



The strangeness form factors of the proton

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Abstract

The present empirical information on the strangeness form factors indicates that the corresponding $uuds\bar{s}$ component in the proton is such that the $uuds$ subsystem has the flavor spin symmetry $[4]_{FS}[22]_F[22]_S$ and mixed orbital symmetry $[31]_X$. This $uuds\bar{s}$ configuration leads to the empirical signs of all the form factors G_E^s , G_M^s and G_A^s . An analysis with simple quark model wave functions for the preferred configuration shows that the qualitative features of the empirical strangeness form factors may be described with a $\sim 15\%$ admixture of $uuds\bar{s}$ with a compact wave function in the proton. Transition matrix elements between the uud and $uuds\bar{s}$ components give significant contributions.

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Recent empirical indications are that the sign of the strangeness magnetic form factor $G_M^s(q^2)$ of the proton is positive [1–4], while the strangeness electric form factor $G_E^s(q^2)$ [4,5] and the strangeness axial form factor [6] are negative. Here it is noted that there is a unique $uuds\bar{s}$ configuration with at most one quark orbitally excited, which is expected to have the lowest energy, and which leads to the same signs, and for which the constituent quark model provides a good qualitative description of the empirical momentum dependence.

In this configuration the \bar{s} antiquark is in the ground state, and the $uuds$ subsystem is in the P -state, such that the flavor-spin symmetry of the $uuds$ system is $[4]_{FS}[22]_F[22]_S$ [7,8]. In this configuration the strangeness magnetic moment is positive, and the strangeness contribution to the proton spin is small and negative. This configuration has the lowest energy of all $uuds\bar{s}$ configurations, under the assumption that the hyperfine interaction between the quarks is spin dependent [7]. Calculation of the momentum dependence of the corresponding form factors calls for a wave function model. For a qualitative analysis the harmonic oscillator constituent quark model should do.

In this model the matrix elements of the vector and axial vector current operators lead to the following form factor contributions for the $uuds\bar{s}$ configuration above:

$$G_E^s(q^2) = -\frac{q^2}{24\omega^2} \frac{e^{-q^2/4\omega^2}}{\sqrt{1+q^2/4m_s^2}} P_{s\bar{s}}, \quad (1)$$

$$G_M^s(q^2) = \frac{m_p}{2m_s} \left(1 - \frac{q^2}{18\omega^2}\right) \frac{e^{-q^2/4\omega^2}}{\sqrt{1+q^2/4m_s^2}} P_{s\bar{s}}, \quad (2)$$

$$G_A^s(q^2) = -\frac{1}{3} e^{-q^2/4\omega^2} P_{s\bar{s}}. \quad (3)$$

Here $P_{s\bar{s}}$ represents the probability of the $uuds\bar{s}$ component in the proton and m_p and m_s are the proton and strange quark masses, respectively. The oscillator parameter ω will be treated entirely phenomenologically. Note that the $q^2 = 0$ limits of these form factors are determined by symmetry alone.

In addition to these “diagonal” matrix elements between the $uuds\bar{s}$ states, there will also arise “non-diagonal” matrix elements between the uud and $uuds\bar{s}$ components of the proton. These will depend both on the explicit wave function model and the model for the $s\bar{s}-\gamma$ vertices. If these vertices are taken to have the elementary forms $\bar{v}(p')\gamma_\mu u(p)$ and $\bar{v}(p')\gamma_\mu\gamma_5 u(p)$ and no account is taken of the interaction between the annihilating $s\bar{s}$ pair and the proton, these transition matrix elements lead

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to the following form factor contributions (in the Breit frame):

$$G_E^s(q^2) = -\delta C_{35} \frac{\sqrt{3}}{6} \frac{q^2}{m_s \omega} \frac{e^{-q^2/4\omega^2}}{\sqrt{1+q^2/4m_s^2}} \sqrt{P_{s\bar{s}} P_{uud}}, \quad (4)$$

$$G_M^s(q^2) = \delta C_{35} \frac{2\sqrt{3}}{3} \frac{m_p}{\omega} \frac{e^{-q^2/4\omega^2}}{\sqrt{1+q^2/4m_s^2}} \sqrt{P_{s\bar{s}} P_{uud}}, \quad (5)$$

$$G_A^s(q^2) = \delta C_{35} \frac{\sqrt{3}}{6} \frac{q^2}{m_s \omega} e^{-q^2/4\omega^2} \sqrt{P_{s\bar{s}} P_{uud}}. \quad (6)$$

