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Finite Element Based Vibration Analysis of a Nonprismatic Timoshenko Beam with Transverse Open Crack

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Abstract

The present day structures and machineries are designed based on optimizing of multi-objectives such as maximum strength, maximum life, minimum weight and minimum cost. Due to this they are flexible and allow having a very high level of stresses. This leads to development of cracks in their elements. Many engineering structures may have structural defects such as cracks due to long-term service. So it is very much essential to know the property of structures and its response in various cases. The present article deals with finite element based vibration analysis of a nonprismatic cracked beam. The beam is modeled using the Timoshenko beam theory. The governing equation of motion is derived by the Hamilton's principle. In order to solve the governing equation two noded beam element with two degrees of freedom (DOF) per node is considered. In this work the effect of structural damping is also incorporated in the finite element model. The dynamic analysis is carried out by using state space model in time domain.

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Keywords: Non-prismatic; finite element method; Euler-Bernoulli beam; Piezoelectricity.

1. Introduction

Presence of crack in a structural member is a serious threat to the performance of the structure. The effects of crack on the dynamic behaviour of the structural elements have been the subject of several investigations for the last few decades. Due to the existence of such cracks the frequencies of natural vibration, amplitudes of forced vibration,

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and areas of dynamic stability change. The introduction of local flexibility due to presence of transverse crack in a structural member whose dimension depends on the number of degrees of freedom considered [1]. It was observed that the local flexibility matrix plays a major role for appropriate analysis of a cracked beam if analytical method used for solving the differential equations piece wisely [2]. The detection of structural damage through changes in frequencies was discussed [3, 4, 5]

A method was used to find the lowest four natural frequencies of the cracked structure by finite element method [6, 7] based on combination of wave-let based elements and genetic algorithm. The experimental investigations of the effects of cracks and damages on the structures were reported [8]. The reduction of eigen frequencies and sensitivity analysis to localize a crack in a non-rotating shaft coupled to an elastic foundation was studied [9]. The different damage scenarios by reducing the local thickness of the selected elements at different locations along with finite element model (FEM) for quantification and localization of damage in beam-like structures were investigated [10,11].The static and dynamic analysis of a cracked prismatic beam on the basis of Hamilton's principle was determined using finite element analysis [12]. The component mode synthesis techniques along with finite element method for free vibration analysis of uniform and stepped cracked beam with circular cross section was studied [13]. The finite element analysis of a cracked cantilever beam and the relation between the modal natural frequencies with crack depth, modal natural frequency with crack location was discussed [14]. An overall flexibility matrix instead of local flexibility matrix find out the total flexibility and stiffness matrices of the cracked beam was considered [15]. It was observed that the consideration of 'overall additional flexibility matrix', due to presence of crack, can indeed give more accurate results than those obtained by using the local flexibility matrix [16]. The present article exclusively focused on vibration analysis of a nonprismatic cracked beam using Timoshenko beam formulations

2. Mathematical formulation

This mathematical formulation deals with finite element modeling of uncracked and cracked nonprismatic beam which are discussed in the following sections.

2.1. *Finite element modeling of uncracked Timoshenko beam*

For modeling the cross section of the beam, the shape function profile can be represented as

$$A_b(x) = A_0\left(1 - c\frac{x}{L_b}\right) \tag{1}$$

Where $A_b(x)$ represents the area of cross section at any position x of the beam. A_0 is the cross section area near the clamped end of the beam. L_b is the length of the beam. c is the taper ratio whose values vary from 0 to 1. Fig.1 shows a cantilever beam of circular cross-section having diameter D with a single transverse crack with constant depth a. The crack is at a distance of Xc from the clamped end of the beam.

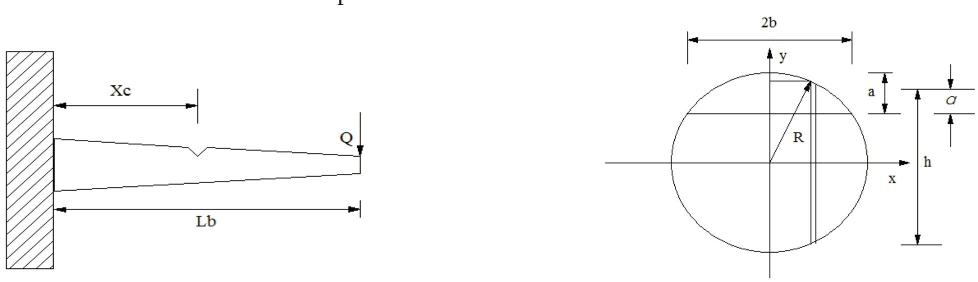

Fig. 1. A cantilever beam with crack subjected to shear force and bending moment

The dynamic equations of motion of system when subjected to point load Q at the free end are derived using Hamilton'sprinciple as

$$\partial\psi = \int_{t_1}^{t_2} \left(KE - PE + W_P\right) dt = 0 \tag{2}$$

Where KE is the total kinetic energy, PE is the total potential energy and W_p is the total work done by the external mechanical force.

