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Testing chiral dynamics in pionic atoms

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Abstract

The energy dependence of chirally expanded πN isoscalar and isovector amplitudes $b_0(E)$ and $b_1(E)$, respectively, for zero-momentum *off-shell* pions near threshold, is used to impose the minimal substitution requirement $E \to E - V_C$ on the properly constructed pion optical potential within a large-scale fit to 100 pionic-atom data across the periodic table which also include the recently established 'deeply bound' pionic atoms of Pb and Sn. This fit cannot be reconciled with the well-known free-space values of the πN threshold amplitudes. In contrast, introducing the empirically known energy dependence for *on-shell* pions leads to a better fit and to satisfactory values for the πN threshold amplitudes. The difference between these two approaches is briefly discussed.

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1. Introduction and methodology

The recent observation of $1s$ and $2p$ 'deeply bound' π^- atomic states in isotopes of Pb [1–3] and very recently also of such $1s$ states in isotopes of Sn [4] has triggered renewed interest in the issue of partial restoration of chiral symmetry in dense nuclear matter [5–13]. In a nutshell, it was argued that since (i) the pion in deeply bound states with relatively large neutron excess charts a fairly dense portion of the nuclear medium, and since (ii) the most influential term of the optical potential $V_{\rm opt}$ for this situation is generated by the s-wave isovector πN threshold amplitude b_1, and since (iii) b_1 in free-space is well approximated in lowest chiral-expansion order by the Tomozawa–Weinberg expression [14]

$$b_1 = -\frac{\mu_{\pi N}}{8\pi f_\pi^2} = -0.08 m_\pi^{-1}, \tag{1}$$

then deeply-bound states could yield valuable information on the dependence of f_π on the density ρ. The pion decay constant f_π serves as an order parameter for the spontaneously broken chiral symmetry in hadronic physics, and its free-space value $f_\pi = 92.4$ MeV should go to zero in dense matter if and when chiral symmetry is restored. Indeed, it has been known for quite some time that the renormalized value of b_1 required to fit pionic-atom data is about $-0.12 m_\pi^{-1}$ [15,16] clearly more repulsive than the free-space value $-0.09 m_\pi^{-1}$ [17].

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Our most extensive recent work [11] has shown, however, that the deeply bound states by themselves on statistical grounds are insufficient to draw firm conclusions about whether or not b_1 is renormalized in dense matter. In fact, contrary to the expectation (i) above, the pion in deeply bound 1s states *does not* chart higher-density regions of the nucleus than it does so in 'normal' 1s states in light nuclei. It was shown in Ref. [11] that only by using a substantially larger data base that includes plenty of normal pionic atom data, and carefully considering uncertainties in the knowledge of neutron density distributions, it becomes possible to make a meaningful statement on the renormalization of the isovector threshold amplitude b_1, i.e., $b_1 = -0.108 \pm 0.007 m_\pi^{-1}$. It is tempting to ascribe this value for b_1, using Eq. (1), to a renormalization of f_π in the nuclear medium.

Recently, Kolomeitsev et al. [12] have suggested that pionic-atom data could be reproduced using a pion optical potential underlain by chirally expanded πN amplitudes, retaining the energy dependence of the amplitudes $b_0(E)$ and $b_1(E)$ for zero-momentum ($\mathbf{q} = 0$) pions in nuclear matter in order to impose the minimal substitution requirement $E \to E - V_C$, where V_C is the Coulomb potential. This has the advantage of enabling one to use a systematic chiral expansion as an input [18], rather than singling out the leading-order term Eq. (1) for b_1. Kolomeitsev et al. applied this programme to study the shifts and widths of the Pb and Sn pionic deeply bound states and reported substantial improvement in reproducing these data with only minimal phenomenological input, mostly limited to the p-wave πN interaction to which allegedly these data are insensitive [12,13]. One could argue, however, that the deeply bound states do not offer sufficient variation over the energy range spanned by the bulk data on pionic atoms which include both 'shallow' states as well as 'deep' states. In the present Letter we test whether or not the energy dependence of this chiral expansion provides a satisfactory description for the bulk of pionic atom data.

