An improved quaternion Gauss–Newton algorithm for attitude determination using magnetometer and accelerometer

Liu Fei, Li Jie *, Wang Haifu, Liu Chang

School of Mechatronical Engineering, Beijing Institute of Technology, Beijing 100081, China

Received 21 August 2013; revised 17 October 2013; accepted 19 December 2013
Available online 24 April 2014

1. Introduction

In many spacecraft attitude systems, multi-vector observations are employed to determine attitude via some measurement sensors, including three-axis magnetometers, accelerometers, sun sensors, Earth-horizon sensors, global positioning system (GPS) sensors and star trackers. The specific choice for the onboard sensor hardware is mostly driven by the individual requirements of the spacecraft mission. In many spacecraft attitude determination methods, only two vector measurements are used. For the accuracy and cost requirements of the small unmanned air vehicles (UAVs), a three-axis magnetometer and accelerometer are often adopted to determine attitude. So it is necessary to explore a useful algorithm and conduct comprehensive analysis for the two vector attitude determination method.

The earliest algorithm for determining spacecraft attitude from two vector measurements was three axis attitude determination (TRIAD) algorithm, which has been applied to both ground-based and onboard. However, TRIAD is suboptimal because it ignores one piece of information from one of the unit vector. As most spacecraft are equipped with sensors able to provide surplus measurements and computers in vehicles are able to work at a negligible additional computational cost, optimal algorithms are employed more frequently than deterministic ones. Almost all single-frame algorithms are based on a problem proposed by Wahba. These algorithms,
which differ from small speed and robustness advantages/disadvantages, include quaternion estimator (QUEST), estimators of the optimal quaternion (ESOQ and ESOQ-2), q-method and singular value decomposition (SVD) method. The basic principles of the above algorithms are to figure out the eigenvector of the maximum eigenvalue.4

Mortari et al.3 proposed an optimal linear attitude estimator (OLAE) and it reformulates the nonlinear constrained problems as a rigorously linear unconstrained problem. But due to the use of the Rodrigues vector, the singularity cannot be avoided. Markley6,7 particularly researched fast quaternion attitude estimation algorithm and optimal attitude matrix algorithm from two vector measurements based on algorithms mentioned above. In this paper, we adopt the Gauss–Newton method to determine the attitude, because it is more efficient than the previous investigations, particularly for estimation involving large dynamic systems where the computational price to compute the system response and their gradients is high. For aircraft parameter estimation purposes the Gauss–Newton method is therefore widely used.8 However, most of the researchers employed the Gauss–Newton method with magnetometer and accelerometer measurement vectors to compute the attitude quaternion, while they treated two vectors the same, which is not reasonable for the sensors with different accuracy.9 In Tanygin’s research,10 the attitude error variance was derived and the weights for each measurement were considered. However, the Gauss–Newton algorithm was just employed to estimate the attitude by GPS antenna baselines only, so it was not essential to normalize the measurements.

The purpose of this paper is to present a corresponding explicit derivation for the unit measurement vector noise form and a variance analysis of the whole algorithm to determine the weights for each measurement. As we all know, these traditional attitude determination algorithms using vector measurements are almost based on the QUEST measurement model which is developed by Shuster11 for the sensors with comparable accuracy. Meanwhile, the approach made the small field-of-view assumption.11 Although this measurement model is simple and convenient for the unit-normalization of the measurement vectors, it is not suitable for our application to the two totally different kinds of sensors without small field-of-view.

The structure of the paper is as follows. First, the measurement model for vector sensors is proposed based on first-order Taylor series expansion and new statistical characteristic of measurement noise is derived. After that, the general variance analysis of the algorithm is performed to quantify the approximation error. According to the variance, the weights for two sensors are established. Then, the details of the improved quaternion Gauss–Newton algorithm are discussed. Finally, simulation and experimental tests are conducted to evaluate the whole algorithm.

