Lattice Boltzmann simulation for magnetic fluids in porous medium

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Abstract

A Lattice Boltzmann method is introduced to simulated magnetic fluids in porous medium. In the simulation, drag force and the other external forces are imported to the momentum equation. The influence of external magnetic field on thermo-sensitive magnetic fluids is expressed as a magnetic potential equation, which will be dispersed depending on the format of density and temperature. To test the reliability of the method, a validation with common fluid in porous medium is compared. Great agreement with previous results verified that the present lattice Boltzmann method is promising for simulating magnetic fluids in porous medium.

Keywords: magnetic fluid; porous medium; Lattice Boltzmann method;

1. Governing equations for magnetic fluid in porous media

In the external magnetic field (H), magnetic fluid is magnetizable, and the magnetization is M, which is assumed to be parallel to H. The magnetic fluid we employed in the present analysis is thermo-sensitive, so that the magnetization depends on the magnetic intensity \( H \) and the temperature \( T \). The magnetic force, which comes into effect due to the interactions between the external magnetic field and magnetization, is a function of \( H \) and \( T \). In the nonuniform temperature field, the magnetic force will be the main driving force for the convection. In porous medium, the fluid also is influenced by drag force and the total forces due to the porous medium.

In the simulation, we assume that the fluid is incompressible, with constant properties and has a higher Curie temperature than the operated temperature. The cavity we model is assumed to be filled with isotropic, homogeneous and non-deformable porous medium. With the assumption, we can get the governing equations as follows:
\[ \nabla \cdot \mathbf{u} = 0 \]
\[ \partial_t (\rho \mathbf{u}) + \nabla \left( \frac{1}{\varepsilon} \rho \mathbf{u} \mathbf{u} \right) = -\nabla (\varepsilon p) + \eta \nabla^2 \mathbf{u} + \mathbf{F} \]
\[ \sigma \partial_t \mathbf{T} + \mathbf{u} \cdot \nabla \mathbf{T} = \frac{\lambda (T_e - T_0)}{[\rho c_p (T_e - T_0) + \mu_e \mathbf{H}^2]} \nabla^2 T + S \]
\[ \frac{1}{\gamma} \partial_t \phi + \mathbf{u} \cdot \nabla \phi = \left[ 1 + \frac{Z_e (T - T_e)}{T_e - T_0} \right] \nabla^2 \phi \]

Where \( \mathbf{F} \) is the total force, including magnetic force, gravity, viscous loss term (Darcy) and geometric loss term in porous medium. It is given as follows:

\[ F = \frac{Z_e \mu_0 H^2}{2(T_e - T_0)} \nabla T - \rho c_{p0} \beta (T - T_0) - \frac{\varepsilon \eta}{K} \mathbf{u} - \frac{\sigma F_e}{\sqrt{K}} |\mathbf{u}| \mathbf{u} \]

\[ F_e = \frac{1.75}{\sqrt{150(1-\varepsilon)^3}} \]

where \( T_c, T_0 \) are the Curie temperature and reference temperature. \( S \) which be obtained from the energy equation, includes the magnetocaloric effects, and it is given as follows:

\[ S = -\frac{Z_e \mu_0 HT}{\rho c_p (T_e - T_0) + Z_e \mu_0 H^2} \]

\[ \sigma = \frac{\rho' C_p'}{\rho C_p} \]

It is noted that Equation (4) is the magnetic potential equation, where \( \phi \) is the scalar potential, the relationship between \( \phi \) and \( H \) is \( H = \nabla \phi \). \( \gamma \) is the adjustable preconditioning parameter in analysis. \( u_T \) is defined as effective velocity, and it is given as:

\[ u_T = \frac{Z_e \sqrt{T_e - T_0}}{T_e - T_0} \]

In all the equations, \( \rho, \lambda, C_p, \eta, \chi_0, \mu_0, \beta \) are the fluid density, thermal conductivity, specific heat, viscosity, magnetization rate, permeability of vacuum and expansion coefficient. \( \rho', C_p', \varepsilon \) are the density, thermal conductivity and porosity for the porous medium.

