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Nuclear and Particle Physics Proceedings 273–275 (2016) 268–274

www.elsevier.com/locate/nppp

Flavour Covariant Formalism for Resonant Leptogenesis

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Abstract

We present a fully flavour-covariant formalism for transport phenomena and apply it to study the flavour-dynamics of Resonant Leptogenesis (RL). We show that this formalism provides a complete and unified description of RL, consistently accounting for three *distinct* physical phenomena: (i) resonant mixing and (ii) coherent oscillations between different heavy-neutrino flavours, as well as (iii) quantum decoherence effects in the charged-lepton sector. We describe the necessary emergence of higher-rank tensors in flavour space, arising from the unitarity cuts of partial self-energies. Finally, we illustrate the importance of this formalism within a minimal Resonant τ-Genesis model by showing that, with the inclusion of all flavour effects in a consistent way, the final lepton asymmetry can be enhanced by up to an order of magnitude, when compared to previous partially flavour-dependent treatments.

Keywords: Flavour Covariance, Discrete Symmetries, Transport Equations, Resonant Leptogenesis

1. Introduction

Leptogenesis [1] is an elegant framework for dynamically generating the observed matter-antimatter asymmetry in our Universe through out-of-equilibrium decays of heavy Majorana neutrinos, whilst simultaneously explaining the smallness of the light neutrino masses by the seesaw mechanism [2]. Resonant Leptogenesis (RL) [3, 4] offers the possibility of realizing this beautiful idea at energy scales accessible to laboratory experiments. In RL, the heavy Majorana neutrino self-energy effects on the leptonic *CP*-asymmetry become dominant [5] and get resonantly enhanced, when at least two of the heavy neutrinos have a small mass difference comparable to their decay widths [3].

Flavour effects in both heavy-neutrino and charged-lepton sectors, as well as the interplay between them, play an important role in determining the final lepton asymmetry in low-scale leptogenesis models [6, 7]. These intrinsically quantum effects can be consistently accounted for by extending the classical flavour-diagonal Boltzmann equations for the number densities of individual flavour species to a semi-classical evolution equation for a *matrix of number densities* [8]. Using this general technique, we present in Section 2 a *fully* flavour-covariant formalism for transport phenomena in the Markovian regime. As an application of this general formalism, we derive a set of flavour-covariant transport equations for lepton and heavy-neutrino number densities with arbitrary flavour content in a quantum-statistical ensemble. We demonstrate the necessary appearance of rank-4 tensor rates in flavour space that properly account for the statistical evolution of off-diagonal flavour coherences. As shown in Section 3, this manifestly flavour-covariant formalism enables us to capture three important flavour effects pertinent to RL: (i) the resonant mixing of heavy neutrinos, (ii) the coherent oscillations between heavy neutrino flavours and (iii) quantum (de)coherence effects in the charged-lepton sector. In Section 4, we present a numerical example to illustrate the importance of these flavour off-diagonal effects on the final lepton asymmetry. Our con-

[1] Speaker.

http://dx.doi.org/10.1016/j.nuclphysbps.2015.09.037
2405-6014/

clusions are given in Section 5. For a detailed discussion of the topics presented here, we refer the reader to [9].

2. Flavour-Covariant Formalism

Let us begin with an arbitrary flavour content for the lepton doublet field operators L_l (with $l = 1, 2, \ldots, \mathcal{N}_L$) and the right-handed Majorana neutrino field operators $N_{\mathrm{R},\alpha} \equiv \mathrm{P_R} N_\alpha$ (with $\alpha = 1, 2, \ldots, \mathcal{N}_N$), where $\mathrm{P_R} = (\mathbf{1}_4 + \gamma_5)/2$ is the right-chiral projection operator. The field operators transform as follows in the fundamental representations of $U(\mathcal{N}_L)$ and $U(\mathcal{N}_N)$:

$$L_l \to L'_l = V_l{}^m L_m\ , \quad L^l \equiv (L_l)^\dagger \to L'^l = V^l{}_m L^m\ , \quad \text{(1a)}$$

$$N_{\mathrm{R},\alpha} \to N'_{\mathrm{R},\alpha} = U_\alpha{}^\beta N_{\mathrm{R},\beta}, \quad N_{\mathrm{R}}^\alpha \to N'^\alpha_{\mathrm{R}} = U^\alpha{}_\beta N^\beta_{\mathrm{R}}, \quad \text{(1b)}$$

where $V_l{}^m \in U(\mathcal{N}_L)$ and $U_\alpha{}^\beta \in U(\mathcal{N}_N)$. In the flavour basis, the relevant neutrino Lagrangian is given by

$$-\mathcal{L}_N = h_l{}^\alpha \overline{L}^l \widetilde{\Phi} N_{\mathrm{R},\alpha} + \frac{1}{2}\overline{N}^C_{\mathrm{R},\alpha}[M_N]^{\alpha\beta} N_{\mathrm{R},\beta} + \mathrm{H.c.}\ , \quad (2)$$

where $\widetilde{\Phi} = i\sigma_2\Phi^*$ is the isospin conjugate of the Higgs doublet Φ. The Lagrangian (2) transforms covariantly under $U(\mathcal{N}_L) \otimes U(\mathcal{N}_N)$, provided the heavy-neutrino Yukawa and mass matrices transform as

$$h_l{}^\alpha \ \to\ h'_l{}^\alpha \ =\ V_l{}^m\, U^\alpha{}_\beta\, h_m{}^\beta\ , \quad \text{(3a)}$$

$$[M_N]^{\alpha\beta} \ \to\ [M'_N]^{\alpha\beta} \ =\ U^\alpha{}_\gamma\, U^\beta{}_\delta\, [M_N]^{\gamma\delta}\ . \quad \text{(3b)}$$

