty$ its form in the coordinate space was established in [8]. The three BFKL Green functions are connected by it as follows (see Fig. 2(a))

$$\frac{2\alpha_s^2N_c}{\pi}\int\frac{d^2r_1\,d^2r_2\,d^2r_3}{r_{12}^2r_{32}^2r_{13}^2}\big(G_{y_1'-y}(r_1',r_3'|r_1,r_3)L_{13}^\dagger\big)G_{y-y_2'}(r_1,r_2|r_1'',r_2')G_{y-y_3'}(r_3,r_2|,r_3'',r_2''). \tag{7}$$

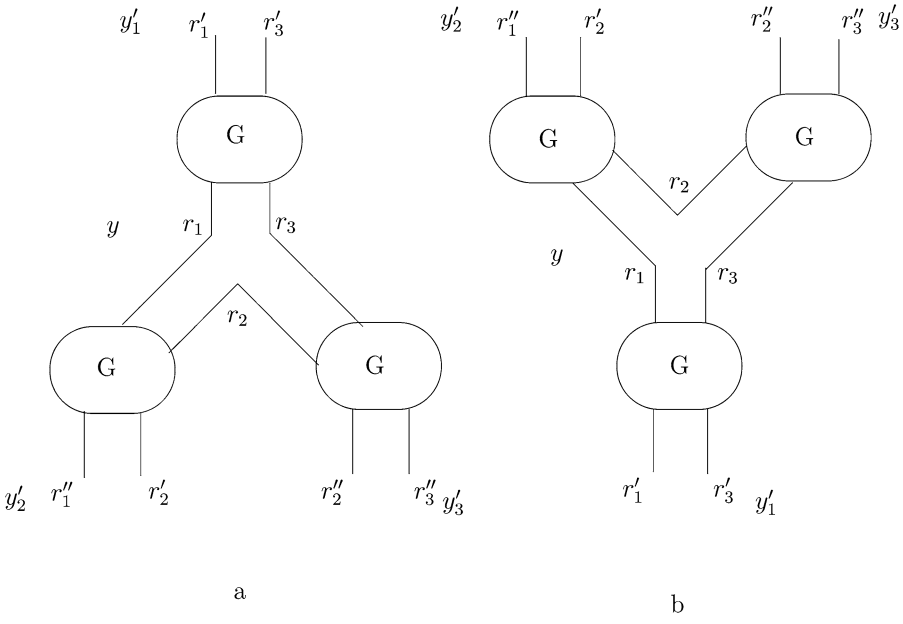

Fig. 2. Splitting of a pomeron into two (a) and fusion of two pomerons into one (b).

Here L_{13} is (up to a numerical factor) the quadratic Casimir operator for the conformal transformations of r_1 and r_3:

$$L_{13} = r_{13}^4 p_1^2 p_3^2, \tag{8}$$

where in the configuration space $p^2 = -\nabla^2$. In (7) $L_{13}^\dagger$ is acting on the left. Note that the triple pomeron vertex is symmetric in the gluons inside the outgoing pomerons (i.e., under $r_1 \leftrightarrow r_2$ and $r_3 \leftrightarrow r_2$). So the outgoing pomerons have to be symmetric in their respective gluons. The form of the vertex for the fusion of two pomerons into one is actually not known. However, the symmetry between target and projectile prompts us to assume that for the inverse process $2+3 \to 1$ the BFKL functions are to be joined as (Fig. 2(b))

$$\frac{2\alpha_s^2 N_c}{\pi} \int \frac{d^2r_1\, d^2r_2\, d^2r_3}{r_{12}^2 r_{32}^2 r_{13}^2} G_{y_2'-y}(r_1'', r_2'|r_1, r_2) G_{y_3'-y}(r_3'', r_2''|r_3, r_2) L_{13} G_{y-y_1'}(r_1, r_3|r_1', r_3'). \tag{9}$$

