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A possible symmetric neutrino mixing ansatz

Wu-zhong Guo [a,*], Miao Li [a,b,c]

[a] Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China
[b] Kavli Institute for Theoretical Physics China, Chinese Academy of Sciences, Beijing 100190, China
[c] National Key Laboratory of Frontiers in Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

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ABSTRACT

Using a recent global analysis result after the precise measurement of θ_{13}, a possible symmetric neutrino mixing ansatz is proposed, the mixing matrix is symmetric and also symmetric with respect to the second diagonal line in the leading order. This leading order ansatz predicts $\theta_{13} = 12.2°$. Next, consider the hierarchy structure of the lepton mass matrix as the origin of perturbation of the mixing matrix, we find that this ansatz with perturbation can fit current data very well.

1. Introduction

Many neutrino experiments have revealed that the three known light neutrinos must have finite but small masses and that different flavor neutrinos oscillate from one to another. The massive neutrino states ν_L are related to the flavor states ν_{fL} by the 3×3 PMNS matrix:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L = U_{PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L . \tag{1}$$

The PMNS matrix is unitary, which can be parameterized in the standard way, with three mixing angles and three CP-related phases.

Before the recent experiment results of T2K [1], MINOS [2] and Double Chooz [3], the best-fit of neutrino oscillation angles are usually taken to be $\theta_{13} \approx 0°$, $\theta_{23} \approx 45°$ and $\theta_{12} \approx 34°$. Many ansatzes are proposed to explain the smallness of $\sin\theta_{13}$ and maximality of $\sin\theta_{23}$ [6]. The well-known tri-bimaximal mixing pattern is consistent with experiment data then [8]. But Daya Bay SBL experiment made a precise measurement of θ_{13} from the $\bar{\nu}_e \to \bar{\nu}_e$ oscillations [4]. The θ_{13} best-fit ($\pm 1\sigma$ range) result is

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst}), \tag{2}$$

which is to say that $\theta_{13} \neq 0°$ at the 5.2σ level. The RENO Collaboration confirmed the Daya Bay result soon [5].

θ_{13} is not really very small. This fact makes many ansatzes face difficulties [6] and opens the door of possible new phenomenological applications. The possible explanation and impact of large θ_{13} have been discussed recently by many authors for example [25–32]. The precise measurement of θ_{13} also has impact on the global analysis of all the mixing parameters. Many groups have already performed global analysis recently [9,10]. And we pay particular attention to the result of [9]. There is an interesting indication that the mixing angle θ_{23} deviates from the maximal mixing, i.e., $\theta_{23} < \frac{\pi}{4}$ (at $\leqslant 3\sigma$ in NH and $\leqslant 2\sigma$ in IH) [9], and a weak hint that the CP-violation phase $\delta \sim \pi$. But a reliable result on δ is not available without the future long-baseline neutrino oscillation experiments.

In the phenomenological view, the new global analysis result may change the form of the PMNS matrix significantly. If the value of θ_{23} is much smaller, the absolute value of $U_{\tau 1}$ ($|U_{\tau 1}| = |s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta}|$) would also decrease. This makes the symmetric ansatz of PMNS matrix seem more possible. In fact, a symmetric mixing pattern has been discussed by some authors [11,15], and the phenomenological view of the Hermitian PMNS matrix also appears in a recent paper [16]. Encouraged by the global analysis result [9], we study the possible view that the PMNS matrix is symmetric in the leading order. Consider the strategy that the PMNS matrix has the following structure [6],

$$U_{PMNS} = \left(U_s^{(0)} + \Delta U\right)K, \tag{3}$$

where $U_s^{(0)}$ is the symmetric mixing pattern in the leading order, and ΔU is the perturbation, $K = diag(e^{i\rho}, e^{i\sigma}, 1)$, which related with Majorana CP phase.

* Corresponding author.
E-mail addresses: wuzhong@itp.ac.cn (W.-z. Guo), mli@itp.ac.cn (M. Li).