2. Measurement model

Three-axis magnetometer and accelerometer are both vector sensors. A conventional sensor includes four sources of errors and uncertainties:

(1) Measurement noise.
(2) Measurement bias.
(3) Quantization errors (i.e. analog-to-digital truncation of the measurement).
(4) Sensors misalignment (i.e. angular errors from the mechanical frame non-orthogonal misalignment).12

We assume that the two sensors have been calibrated so that correlated errors such as null-shift or Markov biases have been removed. Thus, it is reasonable to assume that the remaining measurement noise term ΔW can be modeled as a zero-mean Gaussian noise sequence with variance R and the output error model given as

\[ \dot{W} = W + \Delta W = AV + \Delta W \] (1)

where Civen the true measurement value in the body frame, V is the referenced vector and A is the attitude matrix. In this research, we consider that diagonal elements of the measurement noise variance are not equal after initial calibration in the measured body frame, which is more significant in practice. The true measurement vector can be reconstructed in unit vector form as

\[ w = Av \] (3)

where

\[ w = W/|W| \] (4a)
\[ v = V/|V| \] (4b)

When measurement noise is present,

\[ \dot{w} = Av + \Delta w \] (5a)
\[ \dot{w} = \dot{W}/|W| \] (5b)

to acquire the variance for the actual unit vector measurement noise, the true vector must be replaced with the measured one in Eq. (1). However, Av could not be separated with the noise in the actual model, so the actual noise model contains nonlinear terms coupled with non-Gaussian component.13 In order to derive the variance of unit measurement noise Δw, the new measurement model is obtained based on first-order Taylor series expansion of Eq. (5a), given as

\[ \dot{w} \approx Av + J \cdot \Delta W \] (6)

where J is Jacobian of Eq. (5b):

\[ J = |W|^{-1/2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - |W|^{-3/2} WW^T \] (7)

Meanwhile, the error vector Δw lies in the plane perpendicular to \( \dot{w} \), characterized by

\[ \dot{w} \times \Delta w = 0 \] (8)

Thus, the new statistical property is approximately Gaussian and is given by

\[ E(\Delta w) = 0 \]
\[ R_{\text{unit}} = JRJ^T \] (9)
Though this approach does not make the small field-of-view assumption, it requires that the measurement noise is small compared to the measurement, which is valid for every kind of sensors.

Obviously, \( R_{\text{init}} \) is a singular matrix. The eigenvalue/eigenvector decomposition of \( R_{\text{init}} \) can be indicated as

\[
R_{\text{init}} = TAT^T = \begin{bmatrix}
\lambda_1 & 0 & 0 \\
0 & \lambda_2 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
t_1 \\
t_2 \\
t_3
\end{bmatrix}
\]

where \( t_1, t_2 \) and \( t_3 \) are the eigenvectors and \( \lambda_1, \lambda_2 \) are the non-zero eigenvalues. Compared with \( R_{\text{init}} \), the noise variance of QUSET measurement model has two repeated eigenvalues. The two can be assumed to have the same eigenvectors and the only difference is their nonzero eigenvalues. The purpose to study this unit measurement model is to acquire the accurate noise effects of different kinds of sensors. In this way, the basis of effects could be built to calculate the optimal quaternion.

### 3. Attitude error variance analysis

We select the Wahba problem to find the attitude matrix \( A \) whose determinant is +1:

\[
L(A) = \frac{1}{2} \sum_i (\bar{\omega}_i - A \omega_i)^T a_i (\bar{\omega}_i - A \omega_i) \quad (i = 1, 2)
\]

where subscript \( i \) refers to the \( i \)th sensor vector and \( a_i \) is the non-negative weight. From Eq. (11), we can see that the attitude matrix is not only related to measurements but also with \( a_i \). Hence, it is significant to research the attitude error variance to find the optimal \( a_i \).

