Available online at www.sciencedirect.com
SCIENCE DIRECT
Physics Letters B 587 (2004) 143–149
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Anti-symmetric rank-two tensor matter field on superspace for $N_T = 2$

Wesley Spalenza [a,b,1], Wander G. Ney [a,b,c], J.A. Helayel-Neto [a,b,1]

[a] *Centro Brasileiro de Pesquisas Físicas (CBPF), Rio de Janeiro, RJ, Brazil*
[b] *Grupo de Física Teórica José Leite Lopes (GFT), Petrópoles, RJ, Brazil*
[c] *Centro Federal de Educação Tecnológica (CEFET), Campos dos Goytacazes, RJ, Brazil*

Received 15 December 2003; received in revised form 12 February 2004; accepted 26 February 2004

Editor: L. Alvarez-Gaumé

Abstract

In this work, we discuss the interaction between anti-symmetric rank-two tensor matter and topological Yang–Mills fields. The matter field considered here is the rank-2 Avdeev–Chizhov tensor matter field in a suitably extended $N_T = 2$ SUSY. We start off from the $N_T = 2$, $D = 4$ superspace formulation and we go over to Riemannian manifolds. The matter field is coupled to the topological Yang–Mills field. We show that both actions are obtained as Q-exact forms, which allows us to express the energy–momentum tensor as Q-exact observables.

1. Introduction

Topological field theories such as Chern–Simons and BF-type gauge theories probe space–time in its global structure, and this aspect has a considerable relevance in quantum field theories. On the other hand, there is a great deal of interest in anti-symmetric rank-2 tensor fields that can be put into two categories: gauge fields or matter fields. Some years ago, Avdeev and Chizhov [1–3] proposed a model where the antisymmetric tensor behaves as a matter field.

In a recent work [4], Geyer and Mülsch presented a formulation, until then unknown in the literature, which is a construction of the Avdeev–Chizhov action described in the topological formalism [5]. This was built for $N_T = 1$ and generalized for $N_T = 2$. Known the properties of the anti-symmetric rank-two tensor matter field theory, also referred to as Avdeev–Chizhov field [6], its supersymmetric properties and characteristics are presented in Ref. [7]; following this formalism, we shall write down this action in superfield formalism, in the way presented by Horne [8] in topological theories, as a Donaldson–Witten topological theories [5,9].

E-mail addresses: wesley@cbpf.br (W. Spalenza), wander@cbpf.br (W.G. Ney), helayel@cbpf.br (J.A. Helayel-Neto).
[1] Supported in part by the Conselho Nacional de Desenvolvimento Científico e Tecnológico CNPq, Brazil.

doi:10.1016/j.physletb.2004.02.057

Our goal in this work is to discuss the interaction between matter and topological Yang–Mills fields as presented by Geyer–Mülsch [4] for $N_T = 1$ and $N_T = 2$. The matter field considered here is the rank-2 tensor matter field formulated in terms of a complex self-duality condition [6]. Thus, we now write this field as an anti-symmetric rank-two tensor matter superfield in $N_T = 2$-SUSY in the superspace formalism, discussed also in [7]. The matter field is coupled to the topological Yang–Mills connection by means of the Blau–Thompson action. We express the Yang–Mills superconnection as a 2-superform in a superspace with four bosonic space–time coordinates and two fermionic dimensions described by Grassmann coordinates, and then construct the action in a superfield formalism following the definitions by Horne [8]. Next, we go over to Riemannian manifolds duely described in terms of the vierbein and the spin connection, where we take the gravitation as a background. We introduce and discuss the Wess–Zumino gauge condition induced by the shift supersymmetry, better detailed in [10]. Finally, we arrive at a topological-invariant action as the sum of the Avdeev–Chizhov‘s action coupled to the topological super-Yang–Mills action; both actions are obtained as Q-exact forms, and the energy–momentum tensor is shown to be Q-exact.