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2. The ansatz

To get a proper ansatz, we turn to the experiment data [9]. As observed by Yoni BenTov and Zee [16], the exact Hermitian mixing matrix would predict that the Dirac CP phase is zero. And as we have mentioned above, we expect the CP-violation is very small if the symmetric ansatz is proper. If the CP-violation originates from a small perturbation of an ansatz, it is more natural to assume $\delta \sim 0$. We just ignore the possibility that $\delta \sim \pi$, and use the NH best-fit data [9]

$$\sin^2\theta_{12} = 0.307, \qquad \sin^2\theta_{13} = 0.0241, \qquad \sin^2\theta_{23} = 0.386. \tag{4}$$

And we choose $\delta = 0$. Here we simply take the global analysis best-fit data of the mixing parameters into the standard parametrization. Then the matrix U_{exp}, after a basis transformation, is

$$U_{exp} = \begin{pmatrix} -0.822 & 0.547 & 0.155 \\ 0.514 & 0.599 & 0.614 \\ 0.243 & 0.584 & -0.777 \end{pmatrix}. \tag{5}$$

We notice the assumption that the PMNS matrix is symmetric at the leading order with respect to second diagonal line has been discussed in [12]. Now the data still favor this assumption, and the data also hint the PMNS matrix is possibly symmetric. Encouraged by the data, we put forward a simple ansatz of the matrix U. The form of this matrix is the following:

$$U_s^{(0)} = \begin{pmatrix} -a & b & c \\ b & b & b \\ c & b & -a \end{pmatrix}. \tag{6}$$

This form of the mixing matrix also exhibits symmetry in the lepton sector. The general real and symmetric matrix is

$$U_s = \begin{pmatrix} -a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}. \tag{7}$$

If the Z_2 transformation $\nu_1 \leftrightarrow \nu_3$ and $e \leftrightarrow \tau$ keeps the mixing matrix invariant, we have $-a = f$ and $b = e$. We just need $d = b$ to get the form (6), it turns out that this requirement results from a certain form of the mass matrix, as will be discussed in Section 4. The unitary property of matrix U helps us to fix the parameters a, b and c. The numerical result is

$$U_s^{(0)} = \begin{pmatrix} \frac{1}{\sqrt{3}-3} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}+3} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}+3} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}-3} \end{pmatrix}. \tag{8}$$

$U_s^{(0)}$ has a simple and beautiful structure. Comparing with the standard parametrization, we get $\theta_{13} \approx 12.2^\circ$, deviating from experiment result about 3°, the degeneration of the θ_{23} and θ_{12}, and of course no CP-violation in this simple mixing pattern. According to (3), the deviation of our ansatz from the best-fit data (5) is

$$\Delta U = \begin{pmatrix} -0.0333 & -0.0304 & -0.0563 \\ -0.0634 & 0.0216 & 0.0366 \\ 0.0317 & 0.00660 & 0.0117 \end{pmatrix}. \tag{9}$$

The magnitude of each element of ΔU is of $O(0.01)$ (U_{exp} is of $O(0.1)$). This implies that all the elements of mixing matrix should receive the same order correction in a natural way [6]. The relative deviation of θ_{13} is much larger than others because of the smallness of θ_{13}. We will show in Section 3 that it is easy to make a perturbation to match the experimental data.

3. High order corrections

The lepton flavor mixing matrix U_{PMNS} is directly related with the diagonalization of the charged lepton mass matrix M_l and neutrino mass matrix M_ν. We only consider the low energy theory here, heavy fermions have been integrated out and the neutrino mass term comes from the effective dimension-five Weinberg operator [13]. So there is only the left-handed Majorana mass term. In SM, there is no constraint on the form of the mass matrix of charged leptons. But we can always make arbitrary unitary transformation of the right hand charged lepton in SM and use the freedom to choose the charged lepton mass matrix to be Hermitian [14]. In many special models (for example some left–right symmetric theories), the mass matrix of charged leptons should be Hermitian by the restriction of symmetry [17,18]. We take the charged lepton mass matrix to be Hermitian in this Letter. So we can diagonalize the mass matrix as

$$V_l^\dagger M_l V_l = diag\left(m_e, \quad m_\mu, \quad m_\tau\right) \tag{10}$$

and

$$V_\nu^\dagger M_\nu V_\nu^* = diag\left(m_1, \quad m_2, \quad m_3\right). \tag{11}$$