Conventionally, the attitude error variance matrix is defined as the variance matrix of the errors of Euler angle \([\psi, \theta, \phi]^T\) which parameterizes the attitude. This turns out to be very cumbersome to calculate and less informative, so a set of error angle vector in the body frame is given to analyze the variance. That is

\[
\delta \xi = [\delta \xi_1, \delta \xi_2, \delta \xi_3]^T
\]

The relationship between the Euler angle errors and error angles in the body frame can be derived as:  

\[
\begin{bmatrix}
\delta \psi \\
\delta \theta \\
\delta \phi
\end{bmatrix} = \begin{bmatrix}
-\tan \theta \cos \psi & -\tan \theta \sin \psi & -1 \\
\sin \psi & -\cos \psi & 0 \\
\cos \theta & \sin \theta & 0
\end{bmatrix} \begin{bmatrix}
\delta \xi_1 \\
\delta \xi_2 \\
\delta \xi_3
\end{bmatrix}
\]

It is assumed that \( \delta \xi \) is unbiased and small so that the true attitude matrix \( A_{\text{true}} \) is also the expected mean to first order in the angles. Thus, to first order,

\[
\hat{A} \approx (I - [\delta \xi \times]) A_{\text{true}}
\]

where \( \hat{A} \) is the measured attitude matrix.\(^{15}\) Substitute Eq. (14) into Eq. (11) and we get

\[
L(\hat{A}) = \frac{1}{2} \sum_i (\bar{\omega}_i - (I - [\delta \xi \times]) A_{\text{true}} \omega_i)^T a_i (\bar{\omega}_i - (I - [\delta \xi \times]) A_{\text{true}} \omega_i)
\]

\[
= \frac{1}{2} \sum_i a_i (\Delta \omega_i - [\omega_i \times] \delta \xi)^T (\Delta \omega_i - [\omega_i \times] \delta \xi)
\]

The Gauss–Newton method transforms the nonlinear parameter estimation problem into a linear least squares recursive estimation, which results in a suboptimal solution. Nevertheless, the final results are unbiased estimation. Therefore, the attitude error variance for the general cost function is derived. As a sequence of this, the variance matrix of the Wahba problem can be given directly from the Fisher information matrix and the negative-log-likelihood function is equivalent to the cost function Eq. (11) in our application.

\[
P_{\text{bias}} = \lim_{t \to \infty} E \left( \frac{\partial^2 L}{\partial (\delta \xi) \partial (\delta \xi)^T} \right)
\]

For the measurements given by Eq. (15) are linear form in \( \delta \xi \) vector and Gaussian, Eq. (17) is true for finite \( i \). The simple form for the error angular variance is given as

\[
P_{\text{bias}} = \left( \sum_i a_i [\omega_i \times] [\omega_i \times]^T \right)^{-1} \approx \left( \sum_i a_i [\omega_i \times] [\omega_i \times]^T \right)^{-1}
\]

As we all know, for maximum likelihood estimates of the unknown parameters, the weights should be selected as

\[
a_i = (R_{i\text{init}})^{-1}
\]

And the weight \( a_i \) is a matrix in this situation. However, the normalized noise variance established before is singular, so the weights cannot be acquired by matrix inversion directly. Also, it is extremely complex to find weights by minimizing the trace of the error angular variance.\(^{16}\) Thus, \( a_i \) could be chosen to minimize some matrix 2-norm of the following equations:

\[
\varphi(\kappa_i^2) = \| \kappa_i^2 I_{3 \times 3} - R_{i\text{init}} \|
\]

\[
a_i = \frac{1}{\kappa_i^2}
\]

In this situation, the weight \( a_i \) is recognized as a scalar value. According to Eq. (10), we can rewrite Eq. (19a) as

\[
\varphi(\kappa_i^2) = \| [t_1, t_2, t_3] \| \left( \begin{array}{ccc}
\kappa_i^2 - \lambda_1 & 0 & 0 \\
0 & \kappa_i^2 - \lambda_2 & 0 \\
0 & 0 & \kappa_i^2
\end{array} \right) \| [t_1, t_2, t_3] \| ^2
\]

Now it is clear that \( \varphi(\kappa_i^2) \) is minimized by \( \kappa_i^2(\kappa_i^2 - \lambda_1)(\kappa_i^2 - \lambda_2) \) minimized to the greatest extend. According to Frobenius norm,\(^{17}\) it leads to

\[
\kappa_i^2 = \frac{1}{3} \text{tr}(R_{i\text{init}})
\]

In the case when the noise variance elements \( R_{ii} \), \( R_{ij} \) and \( R_{ji} \) are at equivalent level, \( \kappa_i^2 \) can be obtained as

\[
\kappa_i^2 = \frac{2}{3} \text{det}(W_i)^{-2} R_{ii}
\]

