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Order and chaos in the brain: fractal time series analysis of the EEG activity during a cognitive problem solving task.

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Abstract

The study of brain dynamics has been approached from different mathematical strategies with the aim to obtain more information of the bioelectrical signals coming from an electroencephalogram (EEG). Although several of these tools try to conciliate the fact of using linear approaches to study a non-linear phenomenon, during the last years a set of complementary and alternative approaches has been used to mine deeper into the nature of the brain dynamics and its correlates with experience, thoughts and actions. One of these approaches comes from chaos theory and fractal geometry and considers a model where brain activity, as a time series of an electrical potential variation (EEG) recorded from the scalp, can be assumed as a dynamical state that moves in the range of $1/f^{\alpha}$ (fractal) noise. In this model it is possible to see brain dynamics as a functional state who moves from more chaotic or unpredictable dynamics (Brown or Gaussian uncorrelated noise), to quasi-chaotic or statistical noise (fractal or self-similar noise). The amount of order into the chaotic EEG background must be an indicator of the organization of brain procedural resources dealing with different circumstances. At the same time, it must reflect the degree of inter-individual differences and intra-individual variability in the way each different brain works. To test this hypothesis we implemented a study where EEG activity was recorded during the performing of a simple visual intelligent test (Raven test, abbreviated version of 15 questions) in a set of 10 adults to study their similarities and differences in the rendering of the test (estimated range of IQ) and in the process of solving the easy and the difficult part of the test. We estimated the Hurst exponent and the fractal dimension of the time series for each of the 14 EEG channels (Emotiv-Epoc® BCI headset) and searched for correlations and consistencies in the values of H and the difficulty level of the cognitive task. We found that order tend to emerge from a chaotic background when brain focuses on problem solving, rising the degree of predictability, self-similarity and persistent behavior of the EEG signal.

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Keywords: Order; Chaos; 1/f Noise; Hurst exponent; Fractal dimension; Problem solving; EEG phenomenology.

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1. Introduction

Electroencephalographic (EEG) dynamics has captured our curiosity since Hans Berger in 1929 [1] recorded human brain electrical activity from the scalp revealing that brain cortex manifests a complex oscillatory undercurrents that in a way or another must be correlated with our behavioral and mental activity.

Since then, this source of information about brain electrical phenomena has been the focus of many generations of neuroscientists that has tried to unveil different aspects of the underlying processes that seems to be associated to our motor, perception and cognitive skills. As it is possible to expect, the central attention of these studies has been concentrated in clinical problems, in which fortuitous circumstances have brought some light over very specific problems associated with local brain damages [2]. Thank to this quasi-localized general structure and function correlations has been possible to map into the brain cortex functional areas specifically related with the many ways we human beings deal with the world [3].

Once this structural mapping was done and perfected, the next step has been to understand the functional processing of the brain, meaning how the brain manage its resources to monitor, control and modulate an always behaving animal. In this respect one of the more successful research areas has been the studies conducted by chronobiology [4], which looking at the sleep-awake cycle, has revealed the complex behavior of the brain even in a state we call resting, showing first that the brain never rests and that while that part of our life in which we are sleeping and we have very scarce narrative record of our experiences, the brain works in many different states which bring forth important consequences to our awake daily life. Even though a big deal of findings has been done in this area, there are still some central questions that keep the flame of curiosity and exploration to continue digging in this area for more answers [5].

Until now, electroencephalography has been a powerful tool to ‘see’ what can be happening in the brain. The fact that we can measure electrical fluctuations from the brain cortex allow us to model brain functioning in terms of a communication system that use waves to transport messages and exert control over distant places of the brain. As we can only detect the local whole electrical signal captured by electrodes settled in different parts of the scalp, it is necessary to dissect first this signal in their intensity and frequency components and then to look for consistencies or correlations among these variations according to a particular experimental design built to answer specific questions.

The study of functional organization of the brain involves the analysis of intensity and spectral components of the brain signal in deeper detail, in a way that incorporates new descriptive and quantitative approaches, specially when recent findings and theories development coming from diverse areas of science allow us to look at the electrical brain activity as a case of a set of many other common natural phenomena that has been shown following some natural laws that can be mathematically well described [6].

