$^1S_0$ proton and neutron superfluidity in β-stable neutron star matter

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Abstract

We investigate the effect of a microscopic three-body force on the proton and neutron superfluidity in the $^1S_0$ channel in β-stable neutron star matter. It is found that the three-body force has only a small effect on the neutron $^1S_0$ pairing gap, but it suppresses strongly the proton $^1S_0$ superfluidity in β-stable neutron star matter.
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Superfluidity plays an important role in understanding a number of astrophysical phenomena in neutron stars [1–10]. It is generally expected that the cooling processes via neutrino emission [5–7], the properties of rotating dynamics, the post-glitch timing observations [8,9], the possible vertex pinning [10] of a neutron star are rather sensitive to the presence of neutron and proton superfluid phases as well as to their pairing strength. For instance, since the paired nucleons do not contribute to thermal excitations, the proton superfluidity could suppress considerably the neutrino emis-

sion processes, and consequently affects the neutrino cooling rate of a neutron star remarkably.

Since the neutron and proton superfluidity properties in neutron stars are related only indirectly to the observations, reliable and precise theoretical predictions based on microscopic many-body approaches are highly desirable. In Refs. [11,12] it is reported by using the Brueckner–Hartree–Fock (BHF) calculations based on purely two-body nucleon–nucleon (NN) interactions that the $^1S_0$ neutron superfluid can be formed only in the low-density region ($\rho_B < 0.1$ fm^{-3}) with a maximal gap value of about 2.8 MeV at a Fermi momentum $k_F \simeq 0.8$ fm^{-1}, while due to the small proton fraction in β-stable matter the proton superfluid phase in the $^1S_0$ channel may extend to much

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higher baryon densities up to $\rho_B \sim 0.4$ fm^{-3} with a maximal pairing gap of about 0.9 MeV. In Ref. [13], the relativistic effects on the superfluidity in β-stable neutron star matter have been investigated by using the Dirac–BHF (DBHF) approach and the one-boson-exchange NN interaction (BONN potential) [14]. It was found that the relativistic effect on the proton superfluidity in the $^1S_0$ channel is very weak, while the magnitude of the neutron pairing gap in the $^3P_2$ channel is reduced strongly due to relativistic corrections to the single-particle (s.p.) energies.

The pairing correlations in nuclear medium are essentially related to the underlying NN interaction and their magnitude is determined by the competition between the repulsive short-range and attractive long-range parts of the interaction. The solution of the gap equation is extremely sensitive to the medium modifications of both the bare NN interaction and the s.p. energy. These medium effects on nuclear pairing are extremely difficult to study on a microscopic level and have motivated an extensive investigation by many authors [15,16]. Three-body forces, which turn out to be crucial for reproducing the empirical saturation properties of nuclear matter in a non-relativistic microscopic approach [17–19], are expected to modify strongly the in-vacuum NN interaction, especially the short-range part [17,19,20]. However, their effects on the superfluidity properties in neutron stars have not yet been well investigated. The aim of this Letter is devoted to the influence of three-body forces on the $^1S_0$ neutron and proton superfluid phases in β-stable matter.

For such a purpose, we shall not go beyond the BCS framework. In this case, the pairing gap which characterizes the superfluidity in a homogeneous Fermi system is determined by the standard BCS gap equation [21], i.e.,

$$\Delta_{\vec{k}} = -\sum_{\vec{k}'} v(\vec{k}, \vec{k}') \frac{1}{2E_{\vec{k}'}} \Delta_{\vec{k}'}, \tag{1}$$

where $v(\vec{k}, \vec{k}')$ is the bare NN interaction in momentum space, $E_{\vec{k}} = \sqrt{(\epsilon_{\vec{k}} - \epsilon_F)^2 + \Delta_{\vec{k}}^2}$, $\epsilon_{\vec{k}}$ and ϵ_F being the s.p. energy and its value at the Fermi surface, respectively.