The PMNS matrix is $U_{PMNS} = V_l^\dagger V_\nu$. In general, it is the product of $V_l^\dagger$ and V_ν that determines the mixing pattern. So there are many possible ways to get the PMNS matrix. And the possibility that the leading effect of mixing pattern is caused by V_ν or V_l with the other one as the NLO perturbation has been suggested and studied for example [19–23]. The structure of mass matrix is directly related with the transformation matrix, and the mass spectrum of charged lepton and neutrino would give some information about the mass matrix structure. The much larger mass hierarchy of charged leptons may have some relation with the mass matrix elements hierarchy structure. And the mass hierarchy of neutrino is not so large, so it may be natural to think that the mass matrix elements are not in the hierarchy structure. One possible and simple charged lepton mass matrix structure is that it is nearly diagonal. Then in the leading order the transformation matrix of charged lepton is close to 1. So it is the transformation matrix V_ν that determines the leading mixing pattern.

We start from the charged lepton mass matrix:

$$M_l = \begin{pmatrix} m_1' & \epsilon & \eta \\ \epsilon^* & m_2' & \kappa \\ \eta^* & \kappa^* & m_3' \end{pmatrix}, \tag{12}$$

where m_1', m_2' and m_3' are all real, ϵ, η and κ are complex. Considering the mass hierarchy of charged leptons, we assume the following relation between these parameters:

$$|\epsilon|, m_1' \ll m_2', m_3'; \qquad |\eta|, |\kappa| \ll |\epsilon|, m_1'; \qquad m_1' \ll |\epsilon|. \tag{13}$$

The charged leptons mass matrix is nearly diagonal. In the view of model building, a model based on FN mechanism may produce the mass matrix structure like this. The leading order structure is the following:

$$M_l^{(0)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_2' & 0 \\ 0 & 0 & m_3' \end{pmatrix}. \tag{14}$$

$M_l^{(0)}$ is diagonal with the mass of electron being zero. It is reasonable for the big mass hierarchy between e and μ ($\frac{m_\mu}{m_e} \approx 210$).[1]

[1] Here we use the up-dated values of charged lepton masses renormalized to the M_Z scale in [7], with $M_e(M_Z) \approx 0.4866$ MeV and $M_\mu(M_Z) \approx 102.7181$ MeV.

The unitary transformation matrix V_l is trivial, i.e., $V_l^{(0)} = 1$. So the transformation matrix V_ν is just the leading mixing matrix $U_s^{(0)} K$. And we assume the transformation matrix V_ν stays the same in the NLO and NNLO calculation. The NLO and NNLO correction are only related with the structure of the charged lepton mass matrix. This is a very strong assumption, but still can be realized in some situations. We may expect the high order correction of lepton mass matrix only effect the charged lepton sector because of the special symmetry forbids the correction term of neutrino sector. Or the different origin of the mass matrix of charged leptons and the neutrinos may keep the correction term of neutrinos much smaller than the charged leptons so that we could ignore the variation of the V_ν. It is also possible that the correction term of neutrino mass matrix do not change the structure and symmetry of the leading order mass matrix. All the three situations can lead to the assumption that only the charged lepton mass matrix is the origin of perturbation of the leading mixing matrix. But we may need more special symmetry or mechanism in a model to get any situation mentioned above.