1.1. Non-linear systems and EEG phenomenology.

One of the main advances carried out during the last decades has been the incorporation of mathematical tools that describe non-linear systems [7-8]. A system of this nature is such that as a consequence of its intrinsic complexity it is impossible to accurately describe it with mathematical functions of simple proportionality. During the long history of electroencephalography and until just recently, the only way to explore the brain functioning in terms of the EEG signal has been to see it in a very simplistic but descriptive fashion. By first comparing differences in the intensity of the signals it started to smooth over the arid terrain of a very complex system. First descriptions were enthusiastic in finding correlated changes of intensity of the signal under particular experimental conditions [9] making to believe that the brain were not so complex to understand as we may thought earlier. Further findings that expanded the experimental questions showed a panorama not so simple to conciliate with a simplistic way to approach the problem. Intensity variations were not enough to say something clear concerning to the brain functional dynamics. At first glance it seemed that if we focus on the correlations between signal intensity variations of a single EEG frequency band, associated with a particular behavioral or perceptual task, the more experimental conditions or tasks we tried, the more participation of the same EEG band appear [10].

More success was carried out when studying the spectral distribution of the EEG frequencies. It has been characterized that the whole frequency spectrum of the EEG signal can be subdivided into five frequency ranges in a progression from sleep to awake in an increasing cognitive workload activity. These frequency bands receive the names of Delta (0-4 Hz), Theta (4-8 Hz), Alpha (8-12 Hz), Beta (12-30 Hz) and Gamma (>30 Hz). All after Berger gave the name of Alpha wave to the first clean EEG recording he obtained coming from the occipital region.

Having this spectral classification, intensity variations, correlation, auto-correlation and coherence analysis of the signal acquired, still the approach continued using mainly a linear understanding of the phenomena. As far as we can stretch this view during the last decades, sooner we started to realize that it was a very good starting jump but to mine deeper in the mysteries of the brain we better start considering it as what it is, a system more easily understandable having a non-linear dynamics substrate and a whole new realm of possible interpretations coming from complex and chaotic system theory, having variable stability, volatility, showing phenomena of persistence, long memory and fractal functional structure [11-12].

1.2. The brain as a predictive device.

The model of the brain as a predictive system found appeal when we think about how we make mistakes. For decades perceptual mistakes (or illusions) have been used to understand the underneath processes that allow us to explain these experiential discontinuities that we call errors. From experience, it is fairly evident that we, manipulating perception, can cheat the brain and that, at the same time, the brain, modulating its own activity, can cheat us making us believe that something is there when it is not. Experimental evidences making this clearer are the studies of the blind spot in our retina, the place where neuron axons from photoreceptors joint together to build up the optic nerve. If we close one eye and experiment with the open one, soon we will find a place in our visual field where we do not have visual information. The strange thing is that usually we are not aware of this partial blindness. In part because our eyes are always moving and we count with a pair of these visual devices, the lack of vision in this area is compensated, but if we perform controlled experiments we will find that the brain makes a more impressive adjustment to deal with this situation. If we surround the field of vision around the blind spot with a simple design, say a set of diagonal black and white bars, the brain completes the perception of the blind spot with the surrounding design. In other words, it seems that the brain completes any missing things by assuming anything what it is supposed to be there. That is also the reason because sometimes our attention is not captured by a conspicuous novelty that is in the middle of a very familiar environment. It is necessary to call the attention upon it, to notice it. So, in a very general way we can say the brain itself is kind of blind, meaning by this that it is not aware mainly of the surrounding environment but instead of the internal coherences and experiential regularities experienced through individual and social learning history. Therefore, a fairly valid question arises: How we can predict the behavior of a system that works as a predictive system?

1.3. Order and Chaos in the brain.

Complex non-linear systems move in the verge of instability and for this reason they are difficult to predict. In fact, it is the characteristic of unpredictability what better describes chaotic phenomena which give us a broader field of understanding of these processes far from the more intuitive and narrow view of chaos as only implying disorder. Living beings are very complex systems in such a way that many interacting variables must be taken into account simultaneously. Maybe we are able to predict their behavior under repetitive, controlled or stable conditions, but in front of a new and unknown challenge we will be not very confident about our bet.

Trying to understand how the brain manages to monitor, coordinate and organize itself based on the EEG recording could be very disappointing because interpretation based only on traditional lineal way to understand the problem is incomplete. By the contrary, thinking the brain as a chaotic or quasi-chaotic system gives us the opportunity to model it in a more realistic, even though not simpler, way.

