# Kinematics analysis and simulation of profiling memory cutting for tunnel robot in complex environment 

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#### Abstract

Considering the actual conditions of the roadway drivage for the tunnel robotic, the law of cutting enveloping surface motion in space does not change along with selection of coordinates, roadheader link coordinates and its work coordinates were established. Combined the spatial location of cutting head cutting enveloping surface with the geometric analysis of its mechanism, the mathematical model of profiling memory cutting between enveloping surface and angular sensors detection data for tunnel robot was built. The influence factors of cutting roadway cross-section shape dimension such as the roadway slope, inclination and link mechanism were described. The relation between the spatial location of enveloping surface and angular displacement was described. The real-time and reliability of the model were verified through simulating the motion path of profiling memory cutting. This approach provides a theoretical foundation for the realization of the real-time monitor and auto control of tunnel robot cutting roadway cross-section.


Keywords: tunnel robot; kinematics; coordinate transformation; modeling

## 1. Introduction

Roadheader is the most important equipment of mechanized roadway head ${ }^{[1]}$. Through cutting head drilling and boom swing, roadheader cuts tunnel into requested shape. The profiling memory cutting of roadheader can get the tunnel of uniform geometry and desired size, according to a laneway projected dimension. The useless tunneling and filling quantity is reduced, tunneling efficiency increased, tunnel driving cost reduced maximally, and economic efficiency is dramatic. To implement the automation of drivage, the profiling cutting research of boom-type roadheader has already begun in domestic and overseas ${ }^{[2-7]}$, but is at the beginning stage of theories probing and trial debug phase ${ }^{[8-9]}$ It is an important constituent of robotic roadheader with positive sense on auto-roadheader. Considering the actual conditions of the roadway drivage for the tunnel robotic, and that the law of cutting enveloping surface motion in space does not change along with selection of coordinates, roadheader link coordinates and its work coordinates were established. The mathematical model of the factors of influencing cutting roadway cross-section shape dimension such as the roadway slope, inclination and link mechanism, was built. The relation between the spatial location of enveloping surface and angular displacement was described. The motion trail of longitudinal axis type tunnel robot profiling memory cutting was simulated. This approach provides a scientific foundation for the realization of the real-time monitor and auto control of tunnel robot cutting roadway cross-section.

## 2. Kinematics analysis of tunnel robot profiling memory cutting

According to given laneway dimension and roadheader parameter, the tunnel robot, through selecting the cutting process and the memory storage and autocontrol of computer, realizes the memory cutting of cutterhead. When there is an obvious change in geological condition, the driver carries out the hand operation cutting (as the vernier regulation of position program) and the tunnel robot automatically refreshes memory to store the adjusted running parameters. The computer serves as the work processor of cutter adjustment ${ }^{[5]}$.

### 2.1. Building cutter rotate surface equation

Cutter enveloping surface is an importance parameter in the geometric parameter of cutter. When cutter revolves round its axis, there is an intersection between all tooth cusps of cutter picks and parallel surface of axes. The enveloping curve of tooth cusp of cutter picks is gained through connecting these intersections. The spatial enveloping surface of cutter is obtained when the enveloping curve revolves round its axis. For simplified calculation, it acts as research object to adopt the parabola revolving round $z$ axis to generate the enveloping surface in Fig. 1.
If parabola equation is $z=a x^{2}+c$, then enveloping surface equation of parabola revolves round $z$ axis to get

$$
\left\{\begin{array}{l}
x=r \cos \varphi \\
y=r \sin \varphi \\
z=a r^{2}+c
\end{array}\right.
$$

$M$ is any point on the enveloping surface, and $r$ is the length from $M$ point to $z$ axis.