The data used in the present Letter consist of 100 strong-interaction shifts and widths, stretching from ^{20}Ne to ^{238}U [11]. The Klein–Gordon equation solved for the pionic-atom eigen energies is given by [11,19]:

$$\left[\nabla^2 - 2\mu(B + V_C) + (B + V_C)^2 - \Pi(E)\right]\psi = 0 \quad (\hbar = c = 1), \tag{2}$$

where μ is the pion-nucleus reduced mass, B is the complex binding energy and V_C is the finite-size Coulomb interaction of the pion with the nucleus, including vacuum-polarization terms. The pion-nuclear polarization operator $\Pi(E)$ is given by the standard Ericson–Ericson form [20]

$$\Pi = 2\mu V_{\rm opt} = q(r) + \vec{\nabla} \cdot \alpha(r) \vec{\nabla}, \tag{3}$$

with the s-wave part of $V_{\rm opt}$

$$q(r) = -4\pi\left(1 + \frac{\mu}{M}\right)\left\{\bar{b}_0(r)\left[\rho_n(r) + \rho_p(r)\right] + b_1\left[\rho_n(r) - \rho_p(r)\right]\right\} - 4\pi\left(1 + \frac{\mu}{2M}\right)4B_0\rho_n(r)\rho_p(r). \tag{4}$$

In these expressions ρ_n and ρ_p are the neutron and proton density distributions normalized to the number of neutrons N and number of protons Z, respectively, and M is the mass of the nucleon; $q(r)$ is referred to as the s-wave potential term and $\alpha(r)$ is referred to as the p-wave potential term. The function $\bar{b}_0(r)$ in Eq. (4) is given in terms of the *local* Fermi momentum $k_F(r)$ corresponding to the isoscalar nucleon density distribution:

$$\bar{b}_0(r) = b_0 - \frac{3}{2\pi}\left(b_0^2 + 2b_1^2\right)k_F(r), \tag{5}$$

where the quadratic terms in b_0 and b_1 represent double-scattering modifications of b_0. In particular, the b_1^2 term represents a sizable correction to the nearly vanishing linear b_0 term. Similar double-scattering modifications of b_1, as well as other correction terms to $\Pi(E)$ listed in Refs. [12,18], were found by us to yield negligibly small effects and will not be further discussed below. The complex parameter B_0 is due to s-wave absorption on pairs of nucleons. Its microscopic evaluation is outside the scope of chiral perturbation theory. Finally, the p-wave term $\alpha(r)$ is a standard one with the same form as in Ref. [11].

The chiral expansion of the πN amplitudes for $\mathbf{q} = 0$ at the two-loop level is well approximated by the following expressions [12,18]:

$$4\pi\left(1 + \frac{m_\pi}{M}\right)b_0(E) \approx \left(\frac{\sigma - \beta E^2}{f_\pi^2} + \frac{3g_A^2 m_\pi^3}{16\pi f_\pi^4}\right), \tag{6}$$

$$4\pi\left(1 + \frac{m_\pi}{M}\right)b_1(E) \approx -\frac{E}{2f_\pi^2}\left(1 + \frac{\gamma E^2}{(2\pi f_\pi)^2}\right), \tag{7}$$

where σ is the πN sigma term, $\sigma \sim 50$ MeV [21], g_A is the nucleon axial-vector coupling constant, $g_A = 1.27$, β and γ are tuned to reproduce the threshold values $b_0(m_\pi) \approx 0$ and $b_1(m_\pi) = -0.0885^{+0.0010}_{-0.0021} m_\pi^{-1}$ [17], respectively. For b_0, in view of the accidental cancellations that lead to its near vanishing we limit our discussion to the f_π^{-2} term in Eq. (6), therefore choosing $\beta = \sigma m_\pi^{-2}$. The next, f_π^{-4} term is much bigger than the scale of variation from zero expected for the threshold value and its inclusion here would appear somewhat dubious; if included, it would increase the energy dependence from the conservative estimate adopted by us. Implementing the minimal substitution requirement in the calculation of pionic atom observables, the constant parameters $b_{0,1}$ of the conventionally energy-independent optical potential have been replaced in our calculation by

$$b_{0,1}(r) = b_{0,1} - \delta_{0,1}\big(\mathrm{Re}\, B + V_{\mathrm{C}}(r)\big), \tag{8}$$

where $\delta_{0,1} = \partial b_{0,1}(E)/\partial E$ is the appropriate slope parameter at threshold, $\mathrm{Re}\, B$ is the (real) binding energy of the corresponding pionic atom state and $V_{\mathrm{C}}(r)$ is the Coulomb potential. The constant fit parameters $b_{0,1}$ are then expected to agree with the corresponding free πN threshold amplitudes if the energy dependence is indeed responsible for the renormalized values found in conventional analyses. The added piece proportional to δ in Eq. (8) is dominated by the attractive $V_{\mathrm{C}}(r)$. Since the slope parameters δ from Eqs. (6), (7) are negative, this added piece is always repulsive, in agreement with Refs. [12,22].

