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Higher tetraquark particles

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ABSTRACT

There are strong arguments favoring a four-quark interpretation of sub-GeV light scalar mesons and the diquark–antidiquark body-plan of the tetraquark seems to provide the most convincing picture. The building diquarks of these particles are assumed to be spin zero objects. In this Letter we explore the possibility that radially excited aggregations of spin zero or spin one diquarks might exist and discuss the possibility of the $Y(2175)$ state observed by BaBar and confirmed by BES being one such state.

1. Introduction

It has been shown recently how the diquark-antidiquark interpretation of the sub-GeV scalar meson nonet made of $f_0(980)$, $a_0(980)$, $\kappa(800)$, $\sigma(500)$ can lead to a remarkable description of the decay properties of these particles [1], adding a rather strong confidence that they are indeed tetraquark objects.

In terms of diquarks $\mathbb{q}$, the light scalar nonet can be interpreted as $\mathbb{q}\bar{\mathbb{q}}$ particles made of spin zero diquarks, so-called "good". A good diquark operator in the attractive anti-triplet color channel (Greek letters), antisymmetric in flavor (Latin letters) can be written as [2]:

$$\mathbb{q}_{i\alpha} = \epsilon_{ijk}\epsilon_{\alpha\beta\gamma}\bar{q}_c^{j\beta}\gamma_5 q^{k\gamma}. \tag{1}$$

Also "bad", spin one, diquarks can be conceived. A bad diquark operator can be written as:

$$\mathbb{q}_\alpha^{ik} = \epsilon_{\alpha\beta\gamma}\big(\bar{q}_c^{\beta j}\vec{\gamma}q^{k\gamma} + \bar{q}_c^{\beta k}\vec{\gamma}q^{j\gamma}\big). \tag{2}$$

Both represent positive parity, 0^+ and 1^+, states. Similarly one can construct 0^- and 1^- operators as: $\bar{q}_c q$ and $\bar{q}_c\vec{\gamma}\gamma_5 q$. The latter are identically zero in the 'single mode configuration', quarks that are unexcited with respect to one another. Lattice studies, see, e.g. [3], suggest that diquarks are preferably (energetically) formed in spin zero configurations. In fact the most solid tetraquark candidates are scalars made of good diquarks. Since we intend to study excited tetraquark states along the lines defined in the paper [1], we shall neglect the possibility of having two diquarks both in the repulsive $\mathbf{6}_c$ color channel although they might experience an overall effective attraction.

The next step in building this new spectroscopy is to find states belonging to other multiplets. Recently, the BaBar experiment has observed a new $J^{PC} = 1^{--}$ resonance, the $Y(2175)$, decaying into ϕf_0 [4], later confirmed by BES [5]. The fact that it has been first observed into a mode including a light scalar makes it a good candidate for belonging to a higher tetraquark multiplet.

In this Letter we make a simple ansatz that properly reproduces the light scalar nonet and utilize it to make predictions on possible excitations, focusing on $J^{PC} = 1^{--}$ states and searching for a match with the $Y(2175)$.

2. The model

This is developed in the framework of a non-relativistic Hamiltonian including spin–spin interactions inside a single diquark, spin–spin interactions between quarks and antiquarks belonging to the two diquarks forming the hadron, spin–orbit and a purely orbital term:

$$H = 2m_{\mathbb{q}} + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL}, \tag{3}$$

where:

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Table 1
Estimate, in MeV, of the parameters in the Hamiltonian in Eq. (4) depending on the diquark type $\mathbb{q} = [q_1 q_2]$.

$[q_1q_2]$	$m_{\mathbb{q}}$	k_q	$k_{q_1\bar{q}_2}$	$k_{q_1\bar{q}_1}$	$k_{q_2\bar{q}_2}$
$[qq]$	395	103	315	315	315
$[sq]$	590	64	195	121	315
$[ss]$	740	93	121	121	121

$$
\begin{aligned}
H_{SS}^{(qq)} &= 2\kappa_q(\vec{S}_{q_1}\cdot\vec{S}_{q_2} + \vec{S}_{\bar{q}_1}\cdot\vec{S}_{\bar{q}_2}),\\
H_{SS}^{(q\bar{q})} &= 2\kappa_{q_1\bar{q}_2}(\vec{S}_{q_1}\cdot\vec{S}_{\bar{q}_2} + \vec{S}_{\bar{q}_1}\cdot\vec{S}_{q_2}) + 2\kappa_{q_1\bar{q}_1}\vec{S}_{q_1}\cdot\vec{S}_{\bar{q}_1} + 2\kappa_{q_2\bar{q}_2}\vec{S}_{q_2}\cdot\vec{S}_{\bar{q}_2},\\
H_{SL} &= 2A_{\mathbb{q}}(\vec{S}_{\mathbb{q}}\cdot\vec{L} + \vec{S}_{\bar{\mathbb{q}}}\cdot\vec{L}),\\
H_{LL} &= B_{\mathbb{q}}\frac{L(L+1)}{2}.
\end{aligned} \tag{4}
$$