Finally we have to describe the interaction of the pomerons with the two nuclei. The BFKL Green functions corresponding to the external legs of the diagrams are to be integrated with the colour density of each nucleus. We take the target nucleus (B) at rest, that is, at rapidity zero. Then each outgoing external BFKL Green function is to be transformed into

$$\int d^2r_1'\, d^2r_2'\, G_y(r_1, r_2|r_1', r_2') \rho_B(r_1', r_2') \equiv \int dy'\, d^2r_1'\, d^2r_2'\, G_{y-y'}(r_1, r_2|r_1', r_2') \tau_B(y', r_1', r_2'), \tag{10}$$

where ρ_B is the colour density of the target. If we neglect correlations between the colours of the nucleons in the nucleus B

$$\tau_B(y, r_1, r_2) = g^2 B T_B(b_B) \delta^2\left(b_B - \frac{r_1 + r_2}{2}\right) \rho_N(r_{12}) \delta(y), \tag{11}$$

where T_B is the profile function of the nucleus B and ρ_N is the colour density of the nucleon. Similarly each ingoing BFKL external Green function is to be transformed into

$$\int dy'\, d^2r_1'\, d^2r_2'\, \tau_A(y', r_1', r_2) G_{y'-y}(r_1', r_2'|r_1, r_2), \tag{12}$$

where

$$\tau_A(y, r_1, r_2) = g^2 A T_A(b_A) \delta^2\left(b_A - \frac{r_1 + r_2}{2}\right) \rho_N(r_{12}) \delta(y - Y) \tag{13}$$

and Y is the overall rapidity. If the densities τ_A and τ_B are symmetric in the gluons (which is true for (11) and (13) and will be assumed in the following) then the initial pomerons are also symmetric in the gluons inside them. Together with the mentioned properties of the triple pomeron vertexes it means that only symmetric pomeron states are propagating in the two nuclei.

To find the amplitude, one has to sum over all connected diagrams with M ingoing and N outgoing lines, corresponding to M interactions with the projectile and N interactions with the target, divided by $M!N!$.

It is trivial to see that this sum exactly corresponds to the sum of tree diagrams generated by an effective quantum theory of two pomeronic fields $\Phi(y, r_1, r_2)$ and $\Phi^\dagger(y, r_1, r_2)$, symmetric in r_1, r_2, with the action

$$S = S_0 + S_I + S_E$$

consisting of three terms, which correspond to free pomerons, their mutual interaction and their interaction with external sources (nuclei), respectively.

To give the correct propagators S_0 has to be chosen as

$$S_0 = \int dy\, dy'\, d^2r_1\, d^2r_2\, d^2r_1'\, d^2r_2'\, \Phi^\dagger(y, r_1, r_2) G^{-1}_{y-y'}(r_1, r_2|r_1', r_2')\Phi(y', r_1', r_2') \equiv \langle \Phi^\dagger | G^{-1} | \Phi \rangle, \tag{14}$$

where $\langle|\rangle$ means the integration over y and both gluon coordinates. Note that the sign of S_0 corresponds to the following substitution of the conventionally defined field variables Φ and $\Phi^\dagger$:

$$\Phi \to i\Phi, \qquad \Phi^\dagger \to i\Phi^\dagger, \tag{15}$$

which allows to make all terms of the action real.

According to (7) and (9) the interaction term S_I is local in rapidity

$$S_I = \frac{2\alpha_s^2 N_c}{\pi} \int dy \int \frac{d^2r_1\, d^2r_2\, d^2r_3}{r_{12}^2 r_{32}^2 r_{13}^2} \big\{ \big(L_{13}\Phi(y, r_1, r_3)\big)\Phi^\dagger(y, r_1, r_2)\Phi^\dagger(y, r_3, r_2) + \text{h.c.} \big\}. \tag{16}$$

The overall sign combines the initial factor i and i^3 from the substitution (15).

Finally the interaction with the nuclei is local both in rapidity and coordinates:

$$S_E = -\int dy\, d^2r_1\, d^2r_2 \big\{ \Phi(y, r_1, r_2)\tau_A(y, r_1, r_2) + \Phi^\dagger(y, r_1, r_2)\tau_B(y, r_1, r_2) \big\}. \tag{17}$$

The minus sign comes from the initial i and the substitution (15).

The amplitude $T(Y, b_A, b_B)$ is then expressed through a functional integral

$$Z = \int D\Phi\, D\Phi^\dagger\, e^S. \tag{18}$$

In the tree approximation, corresponding to diagrams of the type shown in Fig. 1, keeping only connected diagrams we find

$$T(Y, b_A, b_b) = -\ln \frac{Z}{Z_0} = -S\big\{\Phi, \Phi^\dagger\big\}, \tag{19}$$

where $Z_0(=1)$ is the value of Z for $S_E = 0$ and the action S is to be calculated for Φ and $\Phi^\dagger$ satisfying the classical equation of motion.