2. The $N_T = 2$ superconnection, supercurvature and shift algebra

Let us now consider the Donaldson–Witten theory, whose space of solutions is the space of self-dual instantons, $F = *F$. To follow our superfield formulation, we shall proceed with the definition of the action of Horne [8] and Blau–Thompson [13,14]. The $N_T = 2$ superfield conventions are the ones of [10]. The superfields superconnection and its associated superghosts are given as below:

$$\hat{A} = \hat{A}^a T_a, \qquad \hat{C} = \hat{C}^a T_a, \text{with } [T_a, T_b] = i f_{ab}{}^c T_c. \tag{2.1}$$

Before the presentation of our superaction formulation, we provide a few results regarding our conventions on the Grassmann coordinates. Thus the topological fermionic index: $I = 1, 2$, is lowered and raised by the anti-symmetric Levi-Civita tensor: ε_{IJ}, ε^{IJ}, with $\varepsilon^{12} = -\varepsilon_{12} = 1$. The θ-coordinate definitions: $\theta^I = \varepsilon^{IJ}\theta_J$, $\theta_I = \varepsilon_{IJ}\theta^J$, and the quadratic forms are:

$$\theta^2 = \theta^I\theta_I = -\frac{1}{2}\varepsilon^{IJ}\theta^2, \qquad \theta_I\theta_J = \frac{1}{2}\varepsilon_{IJ}\theta^2,$$

with $\varepsilon_{IK}\varepsilon^{KJ} = \delta_I{}^J$. The derivatives are:

$$\partial_I = \frac{\partial}{\partial\theta^I}, \qquad \partial^I = \frac{\partial}{\partial\theta_I} \quad \text{and} \quad \partial_I\theta^J \overset{\text{Def}}{=} \delta_I{}^J. \tag{2.2}$$

We still have the integration definition, such that $\int d\theta^I \overset{\text{Def}}{=} \partial_I$. This result is applied to a superfunction $f(x,\theta)$, so that the volume element is $Q^2 f(x,\theta) = \int d^2\theta\, f(x,\theta) = \frac{1}{4}\varepsilon^{IJ}\partial_I\partial_J f(x,\theta)$. A superfield in the topological theory of Witten's type obeys the equation: $Q_I F(x,\theta) \overset{\text{Def}}{=} \partial_I F(x,\theta)$.

We start our topological SUSY formalism by expanding the superforms (2.1) in component superfields; we have:

$$\hat{A} = A(x_\mu, \theta^I) + E_I(x_\mu, \theta^I)\, d\theta^I, \qquad \hat{C} = C(x_\mu, \theta^I), \tag{2.3}$$

with $I = 1, 2$; in component fields, it comes out as below:

$$A(x,\theta) = a(x) + \theta^I\psi_I(x) + \frac{1}{2}\theta^2\alpha(x), \tag{2.4}$$

$$E_I(x,\theta) = \chi_I(x) + \theta^I\phi_{IJ}(x) + \frac{1}{2}\theta^2\eta_I(x), \tag{2.5}$$

$$C(x,\theta) = c(x) + \theta^I c_I(x) + \frac{1}{2}\theta^2 c_F(x). \tag{2.6}$$

The associated supercurvature is defined as $\hat{F} = \hat{d}\hat{A} + \hat{A}^2 = F + \Psi_I\, d\theta^I + \Phi_{IJ}\, d\theta^I\, d\theta^J$, whose components read as follows:

$$F = f - \theta^I D_a\psi_I + \frac{1}{2}\theta^2\left(D_a\alpha + \frac{1}{2}\varepsilon^{IJ}[\psi_I, \psi_J]\right), \tag{2.7}$$

$$\Psi_I = \psi_I + D_a\chi_I + \theta^J\big(\varepsilon_{IJ}\alpha - \theta^J D_a\phi_{IJ} + \theta^J[\psi_J, \chi_I]\big) + \theta^2\left(\frac{1}{2}D_a\eta_I - \frac{1}{2}\varepsilon^{KJ}[\psi_K, \phi_{IJ}] + \frac{1}{2}[\alpha, \chi_I]\right), \tag{2.8}$$

$$\Phi_{IJ} = \frac{1}{2}\bigg\{\phi_{IJ} + \phi_{JI} + [\chi_I, \chi_J] + \theta^K\big(\varepsilon_{KI}\eta_J + \varepsilon_{JK}\eta_I + [\chi_I, \phi_{JK}] + [\phi_{IK}, \chi_J]\big) + \frac{1}{2}\theta^2\big([\chi_I, \eta_J] + [\eta_I, \chi_J] - \varepsilon^{KL}[\phi_{IK}, \phi_{JL}]\big)\bigg\}, \tag{2.9}$$

where $f = da + a^2$ and the covariant derivatives in a being given by $D_a(\cdot) = d(\cdot) + [a, (\cdot)]$; the symbol $(\cdot)$ represents any field which the derivative act upon. This formalism with $N_T = 2$, it can be found as an example in the work [11].

