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Aeroelastic two-level optimization for preliminary design of wing structures considering robust constraints

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Abstract An aeroelastic two-level optimization methodology for preliminary design of wing structures is presented, in which the parameters for structural layout and sizes are taken as design variables in the first-level optimization, and robust constraints in conjunction with conventional aeroelastic constraints are considered in the second-level optimization. A low-order panel method is used for aerodynamic analysis in the first-level optimization, and a high-order panel method is employed in the second-level optimization. It is concluded that the design of the abovementioned structural parameters of a wing can be improved using the present method with high efficiency. An improvement is seen in aeroelastic performance of the wing obtained with the present method when compared to the initial wing. Since these optimized structures are obtained after consideration of aerodynamic and structural uncertainties, they are well suited to encounter these uncertainties when they occur in reality.

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1. Introduction

Aircraft structure design is a complex process which requires a detailed consideration of disciplines such as aerodynamics, structures, and materials. The aircraft design process is in general divided into three phases,¹ i.e., conceptual design phase, preliminary design phase, and detailed design phase. Though

every design phase is very important, the preliminary design phase has a special place since it is the continuation of the conceptual design phase and the base of the detailed design phase.

The earlier appropriate structural layout and sizes can be found, the more economical the whole design process will be, avoiding costly redesign and corrections later. With an increase in flexibility of modern aircraft structures which results in a complex aero-structure coupling, aeroelastic effects must be taken into consideration right from the beginning of a design phase so as to avoid expensive redesign during subsequent design phases or the resulted weight penalties need to satisfy aeroelastic requirements which have been previously unaccounted for.¹ Therefore, aeroelastic optimization design is a necessary way to increase design efficiency in every phase of aircraft design.

Aeroelastic optimization technology has developed very rapidly in last few decades. A considerable amount of research

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has been conducted in aeroelastic optimization of aircraft, and has been used in practice as well.²⁻⁶ The main objective of aeroelastic optimization is to reduce the duration of the design cycle and improve the efficiency of the final product. In aerospace applications, wing design is a very crucial and important part which is considered as a key attribute of aircraft aeroelastic design. Therefore, it is very important to develop a high-efficiency aeroelastic optimization method for wing structure design.

Accurately deciding structural layout and sizes is an important part of wing preliminary design. It is necessary to know the following two issues with considerable accuracy: (1) what are the parameters of wing structural layout including spar position, and (2) what are the parameters of wing structural sizes involving skin thickness and spar section sizes. All these parameters have direct or indirect effects on aerodynamic characteristics, structural stiffness, and structural strength. It has been demonstrated that simultaneous optimization of wing structural layout and sizes results in significant improvement of aircraft performance.^{7,8} Therefore, in order to obtain realistic wing structural layout and sizes, it is necessary to use aeroelastic optimization including aero-structure coupling in conjunction with a complete set of real-world constraints.

Another important issue is the capability to include uncertainties in aerodynamic pressure and structural parameters, because in real operations, these uncertainties may lead to a substantial decline in aircraft performance, causing a catastrophic accident.⁹ The uncertainties in aeroelasticity have been explored in some works.⁹⁻¹⁴ However, much less research has been done in aeroelastic optimization considering aerodynamic and structural uncertainties, and all available procedures for preliminary design do not take into account the abovementioned uncertainties in aeroelastic optimization.

Aeroelastic optimization approaches coupling high-fidelity analysis methods of aerodynamic and structural analysis have been developed step by step.¹⁵⁻¹⁸ Although these high-fidelity approaches are adequate for the analysis of a configuration that can experience complex aero-structure coupling, they are not computationally economical. Therefore, aeroelastic optimization approaches based on low-fidelity methods, such as a linear aerodynamics model coupled with a structural finite element model, are still practiced widely.¹⁹

On the other hand, although there are some existing approaches suitable for aeroelastic optimization of wing preliminary design, there is no approach which can carry out aeroelastic optimization of wing structural layout and sizes, as well as perform aeroelastic robust optimization considering uncertainties of aerodynamic pressure and structural parameters simultaneously.