Here P_{uud} is the probability of the uud component of the proton. The factor C_{35} is the overlap integral of the wave function of the uud and the corresponding component of the $uuds\bar{s}$ configuration. In the oscillator model this factor is

$$C_{35} = \left(\frac{2\omega\omega_3}{\omega^2 + \omega_3^2} \right)^{9/2}. \quad (7)$$

Here ω_3 is the oscillator constant for the uud component of the proton. In the case of compact $uuds\bar{s}$ wave function, for which $\omega \sim 2\omega_3$, the value for C_{35} is $C_{35} \sim 0.4$. The model parameters are the oscillator parameter ω , the probability $P_{s\bar{s}}$ of the $uuds\bar{s}$ component (here $P_{uud} = 1 - P_{s\bar{s}}$) and the phase factor δ in the non-diagonal contribution. The constituent mass of the strange quark will be taken to be $400 \text{ MeV}/c^2$.

The non-diagonal contributions also depend on the relative phase $\delta = \pm 1$ of the uud and $uuds\bar{s}$ components of the wave functions. Below it is shown that a good description of the empirical form factors is obtained with $\delta = +1$.

Most information on the momentum dependence of the strangeness form factors is provided by the G0 experiment [4,9] and indirectly by a combination of extant neutrino scattering data with data on parity violating electron proton scattering [10]. The former gives the momentum dependence of the

combination $G_E^s(q^2) + \eta G_M^s(q^2)$, where η is a combination of kinematical variables and the ratio of nonstrange form factors [4]. The latter phenomenological combination gives values for all the three form factors $G_E^s(q^2)$, $G_M^s(q^2)$ and $G_A^s(q^2)$, albeit with substantial uncertainty margins.

The empirical values for the strangeness form factors given in Refs. [4,9,10] indicate that they all fall slowly with momentum transfer up to $q^2 = 1 \text{ GeV}^2$. This slow falloff indicates that the wave function of the strangeness component is compact relative to the proton radius. Consider first the strangeness electric form factor shown in Fig. 1. The slow falloff with q^2 may be described by taking ω as 1 GeV , which corresponds to a matter radius of $1/\omega \simeq 0.2 \text{ fm}$. A much smaller value for ω the non-diagonal contribution (4) would give rise to a too large value of the strangeness radius. The data favor a positive value for the phase factor δ in the non-diagonal contribution (4). These results were obtained with the overlap factor C_{35} (7) taken to be 0.4 and 1.0, respectively. The values of the probability $P_{s\bar{s}}$ were taken to be 0.1 and 0.15 as indicated in the curves. The calculated strangeness radius is positive, as the s quark is in preferentially in the P -state and the \bar{s} is in the S -state. Therefore the charge distribution of the strange component is positive at short and negative at longer distances.

The calculated values for G_M^s obtained with the same parameter values are shown in Fig. 2. The best description of the data is obtained by taking the probability of the $uuds\bar{s}$ component to be $P_{s\bar{s}}$ in the range 10–15% and the value of the phase factor δ in the non-diagonal contribution (4) to be positive ($\delta = +1$). Here again the slow falloff with q^2 is noteworthy.

The calculated values for $G_A^s(q^2)$ are shown in Fig. 3. The curve qualitatively follows the phenomenological solution given in Ref. [10]. At $q^2 = 0$ G_A^s equals the strangeness contribution to the proton spin. The values obtained for that observ-