Before we go on to consider the NLO calculation, let us compare the leading order prediction with the data. The leading order predicts that $\sin^2\theta_{12}^{(0)} = \sin^2\theta_{23}^{(0)} \approx 0.349$. $\theta_{23}^{(0)}$ is in the 2σ region and $\theta_{12}^{(0)}$ is just in the 3σ region. $\theta_{13}^{(0)}$ is even worse, since $\sin^2\theta_{13}^{(0)} \approx 0.0446$, which is out of the 3σ allowed region of the data, so the NLO calculation is necessary.

Considering (12)–(13), the NLO mass matrix of charged lepton is [24]

$$M_l^{(1)} = \begin{pmatrix} 0 & \epsilon & 0 \\ \epsilon^* & m_2' & 0 \\ 0 & 0 & m_3' \end{pmatrix}, \tag{15}$$

with $\epsilon = |\epsilon| e^{i\lambda}$. This matrix deviates from the diagonal matrix by off-diagonal element ϵ, but it is also easy to be diagonalized by the transformation matrix

$$V_l^{(1)} = \begin{pmatrix} \cos\theta & \sin\theta e^{i\phi} & 0 \\ -\sin\theta e^{-i\phi} & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{16}$$

where $\tan\theta = \sqrt{\frac{m_e}{m_\mu}} \approx 0.0688$, $m_3' = m_\tau$, $m_2' = m_\mu - m_e$, $|\epsilon| = \sqrt{m_e m_\mu}$ and $\phi = \lambda$. Using the relation $U_s^{(1)} K = V_l^{(1)\dagger} V_\nu = V_l^{(1)\dagger} U_s^{(0)} K$, we get the NLO mixing matrix:

$$U_s^{(1)} = \begin{pmatrix} \frac{\cos\theta}{\sqrt{3}-3} - \frac{\sin\theta e^{i\lambda}}{\sqrt{3}} & \frac{\cos\theta}{\sqrt{3}} - \frac{\sin\theta e^{i\lambda}}{\sqrt{3}} & \frac{\cos\theta}{\sqrt{3}+3} - \frac{\sin\theta e^{i\lambda}}{\sqrt{3}} \\ \frac{\cos\theta}{\sqrt{3}} + \frac{\sin\theta e^{-i\lambda}}{\sqrt{3}-3} & \frac{\cos\theta}{\sqrt{3}} + \frac{\sin\theta e^{-i\lambda}}{\sqrt{3}} & \frac{\cos\theta}{\sqrt{3}} + \frac{\sin\theta e^{-i\lambda}}{\sqrt{3}+3} \\ \frac{1}{\sqrt{3}+3} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}-3} \end{pmatrix}. \tag{17}$$

There is only one free parameter λ here. And the possible Dirac CP-violation phase is also included in the NLO. Now after rephrasing (17) and comparing with the standard parametrization, it is straightforward to write down the mixing parameters after NLO correction,

$$\sin^2\theta_{13}^{(1)} \simeq \left(\frac{1}{\sqrt{3}+3}\right)^2 - \frac{2}{3+3\sqrt{3}}\sqrt{\frac{m_e}{m_\mu}}\cos\lambda, \tag{18}$$

$$\sin^2\theta_{23}^{(1)} \simeq \frac{2 + 2(\sqrt{3}-1)\sqrt{\frac{m_e}{m_\mu}}\cos\lambda}{4+\sqrt{3}+2(\sqrt{3}-1)\sqrt{\frac{m_e}{m_\mu}}\cos\lambda}, \tag{19}$$

$$\sin^2\theta_{12}^{(1)} \simeq \frac{2 - 4\sqrt{\frac{m_e}{m_\mu}}\cos\lambda}{4+\sqrt{3}+2(\sqrt{3}-1)\sqrt{\frac{m_e}{m_\mu}}\cos\lambda}, \tag{20}$$

$$\sin\delta^{(1)} \simeq \frac{1}{2}(3\sqrt{3}-1)\sqrt{\frac{m_e}{m_\mu}}\sin\lambda. \tag{21}$$