Therefore, the objective of this research is to develop a two-level aeroelastic optimization method suitable for the preliminary stage of aircraft design. Meanwhile, design efficiency of wing structures using the optimization method should be increased while considering variations in structural layout and sizes. The optimization method can provide a robust structure which is not sensitive to perturbation of structural and aerodynamic parameters. The research focuses on enhancing the preliminary design process and reducing redesign in the subsequent stages by developing an aeroelastic two-level optimization method.

2. Methodology

2.1. Overview of the optimization procedure

The work presented here lies in the field of multilevel and multidisciplinary optimization. It is mainly based on the following remarks: (1) The design variables of structural layout parameters including locations of wing spars and ribs considerably affect the structural and aeroelastic characteristics of aircraft wings. (2) The structural size parameters, such as thickness of a skin panel and sizes of a spar section, also have effects on the structural and aeroelastic characteristics of aircraft wings.

Based on the above factors, the optimization procedure in this paper is divided into two levels. The general layout of the optimization procedure is described in Fig. 1. A genetic algorithm is selected as the optimization algorithm.⁹ The first-level optimization aims to attain a satisfactory global behavior of the wing considering the variations in its structural layout and sizes. The design variables in the first-level optimization include structural layout and size parameters. In the second-level optimization, the parameters of structural layout and sizes are taken as design variables and the aeroelastic robust optimization design is conducted considering uncertainties in aerodynamic loads and structural locations and sizes.

A simplified 2D finite element model is used in the second-level optimization to reduce computational cost. In the 2D model, spars and stringers are simplified as bars, while skins and interspace between upper and lower skins are treated as a multi-layer structure like a composite. After optimization, the 2D model can be transformed into a detailed 3D model with similar characteristics.

2.2. First-level optimization procedure: aeroelastic optimization of structural layout and sizes

2.2.1. Optimization process

A suitable layout of the first-level procedure is shown in Fig. 2. A finite element model is employed. The doublet-lattice method available in MSC/NASTRAN is used for static aeroelastic and flutter constraints calculation during the optimization.^{19,20}

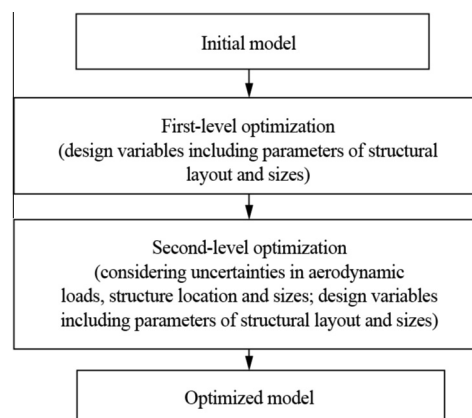


Fig. 1 Aeroelastic two-level design procedure of a wing structure.

