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A new approach for ranking of trapezoidal fuzzy numbers

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ABSTRACT

Ranking fuzzy numbers plays an very important role in linguistic decision making and some other fuzzy application systems. Several strategies have been proposed for ranking of fuzzy numbers. Each of these techniques have been shown to produce non-intuitive results in certain cases. In this paper, we will introduce a new approach for ranking of trapezoidal fuzzy numbers based on the left and the right spreads at some α-levels of trapezoidal fuzzy numbers. The calculation of the proposed method is far simpler and easier. Finally, some comparative examples are used to illustrate the advantage of the proposed method.

1. Introduction

In many applications, ranking of fuzzy numbers is an important component of the decision process. More than 30 fuzzy ranking indices have been proposed since 1976. In 1976 and 1977, Jain [1,2] proposed a method using the concept of maximizing set to order the fuzzy numbers. Jain's method is that the decision maker considers only the right side membership function. A canonical way to extend the natural ordering of real numbers to fuzzy numbers was suggested by Bass and Kwakernaak [3] as early as 1977. Dubios and Prade 1978 [4], used maximizing sets to order fuzzy numbers. In 1979, Baldwin and Guild [5] indicated that these two methods have some disturbing disadvantages. Also, in 1980, Adamo [6] used the concept of α-level set in order to introduce α-preference rule. In 1981 Chang [7] introduced the concept of the preference function of an alternative. Yager in 1981 [8,9] proposed four indices which may be employed for the purpose of ordering fuzzy quantities in $[0, 1]$. Bortolan and Degani have been compared and reviewed some of these ranking methods [10]. Chen and Hwang [11] thoroughly reviewed the existing approaches, and pointed out some illogical conditions that arise among them. Chen [12], Choobineh [13], Cheng [14] have presented some methods, and also more recently numerous ranking techniques have been proposed and investigated by Chu, Tsao [15] and Ma, Kandel and Friedman [16]. Nowadays, many researchers have developed methods to compare and to rank fuzzy numbers. Some of those methods are counter-intuitive and non discriminating [10,17–23]. Recently, a new method based on "Distance Minimization" was introduced [24]. This method has some drawbacks, i.e., for all triangular fuzzy numbers $u = (x_0, \sigma, \beta)$ where $x_0 = \frac{\sigma-\beta}{4}$ and also trapezoidal fuzzy numbers $u = (x_0, y_0, \sigma, \beta)$, such that $x_0 + y_0 = \frac{\sigma-\beta}{2}$, gives the same results. However it is clear that these fuzzy numbers do not place in an equivalence class (numerical examples are illustrated in Section 4).

In this paper, we introduce a new approach which is easy to handle and has a natural interpretation. The rest of this paper is organized as follows: Section 2 contains the basic definitions and notations are used in the remaining parts of the paper; In Section 3, we will introduce a new approach for ranking of trapezoidal fuzzy numbers and investigate some properties of this method; In Section 4 we use some numerical examples to show the advantage of the proposed method and an illogical condition of "Distance Minimization" method. The paper ends with conclusions in Section 5.

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2. Preliminaries

Though there are a number of ways of defining fuzzy numbers, for the purposes of this paper we adopt the following definition, we will identify the name of the number with that of its membership function for simplicity. Throughout this paper, $\mathbb{R}$ stands for the set of all real numbers, E stands the set of fuzzy numbers, u expresses a fuzzy number and $u(x)$ for its membership function, $\forall x \in \mathbb{R}$.

Definition 2.1 ([25]). A fuzzy number is a fuzzy set like $u : \mathbb{R} \to I = [0, 1]$ which satisfies:

1. u is upper semi-continuous,
2. $u(x) = 0$ outside some interval $[a, d]$,
3. There are real numbers a, b such that $a \le b \le c \le d$ and
   a. $u(x)$ is monotonic increasing on $[a, b]$,
   b. $u(x)$ is monotonic decreasing on $[c, d]$,
   c. $u(x) = 1,\ b \le x \le c$.