Before testing the above 'chiral' energy dependence for *off-shell* $\mathbf{q} = 0$ pions we present results for the empirically known *on-shell* πN amplitudes, when the pion energy E and its three-momentum $\mathbf{q}$ are related by $E^2 = m_\pi^2 + \mathbf{q}^2$. This choice corresponds to the original suggestion by Ericson and Tauscher [22] to consider the effect of energy dependence in pionic atoms. Ericson subsequently [23] pointed out that, for strongly repulsive short-range NN correlations, the on-shell requirement follows naturally from the Agassi–Gal theorem [24] for scattering off non-overlapping nucleons. The corresponding πN amplitudes will be denoted below as 'empirical'. Fig. 1 shows the energy dependence of the empirical $b_0(E)$ and $b_1(E)$ on-shell amplitudes as derived from the SAID data base [25]. The value of $b_0(E)$ at threshold is very close to zero and the empirical slope δ_0,

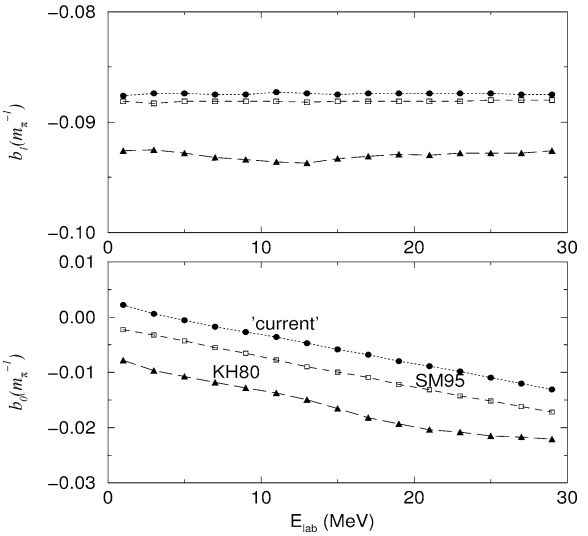

Fig. 1. πN empirical s-wave scattering amplitudes as function of laboratory energy for on-shell pions from the SAID data base [25].

which corresponds to adding repulsion in Eq. (8), is quite well determined over the whole relevant energy range, having changed little since the classical KH80 analysis [26] of the pre pion-factories data to the most recent analyses of modern data. We note that the slope of the $\mathbf{q} = 0$ chiral $b_0(E)$ amplitude of Eq. (6) is larger than the on-shell empirical slope by about 60%. For the empirical $b_1(E)$, its value at threshold has also changed little since KH80 to the present day analysis, and the slope of the empirical amplitude is essentially zero, in contrast to the fairly large slope of the $\mathbf{q} = 0$ chiral amplitude of Eq. (7).

2. Results

The present analysis is based on the 'global 3' data set of Ref. [11] consisting of 100 data points from ^{20}Ne to ^{238}U. For the nuclear density distributions ρ_p and ρ_n we adopt the procedure of Ref. [11] where ρ_p is obtained from the experimental charge distribution by unfolding the finite size of the charge of the proton, and where simple but physical parameterizations are used for ρ_n. A key quantity in this context is the difference $r_n - r_p$ between the root-mean-square radii. Relativistic mean field (RMF) calculations [27] yield

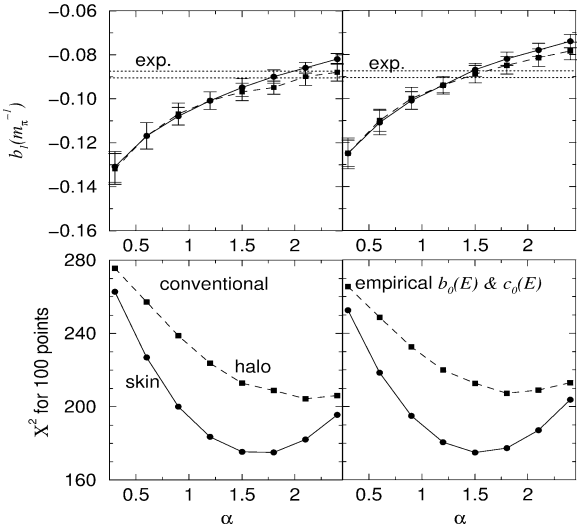

Fig. 2. Pionic-atom fits using 'conventional', energy-independent πN amplitudes (left panels) and 'empirical', energy-dependent SAID amplitudes (right panels) as function of the neutron-excess parameter α, Eq. (9), for two shapes of neutron densities. Lower part: values of χ^2 for 100 data points from ^{20}Ne to ^{238}U. Upper part: best-fit values of b_1 vs. the free πN threshold value [17] marked by 'exp.' within the dotted horizontal lines.

