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# Single transverse spin asymmetry of prompt photon production

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# ABSTRACT

We study the single transverse spin asymmetry of prompt photon production in high energy protonproton scattering. We include the contributions from both the direct and fragmentation photons. While the asymmetry for direct photon production receives only the Sivers type of contribution, the asymmetry for fragmentation photons receives both the Sivers and Collins types of contributions. We make a model calculation for quark-to-photon Collins function, which is then used to estimate the Collins asymmetry for fragmentation photons. We find that the Collins asymmetry for fragmentation photons is very small, thus the single transverse spin asymmetry of prompt photon production is mainly coming from the Sivers asymmetry in direct and fragmentation photons. We make predictions for the prompt photon spin asymmetry at RHIC energy, and emphasize the importance of such a measurement. The asymmetry of prompt photon production can provide a good measurement for the important twist-three quark-gluon correlation function, which is urgently needed in order to resolve the "sign mismatch" puzzle.

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#### 1. Introduction

Single spin asymmetries (SSAs) in transversely polarized proton-proton collisions have provided essential information on the internal partonic structure of the proton, particularly the parton's transverse motion in the transversely polarized proton [1]. Two different yet related QCD factorization formalisms have been proposed to describe the observed asymmetries: the transverse momentum dependent (TMD) factorization [2–6] and the collinear twist-three factorization approaches [7–13].

For processes such as semi-inclusive hadron production in lepton–proton deep inelastic scattering (SIDIS)  $\ell p^{\uparrow} \rightarrow \ell' h X$  which are characterized by both the photon virtuality  $Q^2$  and hadron transverse momentum  $P_{h\perp}$  such that  $Q \gg P_{h\perp} \sim \Lambda_{QCD}$ , one describes the SSAs in the TMD factorization formalism. In this approach the transverse spin effects are associated with naive time-reversal-odd TMDs which represent helicity flip cut quark target scattering amplitudes with a non-trivial color phase [14]. Two well-known TMDs are the quark Sivers function [15] and Collins function [16], which describe the so-called  $\sin(\phi_h - \phi_s)$  and  $\sin(\phi_h + \phi_s)$  modulations in SIDIS on transversely polarized target, respectively. Because of the different angular modulations in the cross section, one can separate Sivers from Collins effect in SIDIS and thus extract them independently from the experimental data

[17–19]. On the other hand, for single inclusive hadron production in proton–proton scattering  $p^{\uparrow}p \rightarrow hX$  where there is a single hard scale given by the hadron's transverse momentum,  $P_{h\perp} \gg \Lambda_{\rm OCD}$ , one can describe the SSAs in the collinear twist-three factorization approach in terms of either the twist-three quark-gluon correlation functions in the transversely polarized proton [20,21], or the twist-three fragmentation functions in the hadronization process [13]. In the twist-three formalism we refer to the former contribution as Sivers effect, and the latter one as Collins effect, since they represent the collinear version of these two effects (based on the operator definitions relating the first  $k_T$ -moments of the Sivers and Collins functions in the collinear twist-three approach [5]). While the most abundant experimental data exist [22,23] on transverse spin effects in SSAs of single inclusive hadron production in proton-proton collisions, disentangling Sivers and Collins contributions presents a significant experimental challenge, thus the true origin (the relative contributions from these two effects) for the inclusive hadron production still remains elusive [24].

Theoretically it has been found that these two formalisms are closely related to each other, and it is shown that they are equivalent in the overlapping transverse momentum region where both can apply [25–27]. However, it has been recently observed that the experimental proton–proton data on the SSAs of the inclusive hadron production appears incompatible with the Sivers data from SIDIS process [28–30], if one assumes that the SSAs of the inclusive hadron production come entirely from the Sivers contribution. This is known as the "sign mismatch". Whether this finding reflects the inconsistency of our theoretical formalism is a very

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important question and needs to be further explored both theoretically and experimentally. Since the inclusive hadron production has the complication from the Collins contribution, the measurement for the SSAs of single inclusive jet and direct photon production in proton–proton collisions [28,31] could be very helpful in studying the sign mismatch, as they are free of complication from the fragmentation process (or the Collins effect).

Even though direct photon production is ideal in the theoretical sense for further exploring this "sign mismatch", there are no true direct photons. Direct photons and fragmentation photons are two indistinguishable contributions in the usual collinear factorization formalism [32], which are designated "prompt" photons. In experiments one might apply the photon isolation cut to reduce the fragmentation contribution, however the asymmetry measurement might suffer from the low photon event rates after such a cut. In any case, it is important to assess how the fragmentation contribution might affect the asymmetry of the prompt photons. This is the main purpose of our Letter. While the direct photons receive only the Sivers type of contribution for the asymmetry, the fragmentation photons could receive both the Sivers and Collins contributions. We perform a model calculation for the quark-tophoton Collins function, which is then used to estimate the Collins asymmetry for fragmentation photons.

