de Sitter vacua from matter superpotentials

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Abstract

Consistent uplifting of AdS vacua in string theory often requires extra light degrees of freedom in addition to those of a (Kähler) modulus. Here we consider the possibility that de Sitter and Minkowski vacua arise due to hidden sector matter interactions. We find that, in this scheme, the hierarchically small supersymmetry breaking scale can be explained by the scale of gaugino condensation and that interesting patterns of the soft terms arise. In particular, a matter-dominated supersymmetry breaking scenario and a version of the mirage mediation scheme appear in the framework of spontaneously broken supergravity.

1. Introduction

Fluxes on an internal manifold allow one to stabilize most moduli [1], but usually not all. In particular, in the KKLT model [2], the overall Kähler modulus $T$ is not fixed by the fluxes and is stabilized by non-perturbative effects such as gaugino condensation [3]. The corresponding superpotential

$$W = W_0 + Ae^{-aT}$$

leads to an AdS supersymmetric minimum. To obtain a realistic vacuum, this minimum has to be uplifted. The original KKLT proposal was to use an explicit SUSY breaking term induced by anti-D3 branes,

$$\Delta V = \frac{k}{(T + \bar{T})^2},$$

(2)

to do the uplifting. A somewhat more appealing possibility is to employ the supersymmetric D-terms for this purpose [4],

$$\Delta V = \frac{1}{2g^2}D^2.$$  

However, a supersymmetric minimum cannot be uplifted by the D-terms [5]. It is possible to uplift non-supersymmetric minima which arise once $\alpha'$ corrections [6] have been included [7]. In any case, this procedure relies on the presence of charged matter in the effective theory [8]. Thus, it appears that the uplifting within the supergravity framework requires extra degrees of freedom in addition to those of a Kähler modulus. This perhaps is not always the case, but at least it is true for simple Kähler potentials. Then one may ask whether it is necessary to use the D-terms at all: de Sitter vacua may simply result from the superpotential interactions with the extra degrees of freedom. We note also that in models with the D-term uplifting it would be very difficult to obtain a hierarchically small SUSY breaking scale [8,9].

In this Letter, we study the possibility that dS and Minkowski vacua arise due to interactions of hidden matter. We identify the local superpotential structures realizing this situation and study the resulting soft SUSY breaking terms. We find that interesting patterns arise. In particular, a matter-dominated SUSY breaking scenario and a version of the mirage mediation scheme appear in the context of spontaneously broken supergravity.

1 We thank K. Choi for pointing out generic difficulties of D-term uplifting with hierarchically small gravitino mass.
2. Minkowski and de Sitter vacua due to matter interactions

Let us start by reviewing the supergravity formalism. The supergravity scalar potential is expressed in terms of the function

\[ G = K + \ln |W|^2, \tag{4} \]

with \( K \) and \( W \) being the Kähler potential and the superpotential, respectively, as

\[ V = e^G (G_i G_{j}G^{ij} - 3). \tag{5} \]

Here the subscript \( i \) denotes differentiation with respect to \( i \)th field and \( G_{i}G_{j}G^{ij} \) is the inverse Kähler metric. The SUSY breaking F-terms are found from

\[ F^{m} = e^{G/2}G^{\bar{n}\bar{m}}G_{\bar{n}}, \tag{6} \]

evaluated at the minimum of the potential. The gravitino mass is

\[ m_{3/2} = e^{G/2}. \tag{7} \]

In what follows, we study under which circumstances de Sitter and Minkowski vacua arise in supergravity models involving a modulus (\( T \)) and a matter field (\( C \)).

2.1. No go with a single modulus

In this subsection, we show that de Sitter vacua are not possible in models with a single modulus as long as the Kähler potential takes on its classical form

\[ K = -a \ln (T + \bar{T}), \tag{8} \]

Here \( 1 \leq a \leq 3 \) depending on the nature of the modulus.