to a good approximation [11]

$$r_n - r_p = \alpha \frac{N-Z}{A} + \eta, \tag{9}$$

with the values $\alpha = 1.51 \pm 0.07$ fm, $\eta = -0.03 \pm 0.01$ fm. A similar expression, but with $\alpha = 1.0$, was obtained by analyzing strong interaction effects in antiprotonic atoms [28]. Owing to the strong correlation between the values assumed for $r_n - r_p$ and the values of b_1 derived from χ^2 fits to pionic atom data, we have varied the neutron-excess parameter α over a wide range, with the expectation that a value in the range of 1.0 to 1.5 will represent *on the average* the 41 nuclei in the present data base.

Fig. 2 shows results for the 'conventional' model (left) for which $\delta_{0,1} = 0$ and for the 'empirical' model (right) as function of the neutron-excess parameter α in Eq. (9) for two shapes of neutron densities. The dependence of the quality of fits on the shape of the neutron density distribution and the vanishingly small sensitivity of the derived values of b_1 to this shape are demonstrated by using either the 'skin' or the 'halo' shape, as discussed in Ref. [11]. The results for the conventional (energy independent) model are practically the same as in Ref. [11] in spite of adopting now the 'current' SAID values [25] for the p-wave parameters $c_0 = 0.21 m_\pi^{-3}$ and $c_1 = 0.165 m_\pi^{-3}$, instead of the values 0.22 and $0.18 m_\pi^{-3}$, respectively, used beforehand. This slight change was made for consistency, since the slope parameters δ for the empirical model were taken from the 'current' SAID analysis. In fact, we also incorporated in the right-hand side (r.h.s.) of the figure the SAID weak energy dependence of c_0 (c_1 is essentially energy independent). The results on the r.h.s. of the figure show that, for the 'skin' shape of ρ_n and with the introduction of the empirical energy dependence of the amplitudes, the minimum in the χ^2 curve has shifted slightly towards the acceptable region of $\alpha = 1.0$ to 1.5, and the value of b_1 for that minimum is in agreement with the free πN value marked by 'exp.' in the upper panels of the figure. It is self-evident that the 'halo' shape for the neutron density distributions cannot be reconciled with the data. Finally, we add that the resulting values of b_0 for both the conventional and empirical models are close to zero, well within the experimental error [17].

A comment on the 'anomalous s-wave repulsion' in pionic atoms is here in order. The net effect of the nearly vanishing parameter b_0, of the repulsive b_1 and of the phenomenological parameter $\mathrm{Re}\, B_0$ has been known [19] to produce a repulsive potential inside nuclei which is twice as large as expected. This is due to the combined action of the too repulsive b_1 and of $\mathrm{Re}\, B_0$ which turns out to be too repulsive compared to the expectations that $|\mathrm{Re}\, B_0| < \mathrm{Im}\, B_0$ (see also Ref. [7]). For the fits mentioned above we obtain for the conventional potential $\mathrm{Im}\, B_0 = 0.053 \pm 0.002 m_\pi^{-4}$ and $\mathrm{Re}\, B_0 = -0.10 \pm 0.03 m_\pi^{-4}$. Although the latter is determined to a moderate accuracy, we note that setting its value to zero while repeating the fits leads to a significant increase in the resulting χ^2 value and to a value for b_0 which is incompatible with experiment (cf. Table 4 of Ref. [11]). Using the empirical $b_{0,1}(E)$ we find $\mathrm{Re}\, B_0 = -0.07 \pm 0.03 m_\pi^{-4}$. We conclude that using the empirical energy dependence the anomaly in $\mathrm{Re}\, B_0$ is reduced, whereas there is essentially no anomaly in the parameter b_1.