In the BCS gap equation, the most important ingredients are the NN interaction $v(\vec{k}, \vec{k}')$ and the neutron and proton s.p. energies $\epsilon_{\vec{k}}$ in β-stable matter.

For the NN interaction, we adopt the Argonne AV18 two-body interaction [22] plus a microscopic three-body force (TBF) [17]. The TBF is constructed self-consistently with the AV18 two-body force by using the meson-exchange current approach [18] and it contains the contributions from different intermediate virtual processes such as virtual nucleon–antinucleon pair excitations, and nucleon resonances (for details, see Ref. [17]). The TBF effects on the equation of state (EOS) of nuclear matter and its connection to the relativistic effects in the DBHF approach have been reported in Ref. [18]. The influence of the TBF on Landau parameters in nuclear matter and on the neutrino mean free path in neutron stars has also been explored [23].

The proton fraction and the s.p. energies in β-stable matter are calculated by using the BHF approach for isospin asymmetric nuclear matter [24]. In solving the Bethe–Goldstone equation for the G-matrix, the continuous choice [25] for the auxiliary potential is adopted since it provides a much faster convergence of the hole-line expansion than the gap choice [26]. The effect of the TBF is included in the self-consistent Brueckner procedure along the same lines as in Refs. [17,18], where an effective two-body interaction is constructed to avoid the difficulty of solving the full three-body problem. A detailed description and justification of the method are discussed in Refs. [17,18]. Here we simply write down the equivalent two-body potential in r-space

$$\begin{aligned}
\langle \vec{r}_1 \vec{r}_2 | V_3 | \vec{r}_1' \vec{r}_2' \rangle \\
= \frac{1}{4} \text{Tr} \sum_n \int d\vec{r}_3 \, d\vec{r}_3' \, \phi_n^*(\vec{r}_3') \phi_n(r_3) \\
\times \left(1 - \eta(r_{23}')\right)\left(1 - \eta(r_{13}')\right) W_3\left(\vec{r}_1' \vec{r}_2' \vec{r}_3' | \vec{r}_1 \vec{r}_2 \vec{r}_3\right) \\
\times \left(1 - \eta(r_{13})\right)\left(1 - \eta(r_{23})\right),
\end{aligned} \tag{2}$$

where the trace is taken with respect to the spin and isospin of the third nucleon, and $\eta(r)$ is the defect function. According to Eq. (2) the effective two-body force is obtained by averaging the three-body force over the wave function of the third nucleon taking into account the correlations between this nucleon and the two others. Due to its dependence on the defect function the effective two-body force is calculated self-consistently along with the G-matrix and the auxiliary potential at each step of the iterative BHF procedure.

Table 1
Proton fractions Y^p in β-stable matter

ρ_B (fm^{-3})	Y^p	
	BHF (AV18)	BHF (AV18 + TBF)
0.005	0.0042	0.0042
0.009	0.0078	0.0077
0.020	0.0093	0.0091
0.030	0.0138	0.0132
0.050	0.0187	0.0181
0.070	0.0225	0.0218
0.085	0.0247	0.0252
0.100	0.0279	0.0280
0.140	0.0332	0.0353
0.170	0.0382	0.0432
0.210	0.0471	0.0570
0.250	0.0558	0.0731
0.300	0.0667	0.0944
0.340	0.0746	0.1162
0.400	0.0872	0.1499
0.450	0.0970	0.1785

Fig. 1. Neutron $^1S_0$ energy gap in β-stable matter.

With the obtained EOS of asymmetric nuclear matter the proton fraction Y^p for a given total baryon density ρ_B in β-stable matter can be calculated according to the charge-neutrality and the equilibrium condition with respect to weak interaction [18]. The calculated proton fractions are listed in Table 1, where the first column gives the total baryon densities, the second column and the third column present the corresponding proton fractions obtained by using the AV18 two-body interaction and the AV18 plus the TBF, respectively. Inclusion of the TBF in the calculations increases the proton fractions at high densities. This has to be attributed to the influence of the TBF on the EOS of asymmetric nuclear matter as verified in Ref. [18], where the possible implications for the neutron star cooling mechanisms were also discussed.