In our application, Eq. (17) can be simplified to

\[
P_{\text{bias}} = \frac{1}{a_1 + a_2} I_{3 \times 3} + \left[ \bar{\omega}_1 \times \bar{\omega}_2 \right]^2
\]

\[
\times \left[ \frac{a_1 \bar{\omega}_1 \bar{\omega}_2^T}{a_2(a_1 + a_2)} + \frac{a_2 \bar{\omega}_2 \bar{\omega}_1^T}{a_1(a_1 + a_2)} + \bar{\omega}_1 \cdot \bar{\omega}_2 \left( \bar{\omega}_1 \bar{\omega}_2^T + \bar{\omega}_2 \bar{\omega}_1^T \right) \right]
\]

\[
(23)
\]
We can see that the form of the variance is consistent with the variance of QUEST algorithm. As the unit noise variance derived from Eq. (9) is different from the QUEST, the weight has been made to be more reasonable. Meanwhile, the quaternion is a four-dimensional vector, defined as
\[ q = [Q, p]^T \]
with
\[ p = [p_1, p_2, p_3]^T \]
The attitude matrix \( A \) is related to the quaternion by
\[ A(q) = (Q^2 - p^2 p^T)I + 2pp^T + 2Q[p \times] \]
This model is quadratic in \( q \).

Also, it is difficult to express the Fisher information matrix in terms of the quaternion directly because the components of the quaternion are not independent. Generally, define \( q_e \) as the small rotation error between the measured quaternion \( \hat{q} \) and the true quaternion \( q \), which is represented as
\[ q = \hat{q} \otimes q_e \]
As \( q_e \) is assumed to be a small amount, it can be approximated as
\[ q_e = \begin{bmatrix} 1 \\ \rho_e_1 \\ \rho_e_2 \\ \rho_e_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \rho_e_1 \\ \rho_e_2 \\ \rho_e_3 \end{bmatrix} \]
Substituting Eq. (29) into Eq. (27), the relationship between \( \rho_e \) and \( \delta x \) could be expressed as
\[ \rho_e = \delta x / 2 \]
As a result, the variance matrix of \( q_e \) can be simply given by
\[ P_{\rho_e, \rho_e} = \frac{1}{2} P_{\text{SINX}} \]
That is, the weights determined above can minimize the angular error variance and quaternion error variance at the same time. Overall, we could note a proper \( \alpha_i \) is determined and the following work will focus on how to determine the attitude quaternion by the improved quaternion Gauss–Newton algorithm.

4. Improved quaternion Gauss–Newton algorithm

The Wahba problem is also adopted as the cost function for developing a quaternion estimator by common Gauss–Newton (CGN) algorithm which treats two vectors in the same way. The attitude quaternion can be solved by linearizing the nonlinear optimization problem using Taylor series expansion which is truncated to the first order, that is,
\[ A(q) r_i \approx A(\hat{q}^{(k)}) r_i + \nabla A(\hat{q}^{(k)}) r_i (q - \hat{q}^{(k)}) \]
where \( \nabla A \) is Jacobian matrix, \( \hat{q} \) is the quaternion to be determined and superscript \( k \) represents the iterations. The stationarity condition to minimize Eq. (11) yields
\[ \partial L(q) / \partial q = -2[\nabla A(\hat{q}^{(k)}) r_i]^T [\omega_t - A(\hat{q}^{(k)}) r_i - \nabla A(\hat{q}^{(k)}) r_i (q - \hat{q}^{(k)})] = 0 \]

So the CGN algorithm has already translated the nonlinear attitude problem into linear problem and employed the principle of least squares to estimate the quaternion. And the quaternion could be acquired by iteration.