The scalar potential reads

\[ V = \frac{1}{(T + \bar{T})^a} \left( \frac{1}{a} |W_T(T + \bar{T}) - a W|^2 - 3|W|^2 \right). \tag{9} \]

The stationary point condition \( \partial V / \partial T = 0 \) is then

\[ \left( \frac{\bar{W}_T(T + \bar{T}) - a\bar{W}}{W_T(T + \bar{T}) + (1 - a)W_T} \right) \left( \frac{T + \bar{T}}{a} - W_T(T + \bar{T}) \right) \left( \frac{3 - a}{a} + \bar{W}(3 - a) \right) = 0. \tag{10} \]

To analyze stability of the stationary point, we need the second derivatives of the potential. Using the above equation, one can write \( \partial^2 V / \partial T \partial\bar{T} \) in a compact form,

\[ \frac{\partial^2 V}{\partial T \partial\bar{T}} = -\frac{2}{(T + \bar{T})^2} \left( V_0 + \frac{3 - a}{(T + \bar{T})^a} |W|^2 \right). \tag{11} \]

For \( a \leq 3 \) and \( V_0 \geq 0 \), this expression is non-positive which implies that at least one of the eigenvalues of the Hessian is negative or zero. Thus realistic dS/Minkowski minima are not possible. This result was also found numerically in [10] (see also [11]).

Our conclusion relies on the classical form of the Kähler potential. In particular, perturbative \( a' \) corrections to the Kähler potential allow for dS vacua [12]. Also, the separation of the \( G \)-function into the Kähler potential and the superpotential is ambiguous. With a fixed Kähler potential, integrating out heavy fields may lead to effects which cannot be described by a holomorphic superpotential [13]. In this work, we will assume that these effects are subdominant in the region of interest. Then extra degrees of freedom are required to obtain dS vacua. In what follows, we will study the case when these additional degrees of freedom are provided by matter fields and analyze the local superpotential structure allowing for dS/Minkowski vacua.

2.2. A modulus and a matter field

Suppose that the low energy theory involves a modulus \( T \) and a matter field \( C \). The corresponding Kähler potential is

\[ K = -3 \ln (T + \bar{T}) + |C|^2, \tag{12} \]

where we have assumed for definiteness that \( T \) is an overall Kähler modulus and \( C \) has an effective “modular weight” zero. Systems of this type arise in type IIB and heterotic string theory. The effective superpotential obtained by integrating out heavy moduli and matter fields is assumed to be of the form

\[ W = \sum_i \omega_i(C)e^{-\omega_i T} + \phi(C), \tag{13} \]

where the sum runs over gaugino condensates [14]. The functions \( \omega_i(C) \) and \( \phi(C) \) arise due to perturbative and non-perturbative interactions in the process of integrating out heavy fields. We will treat them as some generic functions since only their local behaviour is important for our purposes. In particular, we will allow for linear terms \( \alpha C \) which can arise from interactions with heavy matter fields \( s_i, \Delta W \sim C(s_1 \ldots s_N)e^{-\alpha T}. \)

We assume that \( C \) is a singlet under unbroken gauge symmetries.

The supergravity scalar potential is given by

\[ V = e^{C\bar{C}} \left( \frac{1}{3} |W_T(T + \bar{T}) - 3\bar{W}|^2 + |W_C + \bar{W}C|^2 - 3|W|^2 \right). \tag{14} \]

It is convenient to introduce

\[ f^T \equiv \bar{W}_T(T + \bar{T}) - 3\bar{W}, \]
\[ f^C \equiv \bar{W}_C + \bar{W}C, \tag{15} \]

such that

\[ F^T = \frac{T + \bar{T}}{3W} m_{3/2} f^T, \]
\[ F^C = \frac{1}{W} m_{3/2} f^C. \tag{16} \]

Then the stationary point conditions read
\[
\frac{\partial V}{\partial C} = V\tilde{C} + \frac{e^{C\tilde{C}}}{(T + \tilde{T})^3} \left[ \frac{1}{3} (W_{TC}(T + \tilde{T}) - 3W_C)f_T^3 \\
+ (W_{CC} + W_C\tilde{C})f_C + \tilde{W}f_C^3 - 3W_C\tilde{W} \right] \\
= 0,
\]

\[
\frac{\partial V}{\partial T} = -\frac{3}{T + \tilde{T}} V + \frac{e^{C\tilde{C}}}{(T + \tilde{T})^3} \left[ \frac{1}{3} (W_{TT}(T + \tilde{T}) - 2W_T)f_T^3 \\
+ \frac{1}{3} \tilde{W}f_T^3 + (W_{TC} + W_C\tilde{C})f_C^3 - 3W_T\tilde{W} \right] \\
= 0. \tag{17}
\]