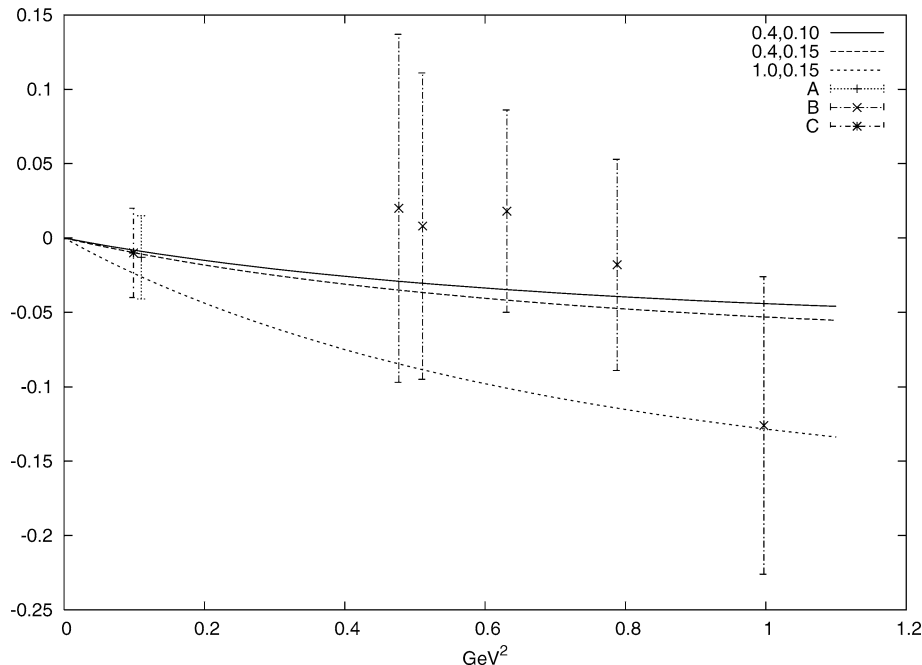


Fig. 1. The strangeness electric form factor for $C_{35} = 0.4$ and $C_{35} = 1.0$ (first number in the brackets in the curves). The second value in the bracket is the value of $P_{s\bar{s}}$. The data points are from [4,9] (A), [10] (B) and [5] (C).

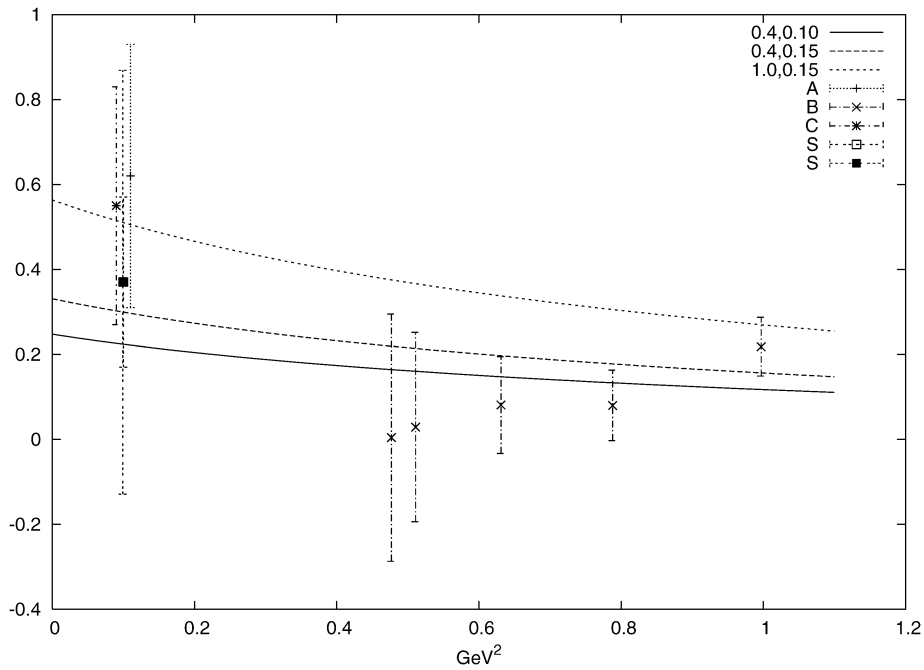


Fig. 2. The strangeness magnetic form factor for $C_{35} = 0.4$ and $C_{35} = 1.0$ (first number in the brackets in the curves). The second value in the bracket is the value of $P_{s\bar{s}}$. The data points are from [1] (S), [4,9] (A), [10] (B) and [5] (C).

