Planck scale boundary conditions in the standard model with singlet scalar dark matter

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Abstract: We investigate Planck scale boundary conditions on the Higgs sector of the standard model with a gauge singlet scalar dark matter. We will find that vanishing self-coupling and Veltman condition at the Planck scale are realized with the $126\,\mathrm{GeV}$ Higgs mass and top pole mass, $172\,\mathrm{GeV} \lesssim M_t \lesssim 173.5\,\mathrm{GeV}$, where a correct abundance of scalar dark matter is obtained with mass of $300\,\mathrm{GeV} \lesssim m_S \lesssim 1\,\mathrm{TeV}$. It means that the Higgs potential is flat at the Planck scale, and this situation can not be realized in the standard model with the top pole mass.

Keywords: Higgs Physics, Beyond Standard Model, Cosmology of Theories beyond the SM

Contents

1 Introduction

The Higgs particle has just been discovered at the Large Hadron Collider (LHC) experiment [1, 2]. In addition, the results from the experiment are consistent with the standard model (SM), and an evidence of new physics such as supersymmetry (SUSY) is not obtained. Currently, the experimental results strongly constrain the presence of SUSY at low energy although the minimal supersymmetric standard model (MSSM) is an attractive candidate for new physics beyond the SM. Thus, a question, "How large is new physics scale?", might become important for the SM and new physics. One can consider several scenarios such as high scale supersymmetric models or a scenario without SUSY in which the SM is valid up to the Planck scale $M_{\rm pl}$, etc.

As an example of the later scenario, it was pointed out that imposing a constraint that the SM Higgs potential has two degenerate vacua, in which one of them is at the Planck scale, leads to the top mass $173 \pm 5\,{\rm GeV}$ and the Higgs mass $135 \pm 9\,{\rm GeV}$ [3]. More recent work [4] showed that an asymptotic safety scenario of gravity predicts $126\,{\rm GeV}$ Higgs mass with a few GeV uncertainty. In these two scenarios, the boundary conditions (BCs) of the vanishing Higgs self-coupling ($\lambda(M_{\rm pl}) = 0$) and its β-function ($\beta_\lambda(M_{\rm pl}) = 0$) are imposed at the Planck scale. In addition to these two BCs, the work [5] also discussed the Veltman condition ($\operatorname{Str} M^2(M_{\rm pl}) = 0$) and the vanishing anomalous dimension of the Higgs mass ($\gamma_{m_h}(M_{\rm pl}) = 0$) at the Planck scale. It was found that the four BCs yield a Higgs mass range of 127–$142\,{\rm GeV}$. Thus, combining these BCs can interestingly predict values of the Higgs and top masses in the SM close to the experimental ones but a slightly heavier Higgs mass and/or lighter top mass than experimental ones are generally predicted from these BCs as shown in [6] (see also [7–13] for the latest analyses). BCs of $\lambda(M_{\rm pl}) = 0$ and $\operatorname{Str} M^2(M_{\rm pl}) = 0$ mean that there exists an approximately flat direction in the Higgs potential,[1] which might be adopted to the Higgs inflation [14–23]. In addition, the

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[1] In order to confirm the existence of the flat direction, one should know the full ultraviolet (UV) completion. In the work, we assume an UV theory yielding the BCs for the flat direction at the Planck scale.

quadratic (logarithmic) divergence for the Higgs mass disappear at the Planck scale under the Veltman condition (the vanishing anomalous dimension $\gamma_{m_h}(M_{\text{pl}}) = 0$). In this work, we investigate the three BCs in a gauge singlet extension of the SM.

One important motivation for the gauge singlet extension of the SM is that the SM does not include a dark matter (DM). In the extension, the gauge singlet scalar can be DM when the scalar has odd parity under an additional Z_2 symmetry [24] (see also [25–27]). Once a gauge singlet scalar is added to the SM, an additional positive contribution from new scalar coupling appears in the β-function of the Higgs self-coupling and the Veltman condition (and the anomalous dimension of the Higgs mass).[2] This means that the discussion of the three BCs at the Planck scale is modified from the SM. Since it actually seems difficult to reproduce 126 GeV Higgs mass and 173.07 ± 1.24 GeV top pole mass [36] (see also [37, 38]) at the same time (i.e. a slightly heavier Higgs and/or a lighter top masses than the experimental center values are required) under the above three BCs at the Planck scale in the SM, it is interesting to investigate if the BCs could be realized with the center values of the Higgs and top masses in the singlet scalar DM extension of the SM. In this work, we take the following setup: (i) we consider a simple framework, in which only one gauge singlet real scalar is added to the SM. (ii) The gauge singlet scalar is DM. (iii) All scalar quartic couplings in the model can be perturbatively treated up to the Planck scale.

The paper is organized as follows: in section 2, we investigate the three BCs at the Planck scale in the above framework. As a result, we will find that the vanishing self-coupling and Veltman condition at the Planck scale are realized with the 126 GeV Higgs mass and top pole mass, $171.8 \, \text{GeV} \lesssim M_t \lesssim 173.5 \, \text{GeV}$, where a correct abundance of scalar dark matter is obtained with mass of $300 \, \text{GeV} \lesssim m_S \lesssim 1 \, \text{TeV}$. It means that the Higgs potential is flat at the Planck scale, and this situation cannot be realized in the SM with the top pole mass. Section 3 is devoted to the summary.