Based on the Wahba problem, the weights are figured out to minimize the angle error variance using the measurement model established in this paper. Subsequently, we combine the CGN algorithm and the results of Sections 2 and 3 above to improve the performance of the attitude determination system. The explicit improved Gauss–Newton (IGN) algorithm process is presented as follows:

1. Initialize the attitude quaternion \( \hat{q}^{(0)} \) as \([1, 0, 0, 0]^T\) or as the results of the last computing period.
2. Normalize the measurement vectors.
3. Calculate the new unit measurement error variance by Eq. (9) and acquire the weights for each sensor by Eq. (21).
4. Use the unit measurement vectors and weights to formulate the correction term \( \Delta q^{(k)} \):
\[ \Delta q^{(k)} = \left( \sum_i a_i (\nabla A(\hat{q}^{(k)}) r_i)^T \nabla A(\hat{q}^{(k)}) r_i \right)^{-1} \left( \sum_i a_i (\nabla A(\hat{q}^{(k)}) r_i)^T (z_i - A(\hat{q}^{(k)}) r_i) \right) \]
5. Update the attitude quaternion as follows:
\[ q^{(k+1)} = q^{(k)} + \Delta q^{(k)} \]
6. Normalize the ultimate updated quaternion.
7. Return to Step (4) and repeat until convergence is achieved.

In general, the whole algorithm has been presented completely. Compare with the CGN algorithm, the IGN algorithm adds the second and third steps. Moreover, the weights for sensors are taken into account during the forth step to obtain the correction term. Given the algorithm developed above, a high iteration to compute the quaternion is unnecessary and the precision is also improved.

5. Simulation results

Static and dynamic simulations are used to evaluate the performance of the results of the IGN algorithm and CGN algorithm developed above. We will compare the results of both algorithms. The zero-mean terms of measurement noise are assumed to be uncorrelated with variances equal to
\[ R_1 = [2^2, 3^2, 2.5^2]^T \times 10^{-14} T^2, R_2 = [10^2, 9^2, 9.5^2]^T (\text{mg/s})^2 \]
These are reasonable error magnitude for the low-cost magnetometer and accelerometer.
5.1. Static simulations

The objective of the static simulation is to analyze the convergence and accuracy performance of the attitude algorithm. In this case, we set the true attitude as

\[ q_{\text{true}} = [0.7595, -0.0934, 0.5858, 0.5242]^T \]  

(37)

The two measurement vectors of magnetometer and accelerometer on the body-fixed directions respectively align with

\[
\begin{align*}
\mathbf{w}_1 &= [-89.3847, -44.7538, 536.8714]^T \times 10^{-7} \mathbf{T} \\
\mathbf{w}_2 &= [-4.9000, 2.9027, 7.9752]^T \mathbf{m/s}^2
\end{align*}
\]  

(38)

Eq. (23) gives the expectation error angle variance for this scenario as

\[
P_{\text{error}} = \begin{bmatrix}
4.24 & -2.35 & -6.82 \\
-2.35 & 1.54 & 3.95 \\
-6.82 & 3.95 & 11.64
\end{bmatrix} \times 10^{-5} \text{rad}^2
\]  

(39)

We simulate \( N = 10,000 \) times tests, randomly produced using sensor noise with the assumptions above. As Euler angle errors \( \delta \psi, \delta \theta \) and \( \delta \phi \) which are yaw, pitch, and roll errors are somewhat intuitive and convenient for presenting, they are employed here to show the simulation results in Fig. 1. The horizontal axis of Fig. 1 represents the simulation test times, and in order to observe the simulation results clearly, only 5000-6000 tests are drawn. The parameters used to quantify the accuracy are the absolute mean value of the errors and the variance of errors. The absolute mean value, variance of the errors and the maximum number of floating point operations to compute a set of angles are summarized in Table 1.

From the Fig. 1, compared with CGN algorithm, we can see that the attitude accuracy of IGN algorithm is raised apparently, especially for the pitch and roll angles. Clearly, \( \delta \psi \) is the largest error, which is because of the degree of observability of certain angles from the vectors selected. It is irrelevant to the inherent limitation of algorithm itself. Fig. 2 shows the trace of the measured attitude error variance matrix \( P_{\text{error}} \). It gives a global picture of the selected error-model quantification. Compared with the results in Eq. (39), we can see that the error is a residual, not the attitude error itself for most of the time. Generally, this parameter can provide the upper bounds of the attitude accuracy for the selected error model.