The membership function u can be expressed as

$$u(x) = \begin{cases} u_L(x), & a \le x \le b, \\ 1, & b \le x \le c, \\ u_R(x), & c \le x \le d, \\ 0, & \text{otherwise,} \end{cases}$$

where $u_L : [a, b] \longrightarrow [0, 1]$ and $u_R : [c, d] \longrightarrow [0, 1]$ are left and right membership functions of fuzzy number u. An equivalent parametric form is also given in [26] as follows:

Definition 2.2 ([26]). A fuzzy number u in parametric form is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(r), \overline{u}(r),\ 0 \le r \le 1$, which satisfy the following requirements:

1. $\underline{u}(r)$ is a bounded monotonic increasing left continuous function,
2. $\overline{u}(r)$ is a bounded monotonic decreasing left continuous function,
3. $\underline{u}(r) \le \overline{u}(r),\ 0 \le r \le 1$.

The trapezoidal fuzzy number $u = (x_0, y_0, \sigma, \beta)$, with two defuzzifier x_0, y_0, and left fuzziness $\sigma > 0$ and right fuzziness $\beta > 0$ is a fuzzy set where the membership function is as

$$u(x) = \begin{cases} \frac{1}{\sigma}(x - x_0 + \sigma), & x_0 - \sigma \le x \le x_0, \\ 1 & x \in [x_0, y_0], \\ \frac{1}{\beta}(y_0 - x + \beta), & y_0 \le x \le y_0 + \beta, \\ 0, & \text{otherwise,} \end{cases}$$

and its parametric form is

$$\underline{u}(r) = x_0 - \sigma + \sigma r, \qquad \overline{u}(r) = y_0 + \beta - \beta r.$$

Provided that, $x_0 = y_0$ then u is a triangular fuzzy number, and we write $u = (x_0, \sigma, \beta)$. The support of fuzzy number u is defined as follows:

$$\text{supp}(u) = \overline{\{x \mid u(x) > 0\}},$$

where $\overline{\{x \mid u(x) > 0\}}$ is closure of set $\{x \mid u(x) > 0\}$.

The addition and scalar multiplication of fuzzy numbers are defined by the extension principle and can be equivalently represented in [27–29] as follows. For arbitrary $u = (\underline{u}, \overline{u})$, $v = (\underline{v}, \overline{v})$ we define addition $(u + v)$ and multiplication by scalar $k > 0$ as

$$(\underline{u + v})(r) = \underline{u}(r) + \underline{v}(r), \qquad (\overline{u + v})(r) = \overline{u}(r) + \overline{v}(r), \tag{1}$$

$$(\underline{ku})(r) = k\underline{u}(r), \qquad (\overline{ku})(r) = k\overline{u}(r). \tag{2}$$

To emphasis the collection of all fuzzy numbers with addition and multiplication as defined by (1) and (2) is denoted by E, which is a convex cone. The image (opposite) of $u = (x_0, y_0, \sigma, \beta)$, can be defined by $-u = (-y_0, -x_0, \beta, \sigma)$ (see [29,30]).

3. New approach for ranking of trapezoidal fuzzy numbers

For an arbitrary trapezoidal fuzzy number $u = (x_0, y_0, \sigma, \beta)$, with parametric form $u = (\underline{u}(r), \overline{u}(r))$, we define the *magnitude* of the trapezoidal fuzzy number u as

$$\mathrm{Mag}(u) = \frac{1}{2}\left(\int_0^1 (\underline{u}(r) + \overline{u}(r) + x_0 + y_0) f(r)\mathrm{d}r\right), \tag{3}$$

where the function $f(r)$ is a non-negative and increasing function on [0, 1] with $f(0) = 0, f(1) = 1$ and $\int_0^1 f(r)\mathrm{d}r = \frac{1}{2}$. Obviously function $f(r)$ can be considered as a weighting function. In actual applications, function $f(r)$ can be chosen according to the actual situation. In this paper we use $f(r) = r$. Obviously, the magnitude of a trapezoidal fuzzy number u which is defined by (3), synthetically reflects the information on every membership degree, and meaning of this magnitude is visual and natural. The resulting scalar value is used to rank the fuzzy numbers. In the other words $\mathrm{Mag}(u)$ is used to rank fuzzy numbers. The larger $\mathrm{Mag}(u)$, the larger fuzzy number. Therefore for any two trapezoidal fuzzy numbers u and $v \in E$, we define the ranking of u and v by the Mag(.) on E as follows:

(1) $\mathrm{Mag}(u) > \mathrm{Mag}(v)$ if and only if $u \succ v$,
(2) $\mathrm{Mag}(u) < \mathrm{Mag}(v)$ if and only if $u \prec v$,
(3) $\mathrm{Mag}(u) = \mathrm{Mag}(v)$ if and only if $u \sim v$.