2   Boundary conditions at the Planck scale

We consider the SM with a gauge singlet real scalar S, and investigate the values of scalar quartic couplings at the Planck scale by solving renormalization group equations (RGEs) in the model. The relevant Lagrangian of the model and the RGEs for the scalar quartic couplings are given by

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_S \,, \tag{2.1}$$

$$\mathcal{L}_{\text{SM}} \supset -\lambda \left(|H|^2 - \frac{v^2}{2} \right)^2 \,, \tag{2.2}$$

$$\mathcal{L}_S = -\frac{\bar{m}_S^2}{2} S^2 - \frac{k}{2} |H|^2 S^2 - \frac{\lambda_S}{4!} S^4 + (\text{kinetic term}) \,, \tag{2.3}$$

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[2]See [28–30] for discussions of the vacuum stability and triviality in the SM with a gauge singlet real scalar. See also [31] and references therein for implications of the LHC data to models with an extra singlet scalar, [32–34] for the classically conformal U(1)$_{B-L}$ extended SM, [35] for a model of electroweak and conformal symmetry breaking.

and

$$(4\pi)^2 \frac{dX}{dt} = \beta_X \quad (X = \lambda, k, \lambda_S),$$ (2.4)

with

$$\beta_\lambda = \begin{cases} 0 & \text{for } \mu < m_H \\ 24\lambda^2 + 12\lambda y^2 - 6y^4 - 3\lambda(g'^2 + 3g^2) + \frac{3}{8}\left[2g^4 + (g'^2 + g^2)^2\right] & \text{for } m_H \leq \mu < m_S \\ 24\lambda^2 + 12\lambda y^2 - 6y^4 - 3\lambda(g'^2 + 3g^2) + \frac{3}{8}\left[2g^4 + (g'^2 + g^2)^2\right] + \frac{k^2}{2} & \text{for } m_S \leq \mu \end{cases}$$ (2.5)

$$\beta_k = \begin{cases} 0 & \text{for } \mu < m_S \\ k\left[4k + 12\lambda + 6y^2 - \frac{3}{2}(g'^2 + 3g^2) + \lambda_S\right] & \text{for } m_S \leq \mu \end{cases},$$ (2.6)

$$\beta_{\lambda_S} = \begin{cases} 0 & \text{for } \mu < m_S \\ 3\lambda_S^2 + 12k^2 & \text{for } m_S \leq \mu \end{cases},$$ (2.7)

respectively, where we assume that the Higgs mass m_H is smaller than DM mass m_S.[3] H is the SM Higgs doublet, v is the vacuum expectation value (VEV) of the Higgs as 246 GeV, y is the top Yukawa coupling, t is defined as $t \equiv \ln \mu$, and μ is a renormalization scale within the range of $M_Z \leq \mu \leq M_{\rm pl}$. We also impose an additional Z_2 symmetry on the model. Only the gauge singlet scalar has odd parity while all the SM fields have even parity under the symmetry. We give some comments on properties of the three scalar quartic couplings obeying eqs. (2.4)–(2.7):

- The right-hand side of eq. (2.6) is proportional to k itself. Thus, if we take a small value of $k(M_Z)$, where M_Z is the Z boson mass, a change of value in the running of $k(\mu)$ is also small and remained in a small value. As a result, the running of λ closes to that of the SM.

- It is known as the vacuum instability that the value of λ becomes negative before the Planck scale in the SM with the experimental center values of the Higgs and top masses. This is due to the negative contribution from the top Yukawa coupling to the β-function of λ as in eq. (2.5). The minimum in the running of λ is around $\mathcal{O}(10^{17})$ GeV. It is also shown from NNLO computations [6] that λ can remain positive up to the Planck scale when $127\,\text{GeV} \lesssim m_h \lesssim 130\,\text{GeV}$ for $M_t = 173.1 \pm 0.6\,\text{GeV}$ (or when $171.3\,\text{GeV} \lesssim M_t \lesssim 171.7\,\text{GeV}$ for $m_h = 126\,\text{GeV}$).

- Once the gauge singlet scalar is added to the SM, the additional contribution of $k^2/2$ with the plus sign appears in the β-function of λ. This contribution can lift the running of λ, and thus, λ can be around zero at the Planck scale.

- The position of the minimum in the running of λ comes to lower energy scale than $\mathcal{O}(10^{17})$ GeV by adding the gauge singlet scalar because the contribution of $k^2/2$ in eq. (2.5) becomes large at a high energy scale compared to the electroweak (EW) scale.

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[3]If $m_S < m_H$, β_λ is zero for $\mu < m_H$ and is given by the third line of right-hand side of eq. (2.5) for $m_H \leq \mu$.

- The realization of the vanishing λ around the Planck scale by adding the gauge singlet scalar means that λ becomes negative before the Planck scale due to the above third and fourth properties of λ. Then, λ returns to zero.

- The running of λ_S is an increasing function of t (or μ). There is not a direct contribution from λ_S to the β-function of λ but the running of λ_S affects on that of λ through the running of k.