For a real-time computer in a spacecraft attitude determined system which must finish all its required tasks in a limited time, the longest computation time is more important than the average time. Therefore, the maximum number of iteration and floating point operations, which is independent of both software and hardware, were used to evaluate the algorithm speed. The census of iterations for each test is presented in Fig. 3. It is acquired from Fig. 3 that IGN algorithm only requires three iterations for each test, but the maximum number of iteration of the IGN algorithm is five.

<table>
<thead>
<tr>
<th>Performance</th>
<th>Yaw</th>
<th>Pitch</th>
<th>Roll</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance of IGN ((^{\circ} ))</td>
<td>0.2822</td>
<td>0.0033</td>
<td>0.0037</td>
</tr>
<tr>
<td>Variance of CGN ((^{\circ} ))</td>
<td>0.3616</td>
<td>0.0380</td>
<td>0.0900</td>
</tr>
<tr>
<td>Mean of IGN ((^{\circ} ))</td>
<td>0.4208</td>
<td>0.0459</td>
<td>0.484</td>
</tr>
<tr>
<td>Mean of CGN ((^{\circ} ))</td>
<td>0.4823</td>
<td>0.1549</td>
<td>0.2386</td>
</tr>
<tr>
<td>Floating point operations (IGN)</td>
<td>1640</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Floating point operations (CGN)</td>
<td>3960</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1  Euler angle errors from static simulation.

Fig. 2  \( \text{tr}(P_{\text{error}}) \) distribution.

Fig. 3  Iterations distribution.
the maximum number of floating point operations for the two algorithms was calculated by the MATLAB function and is listed in Table 1. We could easily conclude that IGN algorithm needs less computation time and is more efficient when high attitude estimation speed is required.

5.2. Dynamic simulations

It is necessary to test the attitude algorithm with a set of simulations of a vehicle in motion. The vehicle is simulated performing maneuvers that include accelerations and attitude changes. We simulate $N=2000$ times tests and both of the sensor triad error statistics are the same as used in the static simulation.

Fig. 4 shows the true attitude history and Fig. 5 show the results for the dynamic simulations. The absolute mean value, variance of the errors and the maximum number of floating point operations to compute a set of angles are summarized in Table 2. It is clear that the IGN algorithm could enhance the accuracy. Meanwhile, the maximum number of floating point operations does not increase. However, the magnitude of the error on the residuals is larger than the static situation. This is due to the inaccurate dynamic acceleration measurements in reference frame used in the test. In a real-time system, maneuvers encountered will be more severe and there will be higher dynamics or frequency content. In this situation, the algorithm will perform poorly in prolonged accelerated maneuvers such as the turns.

**Table 2** Simulation results from dynamic test data.

<table>
<thead>
<tr>
<th>Performance</th>
<th>Yaw</th>
<th>Pitch</th>
<th>Roll</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance of IGN $(\circ^2)$</td>
<td>0.3109</td>
<td>0.0030</td>
<td>0.0175</td>
</tr>
<tr>
<td>Variance of CGN $(\circ^2)$</td>
<td>0.3685</td>
<td>0.0094</td>
<td>0.0980</td>
</tr>
<tr>
<td>Mean of IGN $(\circ)$</td>
<td>0.4459</td>
<td>0.0437</td>
<td>0.0930</td>
</tr>
<tr>
<td>Mean of CGN $(\circ)$</td>
<td>0.4800</td>
<td>0.0750</td>
<td>0.2019</td>
</tr>
<tr>
<td>Floating point operations (IGN)</td>
<td>1640</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Floating point operations (CGN)</td>
<td>3960</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. Experimental validation

The experimental validation of this algorithm was conducted using flight data collected on the research UAV platforms. The research UAV is approximately 2.5 m long with a 2.2 m wing span. It is powered with propeller electric power system and has been utilized for testing the autonomous formation flight control laws several times. It is equipped with XW-ADU 7620 and Analog Device ADIS-16405. XW-ADU 7620 which was adopted to collect the true reference attitude is a high-precision integrated navigation system. ADIS-16405 consists of a triad of rate gyro, accelerometer and magnetometer, and the outputs of accelerometer and magnetometer were used by our algorithm. Furthermore, the accelerometer sensor has its own factory compensation for sensitivity, bias, alignment, linearization and temperature. However, the magnetometer measurements required to calibrate scale factor errors, hard and soft iron bias, which was accomplished after the flight by calibration algorithm. Then the low-pass filter was employed to reduce flutter errors and the measurements were sampled at 50 Hz.

