

Amplitudes for massive vector and scalar bosons in spontaneously-broken gauge theory from the CHY representation

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ABSTRACT: In the formulation of Cachazo, He, and Yuan, tree-level amplitudes for massless particles in gauge theory and gravity can be expressed as rational functions of the Lorentz invariants $k_a \cdot k_b$, $\epsilon_a \cdot k_b$, and $\epsilon_a \cdot \epsilon_b$, valid in any number of spacetime dimensions. We use dimensional reduction of higher-dimensional amplitudes of particles with internal momentum κ to obtain amplitudes for massive particles in lower dimensions. In the case of gauge theory, we argue that these massive amplitudes belong to a theory in which the gauge symmetry is spontaneously broken by an adjoint Higgs field. Consequently, we show that tree-level n -point amplitudes containing massive vector and scalar bosons in this theory can be obtained by simply replacing $k_a \cdot k_b$ with $k_a \cdot k_b - \kappa_a \kappa_b$ in the corresponding massless amplitudes, where the masses of the particles are given by $|\kappa_a|$.

KEYWORDS: Spontaneous Symmetry Breaking, Scattering Amplitudes, Duality in Gauge Field Theories, Field Theories in Higher Dimensions

ARXIV EPRINT: [1506.06134](https://arxiv.org/abs/1506.06134)

In a series of papers over the last few years, Cachazo, He, and Yuan (CHY) have developed a new representation for the tree-level amplitudes of massless particles in a variety of theories, including Yang-Mills theory and Einstein gravity [1–3], as well as the nonlinear σ model, the Dirac-Born-Infeld theory, and more exotic models [4]. In this approach, the ingredients for an n -point amplitude are the positions σ_a of n punctures on a sphere, as well as Lorentz invariants of the momenta k_a^μ and polarizations ϵ_a^μ of the particles. (The graviton polarization is expressed as $\epsilon_a^{\mu\nu} = \epsilon_a^\mu \tilde{\epsilon}_a^\nu$.) The CHY representation involves an integral over the moduli space of the punctured sphere, which can be evaluated [2, 4–6] to yield a rational expression of $k_a \cdot k_b$, $\epsilon_a \cdot k_b$, and $\epsilon_a \cdot \epsilon_b$ for the n -point tree-level amplitude.

A notable feature of this formulation is its independence of the number of spacetime dimensions. One can therefore dimensionally reduce CHY representations of gravity or Yang-Mills amplitudes in $(d + M)$ dimensions to obtain mixed amplitudes in d dimensions containing gravitons, gauge bosons, and massless scalars by choosing the $(d + M)$ -dimensional polarizations \mathcal{E}_a to lie in either the d - or M -dimensional subspace [3, 4].

Dimensional reduction can also be used to endow particles with mass, and therefore to obtain CHY representations for massive particles in d dimensions [7]. (Previous extensions of the CHY approach to massive particles were considered in refs. [8–10].) The momentum of a particle in $(d + M)$ dimensions can be written

$$K_a = (k_a | \kappa_a) \tag{1}$$

where k_a and κ_a are the components of momentum in d - and M -dimensional subspaces respectively; we will refer to κ_a as the *internal momentum* of a particle, and regard it as a fixed quantity. A massless particle in $(d + M)$ dimensions with momentum K_a corresponds to a particle in d dimensions with mass $m_a = |\kappa_a|$. This approach is similar to that used in refs. [11–13], where a fifth dimension was introduced as a massive infrared regulator for planar loop-level $\mathcal{N} = 4$ amplitudes. See also, for example, refs. [14–18], in which massive particles are represented as higher-dimensional massless particles.

