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Application of the covariance on the critical plane for determination of fatigue life under cyclic loading

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Abstract

The paper presents application of the covariance extremum between normal end shear stresses for determination of the critical plane. The fatigue stresses criterion was formulated as a linear combination of normal and shear stresses on the defined critical plane. The weight coefficients occurring in this criterion were determined from fatigue tests in the layer of pure bending and pure torsion. The proposed model was verified while fatigue tests under cyclic proportional and non-proportional bending with torsion (specimens made of aluminium alloy PA6-T4 were tested).

Keywords: critical plane position, fatigue life, multiaxial fatigue

1. Introduction

Multiaxial service loading is usually taken into account while designing of machine elements. In such a case, it is necessary to reduce the complex loading state to the equivalent uniaxial one with use of multiaxial fatigue criteria in order to determine the fatigue life. Three groups of the multiaxial fatigue criteria can be distinguished, namely stress, strain and energy criteria [1]. This division is based on the parameter influencing failure. Some of these criteria are dependent on the critical plane position. There are three methods of critical plane determination: the method of variance, the method of fatigue damage accumulation, and the method of weight functions. The method of variance seems to be the simplest and fastest of them [2]. In this case it is assumed that the critical plane position is defined as the maximum of the variance of normal or shear stresses, depending on the material (elastic-plastic, or brittle) [3]. In some previous papers [4] the critical plane is the plane of the maximum coefficient of correlation between shear and normal stresses. However, this method did not give satisfactory results. The critical plane can be also defined as the plane of the maximum covariance between shear and normal stresses. The aim of this paper is to determine fatigue life of the specimens made of the aluminum alloy PA6 and tested under cyclic loading with use of the covariance extremum on the critical plane according to the stress criterion.

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Nomenclature

α	angle critical plane position
σ_{af}	fatigue limit for bending
$\mu_{\sigma,\tau}$	covariance
σ_{xx}	normal stresses coming from bending
τ_{af}	fatigue limit for torsion
τ_{xy}	shear stresses coming from torsion

2. The criterion of multiaxial fatigue on the critical plane

The equivalent stress according to the criteria of multiaxial random fatigue on the critical plane can be expressed as the sum of normal and shear stresses in this plane

$$\sigma_{eq}(t) = B\tau_{\eta s}(t) + K\sigma_{\eta}(t). \tag{1}$$

The critical plane can be defined by the shear or normal stress, and selection of the coefficients B and K from Eq. (1) depends on the critical plane position and the material.
If the critical plane is defined by the shear stress, Eq. (1) takes the following form

$$\sigma_{eq}(t) = B_1 \tau_{\eta s}(t) + (2\text{-}B_1)\,\sigma_{\eta}(t), \tag{2}$$

where

$$B_1 = \frac{\sigma_{af}}{\tau_{af}} \tag{3}$$

in the case of parallel fatigue characteristics S-N for bending and torsion. This case is typical for elastic-plastic materials (steels, non-ferrous alloys).
When the critical plane is defined by normal stresses, Eq. (1) for the equivalent stress takes the form

$$\sigma_{eq}(t) = B_2\, \tau_{\eta s}(t) + \sigma_{\eta}(t). \tag{4}$$

In this case the weight coefficients B_2 are obtained on the basis of the best correlation between experimental and calculated results for non-proportional loading. This criterion is usually applied for brittle materials.

3. Determination of the plane of maximum covariance between stresses

In order to define the critical plane by the maximum covariance between normal and shear stresses (when the expected value is equal to zero), we should determine covariances on all the possible plane orientations α according to the following equation:

$$\mu_{\sigma,\tau} = \frac{1}{T_0}\int_0^{T_0} \sigma_{\eta}(t)\,\tau_{\eta s}(t)\,dt\,, \tag{5}$$

where T_o is the observation time.

Under combined bending with torsion by the angle α, the normal and shear stresses can be determined from :

$$\sigma_{\eta}(t) = \cos^2\alpha\,\sigma_{xx}(t) + \sin 2\alpha\,\tau_{xy}(t)\,. \tag{6}$$

and

$$\tau_{\eta s}(t) = -\frac{1}{2}\sin 2\alpha\,\sigma_{xx}(t) + \cos 2\alpha\,\tau_{xy}(t)\,, \tag{7}$$

where the stresses $\sigma_{xx}(t)$ and $\tau_{xy}(t)$ are stresses coming from bending and torsion respectively. Interpretation of the critical plane position with the considered shear and normal stresses is shown in Fig. 1.