We investigate the case that the gauge singlet scalar is DM with the three BCs ($\lambda(M_{\rm pl}) = 0$, $\beta_\lambda(M_{\rm pl}) = 0$, and ${\rm Str}\, M^2(M_{\rm pl}) = 0$) in this model. Since we impose the odd-parity on the singlet scalar under the additional Z_2 symmetry, the singlet can be a candidate for DM. Thus, $\Omega_S h^2 = 0.119$ must be reproduced in the case, where Ω_S is the density parameter of the singlet and h is the Hubble parameter.

2.1 Vanishing Higgs self-coupling: $\lambda(M_{\rm pl}) = 0$

First, we consider the BC that λ is zero at the Planck scale $M_{\rm pl} = 10^{18}\,{\rm GeV}$, $\lambda(M_{\rm pl}) = 0$. The BCs of the Higgs self-coupling and top Yukawa coupling at low energy are taken as

$$\lambda(M_Z) = \frac{m_h^2}{2v^2} = 0.131\,, \qquad y(M_t) = \frac{\sqrt{2}m_t(M_t)}{v}\,, \tag{2.8}$$

for the RGEs, where $m_h = 126\,{\rm GeV}$ is taken, M_t is the top pole mass as $173.07 \pm 1.24\,{\rm GeV}$, and m_t is the $\overline{\rm MS}$ mass as $160^{+5}_{-4}\,{\rm GeV}$ [36].[4]

Let us solve the RGEs, eqs. (2.4)–(2.7). Gray dots in figure 1(a) show the region satisfying $|\lambda(M_{\rm pl})| < 10^{-2}$ and $\Omega_S h^2 = 0.119$. In the figure, the horizontal axis is the gauge singlet DM mass defined by $m_S \equiv \sqrt{\bar{m}_S^2 + kv^2/2}$ and the vertical axis is the top pole mass. The bounds of top mass $173.07 \pm 1.24\,{\rm GeV}$ are also depicted by the horizontal dashed lines. Figure 1(b) is a typical example of the runnings of the scalar quartic couplings satisfying the above conditions (and the Veltman condition discussed later). The horizontal and vertical axes are the renormalization scale and the values of scalar quartic couplings, respectively. Black, blue, and red curves indicate the runnings of λ, k, and λ_S, respectively. Initial conditions for the corresponding RGEs are $k(M_Z) = 0.24$, $\lambda_S(M_Z) = 0.34$, $M_t = 173\,{\rm GeV}$, and $m_S = 800\,{\rm GeV}$ with eq. (2.8).

It can be seen from figure 1(a) that $|\lambda(M_{\rm pl})| < 10^{-2}$ can be satisfied in a region of $85\,{\rm GeV} \lesssim m_S \lesssim 1.1 \times 10^3\,{\rm GeV}$ with the corresponding top pole mass, $171.8\,{\rm GeV} \lesssim M_t \lesssim 173.8\,{\rm GeV}$. Such a DM mass region will be checked by the future XENON100 experiment with 20 times sensitivity of the current data [39]. One can also see that a larger top mass requires a larger DM mass in the region of $m_S \gtrsim 10^2\,{\rm GeV}$. This is due to the following reason: a larger top mass needs a larger value of k in order to realize the tiny value of λ at the Planck scale. And a larger k requires a larger DM mass to give the correct abundance in the range of $m_S \gtrsim 10^2\,{\rm GeV}$ (e.g., see [29, 30, 39]).

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[4]We also take the following values as [36], $\sin^2\theta_W(M_Z) = 0.231$, $\alpha_{\rm em}^{-1} = 128$, $\alpha_s(M_Z) = 0.118$ for the parameters in the EW theory, where θ_W is the Weinberg angle, $\alpha_{\rm em}$ is the fine structure constant, and α_s is the strong coupling, respectively.
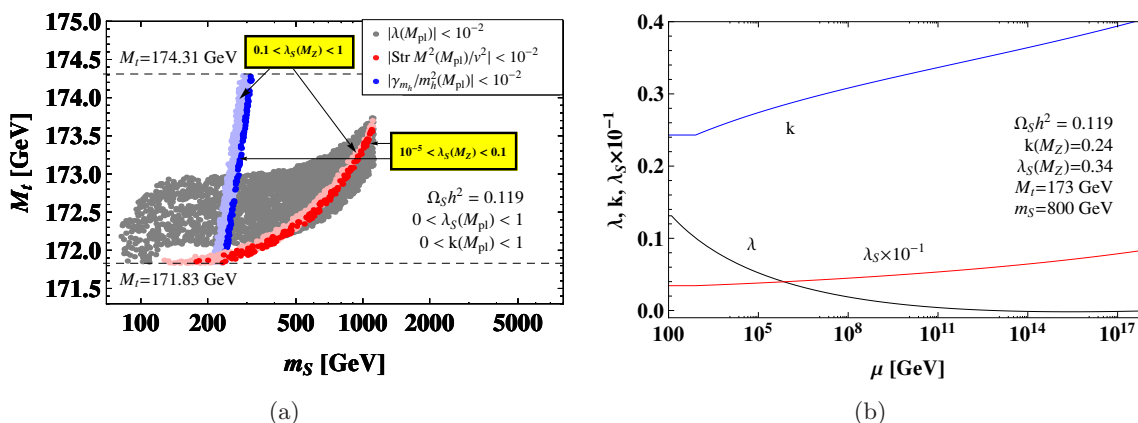
Figure 1. (a) Regions satisfying the BCs at the Planck scale, in which conditions $|\lambda(M_{\rm pl})| < 10^{-2}$, $|\operatorname{Str} M^2(M_{\rm pl})/v^2| < 10^{-2}$, and $|\gamma_{m_h}/m_h^2(M_{\rm pl})| < 10^{-2}$ are depicted as gray, (deep and light) red, and (deep and light) blue, respectively. The deep and light red (blue) dots indicate $10^{-5} < \lambda_S(M_Z) < 0.1$ and $0.1 < \lambda_S(M_Z) < 1$ for $|\operatorname{Str} M^2(M_{\rm pl})/v^2| < 10^{-2}$ ($|\gamma_{m_h}/m_h^2(M_{\rm pl})| < 10^{-2}$), respectively. (b) A typical example of runnings of λ, k, and λ_S, whose initial conditions are specified by $k(M_Z) = 0.24$, $\lambda_S(M_Z) = 0.34$. The parameter set of the figure (b) corresponds to a point of $(m_S, M_t) = (800\,{\rm GeV}, 173\,{\rm GeV})$ in the figure (a).