The amplitude for massless gauge bosons in $(d + M)$ dimensions, which is a rational function of $K_a \cdot K_b$, $\mathcal{E}_a \cdot K_b$, and $\mathcal{E}_a \cdot \mathcal{E}_b$, can be used to obtain an expression for the amplitude for massive gauge bosons in d dimensions by choosing $\mathcal{E}_a = (\epsilon_a | 0)$, so that¹

$$K_a \cdot K_b = k_a \cdot k_b - \kappa_a \cdot \kappa_b, \quad \mathcal{E}_a \cdot K_b = \epsilon_a \cdot k_b, \quad \mathcal{E}_a \cdot \mathcal{E}_b = \epsilon_a \cdot \epsilon_b. \tag{2}$$

In other words, we simply replace $k_a \cdot k_b$ with $k_a \cdot k_b - \kappa_a \cdot \kappa_b$ in the d -dimensional massless gauge boson amplitude. The question remains: to what theory do these massive amplitudes belong?

Not all d -dimensional amplitudes of massive particles can be obtained from dimensional reduction of $(d + M)$ -dimensional massless amplitudes, due to the constraints arising from internal momentum conservation. Overall momentum conservation demands $\sum_{a=1}^n \kappa_a = 0$, placing a restriction on the masses of a dimensionally-reduced n -particle amplitude. Furthermore, internal momentum conservation must be satisfied at each vertex of the

¹The relative minus sign in $K_a \cdot K_b$ arises because we use a mostly-minus metric for K_a and k_a , but an all-plus metric for the internal components κ_a .

tree diagrams that contribute to the amplitude, constraining the mass of any particle propagating in intermediate channels. For example, if particles a and b couple into an intermediate channel, the mass of the particle in that channel must be given by $|\kappa_a + \kappa_b|$. In ref. [7], it was shown that this constraint is automatically satisfied in an amplitude with no more than three massive particles, in which the remaining massless particles are flavor-preserving, and therefore such amplitudes can be given a CHY representation. But for more general massive amplitudes (e.g. in the standard model), the internal momentum constraints cannot be satisfied.

It is the purpose of this note to observe that the internal momentum constraints *can* be satisfied in a gauge theory in which the $U(N)$ symmetry is spontaneously broken by an adjoint Higgs field H . If the Higgs field has vacuum expectation value $\langle H \rangle = \text{diag}(v_1, v_2, \dots, v_N)$, the $U(N)$ symmetry is broken to $U(1)^N$, and the off-diagonal gauge bosons W_i^j obtain masses $g|v_i - v_j|$ from the $\text{Tr}(D_\mu H)^2$ term in the Lagrangian. If some of the v_i are degenerate, then a larger symmetry will be left unbroken. (This is essentially the theory considered in refs. [11, 15, 18–23], where it is embedded in an $\mathcal{N} = 4$ supersymmetric theory on the Coulomb branch. In the string theory picture, the v_i correspond to the positions of D3 branes in the fifth dimension, and massive gauge bosons correspond to strings extending between separated D3 branes. See also refs. [24, 25] for tree-level massive amplitudes on the Coulomb branch of this theory.)

At a triple-gauge-boson vertex, W_i^j and W_j^k couple to W_i^k . If we identify the internal momentum² κ_{ij} of W_i^j with $g(v_i - v_j)$, then internal momentum conservation $\kappa_{ij} + \kappa_{jk} = \kappa_{ik}$ automatically holds at each vertex (and similarly at four-gauge-boson vertices). This suggests that dimensionally-reduced amplitudes with internal momenta κ are equivalent to amplitudes of massive gauge bosons in this spontaneously-broken theory. Moreover, this implies that massive amplitudes in this theory can be obtained from the corresponding massless amplitudes of the unbroken theory, expressed in terms of $k_a \cdot k_b$, $\epsilon_a \cdot k_b$, and $\epsilon_a \cdot \epsilon_b$, by simply replacing $k_a \cdot k_b$ with $k_a \cdot k_b - \kappa_a \kappa_b$. To illustrate this in a simple case, we evaluate the amplitude of four massive gauge bosons in the spontaneously-broken theory