In a previous work [13] some of us showed that in the EEG signal it is possible to find short-term predictable processes arising in the middle of a chaotic background. The predictability and self-similarity of these processes are related with some particular characteristic of these transient activities, specially its auto-correlation parameters. Using a musical transformation and further analysis of the brain signal we found transient harmonic activity arising as ordered and frequency synchronized functional configurations of the eeg signal during a mental task. According to our results it is possible to identify and analyze individually each of these musical self-similar or transient fractal harmonies in the brain. We also found substantial differences in the number of these fractal events when we explore inter- and intra-individual differences, or inter-hemispheric asymmetry.

1.4. Hurst Exponent, Fractal Dimension and 1/f Noise.

A time series of events allows the possibility to study the whole process tendency, and not only the final result or global parameters that by their mathematical nature are obligated to collapse the time giving us a frozen picture of the phenomena. Though this can be no problem while studying and analyzing historically independent samples, it is not the case when the study is focused on chronological processes in which any new event can be dependent in some extent to the previous states. For this kind of phenomena, an average comparison does not give us much information. Using global and time collapsed parameters can help us to look for some research initial directions to decide from where to start digging the data, but afterwards it is necessary to deal with time series analysis.

In studying historical phenomena, time series analysis allowed the discovery of some historical processes, apparently independent of its own past behavior that showed some regularity in their occurrence, as if an inner historical process would be clicking with a specific and regular time-lag

One the first of this characterization came from Harold Edwin Hurst [14] who was studying the hydrology time parameters of the Nile River when he found some strange auto-correlation in the periodicity of the Nile changing flux behavior. The mathematical model of this process gave rise to the Hurst exponent which can be understood as a measure of the long-range dependence that the time series has with respect to its own past history. The H parameter describes the behavior of the underlying physical system over which the observable phenomena arises. The dynamical patterns turn to be self-similar or long-memory, depending on the value of H which measures the intensity of this dependence to the past. Hurst exponent ranges from 0 to 1, having in the 0.5 value a phase shift limit. When $0 < H < 0.5$ indicates a time series with long term switching in sequential values, meaning that a high value in the series will probably followed by a low value, and so on. The result of this kind of behavior is a permanent return to average values without showing a particular trend or tendency. Values of $0.5 < H < 1$ indicates long term positive autocorrelation, meaning that high values are more probably followed by high values, occurring the same when low values appear, being more probably followed by low values. $H = 0.5$ indicates a completely uncorrelated series of Gaussian (Brownian motion) noise. This series is normally distributed and involves no long memory process. The higher the H value is, the more apparent the underlying tendency is, and the time series looks less “jagged” compared with those with low H values. Hurst exponent determines the rate of chaos, in other words, it measures the amount of ordered incidents occurring in time arising from a chaotic or uncorrelated background, and it is useful to distinguishing fractal from random time series. H is also a measure of the ‘mild’ or ‘wild’ randomness of the data series.

The formula $D = 2 - H$, indicates the relationship between Hurst exponent and fractal dimension (D). The concept of fractal dimension was formally introduced by Benoit Mandelbrot [15] and is a parameter indicating how smooth or rough is the time series. Values of fractal dimension are fractional and non-integers, as is common to think about in term of Euclidean dimensions. In the case of our interest here D values ranges between 1 and 2, meaning intermediate dimensions between a line ($D = 1$) and a plane ($D = 2$). Table 1 show the relationship between values of H, D and alpha (α) which is related to H (and D) by the formula $H = (1+ \alpha)/2$. It comes from the fact that the amount of energy that distributes in a particular frequency range of a stochastic process has the form of $S(f) = \text{constant} / f^{\alpha}$. $1/f$ noise is the generalized form to refer to this kind of signals in which its energy distribution decay as a straight line with negative slope in a log-log plot of energy v/s frequency. $1/f$ fluctuations are widely found in nature occurring in many diverse systems both in natural world and man-made procedures. It has been observed in physics, technology, ecology, EEG signals; cardiac response; astrophysics, geophysics, language and even music [16-17-18].

Table 1. Relationship between H, D and α values.

H	D	α
0	2	-1
0.5	1.5	0
1	1	1

Table 2 shows the relationship between these parameters and the three $1/f$ noises.