Then we formulate the order $\succeq$ and $\preceq$ as $u \succeq v$ if and only if $u \succ v$ or $u \sim v$, $u \preceq v$ if and only if $u \prec v$ or $u \sim v$. In the other words, this method is placed in the first class of Kerre's categories [31].

Remark 3.1. If $\inf \mathrm{supp}(u) \geq 0$ or $\inf \underline{u}(r) \geq 0$ then $\mathrm{Mag}(u) \geq 0$.

Remark 3.2. If $\sup \mathrm{supp}(u) \leq 0$ or $\sup \overline{u}(r) \leq 0$ then $\mathrm{Mag}(u) \leq 0$.

Remark 3.3. For two arbitrary trapezoidal fuzzy numbers u and v, we have

$$\mathrm{Mag}(u + v) = \mathrm{Mag}(u) + \mathrm{Mag}(v).$$

Remark 3.4. For all symmetric trapezoidal fuzzy numbers $u = (-x_0, x_0, \sigma, \sigma)$,

$$\mathrm{Mag}(u) = 0.$$

Remark 3.5. For any two symmetric trapezoidal fuzzy numbers $u = (x_0, y_0, \sigma, \sigma)$ and $v = (x_0, y_0, \beta, \beta)$,

$$\mathrm{Mag}(u) = \mathrm{Mag}(v).$$

We consider the following reasonable properties for the ordering approaches, see [31].

A_1: For an arbitrary finite subset Γ of E and $u \in \Gamma$, $u \succeq u$.
A_2: For an arbitrary finite subset Γ of E and $(u, v) \in \Gamma^2$, $u \succeq v$ and $v \succeq u$, we should have $u \sim v$.
A_3: For an arbitrary finite subset Γ of E and $(u, v, w) \in \Gamma^3$, $u \succeq v$ and $v \succeq w$, we should have $u \succeq w$.
A_4: For an arbitrary finite subset Γ of E and $(u, v) \in \Gamma^2$, $\inf \mathrm{supp}(u) > \sup \mathrm{supp}(v)$, we should have $u \succeq v$.
A'_4: For an arbitrary finite subset Γ of E and $(u, v) \in \Gamma^2$, $\inf \mathrm{supp}(u) > \sup \mathrm{supp}(v)$, we should have $u \succ v$.
A_5: Let Γ and Γ' be two arbitrary finite subsets of E also u and v are in $\Gamma \cap \Gamma'$. We obtain the ranking order $u \succ v$ by Mag(.) on Γ' if and only if $u \succ v$ by Mag(.) on Γ.
A_6: Let $u, v, u + w$ and $v + w$ be elements of E. If $u \succeq v$, then $u + w \succeq v + w$.
A'_6: Let $u, v, u + w$ and $v + w$ be elements of E. If $u \succ v$, then $u + w \succ v + w$, when $w \neq 0$.
A_7: For an arbitrary finite subset Γ of E and $u \in \Gamma$, the $\mathrm{Mag}(u)$ must belong to its support.

Theorem 3.1. *The function* Mag(.) *has the properties* $A_1, A_2, A_3, \ldots, A_7$.

Proof. It is easy to verify that the properties A_1–A_5 and A_7 are hold. For the proof of A_6 we consider the trapezoidal fuzzy numbers $u = (x_0, y_0, \sigma_0, \beta_0)$, $v = (x_1, y_1, \sigma_1, \beta_1)$ and $w = (x_2, y_2, \sigma_2, \beta_2)$. Let $u \succeq v$, from the relation (3) we have

$$\mathrm{Mag}(u) \geq \mathrm{Mag}(v),$$

by adding $\mathrm{Mag}(w)$

$$\mathrm{Mag}(u) + \mathrm{Mag}(w) \geq \mathrm{Mag}(v) + \mathrm{Mag}(w),$$

and by Remark 3.3

$$\mathrm{Mag}(u + w) \geq \mathrm{Mag}(v + w).$$

Therefore

$$u + w \succeq v + w,$$

with which the proof is completed. Similarly A'_6 is hold. □

Table 1
Comparative results of Example 4.1.

