## Quasi-Genera of Quadratic Forms

BURTON W. JONES

*Department of Mathematics, University of Colorado, Boulder, Colorado 80302*

Let $f$ be a quadratic $n$-ary form with integer coefficients and determinant $d \neq 0$. Let $R$ be the set of transformations of order $n$ with rational elements and determinant $\pm 1$. Call $E$ the lattice consisting of all $n$-tuples of integers. The denominator of a transformation $T$ in $R$, written den $T$, is the least positive integer $t$ such that $tT(\alpha)$ is in $E$ for all $\alpha$ in $E$. A prime $p$ is called exceptional for $f$ if for every automorph of $f$ with denominator prime to $2pd$, it is true that (den $A \mid p) = 1$ for $p$ odd or den $A \equiv 1 (\mathrm{mod}\ 8)$ for $p = 2$. It is proved that if $p$ is exceptional for $f$ it is exceptional for every form in the genus of $f$ and that for each odd $p$, the genus of $f$ splits into two quasi-genera according to the quadratic character of the denominators of the transformations which take $f$ into the other forms of the genus. For $p = 2$, the genus splits into four quasi-genera. Necessary conditions are given that a prime $p$ be exceptional. In a number of respects these quasi-genera behave like spinor genera. For ternary forms there seems to be some connection between spinor genera and axceptional squares represented by forms of the quasi-genus.

Many of these ideas and results are given in a joint paper of the author with G. L. Watson (*Canad. J. Math.* **8** (1956), 592–608) and two unpublished University of Colorado theses (R. G. Thompson, 1963 and W. C. Ramaley, 1969). The latter of these two theses may be published soon. Most of the results described above will appear in a future issue of Scripta Matematica.

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## The Economics of Quadratic Form Calculations

D. H. LEHMER

*Department of Mathematics, University of California, Berkeley, California 94720*

In planning large scale calculations dealing with quadratic forms one needs to know, at least roughly, the order of magnitude of a long job or, in case of many small jobs, the number of these jobs we can expect to complete in a given time allocation. In case alternative methods are available one would like to make the most economical choice. In order to assist in such planning we give order of magnitude costs of some of the calculations that are basic to our subject. In what follows, $p$ represents an odd prime and $d$ and $m$ represent positive integers whose prime factors are unknown.

| *Problem* | *Cost of Problem* |
|---|---|
| To evaluate a quadratic or higher power residue symbol $(a/p)$ | $O(\log p)$ |
| To solve a quadratic congruence modulo $p$ | $O(\log p)$ |
| To represent $p$ by a binary quadratic form | $O(\log p)$ |
| To represent $m$ by a binary quadratic form | $O(m^{1/2})$ |
| To find the class number $h((-d)^{1/2})$ | $O(\log d)$ |
| To solve the Pell equation $x^2 - my^2 = \pm 1, \pm 4$ | $O(m^{1/2}(\log m)^4)$ |

## The Structure of the Class Group in Orders of Binary Quadratic Forms

John W. Matherne

*Department of Mathematics, Louisiana State University, Baton Rouge, Louisiana 70803*

If $d_0$ is a fundamental discriminant (the discriminant of a quadratic field), then the primitive classes of binary quadratic forms of all the discriminants $d_0s^2$ ($s$ ranging over the positive integers) form a semigroup under composition, and those for a fixed $s$ form a group, and are called an order (Gauss). The problem under study is that of determining the structure of the group $d_0s^2$ when that of discriminant $d_0$ is assumed known.