To solve the gap equation, we follow the scheme given in Ref. [11], where it is shown that the gap equation can be split into two coupled equations,

$$\Delta_{\vec{k}} = - \sum_{k' \leqslant k_c} \widetilde{V}(\vec{k}, \vec{k}') \frac{1}{2E_{\vec{k}'}} \Delta_{\vec{k}'}, \tag{3}$$

$$\widetilde{V}(\vec{k}, \vec{k}') = V(\vec{k}, \vec{k}') - \sum_{k'' \geqslant k_c} \frac{V(\vec{k}, \vec{k}'')}{2E_{\vec{k}''}} \widetilde{V}(\vec{k}'', \vec{k}'), \tag{4}$$

where the effective interaction $\widetilde{V}$, arising from the introduction of a cutoff k_c in momentum space, sums up a series of ladder diagrams analogous to the Bethe–
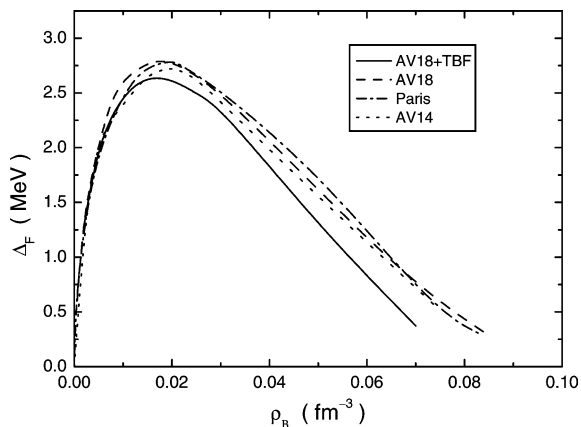
Goldstone equation and it is quite sensitive to the tail ($k > k_c$) of the NN interaction, which reflects the short-range part of the nuclear force.

In order to numerically investigate the effect of the TBF we have solved the gap equation, by adding the effective TBF given in Eq. (2) to the bare AV18 two-body force. At the same time the s.p. energy spectrum $\epsilon_{\vec{k}}$ appearing in the gap equation is computed from the BHF approach by using the AV18 plus the same TBF.

Fig. 1 shows the neutron energy gap in the $^1S_0$ partial wave channel $\Delta_F = \Delta(k_F)$ as a function of the total baryon density ρ_B. The dashed curve is obtained by adopting the pure AV18 two-body interaction only, while the solid curve is predicted by using the AV18 plus the TBF. For comparison, the dotted and dot-dashed curves are the energy gaps reported in Ref. [11] using the Argonne AV14 and the Paris potentials, respectively. It is seen that the results calculated with the three different two-body interactions agree well with each other, i.e., the neutron superfluidity phase in the $^1S_0$ channel can only occur in the low-density region ($\rho_B < 0.1$ fm^{-3}) of neutron stars with a maximal gap value of about 2.8 MeV peaked at a Fermi momentum $k_F \simeq 0.8$ fm^{-1} (the corresponding total baryon density is $\rho_B \simeq 0.02$ fm^{-3}). The TBF effect is quite small, i.e., almost negligible at relatively low density and a slight suppression of the gap as increasing density. This result is expected from the low density for the $^1S_0$ neutron superfluidity, since three-body forces are invented to take, in an effective way, the non-nucleonic degrees of freedom in nuclear medium into account and become significant only at
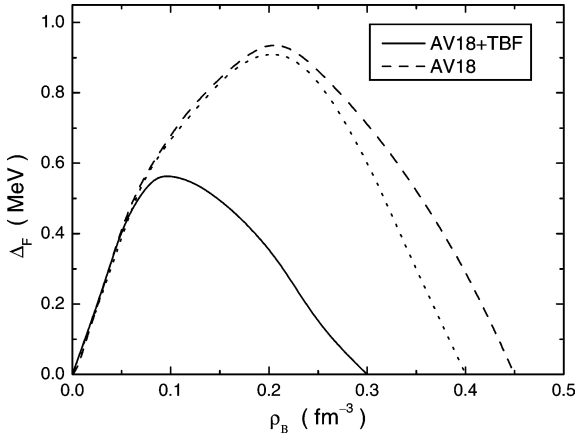
Fig. 2. Proton $^1S_0$ pairing gap in β-stable matter. The solid curve is predicted by using the AV18 plus the TBF, and the dashed curve by using the AV18 two-body force only. The dotted curve is calculated with the pure AV18 for the NN interaction in the gap equation and with inclusion of the TBF in the BHF calculation.