In order to realize the correct abundance of DM in $m_S \lesssim 10^3\,{\rm GeV}$, $k(M_Z) \lesssim 0.3$ is needed. Thus, $k(M_{\rm pl})$ is not also large ($k(M_{\rm pl}) < 1$) for the realization of DM. Since we have also imposed the condition of $0 < \lambda_S(M_{\rm pl}) < 1$ in the analyses, the model can be described by a perturbative theory up to the Planck scale. On the other hand, the value of λ_S does not actually affect on the abundance of DM. Thus, the region described by the gray dots in figure 1(a) is not changed even with the condition of $\lambda_S(M_{\rm pl}) > 1$.

One might suggest the Higgs inflation by the use of the region satisfying $\lambda(M_{\rm pl}) = 0$, which is included in gray dots of figure 1(a), but it is not possible.[5] Since $\lambda(\mu) < 0$ ($10^8\,{\rm GeV} \lesssim \mu < M_{\rm pl}$) and $\lambda(M_{\rm pl}) = 0$, there is a global minimum of the potential between the EW and Planck scales. If one identifies the Higgs with the inflaton, the inflaton rolls downslope to the global minimum not to the EW one. Thus, one must consider the other inflation models.

The smallness of $\lambda(M_{\rm pl})$ predicting close values of the Higgs and top masses to experimental ones at low energy motivates one to investigate the BC of $\lambda(M_{\rm pl}) = 0$ and/or the possibility of the Higgs inflation. On the other hand, the value of λ_S does not strongly affect the SM (Higgs and top masses), DM sectors, and other cosmology compared to that of λ (and k which determines the abundance of DM). Thus, we focus only on the BC of $\lambda(M_{\rm pl}) = 0$ in this work. If there could be phenomenological and/or cosmological motivations to impose $\lambda_S(M_{\rm pl}) = 0$, the discussions of the realization of the BC might also be interesting.

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[5]If one considers $\lambda(\mu) > 0$ and very small $\lambda(M_{\rm pl})$, one would have a successful Higgs inflation with a non-minimal coupling of the Higgs field to the Ricci curvature scalar.

2.2 Veltman condition: $\mathrm{Str}\,M^2(M_{\mathrm{pl}}) = 0$

The Veltman condition, which indicates a disappearance of the quadratic divergence on the 1-loop radiative correction to the bare Higgs mass, is modified to

$$\frac{\mathrm{Str}\,M^2}{v^2} \equiv 6\lambda + \frac{k}{2} + \frac{3}{4}g'^2 + \frac{9}{4}g^2 - 6y^2 = 0\,, \tag{2.9}$$

where the term $k/2$ in eq. (2.9) is new contribution from $k|H|^2S^2/2$ interaction in the SM with gauge singlet scalar. In the SM, the value of the left-hand side of eq. (2.9) without $k/2$ term at the Planck scale is -0.291 when one takes $m_h = 126\,\mathrm{GeV}$ and $M_t = 173.07\,\mathrm{GeV}$.