$$\partial\psi = \int_{t_1}^{t_2} \left[\int_{v_b} \rho_b \delta\dot{q}^T \dot{q} dv_b - \int_{v_b} \delta S^T c^s S dv_b + \sum_{i=1}^{n_f} \delta q(x_i) Q(x_i) \right] \tag{3}$$

Where v is the volume, q is the displacement, x is the position along the beam, ρ is the density and the subscripts b represents the beam material. This equation can now be used to solve for the equations of motion of dynamical mechanical system. By using finite element formulations the displacement field in terms of shape functions can be represented as

$$q(x,t) = [N_W]\{w\} \tag{4}$$

Where $[N_w]$ and $\{w\}$ are the shape function and nodal displacements respectively. Using Eqs. (4), one can simplify the variational indicator to include terms that represent physical parameters. The mass and stiffness matrices matrix for the system can be written as

$$[M_b] = \int_0^{L_b} [N_W]^T \rho_b A_b(x) [N_W] dx \tag{5}$$

$$[K_b] = \int_0^{L_b} \left[\frac{\partial[N_W]}{\partial x}\right]^T E_b I_b(x) \left[\frac{\partial[N_W]}{\partial x}\right] dx \tag{6}$$

Where ρ_b, E_b and $I_b(x)$ represents the density, Young's modulus and bending moment of inertia of the beam. Putting equations (4), (5), (6) into (3) results

$$\partial\psi = \int_{t_1}^{t_2} \left[\left\{\delta\dot{w}^T(t)[M_b]\dot{w}(t)\right\} + \left\{\delta w^T(t)[K_b]w(t)\right\} + \left\{\sum_{i=1}^{n_f} \delta w(t)[N_w]^T Q_i(t)\right\} \right] = 0 \tag{7}$$

Taking the integral of the above, the dynamic equation of motion becomes

$$[M_b]\ddot{w}(t)+[K_b]w(t)=\sum_{i=1}^{n_f}[N_W]^T Q_i(t) \tag{8}$$

Equation (8) now represents the mechanical system and can be used to determine the motion of the beam. By incorporating proportional damping [17] the governing equation of motion become

$$[M_b]\ddot{w}(t)+[C]\dot{w}(t)+[K_b]w(t)=\sum_{i=1}^{n_f}[N_W]^T Q_i(t) \tag{9}$$

2.2. Finite element modeling of cracked Timoshenko beam element

The additional strain energy due to the existence of the crack as shown in Fig. 1 can be expressed as [18] [19]

$$\Pi_c=\int_{Ac} G dA \tag{10}$$

Where G is the strain energy release rate function. The strain energy release rate function can be expressed as

$$G=\frac{1}{E'}\left[(K_{I2}+K_{I3})^2+K_{II2}{}^2\right] \tag{11}$$

Where $E'=E$ for plane stress problem, $E'=E/1-\mu^2$ for plane strain problem. K_{I2}, K_{I3}, K_{II2} are the stress intensity factors. The values of stress intensity factor can be found by using the following equation.

$$K_{ni}=\sigma_i\sqrt{\pi\alpha}F_n\left(\alpha/h\right) \tag{12}$$

The deflection due to the application of load can be obtained by using Paris equation as

$$\delta_i=\frac{\partial \Pi_c}{\partial P_i}\quad(i=2,3) \tag{13}$$

Therefore the overall additional flexibility matrix C_{ij} can be obtained as

$$C_{ij}=\frac{\partial \delta_i}{\partial P_j}=\frac{\partial^2 \Pi_c}{\partial P_i \partial P_j}\,(i=2,3) \tag{14}$$

By using equations (10), (11),(12) and (13) into equation (14) one can obtain

$$C_{ij} = \frac{1}{E'} \frac{\partial^2}{\partial P_i P_j} \int_{\sqrt{Da-a^2}}^{\sqrt{Da-a^2}} \int_0^{\frac{1}{2}\left[\sqrt{(Da-4x^2)}-(D-2a)\right]} \left[\left\{ \frac{32 P_2 L_c h}{\pi D^4} \sqrt{\pi\alpha} F_2\left(\alpha/h\right) + \frac{32 P_3 h}{\pi D^4} \sqrt{\pi\alpha} F_2\left(\alpha/h\right) \right\}^2 + \frac{16 P_2^{\,2}}{\pi^2 D^4} \pi\alpha F_{II}^{\,2}\left(\alpha/h\right) \right] d\alpha dx \tag{15}$$