3. Equations for the classical pomeron fields

The classical equations of motion follow from the variation of the action with respect to Φ and $\Phi^\dagger$:

$$\frac{\delta S}{\delta\Phi(y, r_1, r_2)} = \frac{\delta S}{\delta\Phi^\dagger(y, r_1, r_2)} = 0. \tag{20}$$

We find a pair of equations

$$G^{-1}\Phi(y, r_1, r_2) + \frac{2\alpha_s^2 N_c}{\pi} \int \frac{d^2r_3}{r_{12}^2 r_{32}^2 r_{13}^2} \big\{ \Phi(y, r_1, r_3)\Phi(y, r_2, r_3)L_{12} + 2\,\text{Sym}_{12}\big(\Phi^\dagger(y, r_3, r_2)L_{13}\Phi(y, r_1, r_3)\big)\big\}$$
$$= \tau_B(y, r_1, r_2) \tag{21}$$

and

$$\Phi^\dagger(y, r_1, r_2)G^{-1} + \frac{2\alpha_s^2 N_c}{\pi} \int \frac{d^2r_3}{r_{12}^2 r_{32}^2 r_{13}^2} \big\{ \Phi^\dagger(y, r_1, r_3)\Phi^\dagger(y, r_2, r_3)L_{12} + 2\,\text{Sym}_{12}\big(\Phi(y, r_3, r_2)L_{13}\Phi^\dagger(y, r_1, r_3)\big)\big\}$$
$$= \tau_A(y, r_1, r_2), \tag{22}$$

where the operators L_{12} are assumed to act on the left and Sym_{12} means symmetrization in indexes 1 and 2. These equations have also to be supplemented by conditions

$$\Phi(y, r_1, r_2) = 0 \quad \text{if } y < 0, \qquad \Phi^\dagger(y, r_1, r_2) = 0 \quad \text{if } y > Y. \tag{23}$$

To write the equations in their final form we note that the BFKL Green function G satisfies the equations

$$p_1^2 p_2^2 \left(\frac{\partial}{\partial y} + H\right) G = \left(\frac{\partial}{\partial y} + H^\dagger\right) p_1^2 p_2^2 G = 1, \tag{24}$$

where H is the BFKL Hamiltonian:

$$H = \frac{\bar\alpha}{2}\left(\ln p_1^2 + \ln p_2^2 + \frac{1}{p_1^2}\ln r_{12}^2 \cdot p_1^2 + \frac{1}{p_2^2}\ln r_{12}^2 \cdot p_2^2 - 4\psi(1)\right). \tag{25}$$

From this we conclude that

$$G^{-1} = p_1^2 p_2^2 \left(\frac{\partial}{\partial y} + H\right) = \left(\frac{\partial}{\partial y} + H^\dagger\right) p_1^2 p_2^2. \tag{26}$$

Multiplying Eqs. (21) and (22) by $p_1^{-2} p_2^{-2}$ from the left and from the right respectively we finally find

$$\left(\frac{\partial}{\partial y} + H\right)\Phi(y, r_1, r_2) + \frac{2\alpha_s^2 N_c}{\pi}\int \frac{d^2 r_3\, r_{12}^2}{r_{32}^2 r_{13}^2}\Phi(y, r_1, r_3)\Phi(y, r_2, r_3)$$
$$+ \frac{4\alpha_s^2 N_c}{\pi} L_{12}^{-1}\int \frac{d^2 r_3\, r_{12}^2}{r_{32}^2 r_{13}^2}\,\mathrm{Sym}_{12}\big(\Phi^\dagger(y, r_3, r_2) L_{13}\Phi(y, r_1, r_3)\big) = \tau_B(y, r_1, r_2) \tag{27}$$

and

$$\left(-\frac{\partial}{\partial y} + H\right)\Phi^\dagger(y, r_1, r_2) + \frac{2\alpha_s^2 N_c}{\pi}\int \frac{d^2 r_3\, r_{12}^2}{r_{32}^2 r_{13}^2}\Phi^\dagger(y, r_1, r_3)\Phi^\dagger(y, r_2, r_3)$$
$$+ \frac{4\alpha_s^2 N_c}{\pi} L_{12}^{-1}\int \frac{d^2 r_3\, r_{12}^2}{r_{32}^2 r_{13}^2}\,\mathrm{Sym}_{12}\big(\Phi(y, r_3, r_2) L_{13}\Phi^\dagger(y, r_1, r_3)\big) = \tau_A(y, r_1, r_2). \tag{28}$$