The rest of our Letter is organized as follows. In Section 2, we give the overview on the various sources for the SSAs of prompt photon production. In Section 3, we present our detailed model calculation for the quark-to-photon unpolarized fragmentation function and Collins function, and estimate their relative size. In Section 4, we make phenomenological study for the SSAs of prompt photon production by including all the sources studied in our Letter. We conclude our Letter in Section 5.

# 2. Single transverse spin asymmetry of prompt photon production

#### 2.1. Unpolarized prompt photon production

We consider the prompt photon production in hadronic collisions,  $A(P_A, s_\perp) + B(P_B) \rightarrow \gamma(P_\gamma) + X$ . Here *A* is a transversely polarized proton with spin vector  $s_\perp$ , and *B* is an unpolarized proton. The spin-averaged differential cross section of prompt photon production contains both direct and fragmentation contributions,

$$E_{\gamma} \frac{d\sigma}{d^3 P_{\gamma}} = E_{\gamma} \frac{d\sigma^{\text{dir}}}{d^3 P_{\gamma}} + E_{\gamma} \frac{d\sigma^{\text{frag}}}{d^3 P_{\gamma}}.$$
 (1)

At leading order, the direct contribution is given by

$$E_{\gamma} \frac{d\sigma^{\text{dir}}}{d^{3} P_{\gamma}} = \frac{\alpha_{\text{em}} \alpha_{\text{s}}}{s} \sum_{a,b} \int \frac{dx'}{x'} f_{b/B}(x') \int \frac{dx}{x} f_{a/A}(x) \\ \times H^{U}_{ab \to \gamma}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$
(2)

where  $s = (P_A + P_B)^2$ ,  $f_{a/A}(x)$  and  $f_{b/B}(x')$  are the spin-averaged parton distribution functions,  $\hat{s}$ ,  $\hat{t}$ , and  $\hat{u}$  are the usual Mandelstam variables at the parton level.  $H^U_{ab \rightarrow \gamma}$  are the well-known partonic hard-scattering functions for direct photon production [33,34]. At the leading order, they are calculated from the partonic channels  $qg \rightarrow \gamma q$  and  $q\bar{q} \rightarrow \gamma g$ , and the typical Feynman diagrams are shown in Fig. 1.

For fragmentation photons, in the usual collinear factorization formalism at leading order, we have  $2 \rightarrow 2$  scattering process to produce a parton which then fragments into a photon, with the typical Feynman diagrams shown in Fig. 2. The differential cross section is given by



**Fig. 1.** Typical Feynman diagrams for direct photon production at leading order: left for  $qg \rightarrow \gamma q$  and right for  $q\bar{q} \rightarrow \gamma g$ .



Fig. 2. Typical Feynman diagrams for fragmentation photon production at leading order.



**Fig. 3.** The direct ratio defined in Eq. (4) is plotted as a function of Feynman  $x_F$  at y = 3.5 and  $\sqrt{s} = 200$  GeV. The solid line is for leading order calculation, while the dashed line is for next-to-leading order calculation.

$$E_{\gamma} \frac{d\sigma^{\text{trag}}}{d^{3}P_{\gamma}} = \frac{\alpha_{s}^{2}}{s} \sum_{a,b,c} \int \frac{dz}{z^{2}} D_{c \to \gamma}(z) \int \frac{dx'}{x'} f_{b/B}(x')$$
$$\times \int \frac{dx}{x} f_{a/A}(x) H^{U}_{ab \to c}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}), \qquad (3)$$

where  $D_{c \to \gamma}(z)$  is the quark-to-photon fragmentation function, and  $H^U_{ab\to c}$  are the well-known partonic cross section to produce a parton [33,35].

To see the relative contributions of direct and fragmentation photons, we define the following direct ratio

$$R = \frac{E_{\gamma} \frac{d\sigma^{\text{dir}}}{d^3 P_{\gamma}}}{E_{\gamma} \frac{d\sigma^{\text{dir}}}{d^3 P_{\gamma}} + E_{\gamma} \frac{d\sigma^{\text{frag}}}{d^3 P_{\gamma}}}.$$
(4)

In Fig. 3, we plot the direct ratio *R* as a function of Feynman  $x_F$  at forward rapidity y = 3.5 at RHIC energy  $\sqrt{s} = 200$  GeV. We give the result for both the leading order and next-to-leading order calculations [32]. We find that the fragmentation photons actually contributes to around 50% to the total prompt photon production. Thus it is important to assess the effect of fragmentation photons on the asymmetry of the prompt photon production.