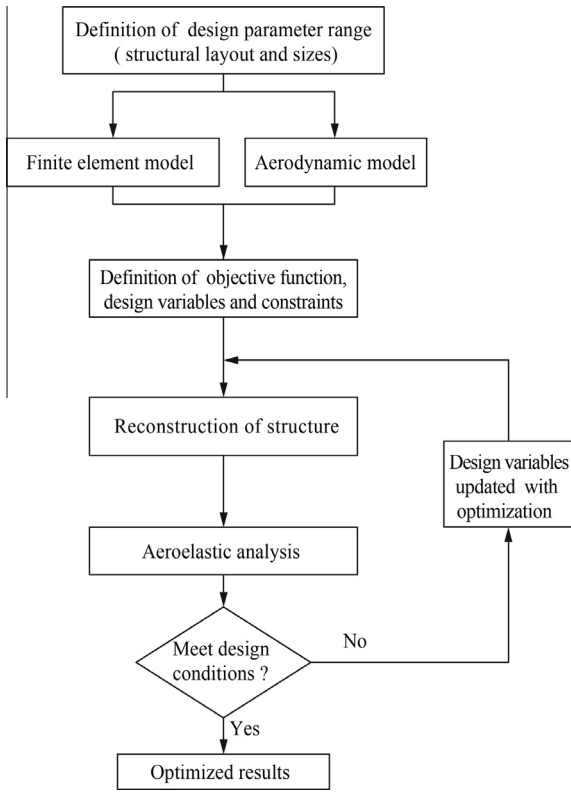


Fig. 2 Flowchart of the first-level optimization.

Model reconstruction will be based on the structural element locations of the model. To improve the efficiency of structural model reconstruction in aeroelastic optimization, a parametric modeling method is employed.

2.2.2. Optimization formulation

The objective function is represented by minimizing the wing structural mass which is a function of structural layout and size parameters in this work.

The constraints include aerodynamic, aeroelastic, and structural constraints. The aerodynamic constraints restrict elastic aerodynamic derivative. The aeroelastic constraints contain the displacement at wing tip, flutter speed, etc. The structural constraints comprise stress or strain of skins and spars.

The structural design variables are skin thickness, section sizes of spars, section areas of stringers, and spanwise and chordwise locations of spars and stringers.

Therefore, the optimization in this study can be formulated as follows:

$$\min F(x_s) \quad (1)$$

$$\text{s.t. } \frac{\partial Q}{\partial \delta} < 0 \quad (2)$$

$$C(w, \tau, V_F) < 0 \quad (3)$$

$$\varepsilon_k \leq \varepsilon_{\text{allow}} \quad (k = 1, 2, \dots, n_s) \quad (4)$$

$$x_{sj}^{\text{lower}} < x_{sj} < x_{sj}^{\text{upper}} \quad (j = 1, 2, \dots, n_d) \quad (5)$$

where Eq. (1) represents minimizing the objective function of $F(x_s)$ in which x_s is the structural design variables, Eq. (2) is the aerodynamic derivation constraint, Eq. (3) is the constraint on aeroelastic characteristics which include the linear displacement w and the angular displacement τ as well as the flutter speed V_F , Eq. (4) is the structural strain constraint in which n_s is the number of constraints, and Eq. (5) is the boundary constraint of structural design variables in which n_d is the number of design variables, both superscript “upper” and subscript “allow” represent the upper limit of constraints, and superscript “lower” is the lower limit of constraints.

2.3. Second-level optimization procedure: robust aeroelastic optimization considering aerodynamic and structural uncertainties

2.3.1. Optimization process

Based on the results of the first level optimization, the second-level aeroelastic optimization is conducted further considering uncertainties in aerodynamic loads, structural layout, and structural sizes. When encountering these aerodynamic and structural uncertainties, a conventional structure is likely to dissatisfy the design requirements, and can even be destroyed. Thus to prevent such a scenario, the aeroelastic optimization should consider the influences of uncertainties in aerodynamic and structural parameters.

Aeroelastic distribution is generally calculated based on the computational fluid dynamics (CFD) or panel method. Differences may exist between aerodynamic forces calculated with computational methods and actual aerodynamic forces. To avoid redesigning a structure, designers need to obtain critical loads, such as critical maneuver loads. Accurate aerodynamic loads are the basis for critical loads selection. Critical design loads are selected and aerodynamic uncertainties can be introduced by the perturbation method of aerodynamic pressure.

The aeroelastic optimization with uncertainties in aerodynamic and structural parameters is conducted in the case of critical design loads. Because of the consideration of uncertainties in aerodynamic and structural parameters, there is no need to perform significant structural redesign for the resulted optimal structure which can sustain itself when encountering these uncertainties in reality. On this basis, it is not necessary to keep a large safety margin in design, and the structure is still credible and safe.