Influence of stresses coming from bending and torsion is not the same, so a correction including the ratio of diameters of Mohr's circles was introduced to Eqs. (6) and (7)

$$\tau'_{xy}(t) = \frac{\sigma_{af}}{2\tau_{af}} \tau_{xy}(t) \tag{8}$$

So, From Eq (5) we obtain

$$\mu'_{\sigma,\tau} = \frac{1}{T_0} \int_0^{T_0} \sigma'_\eta(t) \tau'_{\eta s}(t) dt , \tag{9}$$

Under where :

$$\sigma'_\eta(t) = \cos^2 \alpha \sigma_{xx}(t) + \sin 2\alpha \tau'_{xy}(t) . \tag{10}$$

and

$$\tau'_{\eta s}(t) = -\frac{1}{2} \sin 2\alpha \sigma_{xx}(t) + \cos 2\alpha \tau'_{xy}(t) , \tag{11}$$

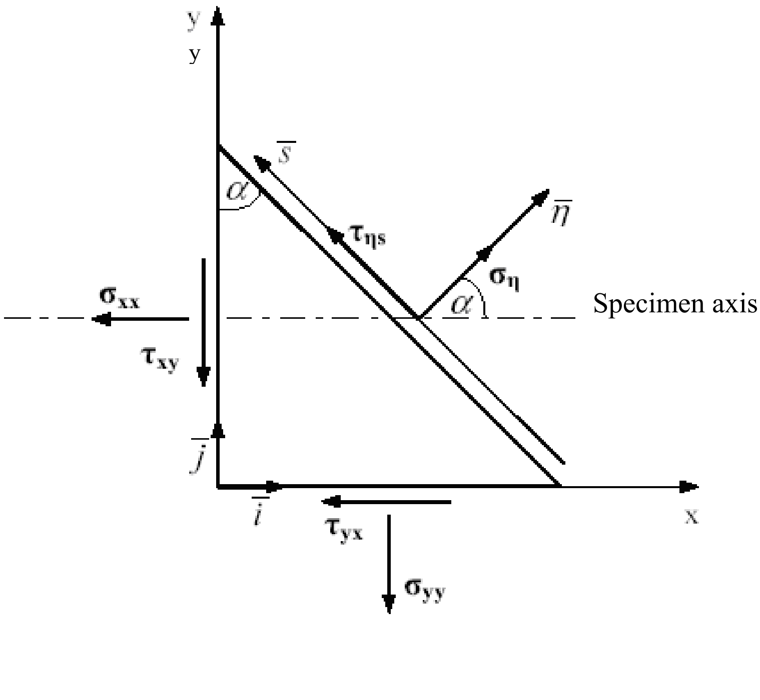

Fig. 1. Interpretation of the critical plane position

The plane in which the covariance according to Eq. (5) takes the extreme values for a given combination of normal stresses σ_η (t) and shear stresses $\tau_{\eta s}$ (t) is defined as the critical plane. Some local extrema are obtained as a result of such calculations.

4. The tests

Duralumin PA6 was tested. This material is also known as AlCuMg1 according to the DIN Standards [5], AlCu4MgSi(A) according to ISO, or 2017(A)–T4 the ASTM Standards. The considered material belongs to zincless aluminium alloys for plastic working. Its main elements are copper, responsible for better strength and hardness, and manganese, improving corrosion resistance. These advantages and a low specific gravity of PA6 cause that the considered alloy is widely applied in aircraft, machine and shipbuilding industries or in building engineering. Chemical composition and mechanical properties of the tested material are shown in Tables 1, 2 and 3 contains cyclic properties of the considered alloy.

Table 1. Chemical composition of the aluminium alloy PA6

Chemical composition of the aluminium alloy PA6			
Copper (Cu)	3.5÷4.5	Zirconium + titanium (Zr + Ti)	<0.25
Magnesium (Mg)	0.4÷1.0	Others together	<0.15
Silicon (Si)	0.2÷0.8	Others separately	<0.05
Manganese (Mn)	0.4÷1.0	Aluminium (Al)	rest
Iron (Fe)	<0.7	Zirconium (Zr)	<0.25

Table 2. Mechanical properties of the aluminium alloy PA6.