The parameters in these equations are fit to data: $m_{\mathbb{q}}$ is the mass of the $[q_1q_2]$ diquark, κ_q is the spin–spin coupling between the quarks inside the diquarks, $\kappa_{q_1q_2}$ are the spin–spin couplings ranging outside the diquark shells, $A_{\mathbb{q}}$ is the diquark spin–orbit coupling, and $B_{\mathbb{q}}$ weights the contribution of the total orbital angular momentum of the $\mathbb{q}\bar{\mathbb{q}}$ system to its mass; the overall factors of two are just conventional notations. We focus on the case where only light u, d, s quarks are involved. The spin–spin interaction Hamiltonian has the form:

$$
H_{SS} = \sum_{\text{pairs}} \frac{\kappa_{ij}}{m_i m_j}(\vec{S}_i\cdot\vec{S}_j)\delta^3(\vec{r}_{ij}), \tag{5}
$$

because the color-magnetic moments are inversely proportional to quark masses. In Eqs. (4) we incorporate the mass dependencies in the κ_{ij} constants. The Hamiltonian (5) describes contact interactions. For this reason we could expect that allowing a relative orbital angular momentum between the diquarks will decrease or switch-off the spin–spin interactions between quarks and antiquarks, namely $H_{SS}^{(q\bar{q})}$. In the following we shall consider both cases.

The values of the couplings appearing in Eqs. (4) were estimated in Ref. [6] from a fit to meson and baryon masses under the assumption that the spin–spin interactions are independent of whether the pair of quarks belong to a meson or a diquark. The estimates are summarized in Table 1.

Extending the same procedure to the $S = 1$, $L = 0, 1$ meson states $\rho(770)$, $a_1(1230)$, $a_2(1320)$, $b_1(1229)$ [7] we also infer the parameters related to the orbital angular momentum: $A_{\mathbb{q}} = 22.5$ MeV, $B_{\mathbb{q}} = 505$ MeV.

To describe a $\mathbb{q}\bar{\mathbb{q}}$ quantum state we adopt the following non-relativistic notation:

$$
|S_{\mathbb{q}}, S_{\bar{\mathbb{q}}}; S_{\mathbb{q}\bar{\mathbb{q}}}\rangle = |s^T\Gamma q, \bar{s}^T\Gamma\bar{q}; S_{\mathbb{q}\bar{\mathbb{q}}}\rangle, \tag{6}
$$

where $\Gamma \propto \sigma_2$ for a spin zero diquark and $\Gamma \propto \sigma_i$ for a spin one diquark. The action of a spin–spin interaction operator, e.g. $\vec{S}_{\bar{s}}\cdot\vec{S}_q$, on (6) is described as follows:

$$
(\vec{S}_{\bar{s}}\cdot\vec{S}_q)|s^T\Gamma q, \bar{s}^T\Gamma\bar{q}; S_{\mathbb{q}\bar{\mathbb{q}}}\rangle = \frac{1}{4}\sum_j |s^T\Gamma\sigma_j q, \bar{s}^T\sigma_j^T\Gamma\bar{q}; S_{\mathbb{q}\bar{\mathbb{q}}}\rangle. \tag{7}
$$

As an example let us diagonalize the Hamiltonian in Eq. (4) between scalars made of diquarks, i.e. $|0_{\mathbb{q}}, 0_{\bar{\mathbb{q}}}; 0\rangle$ with a relative $L_{\mathbb{q}\bar{\mathbb{q}}} = 0$. With an obvious shorthand notation:

$$
|0_{\mathbb{q}}, 0_{\bar{\mathbb{q}}}; 0\rangle = \frac{1}{2}|s^T\sigma_2 q, \bar{s}^T\sigma_2\bar{q}; 0\rangle := \frac{1}{2}\sigma_2\otimes\sigma_2. \tag{8}
$$

