Acknowledgment

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References


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Hausdorff on Ordered Sets

This volume contains an English edition of the papers on order structures written by Felix Hausdorff (1868–1942). Its main part contains a complete collection of Hausdorff’s publications on set theory that appeared before his “magnum opus” Grundzüge der Mengenlehre [Hausdorff, 1914]. They were published between 1901 and 1909 and are presently only accessible in the original journals (in German). They will be contained in a future volume of the ongoing edition of Hausdorff’s Collected Works [Hausdorff, 2001ff., Vol. I]. All of these papers have commentaries by the editor, with the translations undertaken jointly with Marion Scheepers. In the Appendix, an additional translation of a later Hausdorff paper on Sums of $\aleph_1$ sets (1936) is given without a further commentary.

During the first decade of the last century Felix Hausdorff became one of the leading figures of the “second generation of Cantorians,” as Jacob Plotkin rightly comments (p. 181). Hausdorff entered the new field in 1901 by generalizing Cantor’s investigations of order structures from well-orders to what he called “graded” order types, because he hoped for a possible relationship to the continuum problem. The latter had been raised to a first-rank problem of research mathematics by Hilbert’s talk at Paris in 1900. But Hausdorff’s hope did not come true. “Graded types quickly passed from the scene,” as the editor remarks (p. 6).

The next contribution, the Hausdorff recursion formula for transfinite cardinal numbers (denomination due to A. Tarski), arose from a dense and irritating discussion at the 1904 International Congress of Mathematicians at Heidelberg and after. Because of an incorrect formula for transfinite cardinals, due to F. Bernstein, J. König came
to the (relatively convincing) conclusion that Cantor’s continuum hypothesis was wrong. Plotkin gives a detailed historical account, different from the standard one in the older historical literature, of how Bernstein’s error was corrected. In this account Hausdorff plays the crucial part (pp. 24ff., footnote 7). Plotkin arrived at this conclusion independently of W. Purkert, who has recently given a similar evaluation and correction of the standard picture, and he clearly gives credit to Purkert’s earlier findings [Purkert, 2002].

The bulk of Hausdorff’s early papers on set theory deal with investigations of the structure of order types. This is a highly technical matter to which Hausdorff contributed a variety of new concepts (cofinality, cointiality, gap character, element character, regular and singular initial number, species and genus of an order type, type ring, \( \eta_\alpha \)-set, etc.), proposed well-reasoned changes of traditional notions (e.g., “pantachie,” in allusion to P. Du Bois-Reymond), and invented proof methods (Hausdorff maximality principle, back and forth (or zigzag) method). Of Hausdorff’s results we mention here only his general classification of dense order types and his existence proof of a “pantachie” with a special kind of transfinite gap behavior (a so-called \( (\omega_1, \omega^*_1) \)-gap,” still of interest to actual researchers in model theory and order types). Not least, Hausdorff made some fine and far-reaching side remarks, such as the generalized continuum hypothesis or an expected “exorbitant size” of regular initial numbers with limit index, if one wanted to assume them at all (now weakly inaccessible cardinals). These side remarks turned out to be of intriguing importance for the further development of the foundations of set theory, although Hausdorff himself did not actively pursue axiomatic research in that area.

The reviewer might be allowed to add a comment that links Hausdorff’s research activities in order structures during his first decade of studies in set theory back to his earlier philosophical interests. Three years before his first set-theoretic investigation, our author had published a philosophical essay, Das Chaos in kosmischer Auslese [Hausdorff, 1898], in which he had started to use metaphors from transfinite set theory to demolish the neo-Kantian conviction of an \( a \ priori \) or even absolute (“transcendent”) order of time and space. The most general and anti-intuitive order structures available at the time (1898) were used by him to demonstrate the intellectual unfeasibility of such an absolute “transcendent” order from a critical point of view. He was subsequently highly motivated to understand transfinite order structures and possible logical alternatives to the intuitive or even to the standard mathematical continuum (“arithmetized” in Hilbert’s sense by the real numbers). Similarly for the geometrical and topological structures of space [Hausdorff, 2001ff., Vol. VI, forthcoming].