The δ-like dependence of the external sources on y together with conditions (23) imply that one can drop the sources in the equations and substitute them by the boundary conditions at $y = 0$ and $y = Y$:

$$\Phi(y, r_1, r_2)_{y=0} = \rho_B(r_1, r_2), \qquad \Phi^\dagger(y, r_1, r_2)_{y=Y} = \rho_A(r_1, r_2), \tag{29}$$

where $\rho_{A,B}$ are given by (11) and (13) without the δ-functions in rapidity.

From the form of equations one immediately concludes that they are conformal invariant provided the external sources possess this invariance.

These equations can be also written in the form which allows an easy comparison with the BK equation for non-forward fan diagrams. To this aim one rescales the fields putting

$$\Phi(r_1, r_2) = \frac{N(r_1, r_2)}{4\pi\alpha_s}, \qquad \Phi^\dagger(r_1, r_2) = \frac{N^\dagger(r_1, r_2)}{4\pi\alpha_s} \tag{30}$$

and uses a representation for the Hamiltonian H [9]

$$Hf(r_1, r_2) = \frac{\bar\alpha}{2\pi}\int \frac{d^2 r_3\, r_{12}^2}{r_{23}^2 r_{13}^2}\big(f(r_1, r_2) - f(r_1, r_3) - f(r_2, r_3)\big). \tag{31}$$

Then our equations take the form

$$\frac{\partial N(r_1, r_2)}{\partial y} = -\frac{\bar\alpha}{2\pi}\int \frac{d^2 r_3\, r_{12}^2}{r_{23}^2 r_{13}^2}\big\{N(r_1, r_2) - N(r_1, r_3) - N(r_2, r_3) + N(r_1, r_3)N(r_2, r_3)$$
$$+ L_{12}^{-1}\big(N^\dagger(r_3, r_2) L_{13} N(r_1, r_3) + N^\dagger(r_3, r_1) L_{23} N(r_2, r_3)\big)\big\} + 4\pi\alpha_s\tau_B(y, r_1, r_2) \tag{32}$$

and

$$\frac{\partial N^\dagger(r_1, r_2)}{\partial y} = \frac{\bar\alpha}{2\pi}\int \frac{d^2 r_3\, r_{12}^2}{r_{23}^2 r_{13}^2}\big\{N^\dagger(r_1, r_2) - N^\dagger(r_1, r_3) - N^\dagger(r_2, r_3) + N^\dagger(r_1, r_3)N^\dagger(r_2, r_3)$$
$$+ L_{12}^{-1}\big(N(r_3, r_2) L_{13} N^\dagger(r_1, r_3) + N(r_3, r_1) L_{23} N^\dagger(r_2, r_3)\big)\big\} - 4\pi\alpha_s\tau_A(y, r_1, r_2) \tag{33}$$

with the boundary conditions which follow from (29) after rescaling (30).

If one neglects the last terms of the integrand on the r.h.s. in both equations and thus decouples N and $N^\dagger$ the equations turn into a pair of independent BK equations for dipole scattering amplitudes off the nuclei A and B evolving in opposite directions

in rapidity and corresponding to the sum of two sets of fan diagrams starting from the projectile or target. This may be used to give a physical interpretation to pomeronic fields as scattering amplitudes for dipoles from one nucleus off the other one. The last terms in the integrand introduce interaction between these two sets of fans and correspond to diagrams which contain both splitting and fusion of pomerons. The structure of this interaction is rather complicated in both configuration and momentum spaces due to non-locality of the inverse operator L^{-1}. One expects it to be simplified in the conformal basis, which will be the subject of the next section.