Fig. 4 True attitude.  
Fig. 6 Flight test trajectory.  
Fig. 5 Euler angle errors from dynamic simulation.
We selected a 400 s portion of the flight test trajectory shown in Fig. 6 to complete the validation. The start point was established at 200 s of the flight test and the trajectory was depicted in the north-east-up coordinate system. From the trajectory we can see that the UAV experienced climbing, subduction and stationary process. So there were accelerations and deaccelerations as well as changes of the three attitudes. The absolute mean value and the variance of the errors are summarized in Table 3. As the computation has been discussed above, it is unnecessary to demonstrate it here again. According to the nominal performance, IGN algorithm still has higher precision and is more practical. The reference and test attitude of the two algorithms are illustrated in Fig. 7. We could see that the attitude provided by the IGN algorithm generally agrees with the reference attitude. CGN algorithm could not give available attitude when the UAV is in maneuvering condition, especially from about 260 s to 310 s. Consequently, according to the analytical results of the dynamic simulation, although IGN algorithm improves the attitude accuracy during maneuvers, it is more suitable to be employed as an aiding system for a triad of rate gyros.

7. Conclusions

In this paper a new measurement model for vector sensors is proposed based on first-order Taylor series expansion and new statistical characteristic of measurement noise is derived. Based on the Wahba problem, the optimal weights of two sensors are derived to minimize the attitude error variance. Subsequently, the new measurement model and optimal weights are combined with the CGN algorithm. Hence, the performance of the CGN algorithm is improved. According to the simulation tests and experimental validation, the accuracy of the Gauss–Newton algorithm is improved and the algorithm computation amount is reduced. Compared with the traditional algorithms for determining attitude by vector measurements, the algorithm in this paper has also mimimized the Wahba’s loss function, but it is based on different measurement model and can be used more widely. Future researches may include refining and implementing the algorithm to accommodate various practical environments.

Table 3 Results from flight test data.

<table>
<thead>
<tr>
<th>Performance</th>
<th>Yaw</th>
<th>Pitch</th>
<th>Roll</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance of IGN (°²)</td>
<td>0.2099</td>
<td>0.7497</td>
<td>2.1918</td>
</tr>
<tr>
<td>Variance of CGN (°²)</td>
<td>17.3556</td>
<td>3.1948</td>
<td>14.8675</td>
</tr>
<tr>
<td>Mean of IGN (°)</td>
<td>0.2258</td>
<td>0.4003</td>
<td>1.0443</td>
</tr>
<tr>
<td>Mean of CGN (°)</td>
<td>1.446</td>
<td>0.8125</td>
<td>1.0926</td>
</tr>
</tbody>
</table>

Fig. 7 Euler angles from flight test.

References


Liu Fei received the M.S. degree in control science and engineering from Beijing Institute of Technology in 2009. And now she is a Ph.D. candidate at School of Mechatronical Engineering, Beijing Institute of Technology. Her main research interests are attitude estimation and sensor fusion of aerial vehicles.

Li Jie is a professor and Ph.D. supervisor at School of Mechatronical Engineering, Beijing Institute of Technology. He received the Ph.D. degree from the same university in 2001. His current research interest is the design of micro aerial vehicle.

Wang Haifu is a professor and Ph.D. supervisor at School of Mechatronical Engineering, Beijing Institute of Technology. He received the Ph.D. degree from the same university in 1996. His current research interests are the design of micro aerial vehicle and spacecraft protection technology.

Liu Chang received the B.S. degree in mechatronic engineering from Beijing Institute of Technology in 2007. And now is a Ph.D. candidate at School of Mechatronical Engineering, Beijing Institute of Technology. His main research interest is modeling and control of micro aerial vehicles.