Fig. 1. The enveloping surface

### 2.2. Built the kinematical equation of tunnel robot profiling memory cutting

The main mechanisms-boom of tunnel robot is made up of a new line of articulating rods. It is a complicated multiple-joint mechanical arm. These operating mechanisms have the tremendous rigidity, so their distortion which is caused by external force is very small. Therefore, they simplify the rigid arm to be the same length as the actual length. The boom lifting mechanism of roadheader is simplified into the rocker mechanism. The initial condition of roadheader is defined as the state of keeping the boom of longitudinal axis roadheader in horizontal parallel to road central lines.

### 2.2.1. Coordinate transformation and building rotate surface equation

Considering the actual working conditions of the roadheader, the horizontal inclination $\theta_{1}, \theta_{2}$ of any time of anywhere possibility exists at machine body working direction and machine body side direction. The telescopic oil cylinder stretches forward $\underline{\Delta L}$ as cutting. When roadheader swings both right and left under the action of rotating oil
cylinder, supposing swing angle is $\alpha$. When roadheader swings both up and down under the action of lifting oil cylinder, supposing swing angle is $\beta$.

Giving every rod a coordinate, these coordinates are described by using homogeneous coordinates, namely relative position and direction of connecting rod. All coordinates' relative positions of keeping the boom of longitudinal axis roadheader in the point are chosen as the coordinates' origin. The rectangular coordinate $o-x y z$ is set up and plane xoz locates rotary table plane.
$o_{6}$ is the crossing point between boom and rotary table, coordinates $o_{6}-x_{6} y_{6} z_{6}$ is established and the length between $O$ and $o_{6}$ is the rotary table radius $R$ in Fig. 2. $o_{5}$ is the origin from which machine body at working direction generates the inclination $\theta_{1}$ and builds the rectangular coordinate $o_{5}-x_{5} y_{5} z_{5}$. Plane $y_{5} o_{5} z_{5}$ is got by plane $y_{6} o_{6} z_{6}$ around $x_{6}$ axis negative rotation $\theta_{1}$ and the axis $x_{6}$ is paralleled to axis $x_{5} . o_{4}$ is the origin from which machine body at side direction generates the inclination $\theta_{2}$ and builds the rectangular coordinate $o_{4}-x_{4} y_{4} z_{4}$. Plane $x_{4} o_{4} y_{4}$ is got by plane $x_{5} o_{5} y_{5}$ around $z_{5}$ axis negative rotation $\theta_{2}$ and the axis $z_{4}$ is paralleled to axis $z_{5} . o_{3}$ is the origin from which the telescopic oil cylinder stretches forward $\Delta L$ and builds the rectangular coordinate $o_{3}-x_{3} y_{3} z_{3}$. The corresponding coordinate axis of both coordinates $o_{3}-x_{3} y_{3} z_{3}$ and coordinates $o_{4}-x_{4} y_{4} z_{4}$ is parallel. $o_{2}$ is the origin from which the boom cutting both right and left generates the swing angle $\alpha$ and builds the rectangular coordinate $o_{2}-x_{2} y_{2} z_{2}$. Plane $x_{5} o_{5} z_{5}$ is got by plane $x_{3} o_{3} z_{3}$ around $y_{3}$ axis negative rotation $\alpha$ and the axis $y_{2}$ is paralleled to axis $y_{3} . o_{1}$ is the origin from which the boom cutting both up and down generates the swing angle $\beta$ and builds the rectangular coordinate $o_{1}-x_{1} y_{1} z_{1}$. Plane $y_{1} o_{1} z_{1}$ is got by plane $y_{2} o_{2} z_{2}$ around $x_{2}$ axis negative rotation $\beta$ and the axis $x_{1}$ is paralleled to axis $x_{2} . o_{0}$ is the crossing point between boom and cutter, and the point is chosen as the coordinate origin. The cutter rotation surface equation is set up. The corresponding coordinate axis of both coordinates $o-x y z$ and coordinates $o_{1}-x_{1} y_{1} z_{1}$ is parallel.