The field operators in (2) can be expanded in flavour-covariant plane-wave decompositions, e.g.

$$\begin{aligned} L_l(x) \ =\ & \sum_{s=+,-} \int_{\mathbf{p}} \Big[(2E_L(\mathbf{p}))^{-\frac{1}{2}}\Big]_l{}^i \\ & \times \Big([e^{-ip\cdot x}]_i{}^j [u(\mathbf{p},s)]_j{}^k\, b_k(\mathbf{p},s,0) \\ & +\ [e^{ip\cdot x}]_i{}^j\, [v(\mathbf{p},s)]_j{}^k\, d_k^\dagger(\mathbf{p},s,0)\Big)\ , \end{aligned} \quad (4)$$

where we have suppressed the isospin indices. In (4), $\int_{\mathbf{p}} \equiv \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}$, s is the helicity index and $[E_L^2(\mathbf{p})]_l{}^m = \mathbf{p}^2\delta_l{}^m + [M_L^\dagger M_L]_l{}^m$. Notice that the Dirac four-spinors $[u(\mathbf{p},s)]_j{}^k$ and $[v(\mathbf{p},s)]_j{}^k$ transform as rank-2 tensors in flavour space. The lepton creation and annihilation operators $b^k \equiv b_k^\dagger$ and b_k, and the anti-lepton creation and annihilation operators $d^{\dagger,k} \equiv d_k$ and $d_k^\dagger$, satisfy the following equal-time anti-commutation relations

$$\begin{aligned} \{b_l(\mathbf{p},s,\tilde{t}),\, b^m(\mathbf{p}',s',\tilde{t})\} \ &=\ \{d^{\dagger,m}(\mathbf{p},s,\tilde{t}),\, d_l^\dagger(\mathbf{p}',s',\tilde{t})\} \\ &=\ (2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}')\,\delta_{ss'}\,\delta_l{}^m. \end{aligned} \quad (5)$$

Note that for the Dirac field, the lepton annihilation operator $b_k(\mathbf{p},s,\tilde{t})$ and the anti-lepton creation operator $d_k^\dagger(\mathbf{p},s,\tilde{t})$ transform under the *same* representation of $U(\mathcal{N}_L)$.

For the heavy *Majorana* neutrino creation and annihilation operators $a^\alpha(\mathbf{k},r,\tilde{t})$ and $a_\alpha(\mathbf{k},r,\tilde{t})$, with helicities $r = \pm$, it is necessary to introduce the flavour-covariant Majorana constraint

$$d^{\dagger,\alpha}(\mathbf{k},-r,\tilde{t}) \ =\ G^{\alpha\beta}\, b_\beta(\mathbf{k},r,\tilde{t}) \ \equiv\ G^{\alpha\beta} a_\beta(\mathbf{k},r,\tilde{t})\ , \quad (6)$$

where $G^{\alpha\beta} \equiv [U^* U^\dagger]^{\alpha\beta}$ are the elements of a unitary matrix $\boldsymbol{G}$, which transforms as a contravariant rank-2 tensor under $U(\mathcal{N}_N)$. Similar flavour rotations are *forced* by the flavour-covariance of the formalism, when we derive the transformation properties of the discrete symmetries C, P and T. This necessarily leads to the *generalized* discrete transformations

$$b_l(\mathbf{p},s,\tilde{t})^{\widetilde{C}} \ \equiv\ \mathcal{G}^{lm}\, b_m(\mathbf{p},s,\tilde{t})^C \ =\ -i\, d^{\dagger,l}(\mathbf{p},s,\tilde{t})\ , \quad \text{(7a)}$$

$$b_l(\mathbf{p},s,\tilde{t})^P \ =\ -s\, b_l(-\mathbf{p},-s,\tilde{t})\ , \quad \text{(7b)}$$

$$b_l(\mathbf{p},s,\tilde{t})^{\widetilde{T}} \ \equiv\ \mathcal{G}_{lm}\, b_m(\mathbf{p},s,\tilde{t})^T \ =\ b_l(-\mathbf{p},s,-\tilde{t})\ , \quad \text{(7c)}$$

where $\mathcal{G}^{lm} \equiv [V^* V^\dagger]^{lm}$ is the lepton analogue of the heavy-neutrino tensor $\boldsymbol{G}$.

Using a flavour-covariant canonical quantization [9], we may define the matrix number densities of the leptons and heavy neutrinos, as follows:

$$[n^L_{s_1 s_2}(\mathbf{p},t)]_l{}^m \equiv \mathcal{V}_3^{-1}\langle b^m(\mathbf{p},s_2,\tilde{t}) b_l(\mathbf{p},s_1,\tilde{t})\rangle_t\ , \quad \text{(8a)}$$

$$[\bar{n}^L_{s_1 s_2}(\mathbf{p},t)]_l{}^m \equiv \mathcal{V}_3^{-1}\langle d_l^\dagger(\mathbf{p},s_1,\tilde{t}) d^{\dagger,m}(\mathbf{p},s_2,\tilde{t})\rangle_t\ , \quad \text{(8b)}$$

$$[n^N_{r_1 r_2}(\mathbf{k},t)]_\alpha{}^\beta \equiv \mathcal{V}_3^{-1}\langle a^\beta(\mathbf{k},r_2,\tilde{t}) a_\alpha(\mathbf{k},r_1,\tilde{t})\rangle_t\ , \quad \text{(8c)}$$