We can then compute:

$$
\langle 0_{\mathbb{q}}, 0_{\bar{\mathbb{q}}}; 0|\vec{S}_s\cdot\vec{S}_q|0_{\mathbb{q}}, 0_{\bar{\mathbb{q}}}; 0\rangle = -\frac{1}{4}\times 3, \tag{9}
$$

where we have used the fact that $\sigma_j^T\sigma_2 = -\sigma_2\sigma_j$ and $\sigma_j\sigma_j = 3\times\mathbb{1}$. The final result is:

$$
m = 2m_{\mathbb{q}} - 3\kappa_q. \tag{10}
$$

If $\mathbb{q} = [sq]$, then, using the values in Table 1, we get:

$$
m = 988 \text{ MeV}, \tag{11}
$$

reproducing the mass of m_{a_0} and m_{f_0}, considered as $[qs][\bar{q}\bar{s}]$ particles with the two diquarks in spin zero and in S-wave [1]. Repeating the same calculation with $\mathbb{q} = [ud]$ one gets, for the σ-meson mass:

$$
m = 481 \text{ MeV}. \tag{12}
$$

3. Higher mass tetraquark spectrum

The next orbital excitation comes when $L_{\mathbb{q}\bar{\mathbb{q}}} = 1$ and both good and bad diquarks are considered. Among these, also 1^{--} multiplets are generated, which are the main interest of this Letter. To estimate the masses, one needs to repeat the diagonalization with the basis:

Table 2
Eigenvalues of spin–orbit and angular momentum operators in (4). All these combinations of diquark spins and orbital angular momenta allow a $J = L + S_{\mathbb{q}\bar{\mathbb{q}}} = 1$ state.

	$a(S_{\mathbb{q}}, S_{\bar{\mathbb{q}}}, L)$	$b(S_{\mathbb{q}}, S_{\bar{\mathbb{q}}}, L)$
$S_{\mathbb{q}} = 0, S_{\bar{\mathbb{q}}} = 0, L = 1$	0	1
$S_{\mathbb{q}} = 1, S_{\bar{\mathbb{q}}} = 0, L = 1$	−2	1
$S_{\mathbb{q}} = 1, S_{\bar{\mathbb{q}}} = 1, S_{\mathbb{q}\bar{\mathbb{q}}} = 2, L = 1$	−6	1
$S_{\mathbb{q}} = 1, S_{\bar{\mathbb{q}}} = 1, S_{\mathbb{q}\bar{\mathbb{q}}} = 1, L = 1$	−2	1
$S_{\mathbb{q}} = 1, S_{\bar{\mathbb{q}}} = 1, S_{\mathbb{q}\bar{\mathbb{q}}} = 0, L = 1$	−2	1

Table 3
Mass values $m_Y^{(i)}$ in MeV for the 1^{--} states as computed from Eqs. (18) and (19). When applicable, the first value includes spin–spin interactions between diquarks, the second one neglects them: $H_{SS}^{(q\bar{q})} = 0$.

	$m_Y^{(1)}$	$m_Y^{(2)}$	$m_Y^{(3)}$	$m_Y^{(4)}$
$[qq']$	986	1432/1342	1293/1923	1203/1833
$[sq]$	1493	1749/1726	1591/2004	1501/1914
$[ss]$	–	–	2090/2333	2000/2243

$$|1\rangle = |0_{\mathbb{q}}, 0_{\bar{\mathbb{q}}}; 1_J\rangle, \tag{13}$$

$$|2\rangle = \frac{|1_{\mathbb{q}}, 0_{\bar{\mathbb{q}}}; 1_J\rangle + |0_{\mathbb{q}}, 1_{\bar{\mathbb{q}}}; 1_J\rangle}{\sqrt{2}}, \tag{14}$$

$$|3\rangle = |1_{\mathbb{q}}, 1_{\bar{\mathbb{q}}}; 1_J\rangle. \tag{15}$$

Since both the good and the bad diquarks have positive parity, the state $|2\rangle$ has $P = C = -1$, provided that $L_{\mathbb{q}\bar{\mathbb{q}}} = 1$. For the states $|1\rangle$ and $|3\rangle$, since $C_{\mathbb{q}\bar{\mathbb{q}}}(-1)^{L_{\mathbb{q}\bar{\mathbb{q}}}}(-1)^{S_{\mathbb{q}\bar{\mathbb{q}}}} = 1$, $C_{\mathbb{q}\bar{\mathbb{q}}} = -1$ provided that $S_{\mathbb{q}\bar{\mathbb{q}}} = 0, 2$ and $L_{\mathbb{q}\bar{\mathbb{q}}} = 1$.