σ_y, MPa	σ_n, MPa	A_5, %	ρ, g/c	E, MPa	ν
395	545	21	2.8	72060	0.32

Table 3. Cyclic properties of the aluminium alloy PA6

K', MPa	n'	σ'_f, MPa	ε'_f	b	c
489	0.032	642	1.890	-0.065	-1.008

The round drawn bars 15 mm in diameter were tested. The bars were previously subjected to natural ageing in order to precipitation hardening in the ambient temperature. Plastic working (drawing) caused formation of the band structure.

Unnotched cylindrical specimens were tested. A scheme of the tested specimen is shown in Fig. 2. In such specimens it is easy to find the section of the greatest stresses. The specimen diameter *d* in the most narrow place was 10 mm for bending, torsion and proportional bending with torsion. The diameter 8.5 mm was assumed for specimens tested under non-proportional bending with torsion, because the tests were performed at another stand.

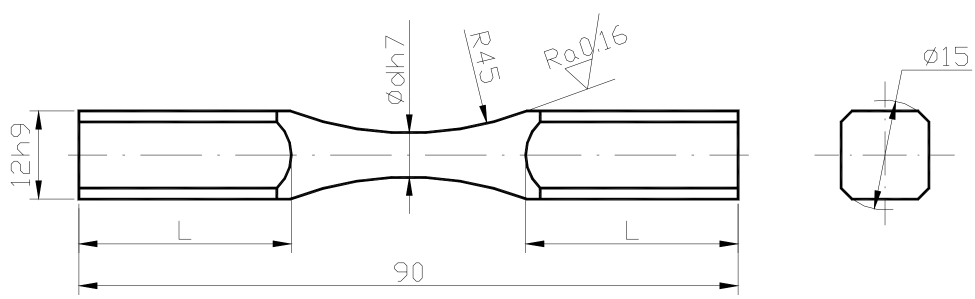

Fig. 2. Geometry of a tested specimen

The tests under simple loading states and combined proportional bending with torsion were performed together at the fatigue test stand MZGS–100. The test stand is shown in Fig. 3.

History of the total moment M_c loading the specimen was controlled while tests. The required loading was obtained by suitable balance of the rotating disks, and set-up of the lever torsion angle β (see Fig. 4). .

Nominal normal stresses coming from bending, and shear stresses coming from torsion on the specimen surface were determined on the assumption of perfect elasticity from:

$$\sigma_{xx}(t) = \frac{M_c(t)}{W_x} \cos\beta, \tag{12}$$

$$\tau_{xy}(t) = \frac{M_c(t)}{2W_x} \sin\beta, \tag{13}$$

where β is location of the head.

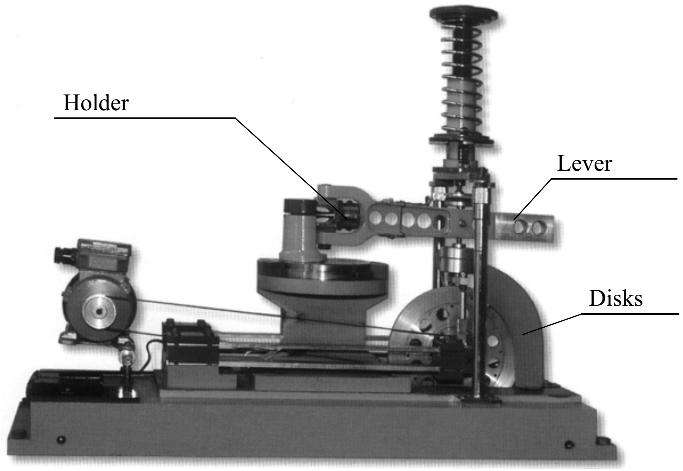

Fig. 3. The fatigue test stand MZGS-100

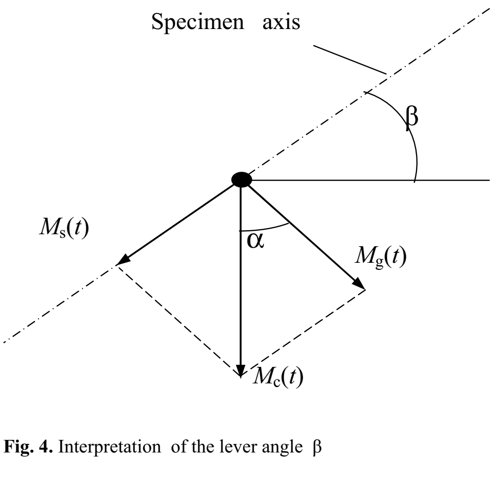

Fig. 4. Interpretation of the lever angle β

Frequency of loading variation was 29 Hz. Increase of the lever vibration amplitude by about 25% was assumed as failure of the specimen. .