where $\mathcal{V}_3 = (2\pi)^3\delta^{(3)}(\mathbf{0})$ is the coordinate three-volume and the macroscopic time $t = \tilde{t} - \tilde{t}_i$, equal to the interval of microscopic time between specification of initial conditions ($\tilde{t}_i$) and subsequent observation of the system ($\tilde{t}$) [10]. Note the relative reversed ordering of indices in the lepton and anti-lepton number densities, which ensures that the two quantities transform in the same representation, so that they can be combined to form a flavour-covariant lepton asymmetry. For the Majorana neutrinos, $\boldsymbol{n}^N$ and $\overline{\boldsymbol{n}}^N$ are not independent quantities and are related by the generalized Majorana condition

$$[\bar{n}^N_{r_1 r_2}(\mathbf{k},t)]_\alpha{}^\beta \ =\ G_{\alpha\mu}\,[n^N_{r_2 r_1}(\mathbf{k},t)]_\lambda{}^\mu\, G^{\lambda\beta}\ . \quad (9)$$

The number density matrices defined above have simple generalized-C transformation properties:

$$[\boldsymbol{n}^X(\mathbf{p},t)]^{\widetilde{C}} \ =\ [\overline{\boldsymbol{n}}^X(\mathbf{p},t)]^{\mathsf{T}}, \quad (10)$$

where T denotes the matrix transpose acting on both flavour and helicity indices. The total number densities $\boldsymbol{n}^X(t)$ are obtained by tracing over helicity and isospin indices and integrating over the three-momenta.

Using the $\widetilde{C}$-transformation relations (10), we can define the generalized $\widetilde{C}P$-"odd" lepton asymmetry

$$\boldsymbol{\delta n}^L \ = \ \boldsymbol{n}^L \ - \ \overline{\boldsymbol{n}}^L \ . \tag{11}$$

In addition, for the heavy neutrinos, we may define the $\widetilde{C}P$-"even" and -"odd" quantities

$$\underline{\boldsymbol{n}}^N \ = \ \frac{1}{2}\Big(\boldsymbol{n}^N \ + \ \overline{\boldsymbol{n}}^N\Big) \ , \qquad \boldsymbol{\delta n}^N \ = \ \boldsymbol{n}^N \ - \ \overline{\boldsymbol{n}}^N \ . \tag{12}$$

We will use these quantities, having definite $\widetilde{C}P$-transformation properties, to write down the flavour-covariant rate equations.

First we derive a Markovian master equation governing the time evolution of the matrix number densities $\boldsymbol{n}^X(\mathbf{p}, t)$. These are defined in terms of the quantum-mechanical number-density operator $\check{\boldsymbol{n}}^X(\mathbf{k}, \tilde{t}; \tilde{t}_i)$ and density operator $\rho(\tilde{t}; \tilde{t}_i)$, as follows:

$$\boldsymbol{n}^X(\mathbf{k}, t) \equiv \langle \check{\boldsymbol{n}}^X(\mathbf{k}, \tilde{t}; \tilde{t}_i)\rangle_t = \mathrm{Tr}\left\{\rho(\tilde{t}; \tilde{t}_i)\, \check{\boldsymbol{n}}^X(\mathbf{k}, \tilde{t}; \tilde{t}_i)\right\} \ , \tag{13}$$

where the trace is over the Fock space. Differentiating (13) with respect to the macroscopic time $t = \tilde{t} - \tilde{t}_i$, and using the Liouville-von Neumann and Heisenberg equations of motion, we proceed via a Wigner-Weisskopf approximation to obtain the leading order Markovian master equation [9]

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{n}^X(\mathbf{k}, t) \ \simeq \ i\langle\, [H_0^X, \ \check{\boldsymbol{n}}^X(\mathbf{k}, t)]\, \rangle_t \\ - \ \frac{1}{2}\int_{-\infty}^{+\infty} \mathrm{d}t' \ \langle\, [H_{\mathrm{int}}(t'), \ [H_{\mathrm{int}}(t), \ \check{\boldsymbol{n}}^X(\mathbf{k}, t)]]\, \rangle_t \ , \tag{14}$$

where H_0^X and H_{int} are the free and interaction Hamiltonians, respectively. The first term on the RHS of (14), involving the free Hamiltonian, generates flavour oscillations in vacuum, whereas the second term in (14), involving the interaction Hamiltonian, generates the collision terms in the generalized Boltzmann equations.

For the system of lepton and Higgs doublets and heavy-neutrino singlets under consideration, we have

$$H_0^L \ = \ \sum_s \int_{\mathbf{p}} [E_L(\mathbf{p})]_m{}^l \Big(b^m(\mathbf{p}, s, \tilde{t})\, b_l(\mathbf{p}, s, \tilde{t}) \\ + \ d_l^\dagger(\mathbf{p}, s, \tilde{t})\, d^{\dagger, m}(\mathbf{p}, s, \tilde{t})\Big) \ , \tag{15a}$$

$$H_0^N \ = \ \sum_r \int_{\mathbf{k}} [E_N(\mathbf{k})]_\beta{}^\alpha \, a^{\dagger, \beta}(\mathbf{k}, r, \tilde{t})\, a_\alpha(\mathbf{k}, r, \tilde{t}) \ , \tag{15b}$$

$$H_{\mathrm{int}} \ = \ \int d^4x\, h_l^{\ \alpha}\, \bar{L}^l\, \widetilde{\Phi}\, N_{\mathrm{R}, \alpha} \ + \ \mathrm{H.c.} \ . \tag{15c}$$