#### 2.2. Spin-dependent cross section for prompt photon production

In order to compute the asymmetry of prompt photon production, we need the spin-dependent cross section  $\Delta\sigma(s_{\perp}) = [\sigma(s_{\perp}) - \sigma(-s_{\perp})]/2$ , which will also contain both direct and fragmentation contributions,

$$E_{\gamma} \frac{d\Delta\sigma}{d^{3}P_{\gamma}} = E_{\gamma} \frac{d\Delta\sigma^{\text{dir}}}{d^{3}P_{\gamma}} + E_{\gamma} \frac{d\Delta\sigma^{\text{frag}}}{d^{3}P_{\gamma}}.$$
(5)

The direct contribution contains only the Sivers type of effect, as given by [20,28]

$$E_{\gamma} \frac{d\Delta\sigma^{\text{dir}}}{d^{3}P_{\gamma}} = \epsilon_{\alpha\beta} s_{\perp}^{\alpha} P_{\gamma\perp}^{\beta} \frac{\alpha_{\text{em}} \alpha_{s}}{s} \sum_{a,b} \int \frac{dx'}{x'} f_{b/B}(x')$$
$$\times \int \frac{dx}{x} \bigg[ T_{a,F}(x,x) - x \frac{d}{dx} T_{a,F}(x,x) \bigg]$$
$$\times \frac{1}{\hat{u}} H_{ab \to \gamma}^{\text{dir}}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}), \tag{6}$$

where the hard-part functions  $H_{ab\to\gamma}^{\text{dir}}$  contain the relevant initialstate interactions between the active parton and the remnant of the proton and have the expressions given in [20,30].  $T_{q,F}(x, x)$  is the twist-three quark-gluon correlation function, and it is related to the quark Sivers function  $f_{1T}^{\perp q}(x, k_1^2)$  as follows [5]

$$T_{q,F}(x,x) = -\int d^2k_{\perp} \frac{|k_{\perp}|^2}{M} f_{1T}^{\perp q}(x,k_{\perp}^2) \big|_{\text{SIDIS}},$$
(7)

where the subscript "SIDIS" here is to emphasize the Sivers function probed in SIDIS process. On the other hand, the spin asymmetry of fragmentation photons can receive both Sivers and Collins contributions,

$$E_{\gamma} \frac{d\Delta\sigma^{\text{frag}}}{d^{3}P_{\gamma}} = E_{\gamma} \frac{d\Delta\sigma^{\text{frag}}_{\text{Sivers}}}{d^{3}P_{\gamma}} + E_{\gamma} \frac{d\Delta\sigma^{\text{frag}}_{\text{Collins}}}{d^{3}P_{\gamma}}.$$
(8)

The Sivers contribution can be written as

$$E_{\gamma} \frac{d\Delta \sigma_{\text{Sivers}}^{\text{rrag}}}{d^{3} P_{\gamma}} = \epsilon_{\alpha\beta} s_{\perp}^{\alpha} P_{\gamma \perp}^{\beta} \frac{\alpha_{s}^{2}}{s} \sum_{a,b,c} \int \frac{dz}{z^{2}} D_{c \to \gamma}(z) \int \frac{dx'}{x'} f_{b/B}(x')$$
$$\times \int \frac{dx}{x} \bigg[ T_{a,F}(x,x) - x \frac{d}{dx} T_{a,F}(x,x) \bigg]$$
$$\times \frac{1}{z\hat{u}} H_{ab \to c}^{\text{Sivers}}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}), \tag{9}$$

where  $H_{ab \rightarrow c}^{\text{Sivers}}$  represents a hard-part functions for the partonic process  $ab \rightarrow cd$ , and it incorporates both the initial and final state interactions and has the expressions given in [20,28]. The Collins contribution for an inclusive hadron production has been calculated in [13], which is related to a convolution of quark transversity and quark-to-hadron twist-three fragmentation function. The only difference for fragmentation photons lies in the quark-to-photon twist-three fragmentation function, and the differential cross section is given by

$$E_{\gamma} \frac{d\Delta\sigma_{\text{Collins}}^{\text{frag}}}{d^{3}P_{\gamma}} = \epsilon_{\alpha\beta} s_{\perp}^{\alpha} P_{\gamma\perp}^{\beta} \frac{\alpha_{s}^{2}}{s} \sum_{a,b,c} \int \frac{dx}{x} h_{a}(x) \int \frac{dx'}{x'} f_{b}(x')$$

$$\times \int \frac{dz}{z} \left[ -z \frac{\partial}{\partial z} \left( \frac{\hat{H}_{c}(z)}{z^{2}} \right) \right]$$

$$\times \left[ \frac{1}{z} \frac{x - x'}{x(-\hat{u}) + x'(-\hat{t})} \right]$$

$$\times H_{ab \rightarrow c}^{\text{Collins}}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}), \qquad (10)$$

where  $h_a(x)$  is the quark transversity, and  $\hat{H}_c(z)$  is the twist-three quark-to-photon fragmentation function and is related to the first  $p_T$ -moment of the Collins function  $H_1^{\perp q}(z, p_T^2)$ ,

$$\hat{H}_{q}(z) = -\frac{1}{z} \int d^{2} p_{T} \, p_{T}^{2} \, H_{1}^{\perp q}(z, p_{T}^{2}), \tag{11}$$

with  $H_1^{\perp q}(z, p_T^2)$  defined in the next section. The relevant hard-part function  $H_{ab\to c}^{\text{Collins}}$  has been computed in [13].