Fig. 2. All coordinates relative position of keeping the boom in horizontal

1) Any point $P\left(x_{0}, y_{0}, z_{0}\right)$ on the rotaion surface translates coordinate transform along $z$ axis to make point $o_{0}$ coincide with point $o_{1}$. Its transformation matrix is

$$
T_{t 1}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & -L & 1
\end{array}\right]
$$

2) Turn plane $y_{1} o_{1} z_{1}$ around $x_{4}$ axis clockwise rotation $\beta$ to be perfectly coplanar to plane $y_{2} o_{2} z_{2}$, then turn plane $x_{2} o_{2} z_{2}$ around $y_{3}$ axis clockwise rotation $\alpha$ to be perfectly coplanar to plane $x_{3} o_{3} z_{3}$. Its transformation matrix is

$$
T_{r x_{2}}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & \cos \beta & \sin \beta & 0 \\
0 & -\sin \beta & \cos \beta & 0 \\
0 & 0 & 0 & 1
\end{array}\right], T_{r y_{3}}=\left[\begin{array}{cccc}
\cos \alpha & 0 & -\sin \alpha & 0 \\
0 & 1 & 0 & 0 \\
\sin \alpha & 0 & \cos \alpha & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

3) By point $O_{3}$, translate the length $\Delta L$ along $z$ axis to $O_{4}$. Its transformation matrix is

$$
T_{t 2}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & -\Delta L & 1
\end{array}\right]
$$

4) Turn plane $x_{4} o_{4} y_{4}$ around $z_{4}$ axis clockwise rotation $\theta_{2}$ to be perfectly coplanar to plane $x_{5} o_{5} y_{5}$, then turn plane $y_{5} o_{5} z_{5}$ around $x_{5}$ axis clockwise rotation $\theta_{1}$ to be perfectly coplanar to plane $y_{6} o_{6} z_{6}$. Its transformation matrix is

$$
T_{r z_{5}}=\left[\begin{array}{cccc}
\cos \theta_{2} & \sin \theta_{2} & 0 & 0 \\
-\sin \theta_{2} & \cos \theta_{2} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right], T_{r x_{6}}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & \cos \theta_{1} & \sin \theta_{1} & 0 \\
0 & -\sin \theta_{1} & \cos \theta_{1} & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

5) Translate point $O_{6}$ along $z_{6}$ axis to the center of rotation $O$. Its transformation matrix is
$T_{t 3}=\left[\begin{array}{cccc}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -R & 1\end{array}\right]$
Then the spatial coordinate equation of cutter enveloping surface is available.
$T_{A R}=\mathrm{T}_{\mathrm{t} 1} \mathrm{~T}_{\mathrm{rx} 2} \mathrm{~T}_{\mathrm{ry}_{3}} \mathrm{~T}_{\mathrm{t} 2} \mathrm{~T}_{\mathrm{rz}_{5}} \mathrm{~T}_{\mathrm{r} \mathrm{z}_{6}} \mathrm{~T}_{\mathrm{t} 3},\left[\begin{array}{ll}x y z & 1]=\left[x_{o} y_{o} z_{o} 1\right] T_{A R} \text {, that is, }\end{array}\right.$