The SUSY weight is defined by attributing -1 to θ. Thus, the supersymmetry generators, Q, exhibit weight 1. The BRST-transformation of the superconnection (2.3) is $s\hat{A} = -\hat{d}\hat{C} - [\hat{A}, \hat{C}] = -\hat{D}_{\hat{A}}\hat{C}$, and, in component superfields, it is given by

$$sA = -dC - [A, C] = -D_A C, \qquad sE_I = -\partial_I C - [E_I, C] = -D_I C, \qquad sC = -C^2; \tag{2.10}$$

the supercovariant derivative is: $\hat{D}_{\hat{A}} = D_A + d\theta^I D_I$, where $D_I(\cdot) = \partial_I(\cdot) + [E_I, (\cdot)]$.

The supersymmetry transformations or shift symmetry transformations are defined as:

$$Q_I A = \partial_I A, \qquad Q_I E_J = \partial_I E_J, \qquad Q_I C = \partial_I C. \tag{2.11}$$

Next, we believe it is interesting to introduce and discuss a sort of Wess–Zumino gauge choice associated to the shift symmetry above, which is the topological BRST-transformation. The Wess–Zumino[2] gauge seen in [10,12], is here defined by the condition

$$\chi_I = 0 \quad \text{and} \quad \phi_{[IJ]} = 0, \tag{2.12}$$

due to the linear shift in the transformations (2.11) for scalar fields χ_I and ϕ_{IJ} respectively, with parameters given by the ghost fields, c_I and c_F. There exists now only the symmetric field $\phi_{(IJ)}$, that we write from now on simply as ϕ_{IJ}. This condition is not SUSY-invariant under Q_I, and it can be defined in terms of the infinitesimal fermionic parameter ϵ^I as: $\tilde{Q} = \epsilon^I\tilde{Q}_I$. This operator leaves the conditions (2.12) invariant, and it is built up by the combinations of Q with the BRST-transformations in the Wess–Zumino gauge, such that

$$\tilde{Q} = (s + Q)\big|_{c_I = \varepsilon^J\phi_{IJ},\ c_F = \frac{1}{2}\varepsilon^J\eta_J}. \tag{2.13}$$

The results in terms of component fields are displayed below:

$$\tilde{Q}a = -D_a c + \epsilon^I\psi_I, \qquad \tilde{Q}\psi_I = -[c, \psi_I] - \epsilon^J D_a\phi_{IJ} + \epsilon_I\alpha,$$
$$\tilde{Q}\alpha = -[c, \alpha] + \varepsilon^{IJ}\epsilon^K[\phi_{Ik}, \psi_J] - \frac{1}{2}\epsilon^I D_a\eta_I, \qquad \tilde{Q}\phi_{IJ} = -[c, \phi_{IJ}] + \frac{1}{2}(\epsilon_I\eta_J + \epsilon_J\eta_I),$$
$$\tilde{Q}\eta_I = -[c, \eta_I] + \varepsilon^{JK}\epsilon^M[\phi_{JM}, \phi_{IK}], \qquad \tilde{Q}c = -c^2 + \epsilon^I\epsilon^J\phi_{IJ}, \tag{2.14}$$

[2] This name is given since we are dealing with a linear gauge and scalar ghost field.

in agreement with the transformation found in the works of [14,15]; the nilpotency reads as

$$(\tilde{Q})^2 \propto \delta_{\phi_{IJ}}, \tag{2.15}$$

that is, an infinitesimal transformation of ϕ_{IJ}. With the result of the previous section, we are ready to write down the Blau–Thompson action, which is the invariant Yang–Mills action for the topological theory.