Fig. 3 shows results for the 'chiral' model, when either b_0 (left) or b_1 (right) is made energy dependent according to Eqs. (6) and (7), respectively. The left-hand side of the figure shows that the quality of the best fit, upon incorporating only the energy de-

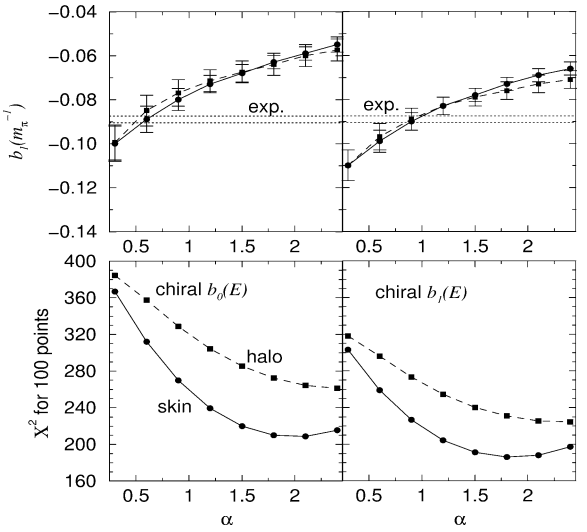

Fig. 3. Same as Fig. 2 but for the energy-dependent 'chiral' amplitudes b_0 (left) and b_1 (right).

pendence of the chiral b_0 amplitude, is significantly inferior to the corresponding best fit obtained using the conventional, energy-independent model (shown in Fig. 2). Furthermore, the value of α at the χ^2 minimum is unacceptably large and the corresponding value of b_1 is in sharp disagreement with the experimental free πN threshold value. The r.h.s. of the figure shows good fits with almost acceptable values for α and for b_1, upon incorporating only the energy dependence of the chiral b_1 amplitude, again for the 'skin' shape of the neutron density. However, incorporating the energy dependence of the chiral $b_0(E)$ (even within the limited scope of using only the f_π^{-2} term on the r.h.s. of Eq. (6)) on top of that for $b_1(E)$, leads to substantial disagreement between the resulting best-fit value for b_1 and the threshold value $b_1(m_\pi)$ which is marked by 'exp.' in Fig. 3. We conclude that, at present, the energy dependence generated by the chirally expanded s-wave πN amplitudes of Eqs. (6), (7) fails badly in reproducing consistently the bulk of pionic-atom data.

3. Discussion and conclusions

In the present Letter we have demonstrated that the consistency between pionic-atom data and the free πN threshold amplitudes is greatly improved by using just the on-shell energy dependence of the πN s-wave amplitudes, in accordance with the original suggestion made by Ericson and Tauscher [22]. The idea behind using this empirical energy dependence is the same one as used for constructing the multiple-scattering series for short-ranged πN interactions occurring within an assembly of largely non-overlapping nucleons [20, 23]. Multiple scattering is naturally described in this idealized limit as occurring *on-shell*. Whereas using off-shell $\mathbf{q} = 0$ pions in chiral expansions is motivated by the ground-state wavefunction description of pions in *nuclear matter*, applying this limitation to the construction of the pion-nuclear optical potential that generates pionic-atom wavefunctions is questionable.

We have also shown that the energy-dependent chiral amplitudes given by Eqs. (6), (7) for $\mathbf{q} = 0$ off-shell pions do not produce consistent or good global fits to pionic-atom data. This conclusion is not at odds with the observation made by Kolomeitsev et al. [12] that the $\mathbf{q} = 0$ off-shell chiral amplitudes work well, and with no need for a dispersive term $\mathrm{Re}\, B_0$ for the few deeply bound states available at present, since partial data sets of this kind do not have sufficient statistical significance to decide one way or another on this issue [10,11]. In fact, as good average reproduction of these deeply-bound data is reached within a wide class of optical potentials, including our 'empirical' energy-dependent potential of the present study. We defer this and other ramifications of the present analysis for a forthcoming detailed publication. Given the fact that the on-shell πN amplitudes provide by far a better description of pionic-atom data than the extremely off-shell $\mathbf{q} = 0$ chiral amplitudes do, we conclude that chiral dynamics is not yet at a stage of being tested in pionic atoms.

Finally, it should be emphasized that we have strictly adhered in the present calculation to imposing minimal substitution, $E \to E - V_{\mathrm{C}}$, on the pion-nuclear polarization operator $\Pi(E)$ within the Klein–Gordon equation (2). Nowhere have we renormalized the threshold value of the πN isovector amplitude b_1 of Eq. (1) by renormalizing the pion decay constant $f_\pi \to f_\pi(\rho)$ in dense matter [5]. This latter prescription which appears to be rooted in the underlying chiral symmetry has been discussed extensively in the context of pionic atoms [6–9,12,13], but according to Refs. [12,13] it need not be applied once the full energy dependence of $\Pi(E)$ is incorporated.

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