2. Lattice Boltzmann methods for magnetic fluids in porous media

Following the lattice Boltzmann method of D2Q9 model with corresponding weight coefficient \( w_f \) and discrete velocity \( \vec{e}_f \), the description of the velocity, temperature and magnetic potential, which is the dispersion for equations (1)-(4), are given by the following equations:

\[ f_f (t + \xi_\delta, t + \delta_t) - f_f (t, t) = \frac{f_f (t, t) - f^{\alpha_\delta} (t, t)}{\tau_f} + w_f \left( \frac{\tau_f - 0.5}{\tau_f} \right) \xi_\delta \cdot \mathbf{u} \]

\[ g_f (t + \xi_\delta, t + \delta_t) - g_f (t, t) = \frac{g_f (t, t) - g^{\alpha_\delta} (t, t)}{\tau_e} + w_e \xi_\delta \]

\[ h_f (t + \xi_\delta, t + \delta_t) - h_f (t, t) = -\frac{h_f (t, t) - h^{\alpha_\delta} (t, t)}{\tau_h} \]
where the equilibrium distribution functions for the above equations are:

\[
f_{j}^{eq}(r, t) = w_{j} \rho \left[ 1 + \frac{\xi_{j} \cdot u}{c_{j}^{2}} + \frac{1}{2c_{j}^{2}} \left( \left( \xi_{j} \cdot u \right)^{2} - u^{2} \right) \right]
\]

\[
g_{ii}^{eq}(r, t) = w_{ii} \frac{DT}{2} \left[ \frac{\xi_{ii}^{2}}{c_{ii}^{2}} \frac{\xi_{ii} \cdot u}{c_{ii}^{2}} \left( \frac{\xi_{ii} \cdot u}{c_{ii}^{2}} - u \right) \right]
\]

\[
h_{iii}^{eq}(r, t) = w_{iii} \phi \left[ 1 + \frac{\xi_{iii} \cdot \gamma a_{r}}{c_{iii}^{2}} + \frac{1}{2c_{iii}^{2}} \left( \left( \xi_{iii} \cdot \gamma a_{r} \right)^{2} - \left( \gamma a_{r} \right)^{2} \right) \right]
\]

The relaxation parameters in the equations are:

\[
\tau_{f} = \frac{\eta}{pc_{s}c_{f} \delta_{f}} + 0.5
\]

\[
\tau_{g} = \frac{1}{\sigma} \frac{D}{D+2} \left( \frac{\lambda(T_{w} - T)}{\rho C_{p}(T_{w} - T) + \chi \mu_{h} H^{2} c_{e} \delta_{c}} \right) + 0.5
\]

\[
\tau_{h} = \frac{\gamma}{c_{h} \delta_{h}} \left[ 1 + \frac{X_{h}(T_{w} - T)}{T_{w} - T} \right] + 0.5
\]

According to the method, the density, velocity and temperature are defined by:

\[
\rho = \sum_{i} f_{i} \quad T = \sum_{i} g_{i} \quad \phi = \sum_{i} h_{i} \quad u = \frac{u'}{c_{0} + \sqrt{c_{0}^{2} + c_{s}^{2} p_{s}^{2}}}
\]

\[
\rho u' = \sum_{i} f_{i} \xi_{i} + 0.5 \delta F \quad c_{0} = 0.5 \times \left( 1 + \frac{\varepsilon \delta \eta}{2K} \right) \quad c_{i} = \frac{a_{s} F}{2 \sqrt{K}}
\]