At the stand MZGS–100 it is possible to perform tests under non-zero mean value of moments. It is not possible, however, to make non-proportional tests. Thus, the tests under the phase displacement were done at the fatigue test stand MZGS–200L.

The fatigue test stand MZGS–200L is equipped with two electromagnetic inductors. Their vibration is transferred by two levers (one for bending and one for torsion) to the tested specimen as histories of bending and torsional moments according to the following equations:

$$M_g(t) = M_{ag} \sin \omega t\,, \tag{14}$$

$$M_s(t) = M_{as} \sin(\omega t - \varphi)\,, \tag{15}$$

where φ is the phase displacement between loadings coming from bending and torsion.

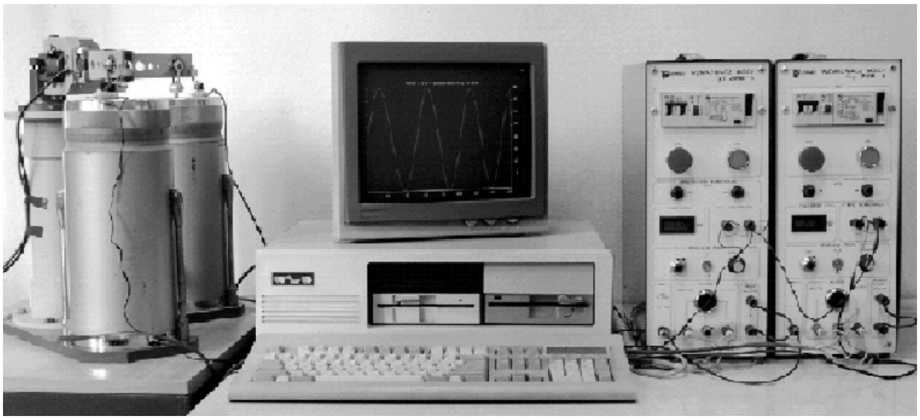

Fig.5. The fatigue test stand MZGS–200L

The tests were performed under the controlled bending and torsional moments. Non-proportionality of loadings was realized by phase displacement of histories for bending and torsion by the angle φ. Frequency of signals was 20 Hz. Increase of the lever vibration amplitude for bending or torsion by about 25% was assumed as the specimen failure. Histories of normal stresses from bending and shear stresses from torsion were determined on the assumption of elasticity of the material from:

$$\sigma_{xx}(t) = \frac{M_g(t)}{W_x} = \frac{M_{ag}}{W_x} \sin \omega t = \sigma_{ax} \sin \omega t\,, \tag{16}$$

$$\tau_{xy}(t) = \frac{M_s(t)}{2W_x} = \frac{M_{as}}{2W_x} \sin(\omega t - \varphi) = \tau_{axy} \sin(\omega t - \varphi)\,. \tag{17}$$

Analysis included pure bending and torsion, proportional loading ($\tau_a=0.25\sigma_a$, $\tau_a=0.5\sigma_a$, $\tau_a=\sigma_a$) and non-proportional loading with phase displacement 60° ($\lambda=0.5$) and 90° ($\lambda=0.25$, $\lambda=0.5$, $\lambda=1$).

The letter λ means the ratio of amplitudes between shear and normal stresses coming from torsion and bending

$$\lambda = \frac{\tau_{axy}}{\sigma_{axx}}\,. \tag{18}$$

From the tests the fatigue characteristics S-N were obtained according to the ASTM Standards [6] for pure bending

$$\log N_f = 21.81 - 7.03 \log\sigma_a \tag{19}$$

and pure torsion

$\log N_f = 19.94\text{-}6.87 \log\tau_a$. (20)

5. Determination of the fatigue life

The algorithm for fatigue life determination is a general model of fatigue life determination in the multiaxial stress state using criteria based on the critical plane concept. The algorithm includes some stages.