Using these expressions in (14), we obtain the following evolution equations for the lepton and heavy-neutrino number densities [9]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\, [n^L_{s_1 s_2}(\mathbf{p}, t)]_l{}^m \ = \ -\, i\Big[E_L(\mathbf{p}), \ n^L_{s_1 s_2}(\mathbf{p}, t)\Big]_l{}^m \\ + \ [C^L_{s_1 s_2}(\mathbf{p}, t)]_l{}^m \ , \tag{16a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\, [n^N_{r_1 r_2}(\mathbf{k}, t)]_\alpha{}^\beta \ = \ -\, i\Big[E_N(\mathbf{k}), \ n^N_{r_1 r_2}(\mathbf{k}, t)\Big]_\alpha{}^\beta \\ + \ [C^N_{r_1 r_2}(\mathbf{k}, t)]_\alpha{}^\beta + \ G_{\alpha\lambda}\, [\overline{C}^N_{r_2 r_1}(\mathbf{k}, t)]_\mu{}^\lambda\, G^{\mu\beta} \ , \tag{16b}$$

where, for instance, the lepton collision terms may be written in the form

$$[C^L_{s_1 s_2}(\mathbf{p}, t)]_l{}^m \ = \ -\, \frac{1}{2}\, [\, \mathcal{F}\cdot\Gamma \ + \ \Gamma^\dagger\cdot\mathcal{F}\,]_{s_1 s_2,\, l}{}^m \ . \tag{17}$$

Here, we have suppressed the overall momentum dependence and used a compact notation

$$[\mathcal{F}\cdot\Gamma\,]_{s_1 s_2,\, l}{}^m \ \equiv \ \sum_{s, r_1, r_2} \int_{\mathbf{k},\, \mathbf{q}} [\mathcal{F}_{s_1 s\, r_1 r_2}(\mathbf{p}, \mathbf{q}, \mathbf{k}, t)]_l{}^n{}_\alpha{}^\beta \\ \times\ [\Gamma_{s\, s_2 r_2 r_1}(\mathbf{p}, \mathbf{q}, \mathbf{k})]_n{}^m{}_\beta{}^\alpha \ . \tag{18}$$

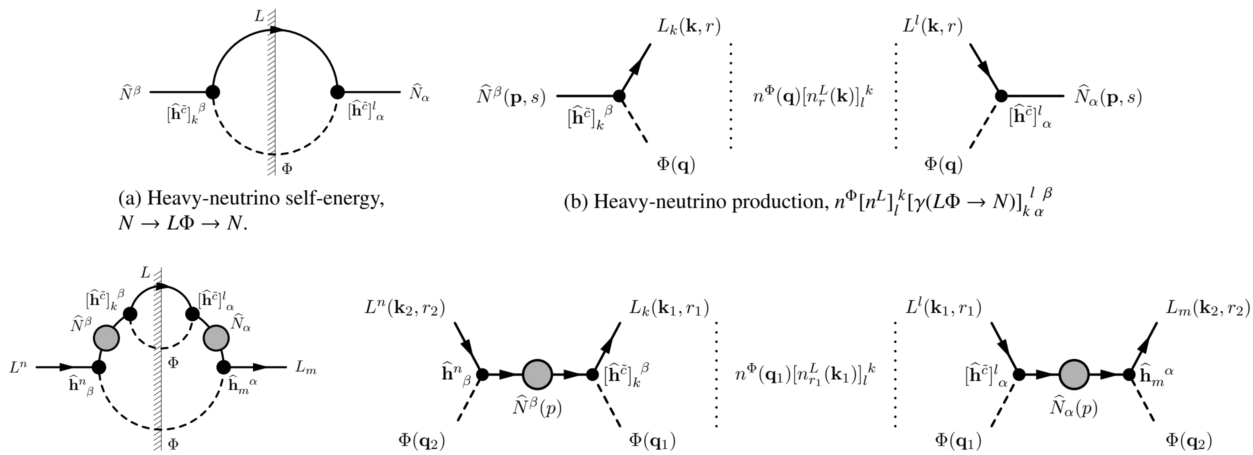
In (18), there are two *new rank-4 tensors* in flavour space, as required by flavour-covariance: (i) the statistical number density tensors

$$\boldsymbol{\mathcal{F}}(\mathbf{p}, \mathbf{q}, \mathbf{k}, t) \ = \ n^\Phi(\mathbf{q}, t)\, \boldsymbol{n}^L(\mathbf{p}, t) \otimes \Big[\mathbf{1} - \boldsymbol{n}^N(\mathbf{k}, t)\Big] \\ - \ \Big[1 + n^\Phi(\mathbf{q}, t)\Big]\Big[\mathbf{1} - \boldsymbol{n}^L(\mathbf{p}, t)\Big] \otimes \boldsymbol{n}^N(\mathbf{k}, t) \ , \tag{19}$$

and (ii) the absorptive rate tensors

$$[\Gamma_{s_1 s_2 r_1 r_2}(\mathbf{p}, \mathbf{q}, \mathbf{k})]_l{}^m{}_\alpha{}^\beta = h^k{}_\nu\, h_i{}^\lambda (2\pi)^4 [\delta^{(4)}(k - p - q)]^j{}_{p\mu}{}^\delta \\ \times\ \frac{1}{2E_\Phi(\mathbf{q})}\ \Big[(2E_L(\mathbf{p}))^{-1/2}\Big]^i{}_j\ \Big[(2E_L(\mathbf{p}))^{-\frac{1}{2}}\Big]_k{}^n \\ \times\ \Big[(2E_N(\mathbf{p}))^{-\frac{1}{2}}\Big]_\lambda{}^\mu\ \Big[(2E_N(\mathbf{k}))^{-1/2}\Big]^\nu{}_\gamma\ \mathrm{Tr}\Big\{[u(\mathbf{k}, r_2)]_\delta{}^\beta \\ \times\ [\bar{u}(\mathbf{k}, r_1)]^\gamma{}_\alpha\, \mathrm{P_L}\, [u(\mathbf{p}, s_2)]_n{}^m\, [\bar{u}(\mathbf{p}, s_1)]^p{}_l\, \mathrm{P_R}\Big\} \ . \tag{20}$$