$$\begin{aligned} & \langle W_{i_1}^{i_2}(k_1, \epsilon_1) W_{i_2}^{i_3}(k_2, \epsilon_2) W_{i_3}^{i_4}(k_3, \epsilon_3) W_{i_4}^{i_1}(k_4, \epsilon_4) \rangle \\ &= g^2 \left[\frac{N_s}{(k_1 + k_2)^2 - m_s^2} + \frac{N_t}{(k_1 + k_4)^2 - m_t^2} + (\text{contact term}) \right] \end{aligned} \quad (3)$$

which have contributions from s - and t -channel exchange as well as a four-gauge-boson contact term. For brevity, we include below only those terms in the numerators that depend on $k_a \cdot k_b$; the omitted terms are the same as those in the unbroken theory:

$$\begin{aligned} N_s &= \frac{(\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot \epsilon_4)}{2} \left[(k_2 - k_1) \cdot (k_4 - k_3) + \frac{(m_2^2 - m_1^2)(m_4^2 - m_3^2)}{m_s^2} \right] + \dots, \\ N_t &= \frac{(\epsilon_1 \cdot \epsilon_4)(\epsilon_2 \cdot \epsilon_3)}{2} \left[(k_2 - k_3) \cdot (k_4 - k_1) + \frac{(m_1^2 - m_4^2)(m_3^2 - m_2^2)}{m_t^2} \right] + \dots \end{aligned} \quad (4)$$

²now restricted to one dimension

Here $m_a = |\kappa_a|$ with $\kappa_a \equiv g(v_{i_a} - v_{i_{a+1}})$, where $i_5 \equiv i_1$ so that $\sum_{a=1}^4 \kappa_a = 0$. Up to this point, we have not made any assumptions about the masses of the s and t -channel intermediate particles. Since the particles propagating in those channels are $W_{i_1}^{i_3}$, and $W_{i_2}^{i_4}$ respectively, we have

$$m_s^2 = g^2(v_{i_1} - v_{i_3})^2 = (\kappa_1 + \kappa_2)^2, \quad m_t^2 = g^2(v_{i_2} - v_{i_4})^2 = (\kappa_2 + \kappa_3)^2 \quad (5)$$

and the numerators above simplify to

$$\begin{aligned} N_s &= \frac{1}{2}(\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot \epsilon_4) \left[(k_2 - k_1) \cdot (k_4 - k_3) - (\kappa_2 - \kappa_1)(\kappa_4 - \kappa_3) \right] + \dots, \\ N_t &= \frac{1}{2}(\epsilon_1 \cdot \epsilon_4)(\epsilon_2 \cdot \epsilon_3) \left[(k_2 - k_3) \cdot (k_4 - k_1) - (\kappa_2 - \kappa_3)(\kappa_4 - \kappa_1) \right] + \dots \end{aligned} \quad (6)$$

We assemble all the pieces and use

$$\begin{aligned} (k_1 + k_2)^2 - m_s^2 &= 2(k_1 \cdot k_2 - \kappa_1 \kappa_2) = 2(k_3 \cdot k_4 - \kappa_3 \kappa_4), \\ (k_1 + k_4)^2 - m_t^2 &= 2(k_1 \cdot k_4 - \kappa_1 \kappa_4) = 2(k_2 \cdot k_3 - \kappa_2 \kappa_3) \end{aligned} \quad (7)$$

to obtain

$$\langle W_{i_1}^{i_2}(k_1, \epsilon_1) W_{i_2}^{i_3}(k_2, \epsilon_2) W_{i_3}^{i_4}(k_3, \epsilon_3) W_{i_4}^{i_1}(k_4, \epsilon_4) \rangle = -g^2 \frac{\mathbf{K}}{(k_2 \cdot k_3 - \kappa_2 \kappa_3)(k_3 \cdot k_4 - \kappa_3 \kappa_4)} \quad (8)$$