A suitable layout of the second-level procedure is shown in Fig. 3. The aeroelastic optimization is conducted by further considering uncertainties in aerodynamic loads, structural layout and structural sizes.

2.3.2. Optimization formulation

The aeroelastic robust optimization can be formulated as follows:⁹

$$\min F(\mathbf{v}) \quad (6)$$

$$\text{s.t. } g_F(\mathbf{v}) = \frac{F_1(\mathbf{v})}{F(\mathbf{v})} \leq \varepsilon \quad (7)$$

$$g_j(\mathbf{v}) + \sum_{i=1}^{n_i} \left| \frac{\partial g_j(\mathbf{v})}{\partial v_i} \right| |\Delta v_i| \leq 0 \quad (j = 1, 2, \dots, n_c) \quad (8)$$

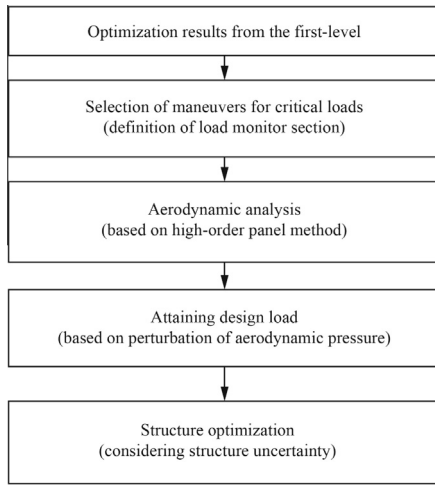


Fig. 3 Second-level procedure layout.

$$(v_i)_{\text{lower}} + \Delta v_i \leq v_i \leq (v_i)_{\text{upper}} - \Delta v_i \quad (i = 1, 2, \dots, n_d) \quad (9)$$

in which

$$F_1(\mathbf{v}) = \sqrt{\sum_{i=1}^{n_c} \left[\left(\frac{\partial F(\mathbf{v})}{\partial v_i} \right)^2 (\Delta v_i)^2 \right]} \quad (10)$$

where Eq. (6) represents minimizing the objective function in which \mathbf{v} is vector of design variables and Eq. (7) reflects an additional constraint in which $F_1(\mathbf{v})$ reflects the magnitude of the relative change in the objective caused by parameter variations. Eq. (7) means that the relative change in the objective is limited to an acceptable range, and ε is the corresponding upper bound defined by users. Therefore, the robust optimization can be formulated as a single-objective problem. Eq. (8) reflects the robust constraints, in which the second item on the left-hand side represents the magnitude of the changes in constraints, assuming the constraints are linearly related to the design variables, and n_c is the number of constraints. $\partial g_j(\mathbf{v})/\partial v_i$ is the sensitivity of the j th constraint with respect to the i th design variable, and Δv_i represents the variation of the i th design variable. Eq. (9) represents the changeable ranges of design variables, which are smaller than the ranges in traditional optimization, and n_d is the number of design variables. Eq. (10) is the particular formulation of the objective change, which is approximately obtained by Taylor series expansion at the design point, and $\partial F(\mathbf{v})/\partial v_i$ is the sensitivity of the objective with respect to the i th design variable.

2.3.3. Uncertainty input of aerodynamic loads

The objectives for design loads are defined on the basis of design requirements and concerned loads. It is an important process to consider uncertainties in aerodynamic loads. In this paper, it is based on the high-order panel method, the static aeroelastic analysis method, and the sequential quadratic programming method.

Considering the uncertainties in aerodynamic loads, the worst critical load can be formulated as follows:

$$\max a(X, P) \quad (11)$$

where $a(X, P)$ is the objective of critical loads including bending moments and torsional moments at different locations, and

represents the design loads, in which X is the air pressures at different positions and P is the perturbation coefficient of aerodynamic loads.

3. Optimization results of the first-level procedure

3.1. Aerodynamic model

The aerodynamic model of wing is shown in Fig. 4. Static aeroelastic responses of the 2D model are studied in longitudinal and lateral critical load states.