Table 2. Relationship between White, Pink and Brown noise with H and α values.

	WHITE NOISE	PINK NOISE	BROWN NOISE
α	$1/f^0$	$1/f$	$1/f^2$
H	-1	0	0.5
D	-	2	1.5

All of these parameters indicate slightly different but complementary characteristics of a time series. They have been widely used testing the value of prediction in the stock market [19-20] and recently has been applied to physiological phenomena like heart beat dynamics [21], epilepsy [22], non-linear analysis of the EEG during mathematical problem solving [23], and body balance strategies [24].

Aimed with these non-linear tools that allow us to explore brain dynamics and describe EEG phenomenology in more heuristic ways taking into consideration the complex nature of brain dynamics, we can be more confident and accurate in trying to unveil and quantitatively describe some aspects of the physical system that underlies the electrical brain activity emerging during a problem solving task.

In this work we explore the hypothesis that brain functional dynamics must change adapting accordingly to the level of difficulty of the faced problem and that this operational change may be related with a process of adequacy and adjust of the use of resources available for information management and problem solving. We expect to find differences at the level of frequency bands in charge of different processes involved in a problem task with increasing level of difficulty, and also inter-individual differences probably due to a different set of strategies and cognitive skills previously learned by experience.

2. Method

2.1. Raven's (abbreviated version) progressive matrices test.

Raven's Progressive Matrices test [25] is a non-verbal test that consists of a number of problems in which the subject selects, from a set of alternatives, missing pieces of a chart with the aim to complete a drawing logical series. The task consists in compare forms and figures reasoning by analogy. The test used in this experiment corresponds to a reduced version (15 questions) of the original Raven test. Sequential figures are presented in a computer screen and the response process recorded in real time, while an electroencephalogram (EEG) is simultaneously recorded from 14 electrodes settled on the subject' scalp.

2.2. EEG recording and analysis.

EEG recording consisted of N=12 subjects in basal (resting conditions) EEG with open and closed eyes, prior to the test and a further EEG recording captured during the whole resolution of the Raven test. We used the non-invasive brain computer device Emotiv EPOC® [26] to record 14 channels from head locations according to the standard 10/20 system. After subtracting the baseline and eliminating common signal artifacts with the assistance of EEGLAB and ADJUST [27] toolboxes running under Matlab, we obtained clean signals for the five EEG bands: delta, theta, alpha, beta and gamma. These signals reflect the ongoing functional dynamics of each subject's brain operation in two conditions of quiet resting awake (eyes open and closed), and during the execution of the abbreviated version of Raven test.

The EEG signal was then analyzed by filtered frequency bands during the execution of the whole test. Hurst exponent was estimated by rescaled range procedure [28], which estimates H by mean of calculating the slope of the curve as indicated in Figure 1. For the final analysis we focused on gamma band (30-64 Hz) expected to be more implicated in cognitive workload.

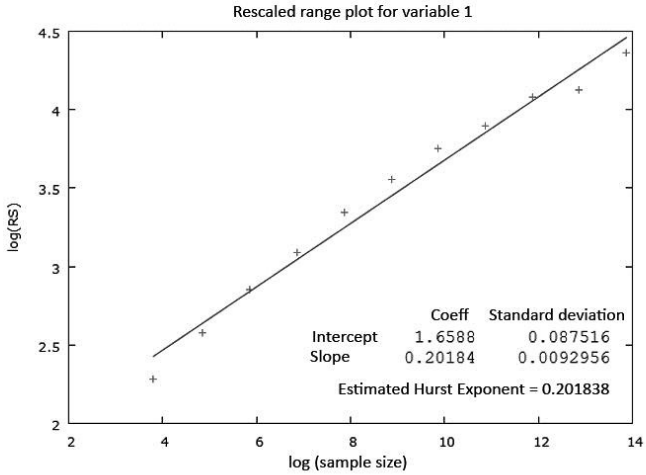

Fig. 1. Pox plot for estimating Hurst Exponent.

3. Results

3.1. Hurst Exponent and EEG bands

Figure 2 shows H values for a representative subject in basal conditions with eyes opened. A strong difference can be seen when comparing H value between 0-4 Hz with rest of the EEG range.