Eventually the single transverse spin asymmetry  $A_N$  is computed from the following definition

$$A_N = \frac{E_{\gamma} \frac{d\Delta\sigma}{d^3 P_{\gamma}}}{E_{\gamma} \frac{d\sigma}{d^3 P_{\gamma}}},\tag{12}$$

where the spin-dependent and spin-averaged cross sections are given in Eqs. (1) and (5), respectively. To calculate  $A_N$  numerically, we need the information for the twist-three quark-gluon correlation function  $\hat{T}_{q,F}(x, x)$  and twist-three quark-to-photon fragmentation function  $\hat{H}_q(z)$ . The information of  $T_{q,F}(x, x)$  has been directly extracted from the proton–proton data [20,21], or indirectly from the SIDIS data by using Eq. (7) [36,37]. However, the information of  $\hat{H}_q(z)$  is completely unknown. To estimate the size of  $\hat{H}_q(z)$  will be the main focus of the next section.

### 3. Quark to photon Collins function

In this section, we perform model calculations for photon fragmentation functions, including both the unpolarized fragmentation function and the Collins function. We first study the transverse momentum dependent quark-to-photon fragmentation functions, and then integrate over the transverse momentum to obtain the relevant unpolarized collinear fragmentation function  $D_{q \rightarrow \gamma}(z)$ and the collinear twist-three fragmentation function  $\hat{H}_q(z)$ .

#### 3.1. Transverse momentum dependent fragmentation functions

Photon fragmentation function can be calculated from the correlation function  $\Delta(z, k_T)$  [4,5,38],

$$\Delta(z, k_T) = \frac{1}{2z} \sum_X \int \frac{d\xi^+ d^2 \xi_T}{(2\pi)^3} e^{ik \cdot \xi} \langle 0|\psi_q(\xi)|\gamma X \rangle$$
$$\times \langle \gamma X|\bar{\psi}_q(0)|0\rangle|_{\xi^-=0}, \qquad (13)$$

where the usual gauge link is suppressed, and we have assumed that the photon is moving in -z direction with momentum  $p^{\mu} = p^{-}n^{\mu}$  and light-cone vector  $n^{\mu} = [0^{+}, 1^{-}, 0_{\perp}]$ . The fragmenting quark has momentum k, with  $k^{-} = p^{-}/z$  and  $k_{T}$  the transverse component with respect to the photon momentum p. We define  $p_{T}$  as the photon transverse momentum with respect to the quark, which is related to  $k_{T}$  as:  $\vec{p}_{T} = -z\vec{k}_{T}$ . Here for our purpose we only keep the terms relevant to the quark to *unpolarized* photon fragmentation. Then the correlation function  $\Delta(z, k_{T})$  is given by [4,5,39],

 $D_{q \to \gamma}(z, p_T^2)$  is the usual unpolarized quark-to-photon fragmentation function, and  $H_1^{\perp q}(z, p_T^2)$  is the quark-to-photon Collins function in agreement with the "Trento conventions" [39]. For the most general case where photon's polarization is also specified, there are more terms in the expansion [38]. We can easily project out these functions

$$D_{q \to \gamma}\left(z, p_T^2\right) = \frac{1}{2} \operatorname{Tr}\left[\Delta(z, k_T) \bar{p}\right],\tag{15}$$

$$\epsilon_T^{\mu\nu} k_{T\nu} H_1^{\perp q} (z, p_T^2) = \frac{1}{2} \operatorname{Tr} [\Delta(z, k_T) i \sigma^{\mu\nu} \bar{n}_\nu \gamma^5], \tag{16}$$

where  $\bar{n}^{\mu} = [1^+, 0^-, 0_{\perp}]$  is a light-cone vector conjugate to  $n^{\mu}$ .



**Fig. 4.** The Feynman diagram which contribute to the unpolarized quark-to-photon fragmentation function  $D_{q \rightarrow \gamma}(z, p_T^2)$ .