To further improve the wing performances, an integrated optimization is conducted considering the interactions between aerodynamics and the structure. The structural layout and size parameters are obtained by optimization.

3.2. Integrated optimization design of structural layout and sizes

3.2.1. Aerodynamic load cases for optimization

The optimization is carried out in the longitudinal and lateral critical load states. The longitudinal condition is an 8.0g pull-up at the sea level with a flight speed of Mach 0.7. The lateral state is a 5.3g roll with the aileron deflected downwards at the sea level and a speed of Mach 0.7. The abovementioned situations represent most severe load states for a wing.

3.2.2. Optimization description

The objective is to minimize wing structural weight.

The constraints include:

- (1) Slope of lift curve in the elastic state $C_{L\alpha} \geq 3.0$.
- (2) Ratio of displacement at the wing tip to the half-span length of the wing $u_{\text{tip}}/B < 8.5\%$.
- (3) Angular deformation at the wing tip $\delta < 1.5^\circ$.
- (4) Flutter speed $V_F \geq 500$ m/s.
- (5) Stresses in the wing-root skin $-160 \text{ MPa} < \sigma < 160 \text{ MPa}$.
- (6) Aileron effectiveness $\eta \geq 50\%$.

The design variables in this work include structural layout and size parameters. The thickness distribution of the upper and lower skins of the wing is divided into five regions.

3.2.3. Optimization results

The structural mass of the optimal wing obtained in this work is 89.1% of that of the initial wing. The optimized structural model of wing is shown in Fig. 5.

The structural dynamic characteristics of the initial model and the optimized model fixed at the wing root are studied. A comparison of the frequencies for the first five modes before and after optimization is shown in Table 1. It shows that the

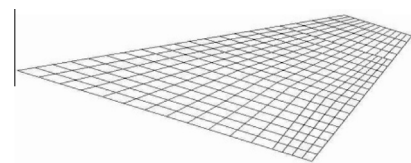


Fig. 4 Aerodynamic model of wing.

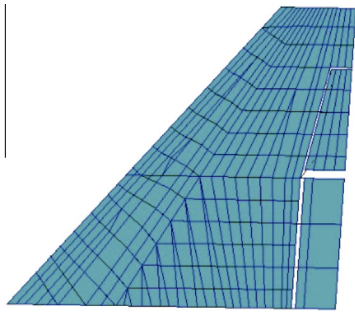


Fig. 5 Optimized structural model of wing.

frequency of the first bending mode of the wing reduces, and the frequency of the aileron deflection mode increases. The changes of the abovementioned two frequencies result in an increase of the flutter speed.

Flutter characteristics of the optimized model are investigated. The first five modes are used in flutter analysis. The flutter speed of the optimized model is 500.3 m/s, and flutter is induced by the coupling between the first bending mode and the first torsion mode. Compared with the initial model, the flutter speed increases and satisfies optimization constraints.

Static aeroelastic responses of the initial wing and the optimized wing are studied in the longitudinal and lateral critical load states, as shown in Table 2. It is clear that the aerodynamic and aeroelastic performances of the wing can satisfy the constraints after optimization. The aileron effectiveness of the wing is greater as compared to the analytical result from the initial model.

4. Optimization results of the second-level procedure

4.1. Structural and aerodynamic models

The distribution of aerodynamics is calculated with the high-order panel method in the second-level optimization. 3D aerodynamic model of the wing with aileron deflection is shown in Fig. 6.

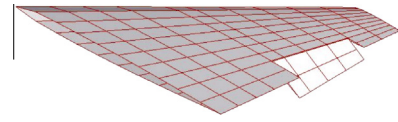


Fig. 6 3D aerodynamic model of the wing with aileron deflection.

The structural layout of the wing is obtained from the first-level optimization. The structure of the wing is made up of shell and beam elements, as shown in Fig. 5.

