Review of *Applied Multidimensional Systems Theory*,* by N. K. Bose and *Multivariable Feedback Systems*,** by F. M. Callier and C. A. Desoer

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Among the areas of applied mathematics that heavily use linear algebra, systems theory is currently one of the most active. The general heading of systems theory includes topics from control theory, circuit theory, and power systems to name a few.

For the person with a strong linear algebra background wanting to learn more about this area, a well written book would be a more efficient way to acquire an understanding than an unguided tour of the literature. Both of the books under review appear to possibly be appropriate for this purpose. This review will only discuss this use for these books and not their suitability as graduate engineering texts.

Both books are quite recent (1982) and aimed at graduate level electrical engineers. They are also both about multivariable systems, concern transfer functions, and are heavily algebraic. Professor Bose has a joint appointment in mathematics and electrical engineering while Drs. Callier and Desoer are in mathematics and electrical engineering departments respectively. The presentation in both volumes is mathematically rigorous with theorems and proofs. The development of some of the theory is left to the exercises. Both books presuppose the reader has some familiarity with undergraduate control/systems theory and a solid course in linear algebra. A familiarity with concepts such as feedback, controllability, and some modern algebra would be helpful. Applications tend to be general rather than specific. Both books heavily use transfer functions. That is, the underlying equations in the time (or spatial) domain have been transformed (Laplace transform for differential systems, z-transform for discrete or difference systems).

On the basis of all these similarities it might seem that these books would be directly comparable. However, this is not the case. The books are quite different in both presentation and content. A comparison of the two should help the reader decide which is the more appropriate for him and also suggest some of the variety in multivariable systems theory.

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*Van Nostrand Reinhold, New York, 1982, 411 pp.

**Springer, New York, 1982, 275 pp.

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The Bose book is typeset and more of a graduate text. There is a great deal of referencing to the literature and discussion of current techniques. The major application the author has in mind is multidimensional filtering. In one two-dimensional variation there is an input array $X(i, j)$ and an output array $Y(i, j)$ related by a linear transformation T. Let I_b, I_a be index sets for the (not necessarily rectangular) arrays $a(i, j)$, $b(i, j)$. The input-out relationship is

$$\sum_{(i_1, j_1) \in I_a} b(i_1, i_1) Y(i - i_1, j - j_1) = \sum_{(i_2, j_2) \in I_b} a(i_2, j_2) X(i - i_2, j - j_2). \tag{*}$$

The relationship (*) reflects the fact that an observation at say (i, j) is a weighted convolution of nearby points in the array.

This application provides the unifying theme for the Bose book. There are several consequences. The emphasis is on the z transform and difference equations. More importantly, the arrays X, Y have several *independent* variables and the entries $X(i, j)$ may lie in a field other than the reals or complexes. For digital filters, the field may be finite. Polynomials in several variables over a finite field form a ring. It becomes necessary in the development of algorithms to do algebra for polynomials over these rings.

The first 74 pages discuss how to do this algebra. Nontrivial examples of factoring polynomials using Euclid's algorithm, Berlekam's algorithm, and the Hensel–Zassenhaus lifting are carefully developed. The next 75 pages discuss non-negativity tests for multivariate polynomials using the ideas of decision algebra. The emphasis throughout is on the development of algebraic algorithms that utilize exact arithmetic and terminate in a finite number of steps. The remaining 240 pages discuss multidimensional filter stability, stabilization, and realization utilizing multiparameter transfer functions. There are numerous exercises. The writing style is not overly terse and the examples are often worked in some detail.

The Callier–Desoer book on the other hand is aimed more at circuit and control theory. The models discussed are differential equations (though the theory can be modified for difference equations). There is a single independent variable in the transfer functions (matrices) and they are taken over the real or complex field. There is more of an emphasis on matrix factorization theorems (over the ring of polynomials) and standard results such as the Smith–McMillan canonical form are frequently used. The theory is applied to interconnected systems, closed loop eigenvalue placement, asymptotic tracking, and design with stable plants. The style of the Callier–Desoer book is readable but very terse. There is extensive use of abbreviations and logical symbols. There is comparatively little referencing to the literature. Callier and Desoer make more frequent reference to, and use of, the original untransformed state equations which may be helpful to some readers.

Both books could be used by a linear algebraist to learn about this area of applied linear algebra. Some readers, of either book, would probably have a little trouble at first getting used to some of the engineering terminology which is not always explicitly defined. Both books are knowledgeable, up to date, and well written with a discussion of open problems. The choice of which to read would depend on the readers preference for filtering or control theory applications.

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