We also show the region satisfying $|\mathrm{Str}\,M^2(M_{\mathrm{pl}})/v^2| < 10^{-2}$ and $\Omega_S h^2 = 0.119$ for the SM with the singlet DM in figure 1(a) by deep and light red dots. The deep and light red dots indicate $10^{-5} < \lambda_S(M_Z) < 0.1$ and $0.1 < \lambda_S(M_Z) < 1$, respectively. One can see that $|\mathrm{Str}\,M^2(M_{\mathrm{pl}})/v^2| < 10^{-2}$ can be satisfied in a region of $180\,\mathrm{GeV} \lesssim m_S \lesssim 1\,\mathrm{TeV}$ and $171.8\,\mathrm{GeV} \lesssim M_t \lesssim 173.6\,\mathrm{GeV}$ with the 126 GeV Higgs mass and the correct abundance of DM. It must be noted that both the vanishing λ and the Veltman condition can be satisfied in the region of

$$300\,\mathrm{GeV} \lesssim m_S \lesssim 1\,\mathrm{TeV}, \qquad 172\,\mathrm{GeV} \lesssim M_t \lesssim 173.6\,\mathrm{GeV}\,. \tag{2.10}$$

We will return to this point later. This DM mass region will also be checked by the future XENON100 experiment with 20 times sensitivity of the current data [39]. One can also see that a larger top mass requires a larger DM mass. The reason is similar to the case of the vanishing λ condition, i.e. a larger top mass needs a larger value of k in order to cancel the negative contribution from $-6y^2$ term at the Planck scale, and thus a larger k requires a larger DM mass to give the correct abundance in the range of $m_S \gtrsim 10^2\,\mathrm{GeV}$.

We also comment on the anomalous dimension for the Higgs mass defined by

$$(4\pi)^2 \frac{dm_h^2}{dt} = \gamma_{m_h}\,, \tag{2.11}$$

which indicates the logarithmic divergence. It is also modified to

$$\gamma_{m_h} = m_h^2\left(12\lambda + 6y^2 - \frac{9}{2}g^2 - \frac{3}{2}g'^2\right) + 2km_S^2\,, \tag{2.12}$$

where the last term of the right-hand side of eq. (2.12) is new contribution from the gauge singlet scalar. The value of the anomalous dimension for the Higgs mass in the SM at the Planck scale is $(\gamma_{m_h}^{\mathrm{SM}}/m_h^2)|_{\mu=M_{\mathrm{pl}}} \simeq -0.695$. It turns naively out that a singlet mass around the EW scale can cancel the negative value of the anomalous dimension from the SM.

In fact, the deep and light blue dots in figure 1(a) show a region satisfying $|\gamma_{m_h}/m_h^2(M_{\mathrm{pl}})| < 10^{-2}$ and $\Omega_S h^2 = 0.119$ at the same time. The deep and light blue dots indicate $10^{-5} < \lambda_S(M_Z) < 0.1$ and $0.1 < \lambda_S(M_Z) < 1$, respectively. One can see that $|\gamma_{m_h}/m_h^2(M_{\mathrm{pl}})| < 10^{-2}$ can be satisfied in a region of $200\,\mathrm{GeV} \lesssim m_S \lesssim 300\,\mathrm{GeV}$ with the corresponding top pole mass, $171.8\,\mathrm{GeV} \lesssim M_t \lesssim 174.3\,\mathrm{GeV}$. A larger top mass leads a smaller value of anomalous dimension due to 12λ term in eq. (2.12). Therefore, a larger

top mass requires a larger value of k (equivalently to m_S). However, the magnitude of the decrease of the anomalous dimension by a larger top mass is smaller than those of λ and $\mathrm{Str}\, M^2/v^2$ because the sign of contribution from the top Yukawa coupling only in the anomalous dimension is positive unlike the λ and $\mathrm{Str}\, M^2/v^2$ cases (see eqs. (2.5), (2.9), and (2.12)). As a result, a top mass dependence of the anomalous dimension is weaker compared to those of the vanishing λ and the Veltman condition. It should be mentioned that there is also a region in which two conditions of the vanishing λ and γ_{m_h} can be realized at the same time.

2.3 Vanishing beta-function of the self-coupling: $\beta_\lambda(M_{\mathrm{pl}}) = 0$

In the SM, the β-function of λ becomes tiny at the Planck scale. The value is about $\beta_\lambda^{\mathrm{SM}}(M_{\mathrm{pl}}) \simeq 8.42 \times 10^{-4}$ when one takes the Higgs and top pole masses as $m_h = 126\,\mathrm{GeV}$ and $M_t = 173.07\,\mathrm{GeV}$ at low energy. Thus, this condition may also be meaningful for theories beyond the SM around the Planck scale.

For instance, when one takes $m_S = 800\,\mathrm{GeV}$, $k(M_Z) = 0.24$, and $\lambda_S(M_Z) = 0.34$ in addition to $m_h = 126\,\mathrm{GeV}$ and $M_t = 173.07\,\mathrm{GeV}$ as an example in the context of the SM with the gauge singlet field, the corresponding values of the β-function at the Planck scale become $\beta_\lambda(M_{\mathrm{pl}}) \simeq 6.32 \times 10^{-2}$. Therefore, the vanishing β-function of λ at the Planck scale in the SM with the gauge singlet cannot be satisfied because the runnings of λ is increasing from a negative value due to the effect of the singlet field as shown in figure 1(b). In this extension of the SM, $\beta_\lambda(\mu)$ becomes zero at $\mu \sim \mathcal{O}(10^{15\text{--}17}\,\mathrm{GeV})$ not the Planck scale.