3. The Blau–Thompson action

The associated action for $N_T = 2$, $D = 4$ is the Witten's action [8,15,16], described in $N_T = 2$ by the Blau–Thompson action [13,14], with gauge completely fixed in terms of the superfield. For the construction of this action, we need a Lagrange multiplier that couples to the topological super-Yang–Mills sector, so as to manifest its self-duality: $F = *F$. We then define a 2-form-superfield Lagrange multiplier, with the property of anti-self-duality and supergauge covariant: $sK = -[C, K]$, such that

$$K(x,\theta) = k(x) + \theta^I k_I(x) + \frac{1}{2}\theta^2 \kappa(x).$$

We still wish a quadratic term in the last component field of K. For that, we still need a 0-form-superfield to complete the gauge-fixing for Ψ_I, which is defined as:

$$H_I(x,\theta) = h_I(x) + \theta^J h_{JI}(x) + \frac{1}{2}\theta^2 \rho_I(x). \tag{3.1}$$

To fix the super-Yang–Mills gauge, we define an anti-ghost superfield for C, being a 0-form-superfield and their Lagrange multiplier

$$\bar{C}(x,\theta) = \bar{c}(x) + \theta^I \bar{c}_I(x) + \frac{1}{2}\theta^2 \bar{c}_F(x), \qquad B(x,\theta) = b(x) + \theta^I b_I(x) + \frac{1}{2}\theta^2 \beta(x). \tag{3.2}$$

Their BRST-transformations are $s\bar{C} = B$, $sB = 0$. Therefore, the complete Blau–Thompson action in superspace takes the form

$$S_{\mathrm{BT}} = \int d^2\theta \sqrt{g}\, \mathrm{Tr}\big\{K * F + \zeta K * D_\theta^2 K + \varepsilon^{IJ} H_I D_A * \Psi_J + s(\bar{C} d * A)\big\}, \tag{3.3}$$

with ζ being constant and g is the background metric of the Riemannian manifold.

In the next section, we shall discuss the Avdeev–Chizhov action in a general Riemannian manifold with the same background metric.

4. Tensorial matter in a general Riemannian manifold

To couple the theory above to the Avdeev–Chizhov model, we start by describing the Avdeev–Chizhov action through the complex self-dual field φ [6], initially written in the 4-dimensional Minkowskian manifold, whose indices are: $m, n, \ldots$. We write this action, according to the work of [6], as

$$S_{\mathrm{matter}} = \int d^4x \,\big\{\big(D^m \varphi_{mn}\big)^\dagger \big(D_p \varphi^{pn}\big) + q\big(\varphi^\dagger_{mn} \varphi^{pn} \varphi^{\dagger mq} \varphi_{pq}\big)\big\}. \tag{4.1}$$

Here, q is a coupling constant for the self-interaction, and the covariant derivative $D_a^m \varphi_{mn} = \partial^m \varphi_{mn} - [a^m, \varphi_{mn}]$; a^m is the Lie-algebra-valued gauge potential and we assume φ_{mn} to belong a given representation of the gauge group G. This action is invariant under the following transformations:

$$\delta_G(\omega) a_m = D_m \omega, \qquad \delta_G(\omega)\varphi_{mn} = \varphi_{mn}\omega, \qquad \delta_G(\omega)\varphi^\dagger_{mn} = -\omega\varphi^\dagger_{mn}, \tag{4.2}$$

with φ given by

$$\varphi_{mn} = T_{mn} + i\tilde{T}_{mn}, \tag{4.3}$$

which exhibit the properties $\varphi_{mn} = i\tilde{\varphi}_{mn}$, $\tilde{\tilde{\varphi}}_{mn} = -\varphi_{mn}$, where the duality is defined by $\tilde{\varphi}_{mn} = \frac{1}{2}\varepsilon_{mnpq}\varphi^{pq}$.