We are interested in local behaviour of the scalar potential. Without loss of generality, assume that the above equations are satisfied at

\begin{equation}
C = 0, \quad T = T_0, \tag{18}
\end{equation}

then Eq. (17) translates into relations among the derivatives of the superpotential at that point. For the analysis of local behaviour of the scalar potential, we only need derivatives of the superpotential up to order three. Then \( W \) can be written as

\[
W = W_0 + W_CC + W_T(T - T_0) + \frac{1}{2} W_{CCC}C^2 \\
+ W_{TCC}(T - T_0) + \frac{1}{2} W_{TTT}(T - T_0)^2 + \frac{1}{6} W_{CCC}C^3 \\
+ \frac{1}{2} W_{TCC}C^2(T - T_0) + \frac{1}{2} W_{TTT}(T - T_0)^2 \\
+ \frac{1}{6} W_{TTT}(T - T_0)^3. \tag{19}
\]

Given vacuum energy \( V_0 \) and supersymmetry breaking parameters \( f_T, f_C \) which measure the balance between modulus and matter SUSY breaking as input, Eq. (17) identifies local superpotentials realizing this situation. Stability considerations impose further constraints on the superpotential structure (see [11,15] on the related discussion).

The superpotential expansion parameters can be expressed in terms of \( f_T, f_C, V_0 \) or, using Eq. (16), in terms of \( f_T, f_C, V_0 \) (up to an irrelevant phase) as

\[
|W_0| = \frac{1}{\sqrt{3}} \left( \frac{1}{3} |f_T|^2 + |f_C|^2 - V_0(T_0 + \tilde{T}_0)^3 \right)^{1/2},
\]

\[
W_C = f_C, \quad W_T = \frac{3W_0 + f_T}{T_0 + \tilde{T}_0}, \quad W_{CCC} = \frac{1}{3} \left( W_{TC}(T_0 + \tilde{T}_0) - 3f_C \right) f_T + 2\tilde{W}_0 f_C f_C, \quad W_{TCC} = \frac{1}{3} \left( W_{TC}(T_0 + \tilde{T}_0) - 3f_C \right) f_T + 2\tilde{W}_0 f_C f_C.
\]

Here the phase of \( W_0 \) is a free parameter. Also, \( W_{TC} \) is a free parameter as long as \( f_T \neq 0 \). If \( f_T = 0 \), \( W_{TT} \) becomes a free parameter and \( W_{TC} \) is found from

\[
W_{TC} = \frac{3}{f_T} \left( (T_0 + \tilde{T}_0)^2 V_0 + W_T \tilde{W}_0 \right). \tag{21}
\]

In this Letter, we will only consider the case \( f_C \neq 0 \).

Higher derivatives of the superpotential remain undetermined at this stage. They are constrained by stability considerations. To analyze stability of the stationary point, one can neglect the vacuum energy, \( V_0 \ll 1 \), and use the following second derivatives of the potential

\[
(W_0 + \tilde{T}_0)^3 V_{CC} \tag{20}
\]

\[
= \frac{1}{3} |W_{TC}(T_0 + \tilde{T}_0) - 3W_C|^2 + |W_{CCC}|^2 + |W_0|^2 - |f_C|^2, \tag{23}
\]

\[
(W_0 + \tilde{T}_0)^3 V_{CC} \tag{22}
\]

\[
= \frac{1}{3} (W_{TCC}(T_0 + \tilde{T}_0) - 3W_{CC}f_T + W_{CCC}f_C - W_{CC}f_C), \tag{24}
\]

\[
(W_0 + \tilde{T}_0)^3 V_{TC} \tag{25}
\]

\[
= \frac{1}{3} (W_{TTC}(T_0 + \tilde{T}_0) - 2W_T f_T + W_{TCC} f_T) f_T \\
+ \frac{1}{3} W_{TTC}(T_0 + \tilde{T}_0) - 3W_{CC} f_T + W_{TCC} f_C, \tag{26}
\]

\[
(W_0 + \tilde{T}_0)^3 V_{TC} \tag{27}
\]

The eigenvalues of \( \partial^2 V/\partial x_i \partial \tilde{x}_j \) must be positive. This constrains \( W_{TC} \) and higher derivatives of the superpotential. The general formulae are unilluminating, so let us focus on the cases of interest, in particular, matter-dominated SUSY breaking: \( 0 \ll |f_T| \ll |f_C| \).