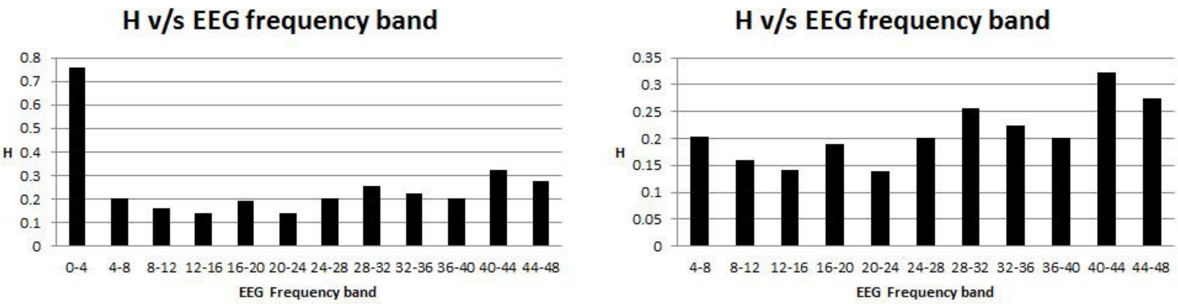

Fig. 2. H values versus EEG frequency bands in 4 Hz steps. Basal condition with eyes opened.
The graph in the left shows the whole EEG range analyzed from 0-4 Hz to 44-48 Hz. In the graph to the right 0-4 Hz range was removed to allow better visualization of H differences in the following EEG ranges.

Although it is expected that delta range (0-4 Hz) had high values of H because of the low frequency and smoother aspect of the EEG trace, what it is not so expected is the sudden decrease when estimating H values for the following range (4-8 Hz). As the shift toward the following higher frequency range (> 4 Hz) is not a proportional shift, this non-linear behavior may confirm the existence of specific processes occurring at low EEG frequencies in the delta range which are substantially different from subsequent ranges of EEG activity. Compared with synthetically generated data showed in Figure 3, it is clear that EEG behaves in a different way.

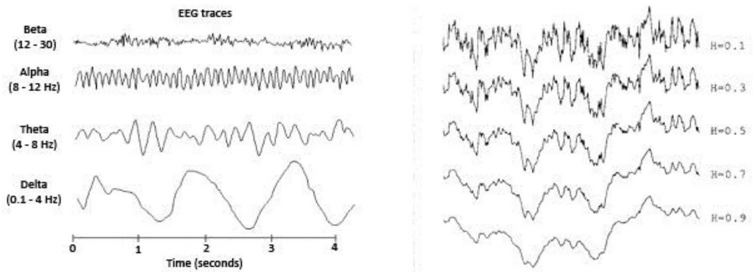

Fig. 3. EEG bands traces comparison (left) with different increasing H values for the same synthetically generated signals (right).

Expressed in terms of the classical EEG bands (Delta, Theta, Alpha, Beta and Gamma), Figure 4 shows the average H values for these frequency ranges, comparing average differences between left and right brain hemispheres.

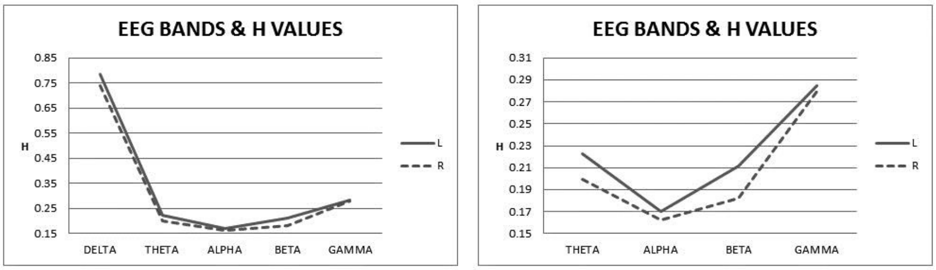

Fig. 4. EEG typical frequency bands comparing average differences between left (L) and right (R) brain hemispheres. Basal condition with eyes opened. The graph in the left shows the whole EEG range analyzed from Delta to Gamma. In the graph to the right Delta range was removed to allow better visualization of H differences in the following EEG bands.

3.2. Hurst Exponent and brain asymmetry.

To explore further into individual brain hemispheric H values asymmetry underlying brain processes occurring at different frequency ranges, Figure 5 depicts these differences for left and right brain hemispheres.