The editor of the book under review, J.M. Plotkin, is a specialist in mathematical logic and set theory and highly knowledgeable in the historical literature. He has managed to integrate the mathematical introduction to each of the articles with a short exposition of the historical context in which the respective paper was written. More historical details are given in extended notes to each of the commentaries. They always stand on the level of the most recent historical research and give full reference and credit to it. Where he deviates from the literature, Plotkin has, and gives, good reasons, for example, when he argues that it was Hausdorff rather than Russell who first proved that the cardinality of the set of subsets of \( M \) is greater than the cardinality of \( M \) (p. 6 and footnote 8), or with respect to the Heidelberg congress event (see above).

The mathematical introductions essentially serve the goal of giving a first orientation to what the reader may expect from reading Hausdorff’s papers. Hausdorff’s brilliant exposition rarely needs additions or corrections. The notes (mostly, although not all of them, of historical content) contain a condensed introduction to the historical literature and original sources. It is outside the range of the reviewer’s competence to judge whether the fineness and subtlety of Hausdorff’s language has survived the transmission of the texts into English. But clearly this volume greatly facilitates the access of the international readership to Hausdorff’s early contributions to set theory and gives detailed information on the history of set theory in general. It is a very welcome, probably even necessary, complement to the ongoing enterprise of editing Hausdorff’s Gesammelte Werke. It will be of great help to anybody interested in the historical and mathematical development of 20th-century set theory and logic.

References


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John von Neumann: Selected Letters

John (Janos, Johannes) von Neumann certainly ranks as one of the greatest polymaths, not just of the 20th century, but of all time. His published mathematical work comprises 150 articles (almost completely contained in the nearly 3800 printed pages of the six volumes of his Collected Works) as well as four books on a large and diverse array of fields. These range from logics and axiomatics for topics in mathematics and physics, operator theory, groups, rings, and continuous geometry, to the theory and development of electronic computers and their applications, to practical problems of all kinds. In addition to this, his work spanned everything from game theory, economic behavior, and human intelligence to the treatment of specific problems in astrophysics, hydrodynamics, and meteorology.

The present volume has been edited by his countryman Miklós Rédei, who also provides some helpful “Introductory Comments” in order to explain the contents of the letters and their personal, scientific and historical background. Preceded by a foreword from Marina von Neumann Whitman, which contributes a personal view of her father, the correspondence as carefully reproduced here basically covers the American period of von Neumann’s life. Consequently the original language is mostly English, with only a few letters having been (well) translated from German and Hungarian by the editor. Unfortunately perhaps, the arrangement of the letters does not follow either historical or topical order—which scientists and historians among the readers probably might have preferred—but is strictly alphabetical according to the name of the recipient.

The contents of the letters fully reflect von Neumann’s wide scope of scientific work and activities, as well as his personal interests in the second half of his career. In a set of “early” letters, von Neumann immediately expounded the seminal importance of Kurt Gödel’s contribution to mathematical logic (even before it was completed), which was to destroy the hope of providing a strict axiomatic foundation for mathematics in David Hilbert’s sense. Five valuable letters to Gödel from 1930 to 1935 are reproduced, as well as a letter to the German philosopher Rudolf Carnap (1931) on “Gödel’s theorem” and a request to Abraham Flexner, the director of the Institute for Advanced Study in Princeton (1939), to secure a U.S. visa for Gödel, by then an Austrian refugee.

Of course, the central topics of von Neumann’s own mathematical work are also addressed in some detail, for instance, by a packet of six letters from 1935 dealing with the “logic of quantum mechanics,” addressed to his collaborator Garrett Birkhoff. Von Neumann conducted a more scattered correspondence with other colleagues and collaborators, such as the Frenchman J. Dixmier (on Hilbert spaces, 1953), the Japanese Kôdi Husimi (on foundations of quantum mechanics, 1937), the student I. Halperin (on operators, 1939–1940), and Irvin Kaplanski (on Banach