To perform the diagonalization we adopt the shorthand notation described above:

$$\begin{aligned}
|0_{\mathbb{q}}, 0_{\bar{\mathbb{q}}}; 1_J\rangle &= \frac{1}{2}\sigma_2 \otimes \sigma_2, \\
|1_{\mathbb{q}}, 0_{\bar{\mathbb{q}}}; 1_J\rangle &= \frac{1}{2}\sigma_2\sigma_i \otimes \sigma_2, \\
|0_{\mathbb{q}}, 1_{\bar{\mathbb{q}}}; 1_J\rangle &= \frac{1}{2}\sigma_2 \otimes \sigma_2\sigma_i, \\
|1_{\mathbb{q}}, 1_{\bar{\mathbb{q}}}; 1_J\rangle &= \frac{1}{2\sqrt{2}}\epsilon^{ijk}\sigma_2\sigma_j \otimes \sigma_2\sigma_k.
\end{aligned} \tag{16}$$

Hence, it is rather straightforward to derive the mass term shift Δm_{SS} due to the part of the Hamiltonian in Eq. (4) constraining only spin–spin interaction terms, H_{SS}:

$$\Delta m_{SS} = \begin{bmatrix} -3\kappa_q & 0 & 0 \\ 0 & -\kappa_q - \kappa_{q_1\bar{q}_2} + (\kappa_{q_1\bar{q}_1} + \kappa_{q_2\bar{q}_2})/2 & 0 \\ 0 & 0 & \kappa_q - \kappa_{q_1\bar{q}_2} - (\kappa_{q_1\bar{q}_1} + \kappa_{q_2\bar{q}_2})/2 \end{bmatrix}. \tag{17}$$

Writing the latter matrix as $\mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)$, the four solutions for states having quantum numbers 1^{--} are:

$$\begin{aligned}
m_Y^{(1)}(S_{\mathbb{q}_1} = 0, S_{\mathbb{q}_2} = 0, S_{\mathbb{q}\bar{\mathbb{q}}} = 0, L_{\mathbb{q}\bar{\mathbb{q}}} = 1) &= 2m_{\mathbb{q}} + \lambda_1 + B_{\mathbb{q}}, \\
m_Y^{(2)}(S_{\mathbb{q}_1} = 1, S_{\mathbb{q}_2} = 0, S_{\mathbb{q}\bar{\mathbb{q}}} = 1, L_{\mathbb{q}\bar{\mathbb{q}}} = 1) &= 2m_{\mathbb{q}} + \delta + \lambda_2 - 2A_{\mathbb{q}} + B_{\mathbb{q}}, \\
m_Y^{(3)}(S_{\mathbb{q}_1} = 1, S_{\mathbb{q}_2} = 1, S_{\mathbb{q}\bar{\mathbb{q}}} = 0, L_{\mathbb{q}\bar{\mathbb{q}}} = 1) &= 2m_{\mathbb{q}} + 2\delta + \lambda_3 - 2A_{\mathbb{q}} + B_{\mathbb{q}}, \\
m_Y^{(4)}(S_{\mathbb{q}_1} = 1, S_{\mathbb{q}_2} = 1, S_{\mathbb{q}\bar{\mathbb{q}}} = 2, L_{\mathbb{q}\bar{\mathbb{q}}} = 1) &= 2m_{\mathbb{q}} + 2\delta + \lambda_3 - 6A_{\mathbb{q}} + B_{\mathbb{q}},
\end{aligned} \tag{18}$$

where $\delta = m_{\mathbb{q}(S=1)} - m_{\mathbb{q}(S=0)}$. Following Jaffe and Wilczeck [8], we will assume for $\mathbb{q} = [qq]$, $\delta \simeq 285$ MeV whereas for $\mathbb{q} = [sq]$, $\delta \simeq 150$ MeV. The numerical values for the coefficients of $A_{\mathbb{q}}$ and $B_{\mathbb{q}}$, call them a, b, are given in Table 2.

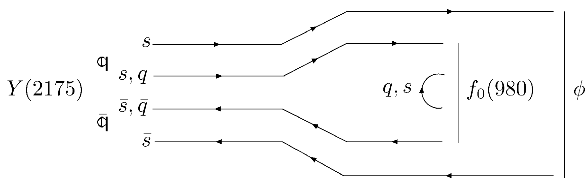
In case $\mathbb{q} = [ss]$, only the last state in Eq. (16) is allowed since only bad diquarks can be formed by Fermi–Dirac. One should therefore consider only the $\langle 1_{\mathbb{q}}, 1_{\bar{\mathbb{q}}}; 1_J | H_{SS} | 1_{\mathbb{q}}, 1_{\bar{\mathbb{q}}}; 1_J\rangle$ correction to the mass, from Eq. (17) is equal to $\kappa_s - 2\kappa_{s\bar{s}}$. We therefore have:

$$\begin{aligned}
m_Y^{(3)}(S_{\mathbb{q}_1} = 1, S_{\mathbb{q}_2} = 1, S_{\mathbb{q}\bar{\mathbb{q}}} = 0, L_{\mathbb{q}\bar{\mathbb{q}}} = 1) &= 2m_{\mathbb{q}} + 2\delta + (\kappa_s - 2\kappa_{s\bar{s}}) - 2A_{\mathbb{q}} + B_{\mathbb{q}}, \\
m_Y^{(4)}(S_{\mathbb{q}_1} = 1, S_{\mathbb{q}_2} = 1, S_{\mathbb{q}\bar{\mathbb{q}}} = 2, L_{\mathbb{q}\bar{\mathbb{q}}} = 1) &= 2m_{\mathbb{q}} + 2\delta + (\kappa_s - 2\kappa_{s\bar{s}}) - 6A_{\mathbb{q}} + B_{\mathbb{q}}.
\end{aligned} \tag{19}$$

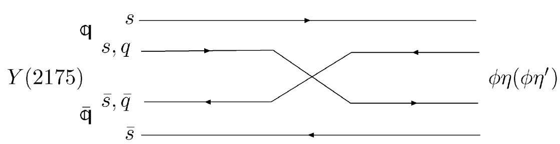
The numerical values for $m_Y^{(i)}$ masses can be found in Table 3. The fact that 1^{--} tetraquark particles require an angular momentum barrier $L_{\mathbb{q}\bar{\mathbb{q}}} \neq 0$ between diquarks must suppress the diquark-antidiquark chromomagnetic interactions. Switching off spin–spin interactions between quarks and antiquarks leads to the second estimates in Table 3.

4. Tetraquark decay modes

From these results, if we want to identify the $Y(2175)$ as a tetraquark state, we have to resort to either a $\mathbb{q} = [qs]$ hypothesis, with spin–spin interactions between diquarks set to zero or to a $\mathbb{q} = [ss]$ hypothesis: indeed we use the hypothesis that $f_0(980)$ in the decay products of $Y(2175)$ is itself a $\mathbb{q}\bar{\mathbb{q}}$ particle with $\mathbb{q} = [qs]$. In order to test the match of the $Y(2175)$ with these assignments, we study its possible decays under both hypotheses, $\mathbb{q} = [qs], [ss]$, and the correspondence with observations. With both assignments, the observed $Y(2175) \to \phi f_0(980)$ decay mechanism would be described by the following diagram:



As for other expected decays, a significant contribution is expected to come from $\phi\eta$ via the diagrams below:



Similar diagrams would also yield $\phi\phi$ and $\eta\eta$ decays, but they are forbidden by charge conjugation and Bose statistics selection rules.

We can also estimate the decay width of the $Y(2175) \to \phi\eta$ channel. The decay proceeds through P-wave and the matrix element is given by:

$$\langle \phi(p', \epsilon^{(\phi)})\eta(q) | Y(p, \epsilon^{(Y)}) \rangle = g_V \epsilon^{\mu\nu\rho\sigma} p_\mu q_\nu \epsilon^{(\phi)}_\rho \epsilon^{(Y)}_\sigma. \tag{20}$$

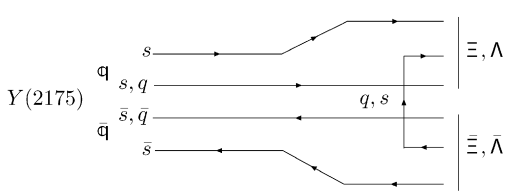
The quark exchange amplitude considered above has been studied first in [9] were a rather good fit of the scalar meson decays to pseudoscalar, $S \to PP$, was obtained associating to this amplitude a coupling strength $A \simeq 2.6$ GeV. Discarding angular momentum barrier effects and following the definition given in [6]:

$$g_V M_V = \frac{A}{\sqrt{2}}, \tag{21}$$

where here $M_V = M_Y$, we get the following estimate for the Y partial width in $\phi\eta$:

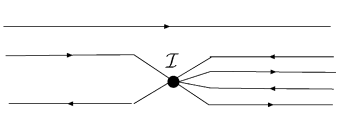
$$\Gamma(Y(2175) \to \phi\eta) = \xi \frac{A^2}{2} \frac{1}{8\pi M_Y^2} \frac{\sqrt{\lambda(M_Y^2, M_\phi^2, M_\eta^2)}}{2M_Y} \frac{(M_Y + M_\eta)^2 - M_\phi^2}{M_Y^2}, \tag{22}$$

where $\xi = \frac{1}{6}$ or $\frac{2}{3}$ depending on the $\mathbb{q} = [qs]$ or $\mathbb{q} = [ss]$ respectively. We therefore estimate $\Gamma(Y(2175) \to \phi\eta) \sim 5$ MeV or ~ 20 MeV under the two hypotheses respectively. The most typical decay mode expected for tetraquarks is the baryon–antibaryon one. Stretching the color string between the diquark and the antidiquark a $q\bar{q}$ pair is formed, $\mathbb{q} \sim q\bar{q} \sim \bar{\mathbb{q}}$, letting two baryons in the decay products. The favored decay diagram is



where the topology is suggested by the fact that the diquark in the Λ baryon could only be of the $[ud]$ type.

Since the $\Xi\bar{\Xi}$ decay mode is phase-space forbidden for the $Y(2175)$, only the $\mathbb{q} = [sq]$ assignation would allow a dominant baryonic decay, $Y(2175) \to \Lambda\bar{\Lambda}$. With the other assignment, $\mathbb{q} = [ss]$, the $Y(2175) \to \Lambda\bar{\Lambda}$ would be made possible by the annihilation of an $s\bar{s}$ pair by, e.g., an instanton interaction giving two pairs of light quarks in the final state: in diagrammatic terms



The 6-fermion instanton interaction has the form $\mathcal{L}_I \propto \det(\bar{q}^i_L q^j_R)$ and its role in scalar meson dynamics has been recently underscored in [1].

Nonetheless, it is known from [1] that the instanton coupling, fitted to explain light scalar meson decays like $f_0(980) \to \pi\pi$, is about ten times smaller that the quark exchange one. Therefore, under the hypothesis that the baryonic mode would be instanton driven in the $\mathbb{q} = [ss]$ case, we would not expect it to be easily visible.

The latter diagram allows also a number of possible decays of the $Y(2175)$ like $p\bar{p}$, $\sigma\pi$, $\pi\pi\pi$, and $\eta\pi^0\pi^0$ in the $\mathbb{q} = [sq]$ hypothesis and $K^-\kappa^+$, $K^-\pi^0K^+$, and $\phi\pi^0\pi^0$ in the $\mathbb{q} = [ss]$ one.

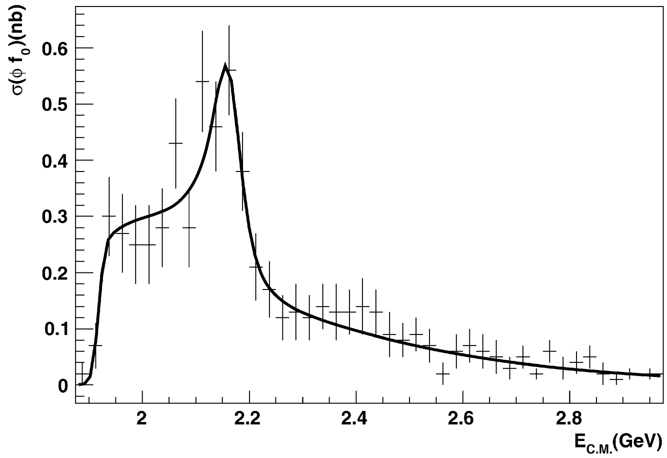


Fig. 1. Fit to the ϕf_0 invariant mass distribution.

Table 4
Fit results to the ϕf_0 invariant mass distribution (Fit A), the ϕf_0 and $\Lambda\Lambda$ invariant mass distributions (Fit B), and the invariant mass distributions of all three modes (Fit C). The meaning of the symbols is explained in the text.

Fit	χ^2/DOF	m_0 (MeV)	Γ_0 (MeV)	$R(\Lambda\bar{\Lambda}, \phi f_0)$	$R(\phi\eta, \phi f_0)$
A	25/36	2167 ± 11	69 ± 21	N/A	N/A
B	47/45	2158 ± 11	66 ± 20	44 ± 19	N/A
C	85/103	2153 ± 9	72 ± 20	6.6 ± 3.5	10 ± 3

5. Observed Y(2175) decays

In order to test the compatibility of the $Y(2175)$ state with a tetraquark interpretation and to discriminate between the two possible diquark compositions, we have reanalyzed the published BaBar data for $e^+e^- \to \phi f_0 \gamma$ [4], $\Lambda\bar{\Lambda}\gamma$ [10], and $\phi\eta\gamma$ [11]. These are initial state radiation processes, where $J^{CP} = 1^{--}$ states are produced together with the initial state photon. The invariant mass of the system produced with the photon is then expected to show a resonant behavior in correspondence to states.