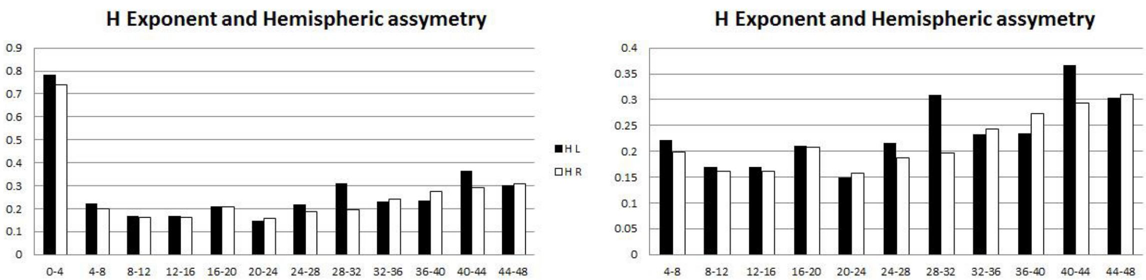

Fig. 5. Individual Hurst Exponent asymmetry by frequency ranges. H from Left (HL) and Right (HR) hemispheres. Left graph shows whole EEG range. In the graph to the right 0-4 Hz range was removed to allow better visualization of H differences in the following EEG ranges. Basal condition with eyes opened.

Figure 6 shows H values asymmetry by EEG channels in basal condition, eyes opened.

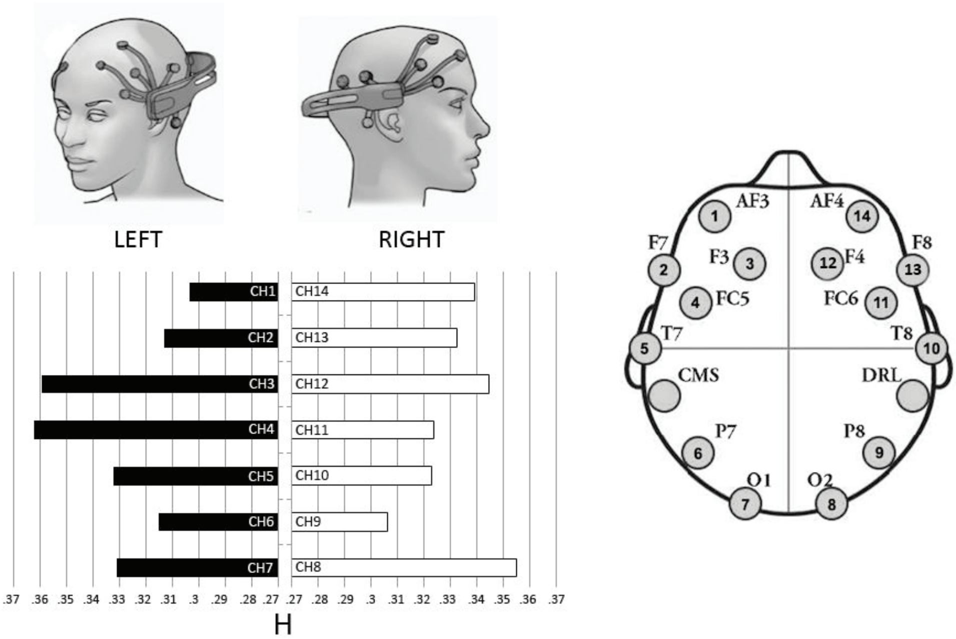

Fig. 6. Emotiv-Epoc® EEG channels configuration and left and right asymmetry for one individual in basal condition, eyes opened.

3.2. Hurst Exponent during execution of cognitive task (Raven test abbreviated version).

Figure 7 shows an individual representative plot of the variation of Hurst exponents during Raven test execution for whole brain data.