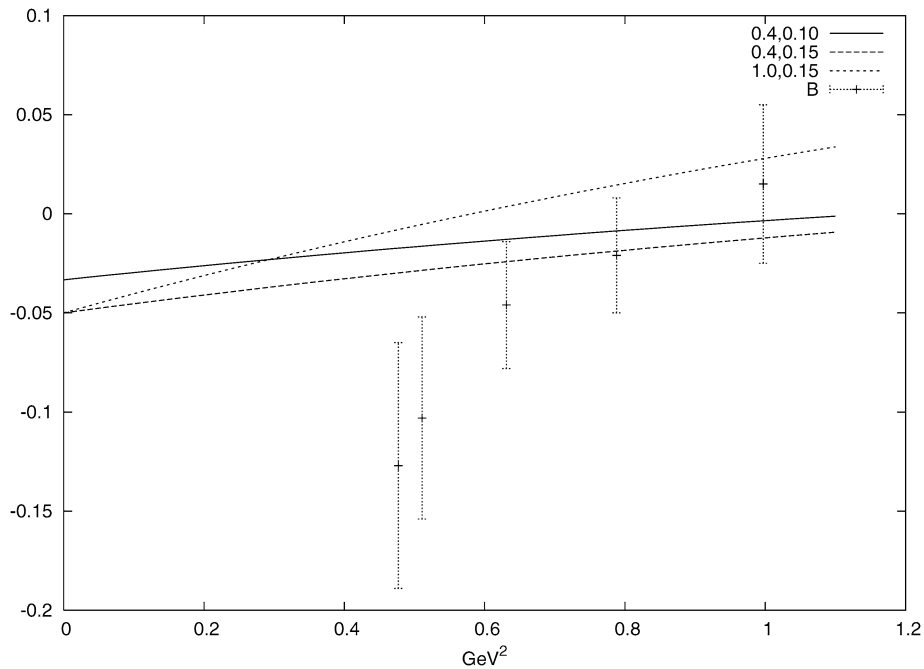


Fig. 3. Strangeness axial form factor for $C_{35} = 0.4$ and $C_{35} = 1.0$ (first number in the brackets in the curves). The second value in the bracket is the value of $P_{s\bar{s}}$. The data points are from [10] (B).

able with the present parameterization are $(-0.03) - (-0.07)$, which fall within the empirical range of values from 0 to -0.10 [11–13].

In Fig. 4 the calculated form factor combination $G_E^s(q^2) + \eta G_M^s(q^2)$ calculated with this parameterization is compared to the results of the A4 [3,16] and G0 experiments [4]. In this case the overall features of the empirical values are best reproduced with $C_{35} = 0.4$ and $P_{s\bar{s}} = 0.10$.

The quality of this comparison with the empirically obtained combination form factor combination $G_E^s + \eta G_M^s$ is not very sensitive to the precise value of the oscillator parameter ω as long as it is larger than ~ 0.7 GeV, which corresponds to a radius of ~ 0.3 fm for the wave function of the $uuds\bar{s}$ component.

Finally, in Fig. 5, we give graphical comparison of the present result for the strange magnetic moment and the strangeness radii (for $C_{35} = 0.4$ and $P_{s\bar{s}} = 0.15$) with previous theoret-