high densities, i.e., around and above the empirical saturation density [19].

In Fig. 2 is reported the proton $^1S_0$ energy gap in β-stable matter. The solid curve is obtained by including the TBF while the dashed one by using the pure AV18 two-body force only. Without the TBF, our result is in good agreement with the predictions by adopting the BONN potential [12,13], the AV14 and the Paris potentials [11]. As compared to the neutron $^1S_0$ superfluidity, the proton $^1S_0$ superfluid phase extends to much higher densities but with a smaller peak gap value around $\rho_B \sim 0.2$ fm^{-3}. The former is a direct consequence of the small proton fraction in β-stable matter, and the latter stems from the different s.p. potentials for neutrons and for protons [11]. As is known, in isospin highly asymmetric nuclear matter like β-stable matter, the proton s.p. potential is much deeper than the neutron one [18,20].

The effects of the TBF are twofold as shown in Fig. 2. One is a strong reduction of the peak value of the gap from ~ 0.95 MeV to ~ 0.55 MeV and a remarkable shift of the peak to a much lower baryon density from ~ 0.2 fm^{-3} to ~ 0.09 fm^{-3}. The another is that the TBF leads to a noticeable shrinking of the density region of the superfluid phase from $\rho_B \leqslant 0.45$ fm^{-3} to $\rho_B \leqslant 0.3$ fm^{-3}. The above predicted TBF suppression of the $^1S_0$ proton superfluidity in β-stable matter appears inconsistent with the small

proton fractions which correspond to small proton densities in the matter. However, since proton pairs are embedded in the medium of neutrons and protons, both the surrounding protons and neutrons contribute to the TBF renormalization of the proton–proton interaction. This means that the relevant density to the TBF effect is the total baryon density, but not the proton one. One can verify from Fig. 2 that the strongest suppression of the energy gap is mainly in the region $\rho_B \geqslant \rho_0$, ρ_0 being the empirical saturation density of nuclear matter, and the reduction of the gap increases rapidly as increasing the total baryon density.

It is worth noticing the discrepancy between the TBF effect and the relativistic effect [13] on the $^1S_0$ proton superfluidity in β-stable matter. The investigation of Ref. [13] shows that the relativistic effect reduces remarkably the $^3P_2$ neutron gap, but the differences between the relativistic and non-relativistic proton gaps in the $^1S_0$ channel are quite small. One possible reason concerns the more or less different mechanisms involved. In the DBHF approach [19,27], the medium renormalization of the bare NN interaction is taken into account via the Dirac spinor which is dressed in nuclear medium (which can be traced to the virtual excitation of nucleon–antinucleon pairs [28]), but the most important effect on the pairing gaps (for instance, the $^3P_2$ neutron gap) comes from the relativistic modification of the s.p. energies [13].

The TBF is expected to influence the superfluidity phases in β-stable matter via three different ways. First it renormalizes the bare nucleon–nucleon interaction. Second it modifies the s.p. energy spectrum in the gap equation and finally the inclusion of the TBF changes the predicted proton fractions as shown in Table 1. To see which is the most important mechanism responsible for the strong suppression of the $^1S_0$ proton energy gaps, we include the TBF in the BHF calculations to obtain the proton fractions and s.p. energies, but adopt only the pure AV18 as the NN interaction $v(\vec{k}, \vec{k}')$ in the gap equation, i.e., Eq. (1). The results are shown by the dotted curve in Fig. 2. One can see that the combined effects via the proton fractions and the s.p. energies are relatively small, i.e., a slight reduction, and mainly in the higher density region ($\rho_B > 0.3$ fm^{-3}) where the TBF modifications of the proton fractions (Table 1) and the proton s.p. energies [18] become appreciably larger. Hence, the

strongest effect stems from the TBF renormalization of the NN interaction in the medium.