Here we only keep the order $O(\sqrt{\frac{m_e}{m_\mu}})$. The mixing parameters after NLO correction as a function of λ are displayed in Fig. 1. If λ is in the first and fourth quadrant, the degeneration of θ_{23} and θ_{12} disappears with θ_{23} becoming larger and θ_{12} becoming smaller, which is in keep with the experiment data. The NLO correction to $\sin^2\theta_{13}^{(0)}$ is about -0.02, which makes θ_{13} fit the data much better. More precisely, $\theta_{13}^{(1)}$ can lie in the 2σ range if $0 \leqslant \lambda \leqslant 0.413$ or $5.870 \leqslant \lambda \leqslant 2\pi$, $\theta_{13}^{(1)}$ can lie in the 3σ range if $0 \leqslant \lambda \leqslant 0.687$ or $5.596 \leqslant \lambda \leqslant 2\pi$. $\theta_{23}^{(1)}$ can be in the 2σ range in a large area of the parameter space of λ. $\theta_{12}^{(1)}$ fits the data very well in the NLO, when λ is small or near 2π, $\theta_{12}^{(1)}$ would lie in the 1σ region. The best-fit point of θ_{12} can also be included. Eq. (21) shows that there are two possible values of Dirac CP-violation phases δ in our NLO prediction, one is near zero, another is near π, depending on the value of λ (see Fig. 1). As we have assumed, δ should be near zero in the natural way, λ is constrained in the first quadrant. And the rephrasing-invariant parameter $\mathcal{J} \approx \frac{\sqrt{3}}{18}\sqrt{\frac{m_e}{m_\mu}}\sin\lambda \sim O(0.001)$.

As we can see, the NLO correction can fit the data. But it is still not very satisfactory, let us consider the NNLO correction now. With the assumptions (12) and (13), the mass matrix of charged lepton would be of the following structure

$$M_l^{(2)} = \begin{pmatrix} m_1' & \epsilon & 0 \\ \epsilon^* & m_2' & 0 \\ 0 & 0 & m_3' \end{pmatrix}. \tag{22}$$

It is also very easy to diagonalize (22) by a similar matrix

$$V_l^{(2)} = \begin{pmatrix} \cos\zeta & \sin\zeta e^{i\rho} & 0 \\ -\sin\zeta e^{-i\rho} & \cos\zeta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{23}$$

where $\tan\zeta = \sqrt{\frac{m_e + m_1'}{m_\mu}}$, $m_3' = m_\tau$, $m_2' + m_1' = m_\mu - m_e$ and $\rho = \lambda$. In addition to the parameter λ, a new parameter appears, i.e. m_1'. We define $m_1' \equiv (C^2 - 1)m_e$ (with $C > 1$), where C is real and positive. Then $\tan\zeta = C\sqrt{\frac{m_e}{m_\mu}}$. As we have assumed $m_1' \ll |\epsilon|$ and know $|\epsilon|$ is of $O(\sqrt{m_\mu m_e}) \sim O(10)m_e$, m_1' should be order m_e, i.e., C is of $O(1)$. It is straightforward to calculate the NNLO correction by the same way as the NLO correction.

$$\sin^2\theta_{13}^{(2)} \simeq \left(\frac{1}{\sqrt{3}+3}\right)^2 - \frac{2}{3+3\sqrt{3}}\sqrt{\frac{m_e}{m_\mu}} C\cos\lambda, \tag{24}$$

$$\sin^2\theta_{23}^{(2)} \simeq \frac{2 + 2(\sqrt{3}-1)\sqrt{\frac{m_e}{m_\mu}} C\cos\lambda}{4+\sqrt{3}+2(\sqrt{3}-1)\sqrt{\frac{m_e}{m_\mu}} C\cos\lambda}, \tag{25}$$

$$\sin^2\theta_{12}^{(2)} \simeq \frac{2 - 4\sqrt{\frac{m_e}{m_\mu}} C\cos\lambda}{4+\sqrt{3}+2(\sqrt{3}-1)\sqrt{\frac{m_e}{m_\mu}} C\cos\lambda}, \tag{26}$$