### 3. Numerical simulation

#### 3.1. Validation for the method

Before the simulation, a validation of natural simulation for the method is assured. Compared with Guo and Nithiarasu’s articles, a same model for a common fluid flowing in a porous medium cavity is used for a sake of comparison. The left wall of the cavity is heated on constant temperature, and the right wall is cooled. Both of the up and down wall are adiabatic. The porosity \( \varepsilon \) and Darcy number \( Da = K/L^{2} \), \( K \) is the permeability of porous medium, and \( L \) is length of the cavity.) is set to be 0.4 and \( 10^{-4} \). In simulations, \( 101 \times 101 \) lattices are meshed. Pr \( (Pr = \nu/k) \) is set to be 1, and Rayleigh number Ra \( (Ra = \rho g \beta T L^{4}/(k \eta)) \) changes from \( 10^{5} \) to \( 10^{6} \). Figure 1. shows the streamlines and isotherms for simulations. In table 1, we compared the average Nusselt numbers with Guo and Nithiarasu’s result, and it is found good agreement.

<table>
<thead>
<tr>
<th>Da</th>
<th>Ra</th>
<th>Present</th>
<th>Guo’s</th>
<th>Nithiarasu’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{-4} )</td>
<td>( 10^{5} )</td>
<td>0.993</td>
<td>1.066</td>
<td>1.067</td>
</tr>
<tr>
<td></td>
<td>( 10^{6} )</td>
<td>2.636</td>
<td>2.603</td>
<td>2.550</td>
</tr>
</tbody>
</table>

Table 1 Comparisons of average numbers
3.2. Modeling and result discussion

The physical model considered in the present study is shown in Figure 2. The cavity with length of $L$ ($L = 0.075m$) is filled with porous medium, and the magnetic fluid in porous medium is magnetized in a uniform magnetic intensity $H_0$. The cavity is heated from the bottom wall and cooled at the upper wall. The initial temperature $T_0$ (reference temperature) is set as 298.15K in whole calculation domain. The boundary conditions are given as follows:

$$\begin{align*}
    u_{x=0,L} &= 0, & u_{y=0,L} &= 0, & T_1 &= T_{\text{heat}}, & T_2 &= T_{\text{cool}} \\
    \frac{\partial T}{\partial x} \bigg|_{x=0,L} &= 0, & \frac{\partial \phi}{\partial x} \bigg|_{x=0,L} &= 0, & \frac{\partial \phi}{\partial y} \bigg|_{y=0,L} &= \frac{H_0}{1 + \chi}
\end{align*}$$

$T_2 = T_{\text{cool}}$  

Figure 2. physical model for magnetic fluid in porous medium.
In this case, the Ra \( (Ra = \frac{\rho g \beta \Delta T L^3}{(k \eta)}) \), Ram \( (Ram = \frac{\mu_0 H_0 M_s L^2}{(k \eta)}) \) are set as \( 10^8, 10^8 \), where \( M_s \) is the saturation magnetization. The porosity and permeability of porous medium are assumed to be 0.4, \( 4.6 \times 10^{-8} \). Figure 3. shows the streamline for this case.

From figure 3, there are two larger circulations between the bottom and upper wall. It is the common result due to external force, magnetic force and gravity. The direction of magnetic force is opposite to gravity, and it is the main force that brings the low-temperature fluid to bottom, while the high-temperature fluid flows upward. External forced due to porous medium is the main force making fluid change its direction, and then circulations appear.

4. Conclusions

In this paper, A Lattice Boltzmann method is introduced to simulate magnetic fluids in porous medium. In the simulation, the magnetic fluid we used in the present study is thermo-sensitive. In the nonuniform temperature field, the magnetic force will be the main driving force for the convection. The influence of external magnetic field on thermo-sensitive magnetic fluids is expressed as a magnetic potential equation which will be dispersed depending on the format of density and temperature. Fluid flow is also influenced by forces in porous medium. Drag force and the other external forces are imported to the momentum equation. To test the reliability of the method, a validation with common fluid in porous medium is operated. Great agreement with previous results shows the present lattice Boltzmann method is promising for simulating magnetic fluids in porous medium.

References