In summary, we have investigated the influence of the TBF on the neutron and proton pairing gaps in the $^1S_0$ channel in β-stable neutron star matter. It is shown that the TBF has only a weak effect on the neutron $^1S_0$ superfluidity phase, i.e., a slight reduction of the energy gap, due to the low-density region concerned. However it suppresses strongly the proton superfluidity in the $^1S_0$ channel induced by the two-body NN interaction. The peak value of the proton $^1S_0$ energy gap is reduced by about 50% from ~ 0.95 MeV to ~ 0.55 MeV and shifted to a much lower density by the inclusion of the TBF. The density region for the superfluid phase is also remarkably shrunken as compared to the pure two-body force prediction. It is shown that this suppression is mainly related to the TBF renormalization of the $^1S_0$ two-body interaction.

Besides the TBF effects, the medium renormalizations of the NN interaction and the s.p. energies, i.e., the screening effects such as the polarization effects and dispersive effects, may also influence largely the superfluidity properties in nuclear medium. Up to now, all investigations in the literature [1,2,15,16] have predicted a reduction of the BCS superfluidity gap in the $^1S_0$ channel. Therefore we expect that the screening effects may further suppress the $^1S_0$ proton superfluidity in β-stable matter.

Since the relativistic effects lead to a strong suppression of the $^3P_2$ neutron superfluidity in β-stable matter [13], the $^1S_0$ proton superfluidity becomes critical in determining the outcome of neutron star cooling. It is expected in Ref. [13] that the main suppression (of the neutrino production) comes from the superfluid proton in the $^1S_0$ state. In this letter, the predicted suppression of the $^1S_0$ proton pairing gap does not favor the proton superfluid phase which is expected to suppress the modified URCA processes in the interior of a neutron star. This is compatible with the recent result of Link [7] derived from the observations of long-period precession in isolated pulsars.

Acknowledgements

References

[1] For a recent review, see, D.J. Dean, M. Hjorth-Jensen, Rev. Mod. Phys. 75 (2003) 607 and references therein.

[2] U. Lombardo, H.-J. Schulze, in: D. Blaschke, N.K. Glendenning, A. Sedrakian (Eds.), Lecture Notes in Physics, vol. 578, Springer, Berlin, 2001, p. 30.

[3] M. Prakash, Phys. Rep. 242 (1994) 191.

[4] C.J. Pethick, Rev. Mod. Phys. 64 (1992) 1133; C.J. Pethick, D.G. Ravenhall, Annu. Rev. Nucl. Part. Phys. 45 (1995) 429.

[5] D. Page, Astrophys. J. 428 (1994) 250.

[6] H. Heiselberg, M. Hjorth-Jensen, Phys. Rep. 328 (2000) 237.

[7] B. Link, Phys. Rev. Lett. 91 (2003) 101101.

[8] S.L. Shapiro, S.A. Teukosky, Black Holes, White Dwarfs and Neutron Stars, Wiley, New York, 1983.

[9] J.A. Sauls, in: H. Ogelman, E.P.J. van den Henvel (Eds.), Timing Neutron Stars, Dordrecht, Kluwer, 1989, p. 457.

[10] D. Pines, M.A. Alpar, Nature 316 (1985) 27.

[11] M. Baldo, J. Cugnon, A. Lejeune, U. Lombardo, Nucl. Phys. A 515 (1990) 409.

[12] Ø. Elgaroy, L. Engvik, M. Hjorth-Jensen, E. Osnes, Nucl. Phys. A 604 (1996) 466.