$$
\begin{gathered}
x=r \cos \varphi \cos \alpha \cos \theta_{2}+r \sin \varphi \sin \beta \sin \alpha \cos \theta_{2}-\left(a r^{2}+c-L\right) \sin \alpha \cos \beta \cos \theta_{2} \\
\quad+r \sin \varphi \cos \beta \sin \theta_{2}+\left(a r^{2}+c-L\right) \sin \beta \sin \theta_{2} \\
y=-r \cos \varphi \cos \alpha \sin \theta_{2} \cos \theta_{1}-r \sin \varphi \sin \beta \sin \alpha \sin \theta_{2} \cos \theta_{1}+\left(a r^{2}+c-L\right) \sin \alpha \\
\cos \beta \sin \theta_{2} \cos \theta_{1}+r \sin \varphi \cos \beta \cos \theta_{2} \cos \theta_{1}+r \cos \varphi \sin \alpha \sin \theta_{1} \\
-r \sin \varphi \sin \beta \cos \alpha \sin \theta_{1}+\left(a r^{2}+c-L\right) \cos \beta \cos \alpha \sin \theta_{1}-\Delta L \sin \theta_{1} \\
z=r \cos \varphi \cos \alpha \sin \theta_{2} \sin \theta_{1}+r \sin \varphi \sin \beta \sin \alpha \sin \theta_{2} \sin \theta_{1}-\left(a r^{2}+c-L\right) \sin \alpha \\
\cos \beta \sin \theta_{2} \sin \theta_{1}-r \sin \varphi \cos \beta \cos \theta_{2} \sin \theta_{1}-\left(a r^{2}+c-L\right) \sin \beta \cos \theta_{2} \sin \theta_{1} \\
+r \cos \varphi \sin \alpha \cos \theta_{1}-r \sin \varphi \sin \beta \cos \alpha \cos \theta_{1}+\left(a r^{2}+c-L\right) \cos \beta \cos \alpha \\
\cos \theta_{1}-\Delta L \cos \theta_{1}-R
\end{gathered}
$$

According to the following formula, any positional value (any point spatial location coordinate of cutter) is directly obtained on the cutter enveloping surface. This provides the shortcut for the relative position conversion as system locating. When the figure moves around the coordinate origin, the clockwise rotation and anticlockwise rotation are respectively defined as positive value and negative value above the transformation. The values of $\theta_{1}, \theta_{2}, \alpha, \beta$ are directly measured through the angular sensors, the value of $\Delta L$ is directly measured through the displacement sensors and the values of $a, c, L, R$ are given in the system design.

### 2.2.2. Amendment quantity of road profiling cutting

1) As $\theta_{1}=0, \theta_{2}=0$, spatial location point is

$$
\left\{\begin{array}{c}
x=r \cos \varphi \cos \alpha+r \sin \varphi \sin \beta \sin \alpha-\left(a r^{2}+c-L\right) \sin \alpha \cos \beta \\
y=r \sin \varphi \cos \beta \\
z=r \cos \varphi \sin \alpha-r \sin \varphi \sin \beta \cos \alpha+\left(a r^{2}+c-L\right) \cos \beta \cos \alpha-\Delta L-R
\end{array}\right.
$$

2) As $\theta_{1}=0, \theta_{2} \neq 0$, spatial coordinate rounds $x$ axis clockwise rotation $\theta_{2}$. Spatial location point is

$$
T_{r x}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & \cos \theta_{2} & -\sin \theta_{2} & 0 \\
0 & \sin \theta_{2} & \cos \theta_{2} & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

$$
\left[\begin{array}{llll}
x & y & z & 1
\end{array}\right]=\left[\begin{array}{ll}
x_{0} & y_{0} \\
z_{0} & 1
\end{array}\right] T_{r x}=\left[x y \cos \theta_{2}+z \sin \theta_{2}-y \sin \theta_{2}+z \cos \theta_{2} 1\right]
$$

3) As $\theta_{1} \neq 0, \theta_{2}=0$, spatial coordinate rounds $z$ axis clockwise rotation $\theta_{1}$. Spatial location point is

$$
T_{r z}=\left[\begin{array}{cccc}
\cos \theta_{1} & -\sin \theta_{1} & 0 & 0 \\
\sin \theta_{1} & \cos \theta_{1} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

$\left[\begin{array}{lll}x & y & z \\ 1\end{array}\right]=\left[x_{0} y_{0} z_{0} 1\right] T_{r z}=\left[x \cos \theta_{1}+y \sin \theta_{1}-x \sin \theta_{1}+\cos \theta_{1} z \quad 1\right]$
4) As $\theta_{1} \neq 0, \theta_{2} \neq 0$, spatial coordinate firstly rounds $x$ axis clockwise rotation $\theta_{2}$, and again rounds $z$ axis clockwise rotation $\theta_{1}$. Spatial location point is