According the above analyses, the BC of $\beta_\lambda(M_{\mathrm{pl}}) = 0$ cannot be realized but two BCs of $\lambda(M_{\mathrm{pl}}) = \mathrm{Str}\, M^2(M_{\mathrm{pl}}) = 0$ can be satisfied in the model. Since the result might indicate that all the Higgs potential is induced from a quantum correction under the current circumstances, one has no warrant for $\beta_\lambda(M_{\mathrm{pl}}) = 0$. Thus, the non-vanishing β-function can be compatibly understood. Furthermore, there are also two additional β-functions (β_k and β_{λ_S}) in this model. Since values of $\beta_k(M_{\mathrm{pl}})$ and $\beta_{\lambda_S}(M_{\mathrm{pl}})$ cannot be zero when we impose $\lambda(M_{\mathrm{pl}}) = 0$ and the correct abundance of DM, the vanishing condition for only $\beta_\lambda(M_{\mathrm{pl}})$ might be meaningless. Thus, in this work we take a stance of giving up the vanishing β-function at the Planck scale to predict the Higgs and top masses, although $\beta_\lambda(M_{\mathrm{pl}}) = \lambda(M_{\mathrm{pl}}) = 0$ condition adopted in [3] predicted the values of the Higgs and top masses roughly close to experimental magnitudes.

2.4 Multi coincidence

It is remarkable that there is a region, given in eq. (2.10), satisfying two independent BCs at the Planck scale ($\lambda(M_{\mathrm{pl}}) \simeq 0$ and $\mathrm{Str}\, M^2(M_{\mathrm{pl}}) \simeq 0$ (or $\gamma_{m_h}(M_{\mathrm{pl}}) \simeq 0$)) at the same time with the correct abundance of the gauge singlet DM, 126 GeV Higgs mass, experimentally allowed top pole mass, and the coupling perturbativity. This double coincidence in the above BCs, $\lambda(M_{\mathrm{pl}}) \simeq 0$ and $\mathrm{Str}\, M^2(M_{\mathrm{pl}}) \simeq 0$, with the correct DM abundance is just a non-trivial result. The double coincidence means that the Higgs potential becomes flat at the Planck scale. The gauge singlet scalar plays a crucial role for the realization, and it becomes DM with the correct abundance in the universe at present. The double coincidence with DM might be an alternative principle to "multiple point criticality principle" discussed
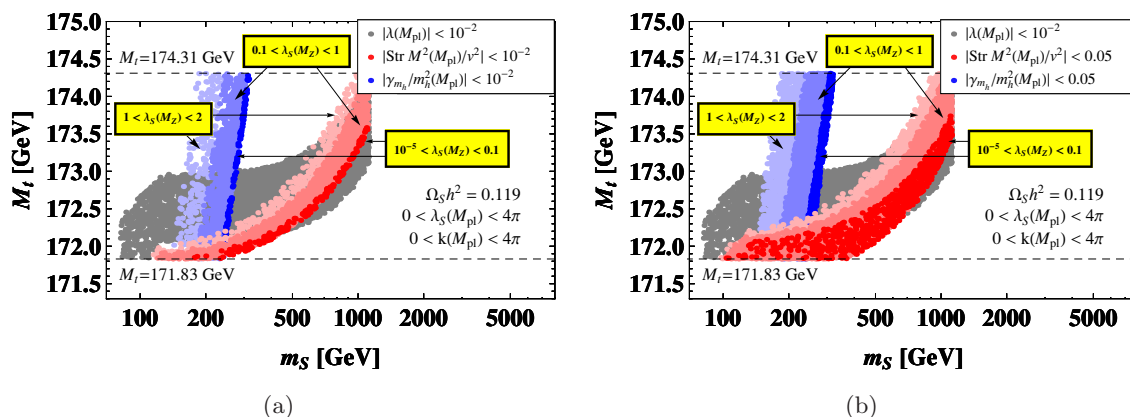
Figure 2. (a) Regions satisfying the BCs with $(k(M_{\rm pl}), \lambda_S(M_{\rm pl})) < 4\pi$, in which conditions $(|\lambda(M_{\rm pl})|, |\operatorname{Str} M^2(M_{\rm pl})/v^2|, |\gamma_{m_h}/m_h^2(M_{\rm pl})|) < 10^{-2}$ are depicted as gray, red, and blue dots, respectively. The deep, light, and the lightest red (blue) dots indicate $10^{-5} < \lambda_S(M_Z) < 0.1$, $0.1 < \lambda_S(M_Z) < 1$, $1 < \lambda_S(M_Z) < 2$ for $|\operatorname{Str} M^2(M_{\rm pl})/v^2| < 10^{-2}$ $(|\gamma_{m_h}/m_h^2(M_{\rm pl})| < 10^{-2})$, respectively. (b) Regions satisfying the BCs with $(k(M_{\rm pl}), \lambda_S(M_{\rm pl})) < 4\pi$, in which conditions $(|\lambda(M_{\rm pl})|, |\operatorname{Str} M^2(M_{\rm pl})/v^2|, |\gamma_{m_h}/m_h^2(M_{\rm pl})|) < (10^{-2}, 0.05, 0.05)$ are depicted as gray, red and blue dots, respectively.

in ref. [3], where a condition that the SM Higgs potential has two degenerate vacua was imposed.[6]