4.2. Uncertainty input in aerodynamic loads

4.2.1. Maneuver selection and air load perturbation

In this research, the design is conducted in a typical longitudinal maneuver. Because optimization in this level focuses on design considering influences of uncertainties, only a typical longitudinal maneuver is selected. The maneuver is an 8g pull-up at the sea level and Mach 0.7.

The perturbation of aerodynamic coefficients is used to predict the actual distribution of aerodynamic pressure considering uncertainties, as follows:

$$C_{pt,i} = \varepsilon_i C_{p,i} \quad (12)$$

where $C_{pt,i}$ and $C_{p,i}$ are the aerodynamic coefficients of the i th aerodynamic pressure center after and before perturbation, ε_i is the perturbation factor of the aerodynamic coefficient of the i th aerodynamic pressure center, and $\varepsilon_i \in [0.9, 1.1]$.

4.2.2. Definition of critical design loads

Three wing sections which refer to the sections at the wing root, the 45% spanwise location, and the 80% spanwise location, are assigned as load monitor sections. Meanwhile, three critical load functions are specified as objectives for the load monitor sections, which are named as Objectives I, II, and III, as shown in Table 3. The three objectives are used to define critical loads as design loads.

In Table 3, M_T represents the torsion moment, M_B represents the bending moment, and Objective III is used to represent the

Table 1 Structural dynamic characteristics of the initial wing and the optimized wing.

Mode	Mode description	Frequency/Hz	
		Initial wing	Optimized wing
1	The first bending	13.29	11.8
2	The first torsion	32.10	32.4
3	The second bending	50.83	47.3
4	Aileron deflection	44.59	56.3
5	The second torsion	64.24	66.2

Table 2 Static aeroelastic responses of the initial model and the optimized model in critical load states.

Table 2 Static aeroelastic responses of the initial model and the optimized model in critical load states.

Parameter	Longitudinal critical load state		Parameter	Lateral critical load state	
	Initial	Optimized		Initial	Optimized
Elastic slope of lift curve	2.9	3.1	Aileron effectiveness (%)	38.5	52.2
Angular deformation of wing tip (°)	0.76	0.14	Angular deformation of wing tip (°)	1.65	1.40
Relative displacement of wing tip (%)	4.6	6.4	Relative displacement of wing tip (%)	1.05	0.83

Table 3 Objectives of critical loads.

Objectives	Expression
I	$(M_T)_{\max}$
II	$(M_B)_{\max}$
III	$\left(\sqrt{M_T^2 + M_B^2}\right)_{\max}$

critical load in the coupling case of the torsion moment and the bending moment. The reference coordinate system used to perform the load analysis of wing sections is defined as the one which has its origin located at the chordwise midpoint of the section, with x -axis pointing to the outboard side along the spanwise direction, y -axis pointing to the forward side along the section chord, and z -axis pointing to the upward side.

On this basis, a comparison between the different load states regarding the monitor sections is performed. Then the design loads can be determined. For the design loads, 9 design load cases obtained from the selections of design load cases in Table 4 are used in the aeroelastic robust optimization.

4.3. Optimization description

The objective is to minimize the wing structural weight. The relative change in the objective is less than 5%.

Structural response constraints during the optimization process which include stress constraint, displacement constraint, angular deformation constraint, and flutter speed constraint are as follows:

- (1) Stress constraint. The element stress is required to satisfy the strength requirement.

- (2) Static aeroelastic constraints. The displacement and angular deformation at the wing tip are less than 9.6% half-span length and 1.8°, respectively.
- (3) Flutter speed constraint. The flutter speed is not less than 500 m/s at the sea level.

The design variables include skin thickness, web thickness, and spar locations. The uncertainties in structural design variables are introduced with non-probability forms. The perturbation factor is assumed as 5% based on engineering experiences.

4.4. Optimization results

The aeroelastic robust optimization of the wing is performed with a consideration of the integrated effects of the 9 design loads in a longitudinal maneuver.