The rate tensor (20) describes heavy neutrino decays and inverse decays, and its off-diagonal components are responsible for the evolution of flavour-coherences in the system. The necessary emergence of these higher-rank tensors in flavour space may be understood in terms of the unitarity cuts of the partial self-energies [9]. This is illustrated diagrammatically in Figure 1 for the in-medium heavy-neutrino production $L\Phi \ \to \ N$ (Figures 1a and 1b) and $\Delta L = 0$ scattering $L\Phi \to L\Phi$ (Figures 1c and 1d) in a spatially-homogeneous statistical background of lepton and Higgs doublets. In Figures 1a and 1c, the cut, across which positive energy flows from unshaded to shaded regions, is associated with production rates in the thermal plasma, as described by a generalization of the optical theorem [9].



(a) Heavy-neutrino self-energy, $N \to L\Phi \to N$.

(b) Heavy-neutrino production, $n^{\Phi}[n^L]_l^{\ k}[\gamma(L\Phi \to N)]_{k\ \alpha}^{\ l\ \beta}$

(c) Charged-lepton self-energy, with $\Delta L = 0$ internally.

(d) $\Delta L = 0$ scattering, $n^{\Phi}[n^L]_l^{\ k}[\gamma(L\Phi \to L\Phi)]_{k\ m}^{\ l\ n}$.

Figure 1: Generalized unitarity cut of the partial heavy-neutrino and lepton self-energies, giving rise to the rank-4 tensor rates for heavy-neutrino production and $\Delta L = 0$ scattering processes. The explicit forms of the thermally-averaged rank-4 rates can be found in [9].

3. Rate Equations for Resonant Leptogenesis

As already mentioned in Section 1, in the limit when two (or more) heavy Majorana neutrinos become degenerate, the ε-type *CP*-violation due to the interference between the tree-level and absorptive part of the self-energy graphs in the heavy-neutrino decay can be resonantly enhanced, even up to order one [3]. In this regime, finite-order perturbation theory breaks down and one needs a consistent field-theoretic resummation of the self-energy corrections. Neglecting thermal loop effects [11], we perform such resummation along the lines of [4] and replace the tree-level neutrino Yukawa couplings by their resummed counterparts in the transport equations given in Section 2. Specifically, for the processes $N \to L\Phi$ and $L^{\tilde{c}}\Phi^{\tilde{c}} \to N$, we have $h_l^{\ \alpha} \to \mathbf{h}_l^{\ \alpha}$ and, for $N \to L^{\tilde{c}}\Phi^{\tilde{c}}$ and $L\Phi \to N$, we have $h^l_{\ \alpha} \to [\mathbf{h}^{\tilde{c}}]^l_{\ \alpha}$, where $\tilde{c}$ denotes the $\widetilde{CP}$-conjugate. The algebraic form of the resummed neutrino Yukawa couplings in the heavy-neutrino mass eigenbasis can be found in [4] and the corresponding form in a general flavour basis may be obtained by the appropriate flavour transformation, i.e. $\mathbf{h}_l^{\ \alpha} = V_l^{\ m} U^{\alpha}_{\ \beta} \widehat{\mathbf{h}}_m^{\ \beta}$, where $\widehat{\mathbf{h}}_m^{\ \beta} \equiv \widehat{\mathbf{h}}_{m\beta}$ in the mass eigenbasis [9].

In order to obtain the rate equations relevant for RL from the general transport equations (16a) and (16b), we perform the following standard approximations:

(i) assume kinetic equilibrium, since elastic scattering processes rapidly equilibrate the momentum distributions for all the relevant particle species on time-scales much smaller than their statistical evolution.

(ii) neglect the mass splittings between different heavy-neutrino flavours inside thermal integrals, and use an average mass m_N and energy $E_N(\mathbf{k}) = (|\mathbf{k}|^2 + m_N^2)^{1/2}$, since the average momentum scale $|\mathbf{k}| \sim T \gg |m_{N_\alpha} - m_{N_\beta}|$.

(iii) take the classical statistical limit of (19).

(iv) neglect thermal and chemical potential effects [12].

With the above approximations, we integrate both sides of (16a) and (16b), and their generalized $\widetilde{CP}$-conjugates, over the phase space and sum over the degenerate isospin and helicity degrees of freedom. The resulting rate equations account for the decay and inverse decay of the heavy neutrinos in a flavour-covariant way [9]. However, in order to guarantee the correct equilibrium behaviour, we must include the washout terms induced by the $\Delta L = 0$ and $\Delta L = 2$ scattering processes, with proper real intermediate state (RIS) subtraction [13, 4, 9] (see e.g., Figure 1d). As illustrated in [9], it is necessary to account for thermal corrections in the RIS contributions, when considering off-diagonal flavour correlations.

In addition to the $2 \leftrightarrow 2$ scatterings, it is also important to include the effect of the charged-lepton Yukawa couplings, which are responsible for the decoherence of the charged leptons towards their would-be mass eigenbasis, as opposed to the interactions with the heavy neutrinos [cf. (2)], which tend to create a coherence between the charged-lepton flavours. Note that, while calculating the reaction rates for the processes involving the charged-lepton Yukawa couplings, it is important to take into account their thermal masses, which control the phase space suppression for the decay and inverse decay of the Higgs boson [14].