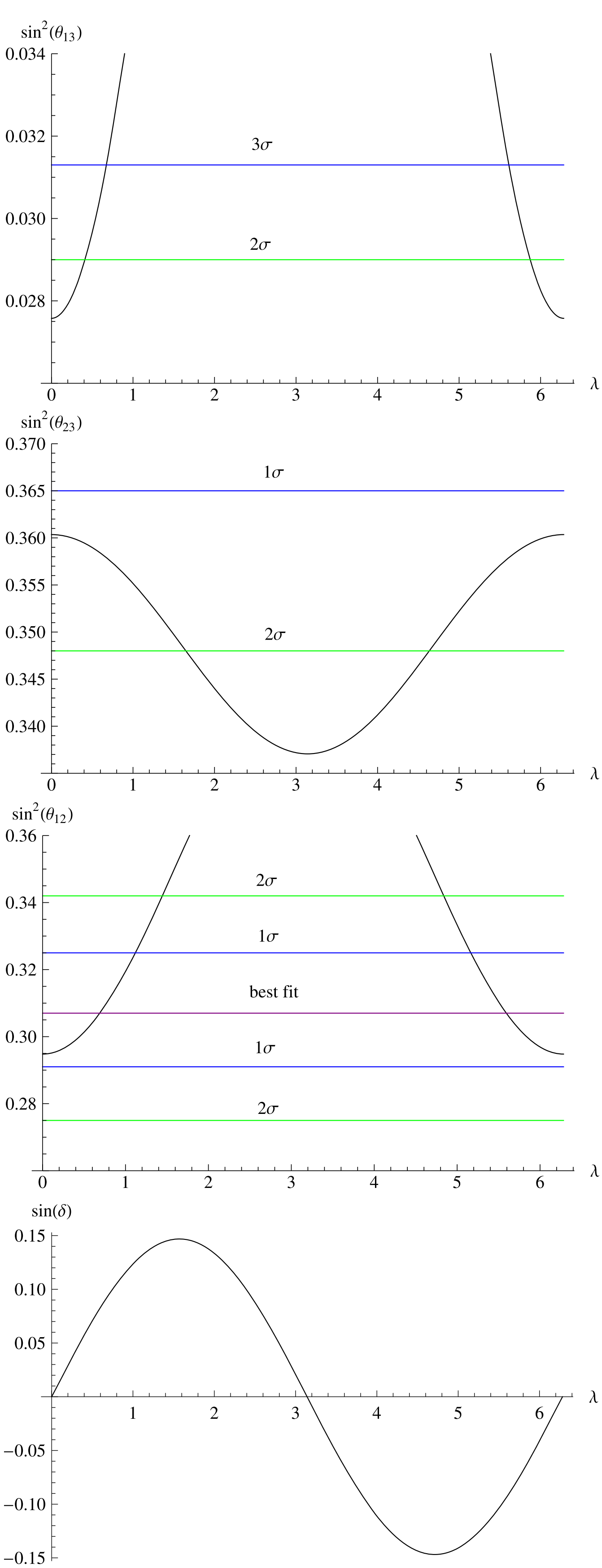

Fig. 1. Behavior of $\sin^2\theta_{13}$, $\sin^2\theta_{23}$, $\sin^2\theta_{12}$, $\sin\delta$ as a function of λ in the NLO.