In our model, the tree-level diagram describing the fragmentation of a quark into a real photon is depicted in Fig. 4. By contrast with the pion fragmentation calculations [40–42], the interaction between quark and the photon is described by a simple point interaction with coupling  $ie_q e \gamma^{\mu}$  and  $e_q$  the quark fractional charge. In the actual calculations, we will choose light-cone gauge  $\bar{n} \cdot A_{\rm em} = 0$  for the photon field [43] to avoid photon eikonal phase [44]. On the other hand, we still use the covariant gauge for the gluon field, thus eikonal phase for gluon field still exists in our calculations. In such a set-up, we have only one Feynman diagram (at leading order) for unpolarized quark-to-photon fragmentation function  $D_{q \to \gamma'}(z, p_T^2)$ , as shown in Fig. 4. Thus, the photon polarization sum is given by

$$\sum_{\lambda} \epsilon^{\mu}(p,\lambda) \epsilon^{*\nu}(p,\lambda) = -g^{\mu\nu} + \frac{p^{\mu}\bar{n}^{\nu} + p^{\nu}\bar{n}^{\mu}}{\bar{n} \cdot p}.$$
 (17)

The calculation is straightforward, and we obtain

$$D_{q \to \gamma}(z, p_T^2) = e_q^2 \frac{\alpha_{\rm em}}{2\pi^2} \frac{1}{z^2(1-z)} \left[ \frac{1+(1-z)^2}{k^2 - m_q^2} - \frac{2zm_q^2}{(k^2 - m_q^2)^2} \right],$$
(18)

where  $\alpha_{em}$  is the electro-magnetic coupling constant,  $m_q$  is the quark mass.  $k^2$  is the virtuality of the fragmenting quark, and it is related to photon transverse momentum  $p_T$  as follows:

$$k^{2} = \frac{p_{T}^{2}}{z(1-z)} + \frac{m_{q}^{2}}{1-z}.$$
(19)

The Collins function receives contributions only from the interference between two amplitudes with different imaginary parts. Since the tree-level amplitude is real, the necessary imaginary parts will be generated by the inclusion of one-loop corrections. Here we study the case of gluon loops [40–42,45]. The relevant Feynman diagrams are given by Fig. 5. The double line in Fig. 5(c) and (d) represents the eikonalized propagator, which give rise to the factor  $1/(-\bar{n} \cdot \ell \pm i\epsilon)$  [40,41]. The calculations are much more involved than the unpolarized fragmentation function, but nevertheless similar to those for the quark-to-pion Collins functions calculated in [40,41]. In particular we note that the contribution from Fig. 5(d) are due to poles on the gluon and incoming quark [40,41, 45] signaling that the photon Collins function is universal [45,46]. Here we give only the final results,

$$H_{1}^{\perp q}(z, p_{T}^{2}) = e_{q}^{2} \frac{\alpha_{\rm em}}{2\pi^{2}} \frac{m_{q}}{k^{2} - m_{q}^{2}} \alpha_{s} C_{F} \Big[ H_{1}^{\perp({\rm fig.a})} + H_{1}^{\perp({\rm fig.b})} \\ + H_{1}^{\perp({\rm fig.c})} + H_{1}^{\perp({\rm fig.d})} \Big],$$
(20)

where the four terms in the bracket correspond to the four diagrams in Fig. 5 and they are given by

$$H_1^{\perp(\text{fig.a})} = \frac{1}{2zk^2} \left( 3 - \frac{m_q^2}{k^2} \right),$$
(21)

$$H_1^{\perp(\text{fig.b})} = -\frac{1}{(1-z)(k^2 - m_q^2)} \left[ \frac{m_q^2}{k^2 - m_q^2} \ln\left(\frac{k^2}{m_q^2}\right) + \frac{1}{2z} \left( 4 - 5z + 3(z-2)\frac{m_q^2}{k^2} + 2\frac{m_q^4}{(k^2)^2} \right) \right], \quad (22)$$

$$H_1^{\perp(\mathrm{fig.c})} = 0, \tag{23}$$

$$H_1^{\perp(\text{fig.d})} = -\frac{1}{(1-z)k^2} \left[ 1 + \frac{(1-z)k^2}{(1-z)k^2 - m_q^2} \ln\left(\frac{(1-z)k^2}{m_q^2}\right) \right].$$
(24)

We note, due to the fundamental quark–photon and quark–gluon interactions that describe the photon Collins function, we find that the overall strength of the various contributions in Eq. (20) are set by both the electro-magnetic and strong coupling. Moreover, we also find that the function vanishes if the quark mass is zero; this is consistent with the chiral-odd property of the Collins function [16].<sup>1</sup> One might expect such behavior in any partonic model description of the photon Collins function.