Meanwhile, using the equations of motion one can simplify the expression for the action S calculated on their solution. Indeed multiplying Eqs. (21) and (22) by $\Phi^\dagger(y,r_1,r_2)$ and $\Phi(y,r_1,r_2)$, integrating over y,r_1,r_2 and summing the results one obtains a relation

$$2S_0+3S_I+S_E=0. \tag{34}$$

This can be used to exclude one of the parts of the action when calculating the amplitude T. Recalling that the fields are discontinuous at the boundaries we obtain from (34)

$$T(Y,b_A,b_B)=\frac{1}{3}(S_E-S_0)=\frac{1}{2}(S_I-S_E). \tag{35}$$

4. Equations in the conformal basis

One may hope that the equations for the pomeron fields may be somewhat simplified in the conformal basis formed by functions $E_\mu(r_1,r_2)$. To this end we present

$$\Phi(y,r_1,r_2)=\sum_\mu E_\mu(r_1,r_2)\Phi_\mu(y). \tag{36}$$

The orthonormalization properties of $E_\mu(r_1,r_2)$ [6] allow to invert this relation and find

$$\Phi_\mu(y)=\int\frac{d^2r_1\,d^2r_2}{r_{12}^4}E^*_\mu(r_1,r_2)\Phi(y,r_1,r_2). \tag{37}$$

Since $\mu=\{n,\nu,r_0\}$, transition to the conformal basis by itself does not change the number of variables (three). However it drastically simplifies the operators L in the mixing term of our equations.

Indeed the mixing term of Eq. (27) can be written as

$$T^{\rm mix}(r_1,r_2)=\frac{4\alpha_s^2N_c}{\pi}L_{12}^{-1}\int\frac{d^2r_3\,r_{12}^2}{r_{32}^2r_{13}^2}\,{\rm Sym}_{12}\sum_{\mu_1,\mu_2}\Phi^\dagger_{\mu_1}(y)\Phi_{\mu_2}(y)\lambda^{-1}_{\mu_2}E^*_{\mu_1}(r_3,r_2)E_{\mu_2}(r_1,r_3), \tag{38}$$

where we have used that

$$L_{13}E_\mu(r_1,r_3)=\lambda_\mu^{-1}E_\mu(r_1,r_3). \tag{39}$$

Expanding the integral over r_3 considered as a function of r_1 and r_2 in the conformal basis we get

$$T^{\rm mix}(r_1,r_2)=\sum_\mu T^{\rm mix}_\mu E_\mu(r_1,r_2), \tag{40}$$

where according to (39)

$$T^{\rm mix}_\mu=\frac{4\alpha_s^2N_c}{\pi}\int\frac{d^2r_1\,d^2r_2}{r_{12}^4}E^*_\mu(r_1,r_2)L_{12}^{-1}\int\frac{d^2r_3\,r_{12}^2}{r_{32}^2r_{13}^2}\,{\rm Sym}_{12}\sum_{\mu_1,\mu_2}\Phi^\dagger_{\mu_1}(y)\Phi_{\mu_2}(y)\lambda^{-1}_{\mu_2}E^*_{\mu_1}(r_3,r_2)E_{\mu_2}(r_1,r_3). \tag{41}$$

We integrate by parts transforming action of L_{12}^{-1} on $E^*_\mu(r_1,r_2)/r_{12}^4$ and use

$$r_{12}^4L_{12}^{-1\dagger}r_{12}^{-4}=L_{12}^{-1} \tag{42}$$

to apply L_{12}^{-1} directly on $E_\mu(r_1,r_2)$ which gives a factor λ_μ. So in the end we get

$$\begin{aligned}T^{\rm mix}_\mu&=\frac{4\alpha_s^2N_c}{\pi}\int\frac{d^2r_1\,d^2r^2\,d^2r_3}{r_{12}^2r_{32}^2r_{13}^2}\lambda_\mu E^*_\mu(r_1,r_2)\,{\rm Sym}_{12}\sum_{\mu_1,\mu_2}\Phi^\dagger_{\mu_1}(y)\Phi_{\mu_2}(y)\lambda^{-1}_{\mu_2}E^*_{\mu_1}(r_3,r_2)E_{\mu_2}(r_1,r_3)\\&=\frac{4\alpha_s^2N_c}{\pi}\lambda_\mu\,{\rm Sym}_{12}\sum_{\mu_1,\mu_2}V_{\tilde\mu,\tilde\mu_1,\mu_2}\lambda^{-1}_{\mu_2}\Phi^\dagger_{\mu_1}(y)\Phi_{\mu_2}(y),\end{aligned} \tag{43}$$