2.3. Stiffness matrix of a cracked Timoshenko beam element

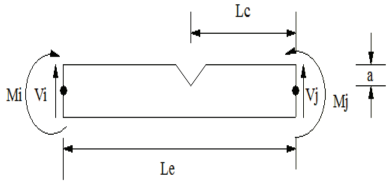

Fig. 2. Cracked Timoshenko beam element

Figure 2 shows a cracked beam element with generalised loading. Under the FEM co-ordinate and notation system the total flexibility matrix of the cracked Timoshenko beam element is obtained by the combination of over-all additional flexibility matrix and flexibility matrix of an intact beam as

$$C_{total} = C_{ovl} + C_{\mathrm{int}act} \tag{16}$$

Through the equilibrium conditions, the stiffness matrix K_c of a cracked beam element can be obtained as follows [20] [21]

$$K_c = L C_{total}^{-1} L^T \tag{17}$$

Where L is the transformation matrix

3. Result and discussion

For the analysis a prismatic cantilever beam of circular cross section is considered. The length and diameter of the beam are 1000mm and 20 mm respectively. The physical properties of the beam are E = 206 GPA, ρ = 7800 kg/m3 and μ = 0.3. The length of beam is divided into finite numbers of small elements. The fundamental frequencies are calculated by using the present MATLAB code developed and compared with the exact solution obtained [22] in Table 1. Tables (2-5) represent the first three natural frequencies of a cracked Timoshenko beam at various crack positions (X_c/L = 0.075, 0.275, 0.475 and 0.675) and relative crack depths (α/D = 0.2, 0.3, 0.4, and 0.5). The taper ratio(c) of the beam is taken as 0.5.

Table 1 Comparison of natural frequencies of uncracked cantilever beam

Natural frequency(rad/sec)	Exact [22]	Present code
ω_1	87.19	88.80
ω_2	556.04	556.92
ω_3	1584.69	1560.46

Table 2 Natural frequencies of cracked Timoshenko beam, X_c/L =0.075

ω (rad/sec)	Xc/L	α/D=0.2	α/D =0.3	α/D =0.4	α/D =0.5
ω_1	0.075	86.77	86.47	86.34	86.28
ω_2	0.075	544.45	542.55	541.70	541.29
ω_3	0.075	1530.05	1524.64	1522.22	1521.07

Table 3 Natural frequencies of cracked Timoshenko beam, X_c/L =0.275.

ω (rad/sec)	Xc/L	α/D=0.2	α/D =0.3	α/D =0.4	α/D =0.5
ω_1	0.275	86.83	86.52	86.37	86.31
ω_2	0.275	551.71	547.07	544.98	543.97
ω_3	0.275	1552.34	1538.58	1532.34	1529.33

Table 4 Natural frequencies of cracked Timoshenko beam, X_c/L =0.475.

ω (rad/sec)	Xc/L	α/D=0.2	α/D =0.3	α/D =0.4	α/D =0.5
ω_1	0.475	86.34	86.21	86.16	86.13
ω_2	0.475	554.31	548.85	546.31	545.08
ω_3	0.475	1555.35	1540.66	1533.90	1530.63

Table 5 Natural frequencies of cracked Timoshenko beam, X_c/L =0.675.

ω (rad/sec)	Xc/L	α/D=0.2	α/D =0.3	α/D =0.4	α/D =0.5
ω_1	0.675	86.06	86.04	86.03	86.02
ω_2	0.675	545.13	543.15	542.20	541.72
ω_3	0.675	1556.73	1542.09	1535.13	1531.71

It is observed from the tables (table 2-5) that natural frequencies of a nonprismatic cracked beam decreases as crack depth increases at a particular position. By using the state space representation the dynamic analysis of the cracked beam has been conducted.1 N load is applied at the free end of cantilever beam. The frequency responses of uncracked and cracked beam (α/D=0.2, 0.3, 0.4, 0.5) at different crack positions (X_c/L=0.075, 0.275, 0.475, 0.675) are shown in fig 3. The displacement responses of cracked beam (α/D=0.2, 0.3, 0.4, 0.5) at different crack positions (X_c/L=0.075, 0.275, 0.475, 0.675) in time domain are shown in fig 4. From the figure it is obtained that the frequency response of cracked beam dies out for all positions of crack. This is due to the presence of structural damping.