Taking into account all these contributions, as well as the expansion of the Universe, we derive the following *manifestly* flavour-covariant rate equations for the normalized $\widetilde{CP}$-"even" number density matrix $\underline{\boldsymbol{\eta}}^N$ and $\widetilde{CP}$-"odd" number density matrices $\boldsymbol{\delta\eta}^N$ and $\boldsymbol{\delta\eta}^L$ (where $\eta^X = n^X/n^\gamma$, n^γ being the photon number density) [9]:

$$\frac{H_N\, n^\gamma}{z}\,\frac{\mathrm{d}[\underline{\eta}^N]_\alpha^{\ \beta}}{\mathrm{d}z} \;=\; -\,i\,\frac{n^\gamma}{2}\Big[\mathcal{E}_N,\,\delta\eta^N\Big]_\alpha^{\ \beta} \,+\, [\widetilde{\mathrm{Re}}(\gamma^N_{L\Phi})]_\alpha^{\ \beta} - \frac{1}{2\,\eta^N_{\mathrm{eq}}}\Big\{\underline{\eta}^N,\,\widetilde{\mathrm{Re}}(\gamma^N_{L\Phi})\Big\}_\alpha^{\ \beta}\,, \tag{21a}$$

$$\frac{H_N\, n^\gamma}{z}\,\frac{\mathrm{d}[\delta\eta^N]_\alpha^{\ \beta}}{\mathrm{d}z} \;=\; -\,2\,i\,n^\gamma\Big[\mathcal{E}_N,\,\underline{\eta}^N\Big]_\alpha^{\ \beta} + 2\,i\,[\widetilde{\mathrm{Im}}(\delta\gamma^N_{L\Phi})]_\alpha^{\ \beta} - \frac{i}{\eta^N_{\mathrm{eq}}}\Big\{\underline{\eta}^N,\,\widetilde{\mathrm{Im}}(\delta\gamma^N_{L\Phi})\Big\}_\alpha^{\ \beta} - \frac{1}{2\,\eta^N_{\mathrm{eq}}}\Big\{\delta\eta^N,\,\widetilde{\mathrm{Re}}(\gamma^N_{L\Phi})\Big\}_\alpha^{\ \beta}\,, \tag{21b}$$

$$\frac{H_N\, n^\gamma}{z}\,\frac{\mathrm{d}[\delta\eta^L]_l^{\ m}}{\mathrm{d}z} \;=\; -\,[\delta\gamma^N_{L\Phi}]_l^{\ m} + \frac{[\underline{\eta}^N]_\beta^{\ \alpha}}{\eta^N_{\mathrm{eq}}}\,[\delta\gamma^N_{L\Phi}]_{l\ \alpha}^{\ m\ \beta} + \frac{[\delta\eta^N]_\beta^{\ \alpha}}{2\,\eta^N_{\mathrm{eq}}}\,[\gamma^N_{L\Phi}]_{l\ \alpha}^{\ m\ \beta} - \frac{1}{3}\Big\{\delta\eta^L,\,\gamma^{L\Phi}_{L^{\tilde c}\Phi^{\tilde c}} + \gamma^{L\Phi}_{L\Phi}\Big\}_l^{\ m} - \frac{2}{3}\,[\delta\eta^L]_k^{\ n}\,[\gamma^{L\Phi}_{L^{\tilde c}\Phi^{\tilde c}} - \gamma^{L\Phi}_{L\Phi}]_{n\ l}^{\ k\ m} - \frac{2}{3}\Big\{\delta\eta^L,\,\gamma_{\mathrm{dec}}\Big\}_l^{\ m} + [\delta\gamma^{\mathrm{back}}_{\mathrm{dec}}]_l^{\ m}\,. \tag{21c}$$

Here $z = m_N/T$, H_N is the Hubble parameter at $z = 1$ and $\boldsymbol{\mathcal{E}}_N$ is the thermally-averaged effective heavy-neutrino energy matrix. $\boldsymbol{\gamma}^N_{L\Phi}$ and $\boldsymbol{\delta\gamma}^N_{L\Phi}$ are respectively the $\widetilde{CP}$-"even" and -"odd" thermally-averaged rate tensors governing the decay and inverse decay of the heavy neutrinos. In (21c), the rates $\boldsymbol{\gamma}_{\mathrm{dec}}$ and $\boldsymbol{\delta\gamma}^{\mathrm{back}}_{\mathrm{dec}}$ govern the charged-lepton decoherence, whereas $\boldsymbol{\gamma}^{L\Phi}_{L\Phi}$ and $\boldsymbol{\gamma}^{L\Phi}_{L^{\tilde c}\Phi^{\tilde c}}$ describe the washout due to $\Delta L = 0$ and $\Delta L = 2$ resonant scattering, respectively. In obtaining (21a) and (21b), we have defined, for a given Hermitian matrix $\boldsymbol{A} = \boldsymbol{A}^\dagger$, its generalized real and imaginary parts, as follows:

$$[\widetilde{\mathrm{Re}}(A)]_\alpha^{\ \beta} \;\equiv\; \frac{1}{2}\Big(A_\alpha^{\ \beta} \,+\, G_{\alpha\lambda}\,A_\mu^{\ \lambda}\,G^{\mu\beta}\Big)\,, \tag{22a}$$

$$[\widetilde{\mathrm{Im}}(A)]_\alpha^{\ \beta} \;\equiv\; \frac{1}{2\,i}\Big(A_\alpha^{\ \beta} \,-\, G_{\alpha\lambda}\,A_\mu^{\ \lambda}\,G^{\mu\beta}\Big)\,. \tag{22b}$$