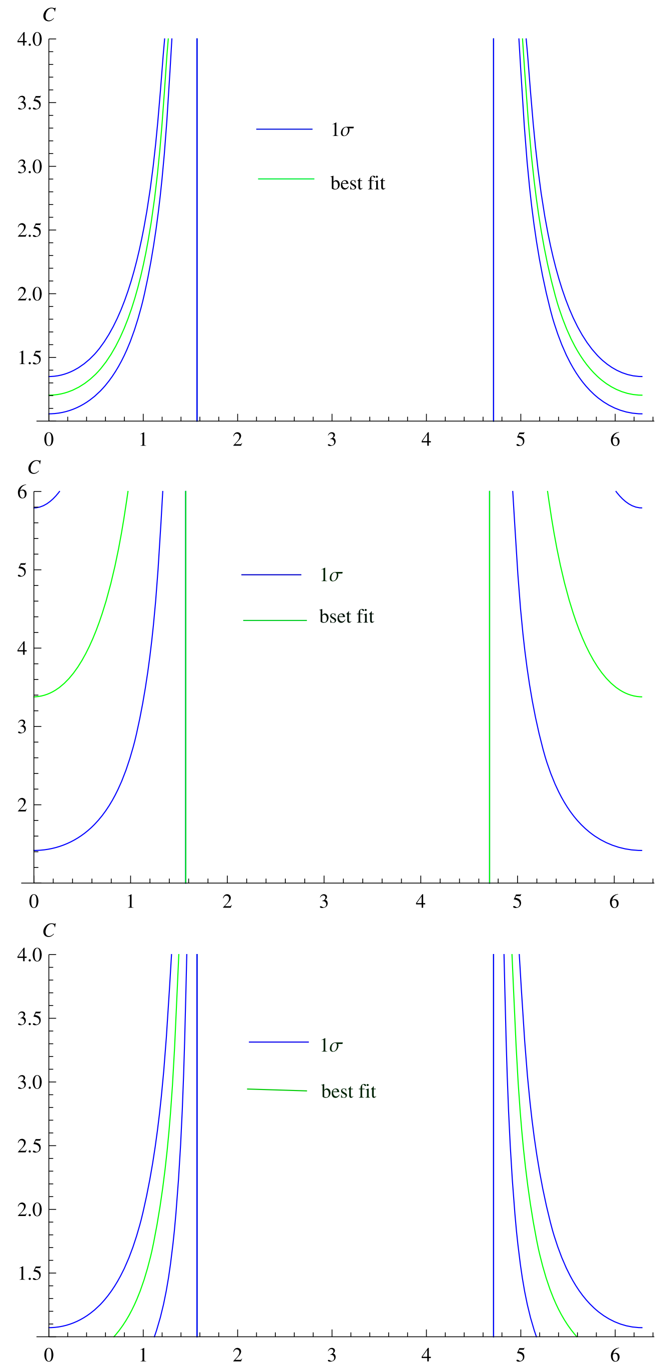

Fig. 2. The parameter space of λ and C allowed by the experiment data in the 1σ region of $\sin^2\theta_{13}$ (top), $\sin^2\theta_{23}$ (middle), $\sin^2\theta_{12}$ (bottom) in the NNLO.

$$\sin\delta^{(2)} \simeq \frac{1}{2}(3\sqrt{3}-1)\sqrt{\frac{m_e}{m_\mu}}C\sin\lambda. \tag{27}$$

If λ is in the first or the fourth quadrant, the new parameter C would be a linear decreasing function of θ_{13}, and θ_{13} is sensitive to the value of C. And it is possible to make $\sin^2\theta_{13}^{(2)}$ lie in the 1σ region. But when C is large, the interval of λ is very narrow (as we can see in Fig. 2). $\sin^2\theta_{12}^{(2)}$ can still be in the 1σ region in a large interval of C and λ. $\sin^2\theta_{23}^{(2)}$ is an increasing function of C, if λ is fixed in the first or fourth quadrant. But $\sin^2\theta_{23}^{(2)}$ increases very slowly, so a large value of C is needed to get the 1σ region data. On the other hand, too large C would make $\sin^2\theta_{12}^{(2)}$

and $\sin^2\theta_{13}^{(2)}$ decrease too much. Though λ could be adjusted to reduce $\sin^2\theta_{12}^{(2)}$, the parameter space of λ and C is expected to be narrow. But when C is near 2 and λ is near 1, all the three mixing parameters would lie in the 1σ region. Fig. 2 shows the parameter space of C and λ allowed by the 1σ data. The CP-violation phase $\delta^{(2)}$ is still near zero with a deviation about 8°. Future measurement of δ will test our theoretical consideration.