# 3.2. Collinear fragmentation functions

The collinear (integrated) unpolarized fragmentation function  $D_{q \rightarrow \gamma}(z)$  is defined as

$$D_{q \to \gamma}(z) = \pi \int_{0}^{p_{T_{\max}}^{2}} dp_{T}^{2} D_{q \to \gamma}(z, p_{T}^{2}).$$
(25)

Following [40,43,47], we take the upper limit  $p_{T \text{ max}}^2$  to be set by a cut-off on the fragmenting quark virtuality  $\mu^2$ , where  $k^2 < \mu^2$ . From Eq. (19), this corresponds to

$$p_{T\,\max}^2 = z(1-z)\mu^2 - zm_q^2.$$
(26)

Then the analytic result for  $D_{q \to \gamma}(z, \mu^2)$  is

$$D_{q \to \gamma}(z, \mu^2) = e_q^2 \frac{\alpha_{\rm em}}{2\pi} \left[ \frac{1 + (1-z)^2}{z} \ln \frac{(1-z)(\mu^2 - m^2)}{zm^2} + 2\left(\frac{m^2}{\mu^2 - m^2} - \frac{1-z}{z}\right) \right].$$
 (27)

We choose a quark mass of  $m_q = 300$  MeV, for  $D_{q \rightarrow \gamma}$  which gives a reasonable estimate for quark-to-photon fragmentation function extracted from phenomenology [49] as indicated in the left panel of Fig. 6. Choosing such a mass value enables us to estimate the possible size of the Collins effect to prompt photon production. We comment more on this in Section 4.

Similarly from Eq. (11), we define the twist-three fragmentation function  $\hat{H}_q(z, \mu^2)$  as

$$\hat{H}_{q}(z,\mu^{2}) = -\frac{\pi}{z} \int_{0}^{p_{T}^{2}} dp_{T}^{2} p_{T}^{2} H_{1}^{\perp q}(z,p_{T}^{2}).$$
(28)

Now let us estimate the relative size of twist-three fragmentation function  $\hat{H}_q(z, \mu^2)$  compared to the unpolarized fragmentation function  $D_{q \to \gamma}(z, \mu^2)$ .

<sup>&</sup>lt;sup>1</sup> Similar quark mass dependence was observed for pion fragmentation in both partonic and effective quark–hadron model calculation of the Collins effect [40,41,48].



**Fig. 5.** The Feynman diagrams which contribute to the quark-to-photon Collins fragmentation function  $H_1^{\perp q}(z, p_T^2)$ . The mirror diagrams with the gluon in the right-hand side of the cut are not shown here, but are included in the calculations.



**Fig. 6.** Left panel: *u*-quark to photon fragmentation function calculated from our model (blue dashed curve) in Eq. (27) with  $m_q = 300$  MeV compared with that extracted from phenomenology in [49] (red solid curve) at  $\mu = 1.5$  GeV. Right panel: The ratio of  $\hat{H}_q(z, \mu^2)/D_{q \to \gamma}(z, \mu^2)$  at scale  $\mu = 1.5$  GeV is plotted as a function of *z*. The magenta dotted curve is the contribution from Fig. 5(a), the green dot-dashed for Fig. 5(b), the blue dashed for Fig. 5(d), and the red solid curve is the sum.

In Fig. 6 (right panel), we present numerical estimates for the analyzing power  $\hat{H}_q(z, \mu^2)/D_{q \to \gamma}(z, \mu^2)$ , separately for each of the diagrams of Fig. 5 at scale  $\mu = 1.5$  GeV as a function of *z*. The magenta dotted curve is the contribution from Fig. 5(a), the green dot-dashed for Fig. 5(b), the blue dashed for Fig. 5(d), and the red solid curve is the sum. We find that there is a strong cancellation between the contribution of diagrams (a) and (b), similar to the quark-to-pion Collins function [40]. Thus the sum is dominantly given by the contribution from diagram (d), the gauge box diagram. We also notice that the quantity  $\hat{H}_q(z, \mu^2)/D_{q \to \gamma}(z, \mu^2)$  for photon case is much smaller than the same quantity for pion case as estimated in Ref. [40]. This leads to a much smaller Collins asymmetry for fragmentation photon production, as shown in the next section.

#### 4. Phenomenology

In this section, we will estimate the SSAs of the prompt photon production in the forward rapidity region at RHIC energy. In order to assess the contributions from the fragmentation photons, beside the overall spin asymmetry  $A_N$  defined in Eq. (12), we will define the following additional asymmetries: the spin asymmetry for direct photon production  $A_N^{\rm dir}$ , the spin asymmetry for fragmentation photons  $A_N^{\rm frag}$ . That is,

$$A_{N}^{\rm dir} = \frac{E_{\gamma} \frac{d\Delta\sigma^{\rm dir}}{d^{3}P_{\gamma}}}{E_{\gamma} \frac{d\sigma^{\rm dir}}{d^{3}P_{\gamma}}}, \qquad A_{N}^{\rm frag} = \frac{E_{\gamma} \frac{d\Delta\sigma^{\rm frag}_{\rm Sivers}}{d^{3}P_{\gamma}}}{E_{\gamma} \frac{d\sigma^{\rm frag}}{d^{3}P_{\gamma}}}.$$
 (29)

