ELSEVIER
Theoretical Computer Science 197 (1998) 241–248
Theoretical Computer Science

Contents and Abstracts of the Electronic Notes in Theoretical Computer Science Vol. 3

A Special Issue on the Linear Logic 96, Tokyo Meeting

Jean-Yves Girard, Mitsuhiro Okada and Andre Scedrov (Guest Editors)

Preface

The field of linear logic has developed very rapidly during the last ten years. Linear logic is now one of the most active research areas in Logic and in Theoretical Computer Science. The Linear Logic 96 Tokyo Meeting provided a forum for an exchange of research results and a discussion of the future directions in the field. The Linear Logic 96 Tokyo Meeting was held from 28 March to 2 April 1996, at the Mita Campus of Keio University (in downtown Tokyo), with the support from Keio University, JSSP-CNRS and the US Office of Naval Research. These proceedings include extended abstracts and preliminary reports by the invited speakers and the contributed speakers. The last four papers in this volume are contributed papers selected by the editors. We plan to publish formal proceedings composed of full-version papers later.

This is the third volume in the new series, *Electronic Notes in Theoretical Computer Science*. This series is published electronically through the facilities of Elsevier Science B.V. and under its auspices. The aim of the series is to provide rapid, electronic publication of conference proceedings and of lecture notes, thematic monographs, and other such publications of interest to the theoretical computer science community. The Managing Editors of the series are Michael Mislove, Maurice Nivat and Christos Papadimitriou. *ENTCS* is affiliated with the journal *Theoretical Computer Science*, and anyone whose home institution has a paid subscription to *TCS* and its electronic components will have access to the *ENTCS* archives. This material and the volumes in the series will be available via Elsevier Science's World Wide Web server, which can be accessed at the URL

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Samson Abramsky and Guy McCusker

Linearity, Sharing and State: a fully abstract game semantics for Idealized Algol with active expressions (extended abstract)

`http://www.elsevier.nl/locate/entcs/volume3.html`

The manipulation of objects with state which changes over time is all-pervasive in computing. Perhaps the simplest example of such objects are the program variables of classical imperative languages. An important strand of work within the study of such languages, pioneered by John Reynolds, focusses on "Idealized Algol", an elegant synthesis of imperative and functional features. We present a novel semantics for Idealized Algol using games, which is quite unlike traditional denotational models of state. The model takes into account the irreversibility of changes in state, and makes explicit the difference between copying and sharing of entities. As a formal measure of the accuracy of our model, we obtain a full abstraction theorem for Idealized Algol with active expressions.

PII S0304-3975(97)00246-6

The aim of this paper is to propose a logical way to handle uncertain knowledge and change. Databases, diagnostic, planification, taxonomy are some of the domains concerned with this problem. This paper focuses on the means Linear Logic offers to represent taxonomical networks and to perform updates of databases containing incomplete information. The two problems are first expressed in graph theory: a taxonomical network is a structure for representing knowledge as a graph whose vertices are concepts, and edges are relations between concepts; a database is specified by facts, deduction rules, i.e. edges between literals, and update constraints. Their formalization in Linear Logic is performed in a very similar way.

The founding idea of linear logic is the duality between A and $A^{\perp}$, with values in $\perp$. This idea is at work in the original denotational semantics of linear logic, coherent spaces, but also in the phase semantics of linear logic, where the "bilinear form" which induces the duality is nothing but the product in a monoid $\mathbb{M}$, $\perp$ being an arbitrary subset $\mathbb{B}$ of $\mathbb{M}$. The rather crude phase semantics has the advantage of being complete, and against all predictions, this kind of semantics had some applications. Coherent semantics is not complete for an obvious reason, namely that the coherent space - interpreting $\perp$ is too small (one point), hence the duality between A and $A^{\perp}$ expressed by the cut-rule cannot be informative enough. But - is indeed the simplest case of a Par-monoid, i.e. the dual of a comonoid, and it is tempting to replace - with any commutative Par-monoid $\mathbb{P}$. Now we can replace coherent spaces with "free $\mathbb{P}$-modules over $\mathbb{P}$", linear maps with "$\mathbb{P}$-linear maps", with the essential result that all usual constructions remain unchanged: technically speaking cliques are replaced with $\mathbb{P}$-cliques and that's it. The essential intuition behind $\mathbb{P}$ is that it accounts for arbitrary contexts: instead of dealing with Γ, A, one deals with A, but a clique of Γ, A can be seen as a $\mathbb{P}$-clique in A. In particular, all logical rules are now defined only on the main formulas of rules, as operations on $\mathbb{P}$-cliques. The duality between A and $A^{\perp}$ yields a $\mathbb{P}$-clique in - , i.e. a clique in $\mathbb{P}$; strangely enough, one must keep the phase layer, i.e. a monoid $\mathbb{M}$ (useful in the degenerated case), and the result of the duality is a $\mathbb{M}\mathbb{P}$-clique. We specify an arbitrary set $\mathbb{B}$ of such cliques as the interpretation of $\perp$. Soundness and completeness are then easily established for closed Π^1-formulas, i.e. second-order propositional formulas without existential quantifiers. We must however find the equivalent of $1 \in \mathscr{F}$ (which is the condition for being a "provable fact"): a $\mathbb{M}\mathbb{P}$-clique is *essential* when it does not make use of $\mathbb{M}$ and $\mathbb{P}$, i.e. when it is induced by a clique in $A^{\bullet}$. We can now state the theorem:

Let A be a closed Π^1 formula, and let a be a clique in the (usual) coherent interpretation $A^{\bullet}$ of A, which is the interpretation of a proof of A; then a (as an essential clique), belongs to the "denotational fact" A° interpreting A for all $\mathbb{M}$, $\mathbb{P}$ and $\mathbb{B}$. Conversely any essential clique with this property comes from a proof of A.

Lincoln, Scedrov and Shankar proved undecidability of intuitionistic second-order multiplicative commutative linear logic by embedding LJ2 into this logic. Emms did the same for intuitionistic second-order non-commutative linear logic. Recently, Lafont and Scedrov demonstrated undecidability of classical second-order multiplicative commutative linear logic. As for classical second-order non-commutative linear logic, its decidability problem remained open. Here we present a direct and natural *encoding* of arbitrary machine computations in minimal fragments of the second-order non-commutative linear logic (for instance, in the product-free Lambek syntactic calculus enriched by only one second-order quantifier $\forall$), and prove the *correctness* and *faithfulness* of this encoding with respect to any second-order system up to the full classical second-order cyclic linear logic. It is thereby proved that any reasonable versions of the second-order non-commutative linear logic are undecidable.

We give a class of proof nets for Intuitionistic Linear Logic with the connectives $\multimap, !$, prove a correctness criterion for them and show that a games semantics can be directly derived from these nets, along with a full completeness theorem.

Proof search in linear logic is known to be difficult: the provability of propositional linear logic formulas is undecidable. Even without the modalities, multiplicative-additive fragment of propositional linear logic, MALL, is known to be PSACE-complete, and the pure multiplicative fragment, MLL, is known to be NP-complete. However, this still leaves open the possibility that there might be proof search heuristics (perhaps involving randomization) that often lead to a proof if there is one, or always lead to something close to a proof. One approach to these problems is to study strategies for proof games. A class of linear logic proof games is developed, each with a numeric score that depends on the number of certain preferred axioms used in a complete or partial proof tree. Using recent techniques for proving lower bounds on optimization problems, the complexity of these games is analyzed for the fragment MLL extended with additive constants and for the fragment MALL. It is shown that no efficient heuristics exist unless there is an unexpected collapse in the complexity hierarchy.

We study three fragments of multiplicative linear logic with circular exchange, respectively, *LLNC* containing all propositional variables, *LLNCa* builded on a single variable and the

constant-only fragment *LLNC0*. By using non-commutative proofnets, we show that the decision problems of these fragments are polynomially equivalent.

Raymond McDowell, Dale Miller and Catuscia Palamidessi
Encoding transition systems in sequent calculus (preliminary report)
`http://www.elsevier.nl/locate/entcs/volume3.html`

Linear logic has been used to specify the operational semantics of various process calculi. In this paper we explore how meta-level judgments, such as simulation and bisimulation, can be established using such encodings. In general, linear logic is too weak to derive such judgments and we focus on an extension to linear logic using definitions. We explore this extension in the context of transition systems.