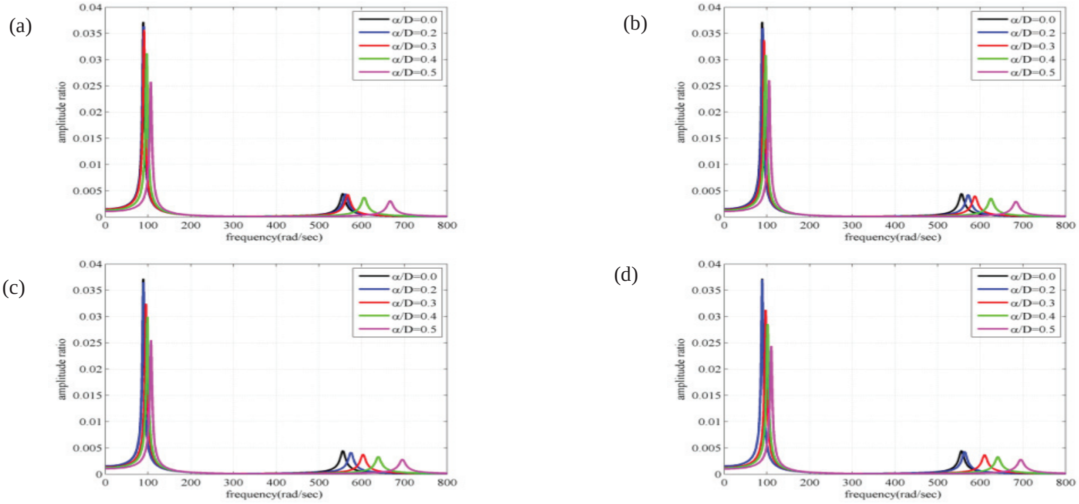

Fig 3 Frequency response of nonprismatic(c=0.5) uncracked (α/D =0.0) and cracked beam (α/D =0.2, 0.3, 0.4, 0.5) for (a) X_c/L =0.075 (b) X_c/L =0.275(c) X_c/L =0.475(d) X_c/L =0.675.

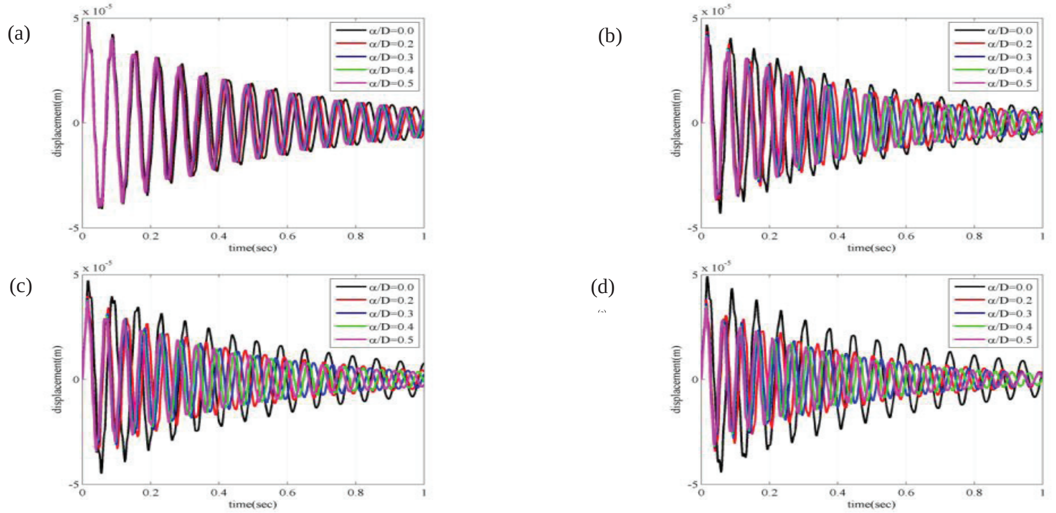

Fig 4 Displacement responses of nonprismatic(c=0.5) uncracked (α/D =0.0) and cracked beam (α/D =0.2, 0.3, 0.4, 0.5) for (a) X_c/L =0.075 (b) X_c/L =0.275(c) X_c/L =0.475(d) X_c/L =0.675 in time domain.

4. Conclusion

The present article focused on the vibration analysis of a nonprismatic cracked beam using finite element formulation. The beam is modeled using Timoshenko beam formulation by considering rotary inertia and shear deformations. Two noded beam elements with two degrees of freedom at each node is considered in order to solve the governing equation. From the analysis it is observed that due to presence of crack in a beam the natural frequencies decrease as the relative crack depth increase. Again from dynamic analysis it is observed that the amplitude of vibration of cracked beam decrease by varying the relative crack depths as compared to uncracked beam for any value of taper ratios.

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