To formulate this theory on a general Riemannian manifold as a topological theory, Geyer–Mülsch [4] rewrite it in a four-dimensional Riemannian manifold, endowed of the vierbein $e_\mu{}^m$ and a spin-connection ω_μ^{mn}, i.e., the tensorial matter read as $\varphi_{\mu\nu} = e_\mu{}^m e_\nu{}^n \varphi_{mn}$, where the action (4.1) is given in terms of the $\varphi_{\mu\nu}$ and $\varphi^\dagger_{\mu\nu}$ now. In this 4-dimensional Riemannian manifold, we find the following properties:

$$\sqrt{g}\,\varepsilon_{\mu\nu\rho\lambda}\varepsilon^{mnpq} = e_{[\mu}{}^m e_\nu{}^n e_\rho{}^p e_{\lambda]}{}^q, \qquad e_\mu{}^m e_\nu{}^n g^{\mu\nu} = \eta^{mn}, \qquad e_\mu{}^m e_\nu{}^n \eta_{mn} = g_{\mu\nu}. \tag{4.4}$$

The covariant derivative in the Riemannian manifold is now written in terms of the spin-connection:

$$\nabla_\mu = D_\mu + \omega_\mu, \tag{4.5}$$

where $\omega_\mu = \frac{1}{2}\omega_\mu{}^{mn}\sigma_{mn}$, being σ_{mn} the generator of the holonomy Euclidean group $SO(4)$, also we have: $D_\mu = (D_a)_\mu$, where, a, is the Yang–Mills connection.

5. Supersymmetrization of the Avdeev–Chizhov action

From now on, we can write the action (4.1) in a Riemannian manifold in terms of superfields, mentioning the conventions of the works [8,10]. The superfield that accommodates the rank-two anti-symmetric tensorial matter field is similar to the one defined in [7], being now expressed as a linear fermionic supermultiplet. This is defined as a rank-two anti-symmetric tensor in the 4-dimensional Riemannian manifold, with the topological fermionic index, I, referring to the topological SUSY index:

$$\Sigma^I_{\mu\nu}(x,\theta) = \lambda^I_{\mu\nu}(x) + \theta^I\varphi_{\mu\nu}(x) + \frac{1}{2}\theta^2\zeta^I_{\mu\nu}(x), \tag{5.1}$$

where $\varphi_{\mu\nu}(x)$ is the Avdeev–Chizhov field. The supermanifold is composed by Riemannian manifold and the $N_T = 2$ topological manifold.

The superfield is defined under the SUSY transformations: $Q_I\Sigma_{\mu\nu J} = \partial_I\Sigma_{\mu\nu J}$, and in components:

$$Q_I\lambda_{\mu\nu J} = \varepsilon_{IJ}\varphi_{\mu\nu}, \qquad Q_I\varphi_{\mu\nu} = -\zeta_{\mu\nu I}, \qquad Q_I\zeta_{\mu\nu J} = 0. \tag{5.2}$$

Based on the work of Ref. [6], we rewrite the BRST-transformations, referring the non-Abelian Avdeev–Chizhov model. We wish to write the BRST-transformation for a supergauge transformation, generalizing the transformations for the Avdeev–Chizhov fields, according to

$$s\big(\Sigma^I_{\mu\nu}\big) = iC\big(\Sigma^I_{\mu\nu}\big), \qquad s\big(\Sigma^I_{\mu\nu}\big)^\dagger = iC\big(\Sigma^I_{\mu\nu}\big)^\dagger. \tag{5.3}$$

The superderivative of (5.1) is covariant under the BRST-transformation. This new covariant derivative reads as below:

$$\mathcal{D}_\mu(\cdot) = (D_A)_\mu(\cdot) + \omega_\mu(\cdot) = \nabla_\mu(\cdot) + \theta^I[\psi_{I\mu},(\cdot)] + \frac{1}{2}\theta^2[\alpha_\mu,(\cdot)],$$

according to (4.5), this yields

$$s\big(\mathcal{D}_\mu\Sigma^I_{\mu\nu}\big) = C\big(\mathcal{D}_\mu\Sigma^I_{\mu\nu}\big) \quad \text{and} \quad s\big(D_I\Sigma^I_{\mu\nu}\big) = C\big(D_I\Sigma^I_{\mu\nu}\big),$$

where we have chosen here, $s\omega_\mu = 0$.