with

$$\begin{aligned} \mathbf{K} &= -\left\{ (k_1 \cdot k_3 - \kappa_1 \kappa_3)(k_2 \cdot k_3 - \kappa_2 \kappa_3) \epsilon_1 \cdot \epsilon_2 \epsilon_3 \cdot \epsilon_4 \right. \\ &\quad + (k_2 \cdot k_3 - \kappa_2 \kappa_3)(k_3 \cdot k_4 - \kappa_3 \kappa_4) \epsilon_1 \cdot \epsilon_3 \epsilon_2 \cdot \epsilon_4 \\ &\quad + (k_1 \cdot k_3 - \kappa_1 \kappa_3)(k_3 \cdot k_4 - \kappa_3 \kappa_4) \epsilon_1 \cdot \epsilon_4 \epsilon_2 \cdot \epsilon_3 \\ &\quad + \left[(k_1 \cdot k_3 - \kappa_1 \kappa_3) \epsilon_1 \cdot k_4 \epsilon_2 \cdot k_3 + (k_2 \cdot k_3 - \kappa_2 \kappa_3) \epsilon_1 \cdot k_3 \epsilon_2 \cdot k_4 \right] \epsilon_3 \cdot \epsilon_4 \\ &\quad + \left[(k_2 \cdot k_3 - \kappa_2 \kappa_3) \epsilon_1 \cdot k_2 \epsilon_3 \cdot k_4 + (k_3 \cdot k_4 - \kappa_3 \kappa_4) \epsilon_1 \cdot k_4 \epsilon_3 \cdot k_2 \right] \epsilon_2 \cdot \epsilon_4 \\ &\quad + \left[(k_1 \cdot k_3 - \kappa_1 \kappa_3) \epsilon_1 \cdot k_2 \epsilon_4 \cdot k_3 + (k_3 \cdot k_4 - \kappa_3 \kappa_4) \epsilon_1 \cdot k_3 \epsilon_4 \cdot k_2 \right] \epsilon_2 \cdot \epsilon_3 \\ &\quad + \left[(k_1 \cdot k_3 - \kappa_1 \kappa_3) \epsilon_2 \cdot k_1 \epsilon_3 \cdot k_4 + (k_3 \cdot k_4 - \kappa_3 \kappa_4) \epsilon_2 \cdot k_4 \epsilon_3 \cdot k_1 \right] \epsilon_1 \cdot \epsilon_4 \\ &\quad + \left[(k_2 \cdot k_3 - \kappa_2 \kappa_3) \epsilon_2 \cdot k_1 \epsilon_4 \cdot k_3 + (k_3 \cdot k_4 - \kappa_3 \kappa_4) \epsilon_2 \cdot k_3 \epsilon_4 \cdot k_1 \right] \epsilon_1 \cdot \epsilon_3 \\ &\quad \left. + \left[(k_1 \cdot k_3 - \kappa_1 \kappa_3) \epsilon_3 \cdot k_2 \epsilon_4 \cdot k_1 + (k_2 \cdot k_3 - \kappa_2 \kappa_3) \epsilon_3 \cdot k_1 \epsilon_4 \cdot k_2 \right] \epsilon_1 \cdot \epsilon_2 \right\}. \end{aligned} \quad (9)$$

As expected, this is simply the four-gluon amplitude with $k_a \cdot k_b$ replaced by $k_a \cdot k_b - \kappa_a \kappa_b$.

Mixed amplitudes containing both massive vector and adjoint scalar bosons in d dimensions can also be obtained via dimensional reduction of massless gauge amplitudes in $d + M$ dimensions [4, 7]. Massive adjoint scalar bosons arise from particles whose $(d + M)$ -dimensional polarization and momentum vectors are given by³

$$\mathcal{E}_a = (0|0, e_a), \quad K_a = (k_a|\kappa_a, 0). \quad (10)$$

³We choose the internal polarization of the would-be scalars to be orthogonal to the internal momenta of all the particles.

Thus, invariants involving the adjoint scalars are given by

$$K_a \cdot K_b = k_a \cdot k_b - \kappa_a \cdot \kappa_b, \quad \mathcal{E}_a \cdot K_b = 0, \quad \mathcal{E}_a \cdot \mathcal{E}_b = e_a \cdot e_b. \quad (11)$$

Again this implies that the amplitudes for massive vectors and scalars can be obtained from the corresponding massless amplitudes by simply replacing $k_a \cdot k_b$ with $k_a \cdot k_b - \kappa_a \cdot \kappa_b$.