Consider the limit \( |f_T| \ll |f_C| \). From Eq. (20), this corresponds to large \( W_{TT} \). Then \( |V_{T\tilde{T}}| \gg |V_{CC}|, |V_{C\tilde{C}}| \). To obtain a particularly simple structure of \( \partial^2 V/\partial x_i \partial \tilde{x}_j \), let us choose (otherwise unconstrained) \( W_{CCC}, W_{TCC}, W_{TTC} \) such that the matrix elements \( W_{TT}, V_{CC}, V_{TC} \) are small. Then we have

\[
\frac{\partial^2 V}{\partial x_i \partial \tilde{x}_j} \approx \frac{1}{(T_0 + \tilde{T}_0)^3} \begin{pmatrix} |A|^2 & 0 & Aa & 0 \\
0 & |A|^2 & 0 & Aa* \\
Aa & 0 & |a|^2 + \Delta & 0 \\
0 & Aa* & 0 & |a|^2 + \Delta \end{pmatrix}, \tag{23}
\]
where

\[ A \equiv \frac{1}{\sqrt{3}} \bar{W}_{TT}(T_0 + \bar{T}_0), \]

\[ a \equiv \frac{1}{\sqrt{3}} \left( W_{TC}(T_0 + \bar{T}_0) - 3W_C \right), \]

\[ \Delta \equiv 2|W_0|^2, \]

so that \(|A| \gg |a|, \Delta\). The order of the indices is defined by \((x_1, x_2, x_3, x_4) = (\bar{T}, T, \bar{C}, C)\). All of the subdeterminants of this matrix are positive, hence the eigenvalues are positive. This proves that the stationary point is a local minimum.

In the case \(f^T = 0\), \(W_{TT}\) is a free parameter and can be taken to be large. Then the same argument applies. In both cases, the spectrum consists of 2 heavy states with masses of order \(|W_{TT}|\) and 2 lighter states with masses of order \(|W_0|\).

The above local structure can be translated into constraints on the parameters of the original superpotential (13). We note that large \(|W_{TT}|\) arises naturally in racetrack models since differentiation by \(T\) brings down the factor \(1/(\beta\text{ function})\) and the moduli are heavy compared to \(m_{3/2}\) (see e.g. [14,16]),

\[ |W_{TT}| \sim \alpha^2 |W_0| \gg |W_0| \]

with \(\alpha \sim 1/(\beta\text{ function})\). This means that \(T\) is stabilized close to a supersymmetric point since \(\partial V/\partial T = 0\) (cf. Eq. (17)) implies

\[ W_{TT} F^T + \text{smaller terms} = 0, \]

such that \(F^T \sim m_{3/2}^2/|W_{TT}| \sim m_{3/2}/\alpha^2\). Further, the scale of SUSY breaking is explained by the scale of gaugino condensation, as long as \(\phi(C)\) is negligible.

2.3. Numerical example

If the observable matter is placed on D7 branes in type IIB constructions, the Kähler modulus \(T\) should be stabilized at \(\text{Re} \, T_0 = 2\) as required by the observed gauge couplings. Consider now an example \(f^C = \epsilon\), \(f^T = 0.03\epsilon\), \(V_0 \simeq 0\) for small \(\epsilon\) generated by gaugino condensation. An example of the local superpotential structure realizing this situation is given by

\[ W \simeq \epsilon \left[ 0.577 + C + 0.441(T - 2) + 0.592C^2 \right. \]

\[ + 9.595(T - 2)^2 + 0.114C^3 + 0.220C^2(T - 2) \]

\[ + 46.451(T - 2)^3 \right]. \]

The shape of the potential around the minimum is shown in Fig. 1.

Similarly, one can construct examples with minima at \(T_0 \approx 100\), where the supergravity approximation is trustable.