[13] Ø. Elgaroy, L. Engvik, M. Hjorth-Jensen, E. Osnes, Phys. Rev. Lett. 77 (1996) 1428.

[14] R. Machleidt, Adv. Nucl. Phys. 19 (1989) 185.

[15] J.W. Clark, C.G. Kallman, C.-H. Yang, D.A. Chakkalakal, Phys. Lett. B 61 (1976) 331; T.L. Ainsworth, J. Wambach, D. Pines, Phys. Lett. B 222 (1989) 173; C.J. Pethick, D.G. Rawenhall, in: Annals of the New York Academy of Sciences, vol. 647, New York Academy of Sciences, New York, 1991, p. 503; J. Wambach, T.L. Ainsworth, D. Pines, Nucl. Phys. A 555 (1993) 128; J.M.C. Chen, J.W. Clark, R.D. Dave, V.V. Khodel, Nucl. Phys. A 555 (1993) 59; A. Rabhi, R. Bennaceur, G. Chanfray, P. Schuck, Phys. Rev. C 66 (2002) 064315; A. Schwenk, B. Friman, G.E. Brown, Nucl. Phys. A 713 (2003) 191.

[16] U. Lombardo, in: M. Baldo (Ed.), Nuclear Methods and the Nuclear Equation of State, World Scientific, Singapore, 1999, p. 458; P. Bozek, Phys. Rev. C 62 (2000) 054316;

U. Lombardo, P. Schuck, W. Zuo, Phys. Rev. C 64 (2001) 021301(R);

M. Baldo, A. Grasso, Phys. Lett. B 485 (2000) 155;

M. Baldo, U. Lombardo, H.-J. Schulze, W. Zuo, Phys. Rev. C 66 (2002) 054304;

C.W. Shen, U. Lombardo, P. Schuck, W. Zuo, N. Sandulescu, Phys. Rev. C 67 (2003) 061303(R).

[17] P. Grangé, A. Lejeune, M. Martzolff, J.-F. Mathiot, Phys. Rev. C 40 (1989) 1040.

[18] A. Lejeune, U. Lombardo, W. Zuo, Phys. Lett. B 477 (2000) 45;

W. Zuo, A. Lejeune, U. Lombardo, J.-F. Mathiot, Nucl. Phys. A 706 (2002) 418.

[19] R. Machleidt, Adv. Nucl. Phys. 19 (1989) 189.

[20] W. Zuo, U. Lombardo, H.J. Schulze, C.W. Shen, Phys. Rev. C 66 (2002) 037303.

[21] P. Ring, P. Schuck, The Nuclear Many Body Problem, Springer, New York, 1980.

[22] R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, Phys. Rev. C 51 (1995) 38.

[23] W. Zuo, C.W. Shen, U. Lombardo, Phys. Rev. C 67 (2003) 037301;

U. Lombardo, C.W. Shen, N. Van Giai, W. Zuo, Nucl. Phys. A 722 (2003) 532c.

[24] W. Zuo, I. Bombaci, U. Lombardo, Phys. Rev. C 60 (1999) 024605.

[25] J.P. Jeukenne, A. Lejeune, C. Mahaux, Phys. Rep. C 25 (1976) 83.

[26] H.Q. Song, M. Baldo, G. Giansiracusa, U. Lombardo, Phys. Rev. Lett. 81 (1998) 1584.

[27] B. ter Haar, R. Malfliet, Phys. Rep. 149 (1987) 208;

M. Hjorth-Jensen, T.T.S. Kuo, E. Osnes, Phys. Rep. 261 (1995) 125;

F. de Jong, H. Lenske, Phys. Rev. C 58 (1998) 890;

C. Fuchs, T. Waindzoch, A. Faessler, D.S. Kosov, Phys. Rev. C 58 (1998) 2022.

[28] G.E. Brown, W. Weise, G. Baym, J. Speth, Comments Nucl. Part. Phys. 17 (1987) 39.