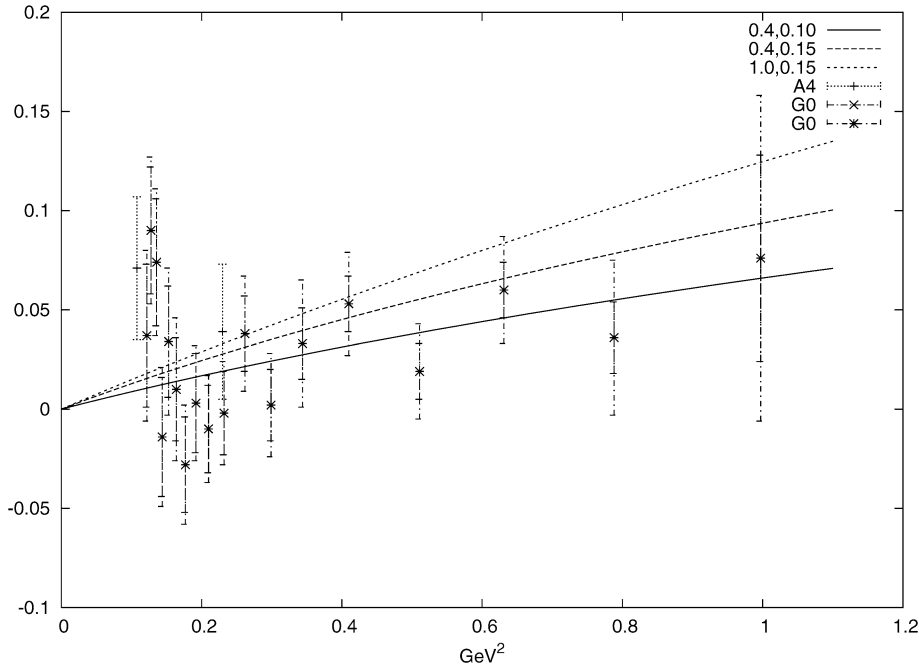


Fig. 4. The strangeness form factor combination $G_E^S + \eta G_M^S$ for 3 values of $P_{s\bar{s}}$ for $C_{35} = 0.4$ and $C_{35} = 1.0$ (first number in the brackets in the curves). The second value in the bracket is the value of $P_{s\bar{s}}$. The data points are from [3,16] (A4) and [4] (G0).

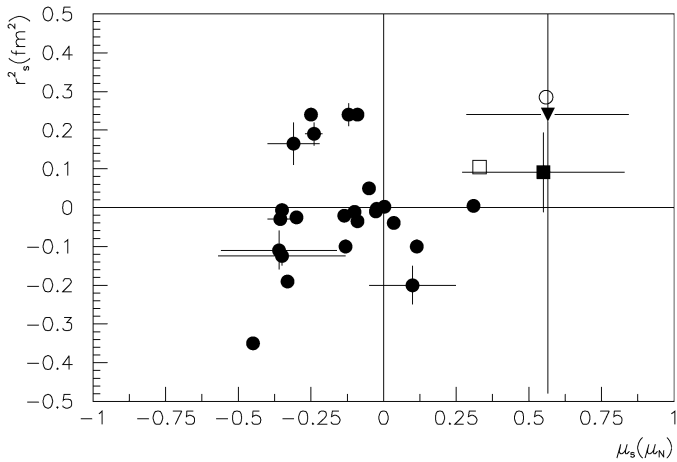


Fig. 5. Calculated values of the strange magnetic moments and the strangeness radii (filled circles) as listed in Ref. [14] and the present values with $C_{35} = 0.4$ and $P_{s\bar{s}} = 0.15$ (open square) and with $C_{35} = 1$ and $P_{s\bar{s}} = 0.15$ (open circle). The data points are from Ref. [2] (solid triangle), [15] (solid square).

ical values [14]. The present result is unique in that it leads to clearly positive values for both μ_s and r_s and thus agrees with the current empirical values for both of these observables.

The results above do not take any account of the interaction between the quarks besides the indirect role of the confining interaction that leads to bound state wave functions. These interactions may also contribute to the vector coupling through interaction currents. In the case of the strangeness magnetic moment the confining interaction leads to an additional term in the annihilation contribution, the magnitude and sign of which depends on the short range part of the confining potential—i.e., to what extent it is negative at short range [17]. In the case of the strangeness magnetic moment an analysis by means of disper-

sion relations it has been shown that such interactions may be significant [18]. This indicates that a considerable uncertainty range should be associated with the numerical estimates above for the annihilation contribution.

In summary, the comprehensive analysis in Refs. [7,8] of all $uuds\bar{s}$ configurations with at most one quark in an orbitally excited state revealed that the configurations, in which the strangeness magnetic moment is positive and the strangeness contribution to the spin is negative, and which has the lowest energy in the case of spin dependent hyperfine interactions, is the configuration $[211]_C[31]_X[4]_{FS}[22]_F[22]_S$ considered above. The present results show that if the wave function is compact in comparison to the proton radius, it also leads to a qualitative description of the extant experimental and phenomenologically extracted momentum dependence of the strangeness form factors.

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