In the above analyses, we have limited the values of $k(M_{\rm pl})$ and $\lambda_S(M_{\rm pl})$ to be less than 1. But, when one allows the values up to 4π, the two regions for the Veltman condition and the vanishing anomalous dimension are changed. We also weaken the conditions of $(|\operatorname{Str} M^2(M_{\rm pl})/v^2|, |\gamma_{m_h}/m_h^2(M_{\rm pl})|) < 10^{-2}$ to < 0.05, the allowed regions for the conditions grow wider. Figure 2 shows the cases, and figure 2(a) shows regions satisfying the BCs with $(k(M_{\rm pl}), \lambda_S(M_{\rm pl})) < 4\pi$ at the Planck scale, in which conditions $|\lambda(M_{\rm pl})| < 10^{-2}$, $|\operatorname{Str} M^2(M_{\rm pl})/v^2| < 10^{-2}$, and $|\gamma_{m_h}/m_h^2(M_{\rm pl})| < 10^{-2}$ are depicted as gray, red, and blue dots, respectively. The deep, light, and the lightest red (blue) dots indicate $10^{-5} < \lambda_S(M_Z) < 0.1$, $0.1 < \lambda_S(M_Z) < 1$, $1 < \lambda_S(M_Z) < 2$ for $|\operatorname{Str} M^2(M_{\rm pl})/v^2| < 10^{-2}$ $(|\gamma_{m_h}/m_h^2(M_{\rm pl})| < 10^{-2})$, respectively. One can see that the region satisfying $|\operatorname{Str} M^2(M_{\rm pl})/v^2| < 10^{-2}$ and $|\gamma_{m_h}/m_h^2(M_{\rm pl})| < 10^{-2}$ grow wider compared to the case shown in figure 1(a) when one allows the value of $\lambda_S(M_Z)$ up to 2, which corresponds to $\lambda_S(M_{\rm pl}) < 4\pi$. Such a relatively large $\lambda_S(M_Z)$ can effectively increase the value of k enough to cancel the negative contribution in the Veltman condition and anomalous dimension at the Planck scale even when one takes a smaller $k(M_Z)$. In this case, the double coincidence of $\lambda(M_{\rm pl}) \simeq 0$ and $\operatorname{Str} M^2(M_{\rm pl}) \simeq 0$ (or $\gamma_{m_h}(M_{\rm pl}) \simeq 0$) still occurs.

Figure 2(b) shows regions satisfying the weaker BCs, $|\lambda(M_{\rm pl})| < 10^{-2}$, $|\operatorname{Str} M^2(M_{\rm pl})/v^2| < 0.05$, and $|\gamma_{m_h}/m_h^2(M_{\rm pl})| < 0.05$ with $(k(M_{\rm pl}), \lambda_S(M_{\rm pl})) < 4\pi$. In the case, the allowed regions become the widest among all cases we have investigated. As

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[6]One vacuum we live is the EW scale, and another one is the Planck scale. Under the condition, the vanishing λ and β_λ are required.

a result, the region satisfying three BCs at the same time appears around

$$150 \, \text{GeV} \lesssim m_S \lesssim 300 \, \text{GeV}, \qquad 172 \, \text{GeV} \lesssim M_t \lesssim 172.2 \, \text{GeV}. \qquad (2.13)$$

This means that the triple coincidence for the three BCs occurs. The triple coincidence requires that the logarithmic divergence of the Higgs mass also disappear at the Planck scale instead of allowing a fine-tuning between the bare Higgs mass and a quadratic correction.

3 Summary and discussions

We have investigated Planck scale BCs on the Higgs sector in the SM with gauge singlet scalar DM. The BCs are the vanishing Higgs self-coupling ($\lambda(M_{\text{pl}}) = 0$), the Veltman condition ($\text{Str} \, M^2(M_{\text{pl}}) = 0$) (and the vanishing anomalous dimension for the Higgs mass parameter, $\gamma_{m_h}(M_{\text{pl}}) = 0$), and the vanishing β-function of the self-coupling ($\beta_\lambda(M_{\text{pl}}) = 0$). If one imposes the BCs in the SM, the Higgs and top masses are predicted to be close to the experimental ones. BCs of $\lambda(M_{\text{pl}}) = 0$ and $\text{Str} \, M^2(M_{\text{pl}}) = 0$ mean that there exists approximately flat direction in the Higgs potential. In addition, the quadratic (logarithmic) divergence for the Higgs mass disappears under the BC of the Veltman condition (and the vanishing anomalous dimension at the Planck scale). However, it actually seems difficult to reproduce 126 GeV Higgs mass an $173.07 \pm 1.24 \, \text{GeV}$ top pole mass at the same time under the three BCs in the SM. Thus, we have investigated these BCs in the context of the SM with the singlet real scalar.

We have taken the setup that the singlet is DM and all scalar quartic coupling in the model can be perturbatively treated up to the Planck scale. And we have utilized the Higgs with 126 GeV mass in the analyses. We could find that the vanishing self-coupling and Veltman condition at the Planck scale can be remarkably realized with the 126 GeV Higgs mass and top pole mass, $172 \, \text{GeV} \lesssim M_t \lesssim 173.6 \, \text{GeV}$ and the coupling perturbativity, where a correct abundance of scalar dark matter is obtained with mass of $300 \, \text{GeV} \lesssim m_S \lesssim 1 \, \text{TeV}$. It means that the Higgs potential is approximately flat at the Planck scale, and this situation cannot be realized in the SM with the top pole mass.