As we can see, the NNLO correction can fit the data very well. According to (12) and (13), the higher order corrections would be much smaller. We do not consider higher order corrections here.

4. The neutrino mass matrix

In this last section, we make some comments on the mass matrix of neutrinos. The hierarchy structure of the mass matrix elements of charged leptons gives the nearly diagonal matrix. The transformation matrix V_ν is just the product of leading order mixing matrix $U_s^{(0)}$ and K, i.e., $V_\nu = U_s^{(0)} K$. Using (11), we can get the mass matrix of neutrinos as the following

$$M_\nu = V_\nu\, diag\left(m_1, \quad m_2, \quad m_3\right) V_\nu^T. \tag{28}$$

Taking $V_\nu = U_s^{(0)} K$ into (28), we get M_ν

$$M_\nu = \begin{pmatrix} x+y & z & y+z \\ z & x & 2y+z \\ y+z & 2y+z & x-y \end{pmatrix}, \tag{29}$$

with

$$x = \frac{1}{3}(\hat{m}_1 + \hat{m}_2 + m_3), \tag{30}$$

$$y = \frac{1}{2\sqrt{3}}(\hat{m}_1 - m_3), \tag{31}$$

$$z = \frac{1}{3}\left(\hat{m}_2 + \frac{1}{\sqrt{3}+1}m_3 - \frac{1}{\sqrt{3}-1}\hat{m}_1\right), \tag{32}$$

where $\hat{m}_1 = m_1 e^{2i\rho}$, $\hat{m}_2 = m_2 e^{2i\sigma}$. From the oscillation experiment and the cosmology observation, we know that the mass spectrum of the three light neutrinos is $m_1 \approx m_2$, $m_1(m_2) \ll m_3$ or $m_3 \ll m_1(m_2)$. So elements of neutrino mass matrix do not show the hierarchy structure as the charged lepton mass matrix, which is what we expect. But (29) has an interesting property. The sums of every column and every row are all equal, i.e., $\sum_{i=1,2,3} M_{\nu,\lambda i} = x + 2y + 2z$ $(\lambda = e, \mu, \tau)$ and $\sum_{\lambda=e,\mu,\tau} M_{\nu,\lambda i} = x + 2y + 2z$ $(i = 1, 2, 3)$. The matrix of this type is called magic matrix [36]. This property also appears in some models with discrete symmetry which lead to the tri-bimaximal mixing pattern for example [33,34]. Comparing with the A_4 models [35], the mass matrix of neutrinos is also invariant by transformation of the following matrix S, i.e., $SM_\nu S = M_\nu$.

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}. \tag{33}$$

S is one of the element of the three-dimensional unitary representation of A_4 group in a suitable basis [35]. But the matrix (29) do not show the μ–τ symmetry, so the mixing pattern is quite different from the tri-bimaximal mixing. We still need another condition (maybe a new symmetry) to fix the form of the mass matrix of neutrinos (29), which is worth to explore in the future. If the mass matrix has the structure as (29), we can diagonalize it and get the mixing ansatz (8). We expect some models with new symmetry or underlying mechanism would give the mass matrix structure (12) and (29). Then we can learn more about the origin of the mass matrix structure, which may be related with the high-scale physics. We also notice the different mass matrix form between charged leptons and neutrinos. But it is not unnatural for the origins of the mass matrix of charged leptons and neutrinos may be due to different mechanisms.

We show in this Letter that a symmetric ansatz can be made to fit the experimental data very well. The mixing ansatz that we propose in Section 2 is just a phenomenological consideration, its theoretical mechanism is unclear yet. But we can observe the symmetry property of the mass matrix of neutrinos, which gives the clue to the model building. The future precise measurements of the mixing parameters, especially of the CP-violation phase δ in the LBL experiments, will provide more information on the mixing pattern and constrain all the parameters. We expect δ be near zero, and then the symmetric ansatz can be an important hint at the underlying physical theory.

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