The main lesson here is that, unlike in the case of a single modulus, dS/Minkowski vacua with interesting SUSY breaking patterns can be realized in the framework of spontaneously broken supergravity.

3. Patterns of the soft masses

Let us now study the emerging patterns of the observable matter soft terms. The scale of the soft terms is set by the gravitino mass

\[ m_{3/2} = \frac{|W_0|}{(T_0 + \bar{T}_0)^{3/2}}, \]

which is in turn generated via gaugino condensation, \(|W_0| \sim \left(\sum_i e^{-\alpha_i T}\right)\), as long as the matter superpotential \(\phi(C)\) is negligible. The tree level soft terms are found from the general formulae,

\[ M_a = \frac{1}{2} (\text{Re} \, f_a)^{-1} F^m \partial_m f_a, \]

\[ m_a^2 = m_{3/2}^2 - \vec{F} \vec{m} F^m \partial_m \ln K_a, \]

\[ A_{a\beta\gamma} = F^m \left[ \hat{K}_m + \partial_m \ln Y_{a\beta\gamma} - \partial_m \ln (K_{a\beta} K_{a\gamma}) \right], \]

where \(m\) runs over SUSY breaking fields, \(f_a\) are the gauge kinetic functions, \(K_a\) is the Kähler metric for the observable sector fields and \(\hat{K}_m \equiv \partial_m \hat{K}\) with \(\hat{K}\) being the Kähler potential for the hidden sector fields. The \(\mu\) and \(B\mu\) terms are not listed as their generation mechanism is strongly model-dependent. These formulae are to be amended by loop-suppressed terms such as the anomaly mediated contributions [17].

The gauge kinetic functions are model dependent quantities. Consider, for definiteness, type IIB string theory. For gauge
fields on D7 branes, we have

\[ f_a = T, \]  

while in the case of D3 branes

\[ f_a = \text{const}. \]

The total Kähler potential is given by

\[ K = -3 \ln(T + \bar{T}) + C\bar{C} 
+ \tilde{Q}_i (T + \bar{T})^{n_i}[1 + \xi_i C\bar{C} + O(C^4)], \]  

where \( Q_i \) are the observable fields with “effective modular weights” \( n_i \). Here we include for generality quartic couplings between observable and hidden sector fields, which can be present at tree level or generated radiatively (see e.g. [18]).

The resulting soft terms are

\[ M_o = (0 \text{ or } 1) \times \frac{F_T}{T_0 + \bar{T}_0} + \text{anomaly}, \]

\[ m_a^2 = m_{T/2}^2 + n_a \left( \frac{|F_T|^2}{(T_0 + \bar{T}_0)^2} - \xi_a \right) |F_C|^2 + \text{anomaly}, \]

\[ A_{\alpha\beta\gamma} = -\frac{F_T}{T_0 + \bar{T}_0} [3 + n_\alpha + n_\beta + n_\gamma] + \text{anomaly}, \]

where we have assumed that \( Y_{a\beta\gamma} \) are independent of \( T \) and \( C \). The “anomaly” contributions generally include various loop-suppressed terms (in addition to those due to the super-Weyl anomaly) which result from regularization of the effective SUGRA [19,20] and string threshold corrections [21]. \( F_T \) and \( F_C \) are subject to the constraint

\[ m_{T/2}^2 = \frac{|F_T|^2}{(T_0 + \bar{T}_0)^2} + \frac{1}{3} |F_C|^2. \]

Below we consider two most interesting special cases: matter domination and mirage mediation. These arise when the T-modulus is heavy,

\[ |W_{TT}| \gg |W_0|, |W_T|, |W_C|, \ldots \]  

such that \( T \) is stabilized close to a supersymmetric point. This situation is rather natural for gaugino condensation models due to the smallness of the beta functions of condensing gauge groups (see e.g. [14,16]).