Fuzzy number	New approach mag	Sign distance $p = 1$	Sign distance $p = 2$	Distance minimization
A	0.6666	3.2000	2.5820	0.0000
B	0.1666	1.5313	1.2417	0.0000
C	1.3334	2.9444	4.0414	0.0000
D	0.3334	3.000	2.3094	0.0000
Results	$B \prec D \prec A \prec C$	$B \prec C \prec D \prec A$	$B \prec D \prec A \prec C$	$A \sim B \sim C \sim D$

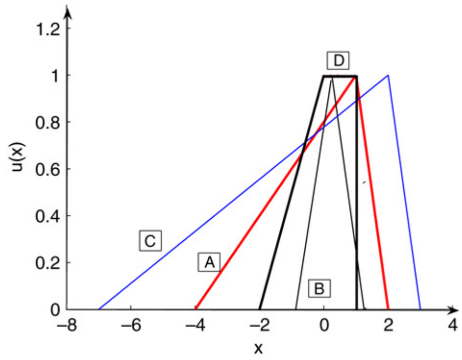

Fig. 1. Fuzzy numbers, $A = (1, 5, 1)$, $B = (\frac{1}{4}, 2, 1)$, $C = (2, 9, 1)$ and $D = (0, 1, 2, 0)$.

4. Numerical examples

Example 4.1. Consider the two triangular fuzzy numbers $A = (0, 1, 1)$ and $B = (1, 5, 1)$.

Intuitively, the ranking order is $A \prec B$. However by Distance minimization, the ranking order is $A \sim B$, which is an unreasonable result. By the proposed method, $\text{Mag}(A) = 0.0000$ and $\text{Mag}(B) = 1.3333$, therefore the ranking order is $A \prec B$. So, our method can overcome the shortcoming of "Distance minimization" method [24].

Example 4.2. Consider the four fuzzy numbers $A = (1, 5, 1)$, $B = (\frac{1}{4}, 2, 1)$, $C = (2, 9, 1)$ and $D = (0, 1, 2, 0)$, Fig. 1.

Intuitively, the ranking order is $B \prec D \prec A \prec C$. By using our new approach $\text{Mag}(A) = 0.6666$, $\text{Mag}(B) = 0.1666$, $\text{Mag}(C) = 1.3334$ and $\text{Mag}(D) = 0.3334$. Hence the ranking order is $B \prec D \prec A \prec C$ too. Obviously, the results obtained by "Distance minimization" method are unreasonable. To compare with some of the other methods in [19,24] the reader can refer to Table 1.

Example 4.3. The three triangular fuzzy numbers $A = (0.3, 0.1, 0.2)$, $B = (0.32, 0.15, 0.26)$ and $C = (0.4, 0.15, 0.3)$ shown in Figs. 2–5, taken from paper [14,15], are ranked by our method.

$\text{Mag}(A) = 0.3084$, $\text{Mag}(B) = 0.3292$ and $\text{Mag}(C) = 0.4126$, producing the ranking order $A \prec B \prec C$. Furthermore $\text{Mag}(-A) = -0.3084$, $\text{Mag}(-B) = -0.3292$ and $\text{Mag}(-C) = -0.4126$, consequently the ranking order of the images of three fuzzy number is $-A \succ -B \succ -C$. Clearly, our method has consistency in ranking fuzzy numbers and their images, which could not be guaranteed by Cheng's CV index.