While fatigue life determination, the initial quantities are histories of normal stresses $\sigma_{xx}(t)$ coming from bending, and shear stresses $\tau_{xy}(t)$ coming from torsion, based on the given sinusoidally variable bending moment $M_g(t)$ and the torsional moment $M_s(t)$. At the next stage, the critical plane position is determined. The plane where covariance takes the extreme value for the given combination of normal and shear stresses, σ_η (t) and $\tau_{\eta s}$ (t), is defined as the critical plane. Then, the equivalent stress history is determined in one of the critical planes defined at the previous stage. The basic relations between normal and shear stresses versus the critical plane position allow to obtain two forms of the multiaxial fatigue criterion [2]

$$\sigma_{eq}(t) = \frac{4\sqrt{3} + 3\sqrt{2}B_1}{3\left(\sqrt{3}+1\right)}\sigma_n(t) + \frac{\sqrt{3}\left(3\sqrt{2}B_1 - 4\right)}{3\left(\sqrt{3}+1\right)}\tau_{ns}(t), \tag{21}$$

$$\sigma_{eq}(t) = \frac{4\sqrt{3} - 3\sqrt{2}B_1}{3\left(\sqrt{3}-1\right)}\sigma_n(t) + \frac{\sqrt{3}\left(4 - 3\sqrt{2}B_1\right)}{3\left(\sqrt{3}-1\right)}\tau_{ns}(t), \tag{22}$$

where the coefficient B_1 is expressed by Eq. (3).

Then, the extrema are determined from the history expressed by Eq. (15) or (16) as well as cycle amplitudes. At the last stage, the fatigue life is determined from the fatigue characteristics S-N for pure bending expressed by Eq. (13). According to the presented procedure, the critical planes are determined for all the considered cases. Under pure bending the planes are inclined at 30° and -30°, and under pure torsion four critical planes located at -22.5°, 22.5°, -67° and 67°. Table 4 presents the determined positions of the critical planes for combined bending with torsion, where the local covariance extrema have been obtained.

Table 4. The critical plane positions for combined bending with torsion under local covariance extrema

	φ=0°		φ=60°		φ=90°	
λ=1	20°	56°			-62°	62°
λ=0.5	-16°	43°	-17°	45°	-36°	36°
λ=0.25	-19°	40°			-31°	31°

Special attention should be paid to a very good agreement between the values obtained for the amplitude coefficient λ=0.5 and phase displacement φ=90° .

From literature it appears that such a good agreement between the results obtained under loading of that type seldom happens [7].

6. Conclusions

The critical plane position can be defined as the plane where there is the extremum of covariance between normal and shear stresses.

Under complex stress states, some critical plane positions can be found. It is reasonable because of the applied method.

From comparison of the calculated and experimental fatigue lives for proportional and non-proportional loading it appears that the applied method is right, because there is a very good agreement between the obtained results.

The extremely good results were obtained for loading with the amplitude ratio λ=0.5 and phase displacement φ=90^0; such good results are seldom obtained because of the problem of determination of the critical plane position according to the standard methods.

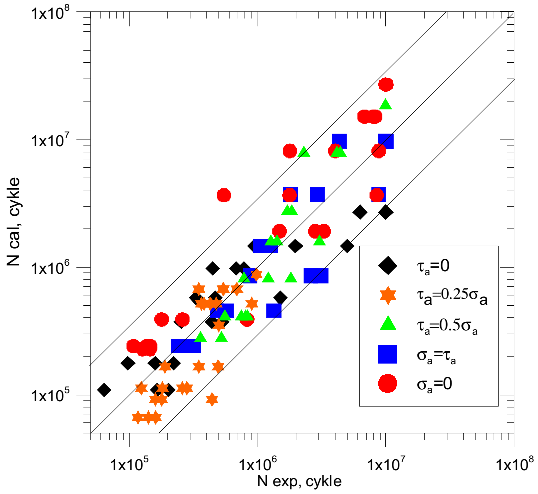

Fig. 6. Comparison of the calculated and experimental fatigue lives for proportional loading

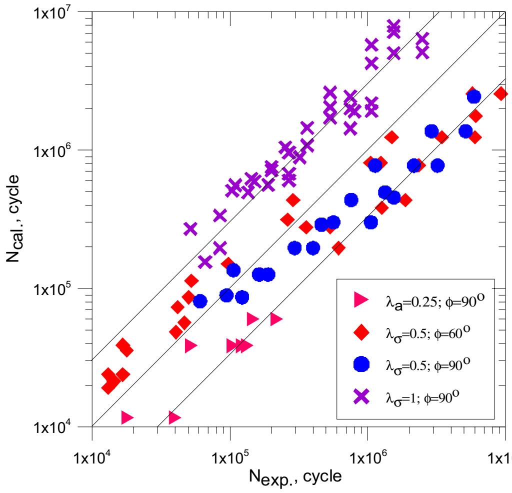

Fig. 7. Comparison of the calculated and experimental fatigue lives for non-proportional loading

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