We perform simultaneous fits applying a consistent notation for the Breit–Wigner and several possible models for the non-resonant component $A_{\rm nr}$. The general notation for the expected cross section as a function of the invariant mass of the system under study is

$$\sigma(m) \propto \Phi^f_{\rm PS}(m)\left|A(m)\right|^2,$$
$$A(m) = e^{i\delta} A_{\rm nr}(m) + \sqrt{\sigma_0 \mathcal{B}(Y \to f)}\frac{m_0 \Gamma_{\rm tot}(m_0)}{m^2 - m_0^2 + i\Gamma_{\rm tot}(m) m_0}, \quad (23)$$

where δ is the relative phase between the two components at the pole; $\Phi^f_{\rm PS}(m) = (p(m)/p(m_0))^{\alpha_f}$ is the final state dependent phase space factor: $\alpha_{\phi\eta} = 3$, $\alpha_{\phi f_0} = \alpha_{\Lambda\bar{\Lambda}} = 1$; $p(m)$ is the momentum of the two particles in the final state when their c.o.m. energy is m; m_0 and σ_0 are the pole mass and production cross section and are independent of the considered final state; $\mathcal{B}(Y \to f)$ is the branching fraction to the specific final state; $\Gamma_{\rm tot}(m)$ is the comoving width, the sum over the considered final states plus a constant term to account for all other decays with thresholds far from the pole mass:

$$\Gamma_{\rm tot}(m) = \Gamma_0\left(1 - \sum_f \mathcal{B}(Y \to f)\left(1 - \xi^f(m)\right)\right), \quad (24)$$
$$\xi^f(m) = \Phi^f_{\rm PS}(m)/F_{\rm BW}(m),$$

where $F_{\rm BW}$ is the Blatt–Weisskopf factor [12] and Γ_0 is the bare width. Note that for masses below the threshold of a given final state the corresponding ξ is imaginary.

Of the three considered modes, in two cases the $Y(2175)$ would decay to states where it is above threshold while for the other ($\Lambda\bar{\Lambda}$) m_0 is below threshold. A fit to the discovery mode, ϕf_0, with $A_{\rm nr} = A \times e^{-k(E-m_\phi-m_{f_0})} \times (1 - e^{-(E-m_\phi-m_{f_0})^4/a_1})$ (see Fig. 1) returns the results listed as “Fit A” in Table 4.

Including the $\Lambda\bar{\Lambda}$ mode in a simultaneous fit, with common bare mass and width and letting the ratio between the branching fractions $R(\Lambda\bar{\Lambda}, \phi f_0) = \mathcal{B}_{\Lambda\bar{\Lambda}}/\mathcal{B}_{\phi f_0}$ float, yields different results depending on the assumptions $A_{\rm nr}$ in the $\Lambda\bar{\Lambda}$ mode (see Fig. 2). In case we assume no non-resonant contribution, $A_{\rm nr} = 0$, we find that the $Y(2175) \to \Lambda\bar{\Lambda}$ decay can explain the whole observed spectrum, with mass and width parameters consistent with the fit to the ϕf_0 mode. The results are reported as “Fit B” in Table 4.

There is an indication that the $\Lambda\bar{\Lambda}$ decay is favored, even by one order of magnitude. If instead we assume $A_{\rm nr}(m) = A \times e^{-km}$, letting k float, we observe no significant decay into $\Lambda\bar{\Lambda}$, but with huge uncertainties: $R(\Lambda\bar{\Lambda}, \phi f_0) = 23 \pm 51$.

Finally, the $\phi\eta$ mass distribution shows a significantly higher background than the other modes and small structure in a position which is lower and narrower than the one observed in the ϕf_0 channel and marginally consistent with it. Fitting the three distributions simultaneously as shown in Table 4, “Fit C” gives an overall good fit (see Fig. 3).