Example 4.4. Consider the following sets, see Yao and Wu [15].
Set 1: $A = (0.5, 0.1, 0.5)$, $B = (0.7, 0.3, 0.3)$, $C = (0.9, 0.5, 0.1)$, (Fig. 2).
Set 2: $A = (0.4, 0.7, 0.1, 0.2)$ (trapezoidal fuzzy number),
$B = (0.7, 0.4, 0.2)$, $C = (0.7, 0.2, 0.2)$, (Fig. 3).
Set 3: $A = (0.5, 0.2, 0.2)$, $B = (0.5, 0.8, 0.2, 0.1)$, $C = (0.5, 0.2, 0.4)$, (Fig. 4).
Set 4: $A = (0.4, 0.7, 0.4, 0.1)$, $B = (0.5, 0.3, 0.4)$, $C = (0.6, 0.5, 0.2)$, (Fig. 5).

To compare with other methods we refer the reader to Table 2 (see [19]).

Example 4.5. Consider the three triangular fuzzy numbers, $A = (6, 1, 1)$, $B = (6, 0.1, 1)$ and $C = (6, 0, 1)$, (see Fig. 6).

By using our method

$$\text{Mag}(A) = 6.00000, \qquad \text{Mag}(B) = 6.0750 \quad \text{and} \quad \text{Mag}(C) = 6.0834.$$

Thus, the ranking order is $A \prec B \prec C$. As you see in Table 3, the results of Chu-Tsao method and Cheng CV index are unreasonable. The results of sign distance method [19] and Cheng distance method, are the same as our new approach.

All the above numerical examples show that the results of the proposed method, similar to the sign distance method, can overcome the drawbacks of Distance Minimization and CV index.

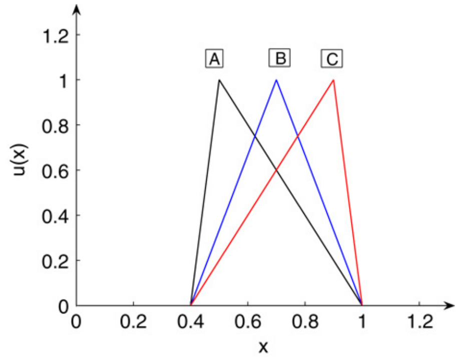

Fig. 2. Set 1.

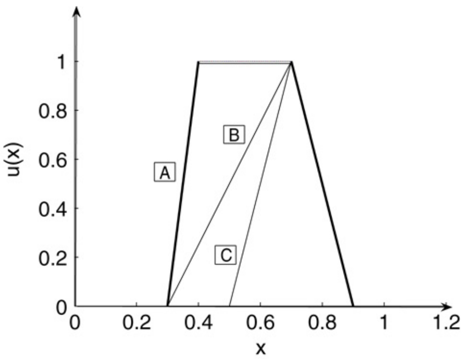

Fig. 3. Set 2.

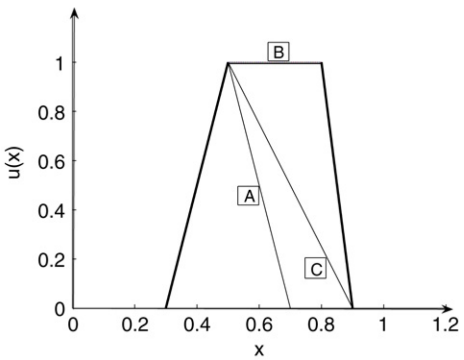

Fig. 4. Set 3.

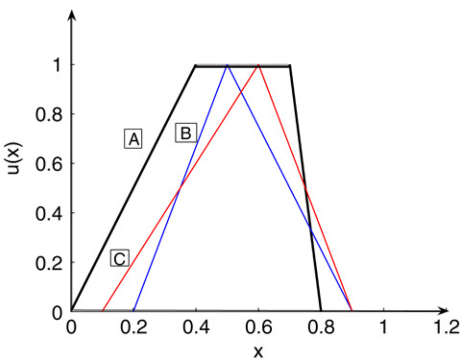

Fig. 5. Set 4.

Table 2
Comparative results of Example 4.3.