In addition, we have used the relations

$$\widetilde{\mathrm{Re}}(\underline{\boldsymbol{n}}^N) \;=\; \underline{\boldsymbol{n}}^N\,, \qquad i\,\widetilde{\mathrm{Im}}(\boldsymbol{\delta n}^N) \;=\; \boldsymbol{\delta n}^N\,. \tag{23}$$

The flavour-covariant rate equations (21a)–(21c) provide a complete and unified description of the RL phenomenon, consistently capturing the following *physically distinct* effects in a single framework, applicable for any temperature regime:

(i) Lepton asymmetry due to the *resonant mixing* between heavy neutrinos, as described by the resummed Yukawa couplings in $\boldsymbol{\delta\gamma}^N_{L\Phi}$, appearing in the first two terms on the RHS of (21c). This provides a flavour-covariant generalization of the mixing effects discussed earlier in [4].

(ii) Generation of the lepton asymmetry via coherent heavy-neutrino *oscillations*. Even starting with an incoherent diagonal heavy-neutrino number density matrix, off-diagonal $\widetilde{CP}$-"even" number densities will be generated at $O(h^2)$ due to the CP-conserving part of the coherent inverse decay rate $\boldsymbol{\gamma}^N_{L\Phi}$ in the last two terms on the RHS of (21a). Heavy-neutrino oscillations will transfer these coherences to the $\widetilde{CP}$-"odd" number densities $[\delta\eta^N]_\alpha^{\ \beta}$ due to the commutator terms in (21a)) and (21b)). Finally, a lepton asymmetry is generated at $O(h^4)$ by the $\widetilde{CP}$-"even" coherent off-diagonal decay rates in the first term on the second line of (21c). Notice that the novel rank-4 rate tensor $[\gamma^N_{L\Phi}]_{l\ \alpha}^{\ m\ \beta}$, required by flavour covariance, plays an important role in this mechanism, along with the $\widetilde{CP}$-"odd" number density $[\delta\eta^N]_\alpha^{\ \beta}$, which is purely off-diagonal in the heavy-neutrino mass eigenbasis. We stress here that this phenomenon of coherent oscillations is an $O(h^4)$ effect on the *total* lepton asymmetry, and so differs from the $O(h^6)$ mechanism proposed in [15]. The difference is due to the fact that the latter typically takes place at temperatures much higher than the sterile neutrino masses in the model (see e.g. [16]), where the total lepton number is not violated at leading order. On the other hand, the $O(h^4)$ effect identified here is enhanced in the same regime as the resonant $T = 0$ ε-type CP violation, namely, for $z \approx 1$ and $\Delta m_N \sim \Gamma_{N_\alpha}$ [9].

(iii) *Decoherence* effects due to charged-lepton Yukawa couplings, described by the last two terms on the RHS of (21c). Our description of these effects is similar to the one of [6], which has been generalized here to an arbitrary flavour basis.

4. A Numerical Example

To illustrate the importance of the flavour effects captured *only* by the flavour-covariant rate equations (21a) - (21c), we consider a minimal *Resonant ℓ-Genesis* (RL_ℓ) scenario in which the final lepton asymmetry is dominantly generated and stored in a *single* lepton flavour ℓ [17]. In this case, the heavy neutrino masses could be as low as the electroweak scale [12], still with sizable couplings to other charged-lepton flavours $\ell' \neq \ell$, whilst being consistent with all current experimental constraints [18]. This enables the modelling of minimal RL_ℓ scenarios [19] with electroweak-scale heavy Majorana neutrinos that could be *tested* during the run-II phase of the LHC [20].

The basic assumption underlying the minimal RL_ℓ model is an $O(\mathcal{N}_N)$-symmetric heavy-neutrino sector at some high scale μ_X, with degenerate heavy neutrinos of mass m_N. At the phenomenologically-relevant low-energy scale, small mass splittings between them, as required by the RL mechanism, may be naturally induced by the RG evolution. In the heavy-neutrino mass eigenbasis, the RG effects consistently break the degeneracies of the $O(\mathcal{N}_N)$-symmetric heavy-neutrino parameter space, thereby justifying the definition of the resummed Yukawa couplings in this basis [9].

As an explicit example of RL_ℓ, we consider an RL_τ model with $O(3)$ symmetry at the grand unification scale, $\mu_X \sim 2 \times 10^{16}$ GeV, which is explicitly broken to the $U(1)_{L_e+L_\mu} \times U(1)_{L_\tau}$ subgroup of lepton-flavour symmetries by a neutrino Yukawa coupling matrix [19]

$$\boldsymbol{h} \ = \ \begin{pmatrix} 0 & ae^{-i\pi/4} & ae^{i\pi/4} \\ 0 & be^{-i\pi/4} & be^{i\pi/4} \\ 0 & 0 & 0 \end{pmatrix} + \boldsymbol{\delta h} \, , \qquad (24)$$

where a, b are arbitrary complex parameters, and the perturbation matrix $\boldsymbol{\delta h}$ vanishes in the flavour-symmetric limit, thereby making the light neutrinos massless to all orders in perturbation theory [21]. In order to be consistent with the observed neutrino oscillation data, we consider a minimal deviation of the following form from the flavour-symmetric limit [19]:

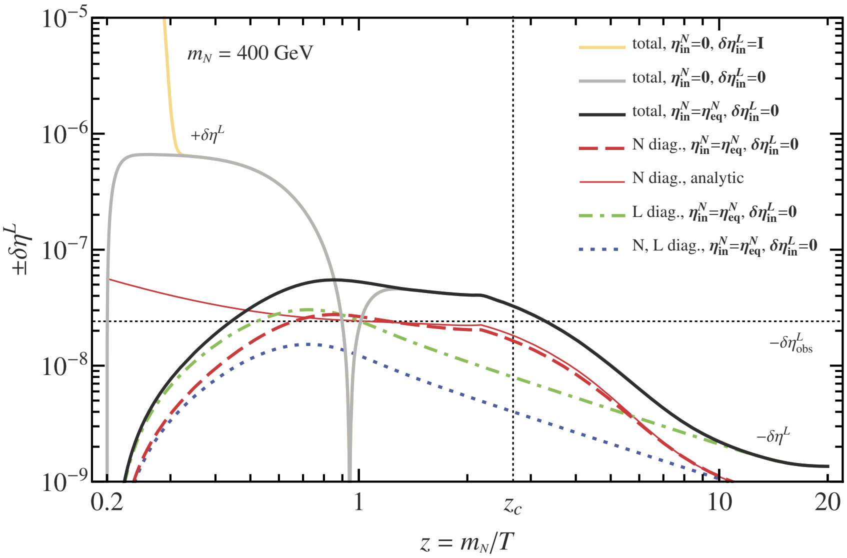
$$\boldsymbol{\delta h} \ = \ \begin{pmatrix} \epsilon_e & 0 & 0 \\ \epsilon_\mu & 0 & 0 \\ \epsilon_\tau & \kappa_1 e^{-i(\pi/4-\gamma_1)} & \kappa_2 e^{i(\pi/4-\gamma_2)} \end{pmatrix} , \qquad (25)$$

where $|\epsilon_{e,\mu,\tau}|, \kappa_{1,2} \ll |a|, |b|$, and $\gamma_{1,2}$ are arbitrary phases. A choice of benchmark values for these parameters, satisfying all the current experimental constraints, is given in Figure 2b. The corresponding numerical solution for the total lepton asymmetry $\delta\eta^L \equiv \mathrm{Tr}(\boldsymbol{\delta\eta}^L)$ in our flavour-covariant formalism is shown in Figure 2a. Here the horizontal dotted line shows the value of $\delta\eta^L$ required to explain the observed baryon asymmetry in our Universe, whereas the vertical line shows the critical temperature $z_c = m_N/T_c$, beyond which the electroweak sphaleron processes become ineffective in converting lepton asymmetry to baryon asymmetry. The thick solid lines show the evolution of $\delta\eta^L$ for three different initial conditions, to which the final lepton asymmetry $\delta\eta^L(z \gg 1)$ is shown to be insensitive. This is a general consequence of the RL mechanism in the strong washout regime [12].

For comparison, we also show in Figure 2a various partially flavour-dependent limits, i.e. when either the heavy-neutrino (dashed line) or the lepton (dash-dotted line) number density or both (dotted line) are diagonal in flavour space. Also shown is the approximate analytic solution obtained in [9] for the case of a diagonal heavy-neutrino number density (thin solid line). The enhanced lepton asymmetry in the *fully* flavour-covariant formalism is mainly due to (i) coherent oscillations between the heavy-neutrino flavours, leading to an enhancement by a factor of two, and (ii) flavour coherences in the charged-lepton sector, generated through the heavy-neutrino Yukawa couplings and destroyed through the charged-lepton Yukawa couplings. The latter gives rise to an distinctive 'plateau' at intermediate z values, which happens to occur before z_c for the chosen model parameters, and hence, leads to an additional enhancement of a factor ~ 5 in the lepton asymmetry.

5. Conclusions

We have presented a *fully* flavour-covariant formalism for transport phenomena by deriving Markovian master equations that describe the time-evolution of particle number densities in a quantum-statistical ensemble with arbitrary flavour content. As an application, we have studied the flavour effects in RL and have obtained *manifestly* flavour-covariant rate equations for heavy-neutrino and lepton number densities. This provides a complete and unified description of RL, capturing three *distinct* physical phenomena: (i) resonant mixing between the heavy-neutrino states, (ii) coherent oscillations between different heavy-neutrino flavours and (iii) quantum decoherence effects in the charged-lepton sector. The quantitative importance of this formalism is illustrated for a minimal RL_τ model, where the total lepton asymmetry obtained by solving the fully flavour-covariant rate equations is enhanced by up to an order of magnitude, as compared to the predictions from partially flavour-dependent limits.



(a)

Parameter	Value
m_N	400 GeV
γ_1	$\pi/3$
γ_2	0
κ_1	2.4×10^{-5}
κ_2	6×10^{-5}
a	$(4.93 - 2.32\,i) \times 10^{-3}$
b	$(8.04 - 3.79\,i) \times 10^{-3}$
ϵ_e	5.73×10^{-8}
ϵ_μ	4.3×10^{-7}
ϵ_τ	6.39×10^{-7}

(b)

Figure 2: (a) Total lepton asymmetry as predicted by the minimal RL$_\tau$ model with benchmark parameters given in (b). We show the comparison between the total asymmetry obtained using the fully flavour-covariant formalism (thick solid lines, with different initial conditions) with those obtained using the flavour-diagonal formalism (dashed lines). Also shown (thin solid line) is an approximate analytic result discussed in [9].

Acknowledgments

The work of P.S.B.D., P.M. (in part) and A.P. is supported by the Lancaster-Manchester-Sheffield Consortium for Fundamental Physics under STFC grant ST/J000418/1. P.M. is also supported in part by the IPPP through STFC grant ST/G000905/1. The work of D.T. has been supported by a fellowship of the EPS Faculty of the University of Manchester.

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