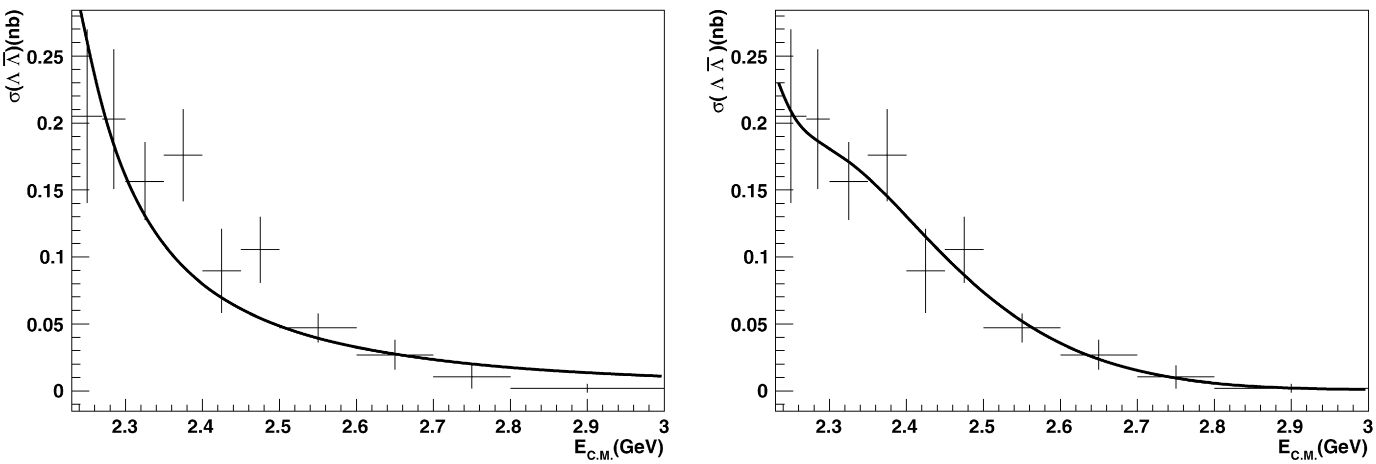


Fig. 2. Fit to the $\Lambda\bar{\Lambda}$ invariant mass distribution under the assumption of no non-resonant contribution (left), or assuming an exponential amplitude for it (right).

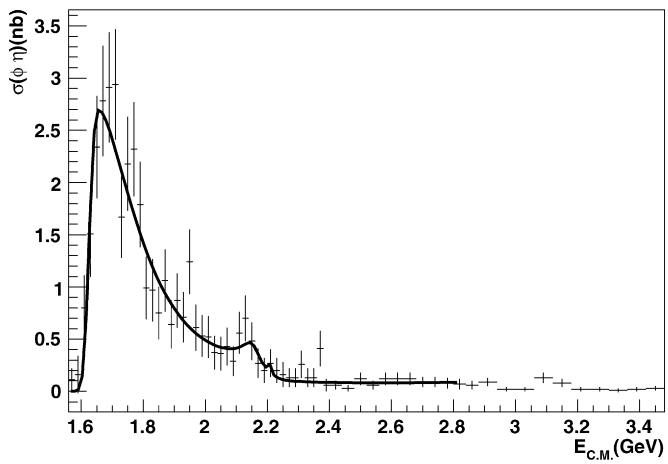


Fig. 3. ϕf_0, $\Lambda\bar{\Lambda}$, and $\phi\eta$ invariant mass distributions with the simultaneous fit overlaid.

We can then conclude that the presence of the $Y(2175) \to \Lambda\bar{\Lambda}$ is suggested by the fact that its existence would explain the whole $\Lambda\bar{\Lambda}$ mass spectrum. Under this hypothesis the baryonic decay mode would be dominant ($\mathcal{B}_{\Lambda\bar{\Lambda}}/\mathcal{B}_{\phi f_0} = 44 \pm 19$), thus favouring $\mathbb{q} = [qs]$ for the $Y(2175)$. As for the $\phi\eta$ mass distribution, uncertainties are large, but the case of a relatively large decay amplitude into it is not disfavored.

Both the calculation of the mass spectrum and the reanalysis of the experimental data tend to favor the assignment of the $Y(2175)$ to a $[sq][\bar{s}\bar{q}]$ state with both diquarks in the $S = 1$ state and with one unit of relative orbital angular momentum.

6. Conclusions

In this Letter we have studied the consequences of allowing spin one diquarks to build 1^{--} $(\mathbb{q}\bar{\mathbb{q}})$ orbitally excited tetraquark states potentially visible in processes with initial state radiation at BaBar and Belle. In particular we have focused on the $Y(2175)$ resonance recently discovered by BaBar. This particle could be the first tetraquark state showing the expected baryon–antibaryon decay. Indeed, reanalyzing BaBar data, we find that if we set to zero the non-resonant contributions, the $\Lambda\bar{\Lambda}$ decay mode is the prominent one, indicating a $\mathbb{q} = [qs]$ assignation for the $Y(2175)$. Under this hypothesis we would also expect to have a dominant $K\kappa$ decay mode and visible $p\bar{p}$ and $\sigma\pi$ decays, suppressed by about two orders of magnitude.

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