3.1. Matter dominated SUSY breaking

This corresponds to \( F_T = 0 \) such that

\[ M_o = \text{anomaly}, \]

\[ m_a^2 = m_{T/2}^2 (1 - 3\xi_a) + \text{anomaly}, \]

\[ A_{\alpha\beta\gamma} = \text{anomaly}. \]

A particularly simple case is \( \xi_a \sim 0 \). This provides an interesting “regularization” of the traditional anomaly mediation scheme in the sense that it inherits main features of the latter while avoiding tachyonic sfermions. We note that this scenario is different from the moduli-dominated models in the heterotic string in two aspects. First, the cosmological constant here can be made arbitrarily small and positive. Second, the string threshold corrections to the gauge kinetic functions are independent of \( C \) (or, at least, negligible at \( C = 0 \)) and the Kähler anomalies [19,20] do not contribute to the gaugino masses. Therefore, \( M_o \) receive a leading contribution from the super-Weyl anomaly, as in the original version of anomaly mediation [17].

The soft terms exhibit the following hierarchy

\[ M_o, A \ll m_{\text{scalar}}, m_{T/2}, \]  

while for the T-modulus and the hidden matter we have \( m_T \gg m_{T/2} \) and \( m_C \sim m_{T/2} \).

A solid feature of this SUSY breaking scenario is that the LSP is predominantly a wino and the mass splitting between the chargino and the neutralino is small. This leads to spectacular collider signatures such as long lived charged particle tracks [22].

3.2. Mirage mediation

This scenario appears in the case \( F_T/(T_0 + \bar{T}_0) \sim F_C/4\pi^2 \) [5,23]. The modulus and the anomaly contribute to the gaugino masses and the A-terms in comparable proportions. Then the gaugino masses unify at an intermediate “mirage” scale. This is because the Kähler anomalies contributions [19,20] are suppressed at small \( F_T \) and \( C = 0 \) such that the gaugino mass splitting at the high energy scale is proportional to the beta functions. Since the RG running is governed by the same beta functions, this splitting disappears at some intermediate scale.

The resulting soft terms are

\[ M_o = \frac{F_T}{T_0 + \bar{T}_0} + \text{anomaly}, \]

\[ m_a^2 = \Delta_a + (n_a + 3\xi_a) \left( \frac{|F_T|^2}{(T_0 + \bar{T}_0)^2} \right) + \text{anomaly}, \]

\[ A_{\alpha\beta\gamma} = -\frac{F_T}{T_0 + \bar{T}_0} [3 + n_\alpha + n_\beta + n_\gamma] + \text{anomaly}, \]

where \( \Delta_a \equiv (1 - 3\xi_a)m_{T/2}^2 \) and the “anomaly” contribution to the scalar masses subsumes possible 1-loop contributions [20] as well as a mixed modulus-anomaly and 2-loop contributions. In the case \( \xi_a \sim 1/3 \), the scalar masses are also suppressed resulting in the hierarchy

\[ m_{\text{soft}} \ll m_{T/2}, \]

and \( m_T \gg m_C \sim m_{T/2} \). Heavy gravitinos and moduli (\( \gtrsim 30 \) TeV) are desirable from the cosmological perspective since they decay before the nucleosynthesis and do not affect the abundances of light elements [23]. In addition, this scheme avoids the problem with overproduction of gravitinos by heavy moduli [24]. The reason is that, at late times, the energy density of the Universe is dominated by the C field which has a mass \( \sim m_{T/2} \). Therefore, the branching ratio for the C decays into gravitinos is suppressed and the “moduli-induced” gravitino problem [24] is absent.

\[ \text{FT}, \text{C} \]
4. Conclusions

Uplifting AdS vacua in string theory has been a difficult issue. One of the popular proposals consistent with spontaneous SUSY breaking is to use the supersymmetric D-terms. This requires proper consideration of the effects due to charged matter which complicates the analysis.

In this Letter, we have taken an alternative route. Since one has to include matter effects anyway, one may as well consider the possibility that dS and Minkowski vacua arise due to superpotential interactions involving hidden matter. In this Letter, we have identified the local superpotential structures realizing this situation and studied the resulting SUSY breaking.

We find that, within this scheme, the SUSY breaking scale can be explained by the scale of gaugino condensation. We also find that, when the T-modulus is heavy, interesting patterns of the soft terms occur. In particular, a matter-dominated SUSY breaking scenario arises. It provides a “regularization” of the traditional anomaly mediation scheme as it has most features of the latter while avoiding tachyonic sfermions. Finally, we have shown how mirage mediation is realized in the context of spontaneously broken supergravity.

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References