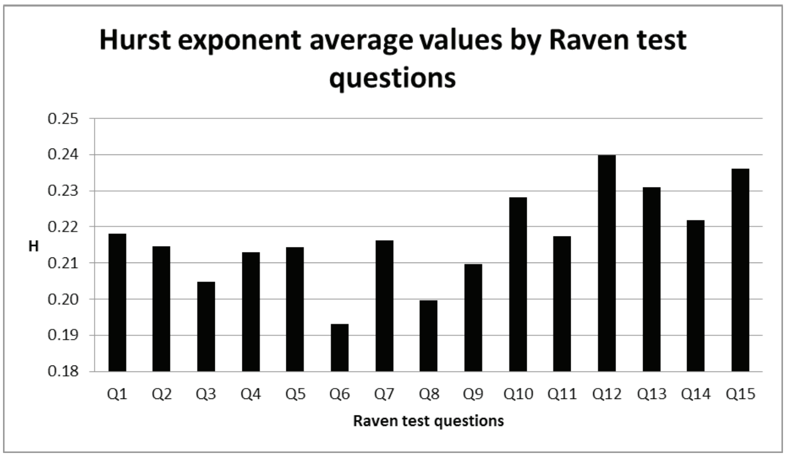

Fig. 7. Hurst Exponent variation according execution of Raven test. Questions 1 to 15 (Q1 to q15). Whole brain data.

Figure 8 shows a representative plot of the variation of Hurst exponent values during Raven test execution for brain data clustered in left and right hemispheres to show asymmetric behavior of H during problem solving processing.

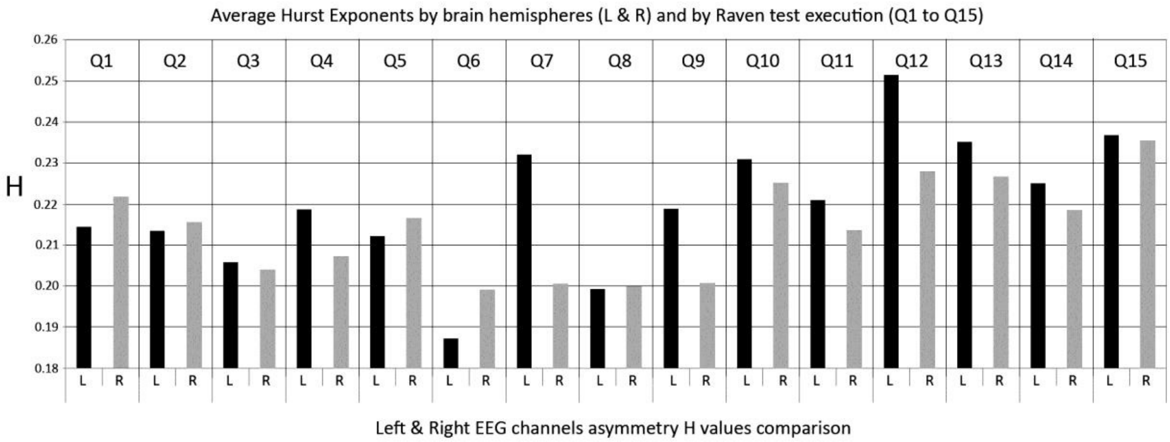

Fig. 8. Hurst Exponent variation according execution of Raven test. Questions 1 to 15 (Q1 to Q15).
Left (L) and right (R) brain hemispheres asymmetry.

Conclusions

Non-linear analysis of the EEG through fractal modeling and chaos theory gives a deeper picture and understanding of the underlying physical processes that must be involved in the generation of EEG phenomenology. Our first exploratory results confirm the idea that the rate of chaos and order emerging from brain functional processing varies according to the difficult of the cognitive task executed, revealing as well individual particularities in the asymmetric processing of the information that brain performs when is faced to a cognitive challenge.

Individual differences were also seen in basal resting condition with eyes opened revealing that a default state of the brain could be setting a background status from where to start processing. This individual default state should be the result of individual differences stabilized over the years through learning.

Interestingly, alpha EEG band in resting condition resulted in the lower values of H, meaning that at this frequency the brain functional state is in its more unpredictable fashion, short memory, anti-persistence and high volatility tendency. By the contrary, while scaling into processing problems with increasing level of difficulty, H values increase toward the direction of less complexity and more predictability.

As the increase of H values are not perfectly proportional with the increase of EEG frequency range, this suggests that each range of EEG frequency can be characterized by a different level of self-similarity and randomness, in relation with the experimental protocol tested.

Above delta band and specially related with those EEG bands presumably involved in cognitive processing it is expected that H values tend to increase, revealing underlying mechanisms that attract the system to less chaotic, more ordered, self-similar, fractal and predictable system.

At the range of delta frequencies, it is clear that the brain shows a more structured and predictable dynamics maybe having to do with task unrelated with perceptual or behavioral performance and more associated with *housekeeping* less volatile processes.

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