By now performing BRST-transformations on the components that survive in the $N_T = 2$ Wess–Zumino gauge (2.13), we find:

$$\begin{aligned}
&\tilde{Q}\lambda_{\mu\nu I} = \epsilon^J \varepsilon_{JI}\varphi_{\mu\nu} + ic\lambda_{\mu\nu I}, \qquad \tilde{Q}\lambda^\dagger_{\mu\nu I} = \epsilon^J \varepsilon_{JI}\varphi^\dagger_{\mu\nu} - ic\lambda^\dagger_{\mu\nu I},\\
&\tilde{Q}\varphi_{\mu\nu} = ic\varphi_{\mu\nu} + i\epsilon^I\zeta_{\mu\nu I} + i\epsilon^I\phi_{IJ}\lambda^J_{\mu\nu}, \qquad \tilde{Q}\varphi^\dagger_{\mu\nu} = -ic\varphi^\dagger_{\mu\nu} - i\epsilon^I\zeta^\dagger_{\mu\nu I} - i\epsilon^I\phi_{IJ}\lambda^{\dagger J}_{\mu\nu},\\
&\tilde{Q}\zeta_{\mu\nu I} = ic\zeta_{\mu\nu I} - i\epsilon^J\phi_{JI}\varphi_{\mu\nu} + i\epsilon^J\eta_J\lambda_{\mu\nu I}, \qquad \tilde{Q}\zeta^\dagger_{\mu\nu I} = -ic\zeta^\dagger_{\mu\nu I} + i\epsilon^J\phi_{JI}\varphi^\dagger_{\mu\nu} - i\epsilon^J\eta_J\lambda^\dagger_{\mu\nu I},
\end{aligned} \tag{5.4}$$

in agreement to (2.15).

We build up rank-two anti-symmetric tensorial matter field in a superspace formulation, leaving the superfield with the same properties as shown in [7]; this is invariant under gauge transformations (5.4) and SUSY transformations. The total action is finally expressed by the equation that follows:

$$S_{\mathrm{AC}} = -\int d^2\theta\,\sqrt{g}\big\{\varepsilon^{IJ}\big(\mathcal{D}_\mu\Sigma^{\mu\nu}_I\big)^\dagger\big(\mathcal{D}_\rho\Sigma^\rho{}_{\nu J}\big) + q\varepsilon^{IJ}\varepsilon^{LM}(\Sigma_{\mu\nu I})^\dagger D^K\big(\Sigma^{\rho\nu}_J\big)\big(\Sigma^{\mu\lambda}_L\big)^\dagger D_K(\Sigma_{\rho\lambda M})\big\}, \tag{5.5}$$

where q is a quartic coupling constant. It is invariant under conformal transformations.

Therefore, the total gauge-invariant action can be written as: $S_{\mathrm{AC}} + S_{\mathrm{BT}}$. We could also have replaced S_{BT} by the super-BF action described in the work of Ref. [11].

The Q-exactness of the total action above is also true for $N_T = 2$ SUSY, as in [4]; this is so because the fermionic volume element reads as $Q^2 \propto Q_1Q_2$, which means the exactness in the charges Q_1, Q_2 of this action. This proof is true for $N_T = 1$ and it can be extended to a general N_T, as it can be seen in the works of Ref. [10], where the total action is also s-exact. According to Blau–Thompson in their review [17], the energy–momentum tensor $\Theta_{\mu\nu}$ is also Q-exact,

$$\mathcal{O} = \langle 0|\Theta_{\mu\nu}|0\rangle = \langle 0|\frac{2}{\sqrt{g}}\frac{\delta}{\delta g^{\mu\nu}}(S_{\mathrm{BT}} + S_{\mathrm{AC}})|0\rangle = \langle 0|Q\Upsilon_{\mu\nu}|0\rangle, \tag{5.6}$$

ensuring the topological nature of the theory, where we shall just use the Avdeev–Chizhov kinetic term, because the interaction term carries the coupling constant q, which is irrelevant for the attainment of the observables of the theory [4].

6. Concluding remarks

The main goal of this Letter is the settlement of a topological superspace formulation for the investigation of the coupling between the rank-two Avdeev–Chizhov matter field and Yang–Mills fields. It comes out that the stress tensor is Q-exact. This opens us the way for the identification of a whole class of observables that we are trying to classify [19].