For the fragmentation photons, there are both Sivers and Collins contributions for the spin asymmetry, we thus further define the Sivers asymmetry for fragmentation photons  $A_{N,\text{Sivers}}^{\text{frag}}$ , and the Collins asymmetry for fragmentation photons  $A_{N,\text{Collins}}^{\text{frag}}$ ,

$$A_{N,\text{Sivers}}^{\text{frag}} = \frac{E_{\gamma} \frac{d\Delta \sigma_{\text{Sivers}}^{\text{frag}}}{d^3 P_{\gamma}}}{E_{\gamma} \frac{d\sigma^{\text{frag}}}{d^3 P_{\gamma}}}, \qquad A_{N,\text{Collins}}^{\text{frag}} = \frac{E_{\gamma} \frac{d\Delta \sigma_{\text{Collins}}^{\text{collins}}}{d^3 P_{\gamma}}}{E_{\gamma} \frac{d\sigma^{\text{frag}}}{d^3 P_{\gamma}}}.$$
 (30)

Note that the spin asymmetry for fragmentation photons is the sum of Sivers and Collins asymmetry as  $A_N^{\text{frag}} = A_{N,\text{Sivers}}^{\text{frag}} + A_{N,\text{Collins}}^{\text{frag}}$ . However, the overall spin asymmetry for prompt photon  $A_N \neq A_N^{\text{dir}} + A_N^{\text{frag}}$ .



**Fig. 7.** Single transverse spin asymmetry for prompt photon production,  $p^{\uparrow} + p \rightarrow \gamma + X$ , is plotted as a function of Feynman  $x_F$  at rapidity y = 3.5 and center-of-mass energy  $\sqrt{s} = 200$  GeV. Left panel: the asymmetry for the fragmentation photons. The black solid curve is the Collins asymmetry  $A_{N,Collins}^{frag}$ . Dashed curves are the Sivers asymmetry  $A_{N,Sivers}^{frag}$ , with the red curve for "KQVY" parametrization, the blue curve for "new" parametrization, and the green curve for "old" parametrization for  $T_{q,F}(x, x)$ . For each set, the solid curve is the asymmetry for fragmentation photons  $A_N^{frag}$ , which is the sum of  $A_{N,Collins}^{frag}$  and  $A_{N,Sivers}^{frag}$ . Right panel: the asymmetry for the prompt photons. For each set, the dashed curve is the direct asymmetry  $A_N^{dir}$ , the dotted curve is the fragmentation asymmetry  $A_N^{frag}$ , and the solid curve is the overall spin asymmetry  $A_N$ . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

The quark-to-photon fragmentation function has been extracted from the phenomenological study, see e.g., Ref. [49]. This parametrization has been used to describe the unpolarized prompt photon production at RHIC energy [50]. Thus, to compute the fragmentation photon cross section in spin-averaged proton-proton collisions, we will use this phenomenological parametrization instead of the model result in Eq. (27). On the other hand, since there is no experimental information at all for the quark-to-photon twist-three fragmentation function  $\hat{H}_q(z)$ , we rely on our model calculation in order to estimate the Collins contribution to the asymmetry of fragmentation give a reasonable estimate on the relative size for  $\hat{H}_q(z, \mu^2)$  and  $D_{q \to \gamma}(z, \mu^2)$ . Thus we will use the following approximation,

$$\frac{\hat{H}_q(z,\mu^2)}{D_{q\to\gamma}(z,\mu^2)}\Big|_{\text{phenomenology}} = \frac{\hat{H}_q(z,\mu^2)}{D_{q\to\gamma}(z,\mu^2)}\Big|_{\text{model}},$$
(31)

where  $\hat{H}_q(z, \mu^2)$  and  $D_{q \to \gamma}(z, \mu^2)$  on the right-hand side are given by the expressions in Eqs. (27) and (28) in our model calculations,  $D_{q \to \gamma}(z, \mu^2)$  on the left-hand side is the phenomenological parametrization from Ref. [49], and  $\hat{H}_q(z, \mu^2)$  in the numerator on the left-hand side will be the quark-to-photon twist-three fragmentation function to be used in our calculation for the asymmetry  $A_{N,\text{Collins}}^{\text{frag}}$  of fragmentation photons. For quark transversity distribution  $h_a(x)$ , we take the parametrization from Ref. [51].