where V_{μ,μ_1,μ_2} is the triple pomeron vertex in the conformal basis

$$V_{\mu,\mu_1,\mu_2} = \int \frac{d^2r_1\,d^2r^2\,d^2r_3}{r_{12}^2 r_{32}^2 r_{13}^2} E_\mu(r_1,r_2) E_{\mu_1}(r_3,r_2) E_{\mu_2}(r_1,r_3) \tag{44}$$

and $\tilde{\mu}$ corresponds to the complex conjugate basis function: if $\mu = \{h, r_0\}$ then $\tilde{\mu} = \{1-h, r_0\}$ (and always $\bar{h} = 1 - h^*$).

A similar transformation of the first integral term in Eq. (27) is straightforward and leads to the result which is different from (43) by the absence of conjugate Φ's and factors λ. So we find the first equation in the conformal basis as (suppressing the common argument y and dropping the source term)

$$\frac{\partial \Phi_\mu}{\partial y} = \omega_\mu \Phi_\mu - \frac{2\alpha_s^2 N_c}{\pi} \mathrm{Sym}_{12} \sum_{\mu_1,\mu_2} \Phi_{\mu_2} \left(V_{\tilde{\mu},\mu_1,\mu_2} \Phi_{\mu_1} + 2\frac{\lambda_\mu}{\lambda_{\mu_2}} V_{\tilde{\mu},\tilde{\mu}_1,\mu_2} \Phi^\dagger_{\mu_1} \right). \tag{45}$$

The second equation can be obtained by reversing the direction of propagation in rapidity and passing to conjugate fields:

$$\frac{\partial \Phi^\dagger_\mu}{\partial y} = -\omega_\mu \Phi^\dagger_\mu + \frac{2\alpha_s^2 N_c}{\pi} \mathrm{Sym}_{12} \sum_{\mu_1,\mu_2} \Phi^\dagger_{\mu_2} \left(V_{\mu,\tilde{\mu}_1,\tilde{\mu}_2} \Phi^\dagger_{\mu_1} + 2\frac{\lambda_\mu}{\lambda_{\mu_2}} V_{\mu,\mu_1,\tilde{\mu}_2} \Phi_{\mu_1} \right). \tag{46}$$

The triple pomeron vertex V_{μ,μ_1,μ_2} was studied in [10]. It depends on three conformal weights h, h_1 and h_2 and three center-of-mass vectors $\{r_0, r_{01}, r_{02}\} \equiv \{\rho_0, \rho_1, \rho_2\}$. The dependence on the letters is determined by the conformal invariance, so that (in complex notation)

$$V_{\mu,\mu_1,\mu_2} = \Omega_{h,h_1,h_2} \prod_{i<j} \rho_{ij}^{-\Delta_{ij}} \rho_{ij}^{*\,-\bar{\Delta}_{ij}}, \tag{47}$$

where $i = 0, 1, 2$, $\Delta_{01} = h_0 + h_1 - h_2$, $\bar{\Delta}_{01} = \bar{h}_0 + \bar{h}_1 - \bar{h}_2$, etc. The part of the vertex depending on conformal weights Ω_{h,h_1,h_2} was found in [10] for arbitrary conformal weights in terms of the Meijer function G^{pq}_{44}. The complicated form of the vertex together with the use of complex variables make Eqs. (45), (46) for the pomeron fields in the general case not very suitable for practical calculations, in spite of the simplification for the action of operators L. However they may serve as a starting point for further simplifications realized by truncating the equations by certain low values of conformal weights. In the next section we consider a most drastic example of such a truncation.