It is worthwhile to draw the attention here to the shift symmetry that allows us to detect the ghost character of the Avdeev–Chizhov field. On the other hand, it is known that there appears a ghost mode in the spectrum of excitations of our tensor matter field [1]. The connection between these two observations remain to be clarified. The fact that the Avdeev–Chizhov field manifests itself as a ghost guides future developments in the quest for a consistent mechanism to systematically decouple the unphysical mode mentioned above.

We are also trying to embed the tensor field in the framework of a gauge theory with Lorentz symmetry breaking [18]. We expect that this breaking may identify the right ghost mode present among the two spin-1 components of the Avdeev–Chizhov field.

A relevant question which remains to be answered concerns the eventual appearance of new topological observables, once the Avdeev–Chizhov matter field is coupled to the Yang–Mills sector. In this coupled model, an inspection of the Witten's descent equations would reveal the emergence of new observables, which would give even more legitimacy to the coupling we have written down. We should seek, in the Avdeev–Chizhov sector, a

BRST-invariant field which would correspond to the Donaldson–Witten theory. This is the subject of a present investigation and it will be reported on in a forthcoming paper [20].

Acknowledgements

We thank Álvaro Nogueira, Clisthenis P. Constantinidis, José L. Boldo, and Daniel H.T. Franco for many useful discussions, and Prof. Olivier Piguet for the great help and encouragement.

References

[1] L.V. Avdeev, M.V. Chizhov, A queer reduction of degrees of freedom, preprint JINR Dubna, hep-th/9407067.
[2] M.V. Chizhov, Phys. Lett. B 381 (1996) 359, hep-ph/9511287.
[3] L.V. Avdeev, M.V. Chizhov, Phys. Lett. B 321 (1994) 212, hep-th/9312062.
[4] B. Geyer, D. Mülsch, Phys. Lett. B 535 (2002) 349.
[5] E. Witten, Int. J. Mod. Phys. A 6 (1991) 2775;
E. Witten, Commun. Math. Phys. 117 (1988) 353.
[6] V.E.R. Lemes, R. Renan, S.P. Sorella, Phys. Lett. B 352 (1995) 37.
[7] V.E.R. Lemes, A.L.M.A. Nogueira, J.A. Heläyel-Neto, Int. J. Mod. Phys. A 13 (18) (1998) 3145, hep-th/9508045.
[8] J.H. Horne, Nucl. Phys. B 318 (1989) 22.
[9] S. Donaldson, J. Diff. Geom. 30 (1983) 289;
S. Donaldson, Topology 29 (1990) 257.
[10] J.L. Boldo, C.P. Constantinidis, F. Gieres, M. Lefrançois, O. Piguet, hep-th/0303053;
J.L. Boldo, C.P. Constantinidis, F. Gieres, M. Lefrançois, O. Piguet, hep-th/0303084, Int. J. Mod. Phys. A 18 (2003) 2119.
[11] C.P. Constantinidis, O. Piguet, W. Spalenza, hep-th/0310184.
[12] J. Wess, J. Bagger, Supersymmetry and Supergravity, second ed., Princeton Univ. Press, Princeton, NJ, 1992.
[13] M. Blau, G. Thompson, Commun. Math. Phys. 152 (1993) 41, hep-th/9112012.
[14] M. Blau, G. Thompson, Nucl. Phys. B 492 (1997) 545, hep-th/9612143.
[15] B. Geyer, D. Mülsch, Nucl. Phys. B 616 (2001) 476.
[16] C. Vafa, E. Witten, Nucl. Phys. B 431 (1994) 3, hep-th/9408074.
[17] D. Birmingham, M. Blau, M. Rakowski, G. Thompson, Phys. Rep. 209 (1991) 129.
[18] J.A. Helayel-Neto, W.G. Ney, W. Spalenza, in preparation.
[19] J.L. Boldo, J.A. Nogueira, C.P. Constantinidis, O. Piguet, W. Spalenza, in preparation.
[20] J.A. Helayel-Neto, W.G. Ney, W. Spalenza, in preparation.