Authors	Fuzzy number	Set 1	Set 2	Set 3	Set 4
New approach (mag)	A	0.5334	0.5584	0.5000	0.5250
	B	0.7000	0.6334	0.6416	0.5084
	C	0.8666	0.7000	0.5166	0.5750
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$B \prec A \prec C$
Sign distance method $p = 1$	A	1.2	1.15	1	0.95
	B	1.4	1.3	1.25	1.05
	C	1.6	1.4	1.1	1.05
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \sim C$
Sign distance method $p = 2$	A	0.8869	0.8756	0.7257	0.7853
	B	1.0194	0.9522	0.9416	0.7958
	C	1.1605	1.0033	0.8165	0.8386
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \prec C$
Choobineh and Li	A	0.333	0.458	0.333	0.50
	B	0.50	0.583	0.4167	0.5833
	C	0.667	0.667	0.5417	0.6111
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Yager	A	0.60	0.575	0.5	0.45
	B	0.70	0.65	0.55	0.525
	C	0.80	0.7	0.625	0.55
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Chen	A	0.3375	0.4315	0.375	0.52
	B	0.50	0.5625	0.425	0.57
	C	0.667	0.625	0.55	0.625
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Baldwin and Guild	A	0.30	0.27	0.27	0.40
	B	0.33	0.27	0.37	0.42
	C	0.44	0.37	0.45	0.42
Results		$A \prec B \prec C$	$A \sim B \prec C$	$A \prec B \prec C$	$A \prec B \sim C$
Chu and Tsao	A	0.299	0.2847	0.25	0.24402
	B	0.350	0.32478	0.31526	0.26243
	C	0.3993	0.350	0.27475	0.2619
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec C \prec B$
Yao and Wu	A	0.6	0.575	0.5	0.475
	B	0.7	0.65	0.625	0.525
	C	0.8	0.7	0.55	0.525
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \sim C$
Cheng distance	A	0.79	0.7577	0.7071	0.7106
	B	0.8602	0.8149	0.8037	0.7256
	C	0.9268	0.8602	0.7458	0.7241
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec C \prec B$
Cheng CV uniform distribution	A	0.0272	0.0328	0.0133	0.0693
	B	0.0214	0.0246	0.0304	0.0385
	C	0.0225	0.0095	0.0275	0.0433
Results		$B \prec C \prec A$	$C \prec B \prec A$	$A \prec C \prec B$	$B \prec C \prec A$
Cheng CV proportional distribution	A	0.0183	0.026	0.008	0.0471
	B	0.0128	0.0146	0.0234	0.0236
	C	0.0137	0.0057	0.0173	0.0255
Results		$B \prec C \prec A$	$C \prec B \prec A$	$A \prec C \prec B$	$B \prec C \prec A$

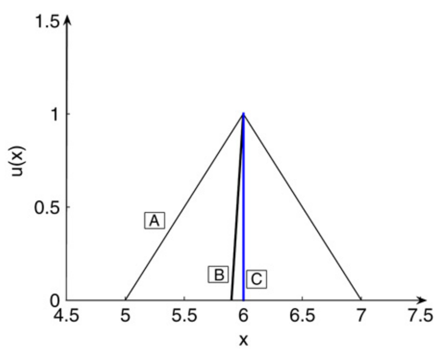

Fig. 6. Triangular fuzzy numbers, $A = (6, 1, 1)$, $B = (6, 0.1, 1)$ and $C = (6, 0, 1)$.

Table 3
Comparative results of Example 4.4.

Fuzzy number	New approach mag	Sign distance $p = 1$	Sign distance $p = 2$	Chu and Tsao	Cheng distance	CV index
A	6.0000	6.12	8.52	3	6.021	0.028
B	6.0750	12.45	8.82	3.126	6.349	0.0098
C	6.0834	12.5	8.85	3.085	6.3519	0.0089
Results	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \prec C$	$C \prec B \prec A$

5. Conclusions

In spite of many ranking methods, no-one can rank fuzzy numbers with human intuition consistently in all cases. Here, we pointed out the shortcomings of "Distance minimization" and in order to solve the problems we have presented a simple ranking method for trapezoidal fuzzy numbers. The proposed method can effectively rank various fuzzy numbers and their images. Our new ranking method has some mathematical properties. It does not imply much computational effort and does not require a priori knowledge of the set of all alternatives. We also used comparative examples to illustrate the advantages of the proposed method.

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