5. Lowest conformal weights

As well known from the study of the linear BFKL equation in the high-energy limit, the leading contribution comes from the minimal conformal weight in the expansion (36), namely $h = \bar{h} = 1/2$, which corresponds to $n = 0$ and $\nu = 0$. So the simplest case which may be of interest for our problem is to put $n = 0$ in all places and $\nu = 0$ whenever this is allowed by the equations, that is in Φ_μ, $\Phi^\dagger_\mu$ and Ω_{h,h_1h_2}. Then one finds for the triple pomeron coupling [10]

$$\Omega_{1/2,1/2,1/2} \equiv \Omega_0 = 2\pi^7\, {}_4F_3(1/2)_6F_5(1/2) = 7766.679. \tag{48}$$

The unknown fields Φ_μ and $\Phi^\dagger_\mu$ become functions of rapidity y and center-of-mass vector ρ_0 (actually of ρ_0^2 due to rotational invariance). Two Eqs. (45) and (46) simplify to

$$\frac{\partial \Phi(\rho_0)}{\partial y} = \Delta\Phi(\rho_0) - \frac{\alpha_s^2 N_c}{8\pi^7} \Omega_0 \int \frac{d^2\rho_1\,d^2\rho_2}{\rho_{01}\rho_{02}} \delta''\left(\ln\frac{\rho_{01}}{\rho_{02}\rho_{12}}\right) \delta''\left(\ln\frac{\rho_{02}}{\rho_{01}\rho_{12}}\right) \Phi(\rho_1)\big(\Phi(\rho_2) + 2\Phi^\dagger(\rho_2)\big) \tag{49}$$

and

$$\frac{\partial \Phi^\dagger(\rho_0)}{\partial y} = -\Delta\Phi^\dagger(\rho_0) + \frac{\alpha_s^2 N_c}{8\pi^7} \Omega_0 \int \frac{d^2\rho_1\,d^2\rho_2}{\rho_{01}\rho_{02}} \delta''\left(\ln\frac{\rho_{01}}{\rho_{02}\rho_{12}}\right) \delta''\left(\ln\frac{\rho_{02}}{\rho_{01}\rho_{12}}\right) \Phi^\dagger(\rho_1)\big(\Phi^\dagger(\rho_2) + 2\Phi(\rho_2)\big), \tag{50}$$

where $\Delta = \omega_{n=0,\nu=0}$ is the BFKL intercept. We have taken into account that due to the presence of the δ-functions we have $\rho_{12} = 1$ (in the chosen scale, determined by the sources).

From the assumed independence of the fields of ν it follows that the boundary conditions for these equations have to belong to the class of functions of the form

$$f(r_1,r_2) = \int \frac{r_{12}\,d^2r_3}{r_{12}r_{13}} \delta''\left(\ln\frac{r_{12}}{r_{13}r_{23}}\right) g(r_3), \tag{51}$$

where $g(r)$ is an arbitrary function. Obviously this restricts the sources to be of a very special sort, with the dependence on two vectors r_1 and r_2 and thus on three variables r_1^2, r_2^2 and $\mathbf{r}_1\mathbf{r}_2$ determined by a function of a single variable r_3^2. So any practical use of the thus simplified system of equations is questionable. At most it may serve to study the qualitative features of the solution in

the limit of high energies, when one may hope that the influence of the choice of the boundary conditions becomes negligible (as it happens with the BK equation). Still even Eqs. (49) and (50) do not look easily solvable. We reserve their study for a separate publication.

6. Conclusions

We have derived a pair of equations for the pomeron fields in two heavy nuclei, which describe nucleus–nucleus scattering in the perturbative QCD with a large number of colours. The equations contain mixing terms which are both non-linear and non-local. In absence of mixing the equations decouple into a pair of BK equations for the projectile and target.

In contrast to the hA case the equations are to be solved with given boundary conditions at rapidities of the projectile and target, which complicates their solution enormously. The equations themselves are conformal invariant. This invariance is naturally broken by the sources. However use of the conformal basis may open ways for various simplifications of the equations, which may facilitate their solution, if only on the qualitative level.

Our equations have been obtained as a result of classical approximation to an effective non-local quantum field theory constructed to give rise to all Feynman diagrams for propagating and interacting pomerons. This approximation contains both splitting and merging vertexes for transition of one pomeron into two but it does not contain pomeron loops. To include loops one has to consider our effective field theory as a full-fledged quantum theory. Then one has to deal with an infinite system of equations for pomeron Green functions rather than with a pair of equations derived in this Letter. This raises the complexity of the problem to an incomparable level. The author intends to discuss some aspects of taking pomeron loops into account in a forthcoming publication.

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