Misao Nagayama and Mitsuhiro Okada
A graph-theoretic characterization theorem for multiplicative fragment of noncommutative linear logic (extended abstract)
`http://www.elsevier.nl/locate/entcs/volume3.html`

It is well-known that every proof net of a non-commutative version of MLL (multiplicative fragment of commutative linear logic) can be drawn as a plane Danos–Regnier graph (drawing) satisfying the switching condition of Danos–Regnier. In this paper, we study the reverse direction; we introduce a system MNCLL logically equivalent to the multiplicative fragment of cyclic linear logic introduced by Yetter, and show that any plane Danos–Regnier graph drawing satisfying the switching condition represents a unique non-commutative proof net (i.e., a proof net of MNCLL) modulo cyclic shifts. In the course of proving this, we also give the characterization of the non-commutative proof nets by means of the notion of strong planity, as well as the notion of a certain long-trip condition, called the stack-condition, of a Danos–Regnier graph, the latter of which is related to Abrusci's balanced long-trip condition.

Mitsuhiro Okada
Phase semantics for higher-order completeness, cut-elimination and normalization proofs (extended abstract)
`http://www.elsevier.nl/locate/entcs/volume3.html`

We give a natural extension of Girard's phase semantics of the linear logic to the classical and intuitionistic higher-order linear logics and give a uniform phase-semantic proof of the higher-order cut-elimination theorem as well as the completeness theorem. Although our proof in this paper is mainly concentrated on the framework of linear logic, the proof technique works for various different logical systems *uniformly*, too. We also extend our phase semantics for provability to phase semantics for *proofs*, by modifying the phase space of monoid domain to that of *proofs* domain, in a natural way. The resulting phase semantics for proofs provide various versions of proof-normalization theorem. The details of this extension will appear in the full-version of this paper.

The proof-theoretic origins and specialized models of linear logic make it primarily operational in orientation. In contrast, first-order logic treats the operational and denotational aspects of general mathematics quite evenhandedly. Here we show that linear logic has models of even broader denotational scope than those of first-order logic, namely Chu spaces, the category of which Barr has observed to form a model of linear logic. We have previously argued that every category of n-ary relational structures embeds fully and concretely in the category of Chu spaces over 2^n. The main contributions of this paper are improvements to that argument, and an embedding of every small category in the category of Chu spaces via a symmetric variant of the Yoneda embedding.

A graph-theoretical look at multiplicative proof nets lead us to two new descriptions of a proof net, both as a graph endowed with a perfect matching. The first one is a rather conventional encoding of the connectives which nevertheless allows us to unify various sequentialisation techniques as the corollaries of a single graph theoretical result. The second one is more exciting: a proof net simply consists in the set of its axioms – the perfect matching – plus one single series–parallel graph which encodes the whole syntactical forest of the sequent. We thus identify proof nets which only differ because of the commutativity or associativity of the connectives, or because final *par* have been performed or not. We thus push further the program of proof net theory which is to get closer to the proof itself, ignoring as much as possible the syntactical "bureaucracy".

We prove that any classical equivalence – not using structural rules – between two classical first-order formulas – with no repetition of predicate symbols – if proved in an appropriate complete and closed fragment of classical sequent calculus LK, define a computational isomorphism up to an equivalence on derivations which builds on the reversibility properties of classical rules.

We feel this result gives the rationale behind the success story of Girard's denotational semantics for classical logic where all the standard 'linear' boolean equations are caught.

When Miller introduced Forum he called it a specification logic, rather than a logic programming language. In this paper we outline those features that create problems in attempting to implement an interpreter for the language, and describe solutions to those problems. We show how techniques used in the implementation of Lolli can be extended naturally to Forum. Finally,

we show two Forum programs in order to demonstrate some of the paradigms that arise in using the language.

Marco Pedicini
Remarks on elementary linear logic (preliminary report)
`http://www.elsevier.nl/locate/entcs/volume3.html`

Last works in linear logic mark a trend that can be summarized as: *studying of fragments* of calculi which have good *logical properties* and a quite *powerful class of representable functions.* **ELL** is a system derived from linear logic which presents several of such good properties: first of all a bound of the cut-elimination procedure which is lower w.r.t. the usual one.

Lorenzo Tortora de Falco
Generalized standardization lemma for the additives (preliminary report)
`http://www.elsevier.nl/locate/entcs/volume3.html`

This paper is concerned with the additive connectives of linear logic. We show that a careful analysis of the duplication involved in (the natural generalization of) the additive commutative elementary reduction step gives a complete proof of the standardization lemma for the additives. The same analysis suggests also some remarks on the lack of confluence of full linear logic.