The aeroelastic robust optimization of the wing is conducted for two cases. In Case A, uncertainties in aerodynamic loads, structural layout parameters, and structural size parameters are considered. In Case B, no aerodynamic and structural uncertainty is considered.

The structural weights of the optimized wing for Cases A and B are shown in Table 5 in which M_0 is the mass ratio of the optimized structures to the initial ones. The results show that the structural weight for case A is greater than that in Case B, and the structural weights in Case A and Case B are smaller than that of the initial model. The results also indicate that, when encountering these uncertainties in reality, additional structural weight is required to maintain reliability, safety, and performance of the wing. Table 5 also shows the aeroelastic responses of the optimized wings for Case A and Case B. It is clear from Table 5 that the optimized wings meet all constraints.

Table 4 Objectives of critical loads

Objective	10 ⁴ N·m.								
	Wing root section			45% Spanwise section			80% Spanwise section		
	$ M_T $	$ M_B $	$\sqrt{M_T^2 + M_B^2}$	$ M_T $	$ M_B $	$\sqrt{M_T^2 + M_B^2}$	$ M_T $	$ M_B $	$\sqrt{M_T^2 + M_B^2}$
I	15.618*	41.637	44.470	2.401*	10.060	10.343	0.291*	0.689	0.748
II	14.583	42.761*	45.179	1.977	10.486*	10.671	0.249	0.752*	0.792
III	15.010	42.694	45.256*	2.109	10.473	10.683*	0.257	0.751	0.794*

Note: * represents the design load.

Table 5 Aeroelastic responses of the optimized structures in Case A and Case B.

Design loads number	Displacement at wing tip (%)		Angular deformation at wing tip (absolute value) (°)		Flutter speed (m/s)		Weight/ M_0	
	Case A	Case B	Case A	Case B	Case A	Case B	Case A	Case B
	1–9	6.22–6.82	6.79	0.745–0.928	0.911	503.7	502.8	0.954

Table 6 Aeroelastic responses of the optimized wing structures in Cases C and D.

Design loads number	Displacement at wing tip (%)		Angular deformation at wing tip (absolute value) (°)		Flutter speed (m/s)		Weight/ M_0	
	Case C	Case D	Case C	Case D	Case C	Case D	Case C	Case D
	1–9	6.35	6.58–7.21	0.865	0.854–1.04	501.6	501.2	0.943

4.5. Influences of uncertainties in different disciplines on the optimization results

To further investigate the influences of uncertainties in different disciplines on the optimization results, Case C with uncertainties in structural layout and size parameters, as well as Case D with uncertainties in aerodynamic loads, are studied. The structural weights and aeroelastic responses of the optimized wings in Case C and Case D are shown in Table 6.

Table 6 shows that the optimized weight of the wing structure in Case C is slightly greater than that in Case D. It is also indicated that static aeroelastic responses of the optimized wing structure in Case C are smaller than those in Case D. Moreover, the difference in the flutter speed between Case C and Case D is small. Both the results in Case C and in Case D meet all constraints. Therefore, both structural and aerodynamic uncertainties have great effects on aeroelastic behaviors of the aircraft wing. It is indicated that considering uncertainty in just a certain discipline is incomprehensive. Uncertainties in different disciplines should be considered in the aeroelastic robust optimization simultaneously.

5. Conclusions

An aeroelastic two-level optimization procedure suitable for the preliminary wing design has been presented. The first-level procedure is an aeroelastic optimization of structural layout which considers variations of structural layout and size parameters. The second-level procedure is a robust aeroelastic optimization considering uncertainties in aerodynamic loads, structural layout parameters, and structural size parameters. The optimization method can provide optimal structural layout and structural sizes for a wing in the preliminary design stage. Furthermore, there is no need to impose significant structural redesign for the resulted optimal structure which can handle aerodynamic and structural uncertainties in reality, because of the consideration of uncertainties in optimization.

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