When one takes weaker conditions for the BCs, $(k(M_{\text{pl}}), \lambda_S(M_{\text{pl}})) < 4\pi$ and $(|\text{Str} \, M^2(M_{\text{pl}})/v^2|, \ |\gamma_{m_h}/m_h^2(M_{\text{pl}})|) < 0.05$, the triple coincidence ($\lambda(M_{\text{pl}}) \simeq 0$, $\text{Str} \, M^2(M_{\text{pl}}) \simeq 0$, and $\gamma_{m_h}(M_{\text{pl}}) \simeq 0$) can be realized.

Next, we discuss some points, which are related with this work and an extension. The BC of $\lambda(M_{\text{pl}}) = 0$ implies that our EW vacuum is false and the true vacuum is at a high energy scale slightly smaller than the Planck scale like the SM with the center values of the Higgs and top masses. We have checked that the quantum tunnelling probability p through out the history of the universe, which is estimated by $p \simeq V_U H^4 \exp(-8\pi^2/(3|\lambda(H)|))$ (e.g., see [40]), can be much smaller than 1, where $V_U = \tau_U^4$, τ_U is the age of the universe as $\tau_U \simeq 13.7 \, \text{Gyrs}$, and we took $|\lambda(H)| \simeq 9.59 \times 10^{-5}$ with $H \simeq 8 \times 10^{17} \, \text{GeV}$ for the true vacuum of our sample point of $m_H = 126 \, \text{GeV}$, $M_t = 173 \, \text{GeV}$, $m_S = 800 \, \text{GeV}$, $k(M_Z) = 0.24$, and $\lambda_S(M_Z) = 0.34$.

We comment on the realization of the BCs of $\beta_\lambda(M_{\text{pl}}) = 0$ with $\lambda(M_{\text{pl}}) = 0$, which were firstly considered in [3], in this single extension of the SM. $\beta_\lambda(\mu)$ cannot be zero at

the Planck scale with $\lambda(M_{\mathrm{pl}}) = 0$ in the extension because there is an additional positive contribution from $kS^2|H|^2$ interaction to $\beta_\lambda(\mu)$. $\beta_\lambda(\mu)$ becomes zero at $\mu \sim \mathcal{O}(10^{15\text{–}17}\,\mathrm{GeV})$ (not the Planck scale) with $\lambda(M_{\mathrm{pl}}) \simeq 0$ and experimental center values of the Higgs and top masses. If one respects both BCs of $\lambda(M_{\mathrm{pl}}) = \beta_\lambda(M_{\mathrm{pl}}) = 0$ in this singlet extension of SM, the BCs predict about $145\,\mathrm{GeV}$ Higgs mass and $175\,\mathrm{GeV}$ top pole mass at M_Z scale, which are ruled out by experiments.

Finally, we also comment on other issues such as the existence of the tiny neutrino mass and the baryon asymmetry of the universe (BAU), which cannot be explained in the SM. One popular explanation is given by adding heavy right-handed Majorana neutrinos into the SM. These are known as the seesaw mechanism and the leptogenesis for generating the tiny neutrino mass and BAU, respectively. In this example of the extension, there exist additional contributions from the neutrino Yukawa couplings to β_λ, $\mathrm{Str}\,M^2$, and γ_{m_h}. If the magnitude of the neutrino Yukawa couplings is smaller than $\mathcal{O}(10^{-2})$, which corresponds to the right-handed neutrino Majorana neutrino mass smaller than $\mathcal{O}(10^{10})\,\mathrm{GeV}$, the contributions are negligible in the BCs like the Yukawa couplings of the bottom quark and tau. On the other hand, if the neutrino Yukawa couplings are larger than $\mathcal{O}(0.1)$, the contributions should be taken account in the BCs. For the BC of $\lambda(M_{\mathrm{pl}}) = 0$, a larger $k(M_Z)$ (equivalently a heavier DM mass) is required because of a negative contribution from the neutrino Yukawa coupling to β_λ. Such a negative contribution may well cancel other positive contributions in β_λ such that $\beta_\lambda(M_{\mathrm{pl}}) = 0$ can be realized at the same time. A larger $k(M_Z)$ is needed also for the BC of $\mathrm{Str}\,M^2(M_{\mathrm{pl}}) = 0$ because the contribution from the neutrino Yukawa to $\mathrm{Str}\,M^2$ is negative. Finally, an effect of the neutrino Yukawa coupling for γ_{m_h} is relatively non-trivial compared to the other BCs because the positive contributions from (top and neutrino) Yukawa couplings to γ_{m_h} compete with the negative one from 12λ term, i.e. larger (positive) Yukawa couplings lead smaller (negative) value of λ at the Planck scale. Thus, an accurate numerical analysis is required. Effects in the BCs from additional particles and their mass scales strongly depend on a model for generating the tiny neutrino mass and BAU, e.g. adding right-handed neutrinos, but such a model dependent analysis of the BCs with explanations of the neutrino mass and BAU in addition to DM might also be interesting.

Acknowledgments

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