The field theory corresponding to these dimensionally-reduced amplitudes contains adjoint scalar fields Φ_i^j , whose masses $g|v_i - v_j|$ arise from the $g^2 \text{Tr}([\Phi, H]^2)$ term in the Lagrangian [11]. As described above, the couplings in the theory are consistent with internal momentum conservation at each vertex. We have computed various mixed gauge-field and scalar four- and five-point amplitudes in the spontaneously-broken theory, and verified in all cases that the massive amplitudes are equivalent to the corresponding massless amplitudes with the replacement $k_a \cdot k_b \rightarrow k_a \cdot k_b - \kappa_a \cdot \kappa_b$, where $\kappa_a = g(v_{i_a} - v_{i_{a+1}})$. This is not initially obvious from the Feynman diagram calculation in the spontaneously-broken theory, in which several different diagrams conspire to give the appropriate expressions, but is manifest from the dimensional reduction of the CHY representation of the amplitudes.

What if the internal momenta κ_a of the particles span more than one dimension of the M -dimensional internal space? In this case, the dimensionally-reduced amplitudes appear to correspond to massive amplitudes in a spontaneously-broken gauge theory in which several adjoint Higgs fields H^I , $I = 1, \dots, h$ have nonzero vacuum expectation values $\langle H^I \rangle = \text{diag}(v_1^I, v_2^I, \dots, v_N^I)$ that are mutually commuting. In this theory, the off-diagonal gauge bosons W_i^j obtain masses $g[\sum_{I=1}^h (v_i^I - v_j^I)^2]^{1/2}$. We identify the internal momentum κ_{ij}^I of W_i^j with $g(v_i^I - v_j^I)$, automatically satisfying the triple vertex constraint $\kappa_{ij}^I + \kappa_{jk}^I = \kappa_{ik}^I$. Although momentum conservation still requires $\sum_{a=1}^n \kappa_a^I = 0$, the constraint on the masses of the external particles is considerably weakened compared to the case in which only one Higgs field gets a vev.

Finally, we recall that momentum conservation can be used to express all kinematic invariants $k_a \cdot k_b$ of an n -point amplitude in terms of an independent set of $n(n-3)/2$ invariants

$$k_1 \cdot k_c, \quad k_2 \cdot k_c, \quad k_c \cdot k_d, \quad c, d \in \{3, \dots, n-1\}. \quad (12)$$

If we consider n -point amplitudes with at most three massive particles ($a = 1, 2$, and n), then $\kappa_c = 0$ for $c \in \{3, \dots, n-1\}$ and all of the invariants in eq. (12) are unchanged under $k_a \cdot k_b \rightarrow k_a \cdot k_b - \kappa_a \cdot \kappa_b$. Consequently, the expressions for amplitudes with at most three massive particles, when expressed in terms of $\epsilon_a \cdot k_b$, $\epsilon_a \cdot \epsilon_b$, and the invariants (12), are identical to the corresponding massless amplitudes, as found in ref. [7]. For example, eqs. (8) and (9) reduce to the massless amplitude when m_3 , and therefore κ_3 , vanishes.

In this note, we have argued that dimensionally-reduced tree-level gauge-theory amplitudes of massless particles possessing internal momentum κ are equivalent to tree-level amplitudes of massive particles in a lower-dimensional gauge theory in which the gauge symmetry is spontaneously broken by an adjoint Higgs field. The same procedure can be applied to obtain amplitudes of massive particles in other theories with a CHY representation, including gravity, Dirac-Born-Infeld theory, and the nonlinear σ model. It will be intriguing to determine the lower-dimensional theories to which these massive amplitudes belong.

Acknowledgments

The author wishes to thank Freddy Cachazo, Song He, and Ellis Yuan for fruitful discussions. This material is based upon work supported in part by the National Science Foundation under Grant No. PHY14-16123.

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