On the other hand, to calculate  $A_N^{\text{dir}}$  and  $A_{N,\text{Sivers}}^{\text{frag}}$ , we need the twist-three quark-gluon correlation functions  $T_{q,F}(x, x)$ . This function has been extracted directly from the inclusive hadron production in proton-proton collisions [20], which will be labeled as "KQVY" parametrization in our plots.  $T_{q,F}(x, x)$  can also be computed indirectly from Eq. (7) with the quark Sivers function extracted from SIDIS process [36,37]. Such indirectly obtained parametrization for  $T_{q,F}(x, x)$  from [36] will be called "old" parametrization, while that from [37] will be labeled as "new" parametrization in our plots. It has been found in [28] that the *directly* and *indirectly* obtained  $T_{q,F}(x, x)$  have conflicting signs, for both *u* and *d* quark flavors. The future prompt photon production hopefully could help us pin down the sign and magnitude of  $T_{q,F}(x, x)$ . In Fig. 7 (left), we plot the spin asymmetry for fragmentation photons as a function of Feynman  $x_F$  at forward rapidity y = 3.5 and RHIC energy  $\sqrt{s} = 200$  GeV. The black solid curve is the Collins asymmetry  $A_{N,Collins}^{frag}$ . We find that the Collins asymmetry is very small in the whole  $x_F$  region, less than 1%. Dashed curves are the Sivers asymmetry  $A_{N,Sivers}^{frag}$ , with the red curve for "KQVY" parametrization, the blue curve for "new" parametrization, and the green curve for "old" parametrization for  $T_{q,F}(x, x)$ . For each set, the solid curve is the asymmetry for fragmentation photons  $A_N^{frag}$ , which is the sum of  $A_{N,Collins}^{frag}$  and  $A_{N,Sivers}^{frag}$ . In Fig. 7 (right), we plot the spin asymmetry  $A_N$  the prompt photons. For each set, the dashed curve is the direct asymmetry  $A_N^{frag}$ , and the solid curve is the overall spin asymmetry  $A_N$  has the same sign as the direct asymmetry  $A_N^{frag}$  has the same sign as the direct asymmetry  $A_N^{frag}$  has the same sign as the same sign as  $A_N^{dir}$  and  $A_N^{frag}$ .

Some comments are in order on the reliability our model estimate of the photon Collins contribution in prompt photon production. We re-emphasize, in this partonic model picture the quark-tophoton Collins function is set by the electro-magnetic and strong couplings, as well as the chiral-symmetry breaking quark mass [40, 41,48]. Further, fixing the guark mass by making a best estimate to phenomenological extraction of the unpolarized photon fragmentation function, we then find that the photon Collins contribution is relatively small. While this estimate of the Collins effect is derived from a specific model calculation we expect this behavior from any partonic description of the photon Collins function. Thus, within this partonic framework the Collins asymmetry for fragmentation photons is very small, and the asymmetry of prompt photon production can possibly be a very good probe for the twist-three quark-gluon correlation functions  $T_{q,F}(x, x)$ . We urge the experiments to measure the asymmetry of prompt photon production at RHIC. It will provide important information on the twist-three quark-gluon correlation functions, a quantity much needed to verify our current theoretical formalism for describing single transverse spin asymmetry in proton-proton scatterings. The measurement can go a long way to resolving the so called "sign mismatch" [28-30].

#### 5. Summary

We have studied the single transverse spin asymmetry of prompt photon production in high energy proton-proton scattering including the contributions from both the direct and fragmentation photons. While the asymmetry for direct photon production receives only the Sivers type of contribution, the asymmetry for fragmentation photons receives both the Sivers and Collins types of contributions. In order to estimate the Collins asymmetry for fragmentation photons, we perform a model calculation for the chiral-odd quark-to-photon Collins function. Our estimate of the Collins asymmetry is derived from a partonic model calculation extended from that for quark-to-pion fragmentation [40-42,45]. In order to obtain a non-trivial Collins effect in this framework we estimate the chiral-odd property of the Collins effect by choosing a non-zero quark mass of  $m_q = 300$  MeV. This framework has been shown to give reasonable estimate of unpolarized quark-to-photon fragmentation function. Further based on the fundamental quarkphoton and quark-gluon interactions we expect it characterizes the dynamics of the photon Collins function and in turn yields a reasonable estimate of the photon Collins contribution to the prompt photon production. We find that the Collins asymmetry for fragmentation photons is very small in the whole kinematic region, thus the single transverse spin asymmetry of prompt photon production is mainly coming from the Sivers asymmetry in direct and fragmentation photons. We hope the experiments in the future could constrain the different contributions to the prompt photon production, e.g., through an isolation cut. We further make predictions for the prompt photon spin asymmetry at RHIC energy, and find that the asymmetry is sizable. The asymmetry of prompt photon production should then provide a good measurement for the important twist-three quark-gluon correlation function, which is urgently needed in order to resolve the "sign mismatch" puzzle. We urge the experiments to measure the asymmetry of prompt photon production at RHIC in the near future.

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