$$
\begin{aligned}
& {\left[\begin{array}{lll}
x y z & z & 1]
\end{array}\right]\left[\begin{array}{lll}
x_{0} y_{0} z_{0} & 1
\end{array}\right] T_{r x} T_{r z}=\left[x y \cos \theta_{2}+z \sin \theta_{2}-y \sin \theta_{2}+z \cos \theta_{2} 1\right] T_{r z} } \\
&=\left[x \cos \theta_{1}+\left(y \cos \theta_{2}+z \sin \theta_{2}\right) \sin \theta_{1}-x \sin \theta_{1}+\left(y \cos \theta_{2}+z \sin \theta_{2}\right) \cos \theta_{1}-y \sin \theta_{2}+z \cos \theta_{2} 1\right]
\end{aligned}
$$

### 2.3. Tunnel robot drivage control

Because the environment of drivage face is complicated and the illumination is very bad, it is better suited to adopt the non-visual sensor locating technology. From the kinematical equation of tunnel robot, the pose coordinate of tunnel robot is described as $x, y, z, \theta_{1}, \theta_{2}, \alpha, \beta, \square L$, etc. There are two situations of the tunnel robot drivage operation control.

1) Knowing the pose of tunnel robot responding to geodetic coordinate system, the position space coordinate of cutting envelope surface is solved through the coordinate transformation. This is called the positive solution of drivage control.

Machine inclination and connecting rod inclination are directly measured to adopt the tilt sensor. Moreover, the connecting rod inclination is indirectly got through the displace sensor ${ }^{[7]}$.
2) Knowing the position space coordinate of the designed road responding to geodetic coordinate system, the pose of tunnel robot is inversely solved through the coordinate inverse transformation. This is called the inverse solution of drivage control.

Therefore, whether up and down or left and right swing, tunnel robot is interpreted as three-link structure. Any spatial location of cutter reaching is calculated through the measured inclination. Thus, using computer to compile the determinate program, and getting through controlling the cylinder flexing, the tunnel robot profiling memory cutting is realized.

## 3. Simulation analysis

EBZ-160 type boom roadheader and excavated section of road parameter in a mine act as the sample. In order to show the cutter motion trail and cutting road cross-section clearly, the cutter motion situation was simulated by using MATLAB. The outside envelope curve of roadheader cutting semicircle arch road was gotten in Fig. 3. Through the research of its kinematics simulation, the simulation results curve was in accord with actual conditions. This showed the accuracy of tunnel robot profiling memory cutting kinematical model.


Fig. 3. Outside envelope curve of profiling cutting (a) working outside envelope curve (b) profiling cutting of road flitching

## 4. Conclusions

1) Using coordinate transformation, the kinematical equation of tunnel robot profiling memory cutting was built in complex environment. The research aimed to lay foundation for studying the auto control of roadheader drivage.
2) Adopting the correlation parameter of detection sensor measured, the positive solution of kinematical equation was worked out. It is the foundation of roadway automatic monitoring to get the pose of tunnel robot. The inverse solution of drivage control is the foundation of roadheader orientation and advancing cross-section auto control.
3) The simulation results have verified the consistency between the research of tunnel robot profiling memory cutting and actual conditions in complex environment.

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## References

[1] R.H. Huang, Boom-type Roadheader. Xuzhou: China University of Mining and Technology Press, 1996.
[2] J.J. Ortea, J.C. Catalina and M. Devy, Perceptionfor a roadheader in automatic selective cutting operation. International Conference on Robotic and Automation. 5 (1992) 626-632.
[3] O.Z. Hekim and M. Ayhan, Effect of cutting head motion on the boom length of rock and coal cutting operation. Mineral Resources Engineering. 87 (1999) 381-389.
[4] Bever Control AS Company. Computer control of tunnel profile. Tunnels \& Tunnelling. 128 (1990) 47-48.
[5] J. Mao, J. Li and W. Li, Building Model and Virtual Simulation of Profiling Memory Cutting for Tunnel Robot. Journal of System simulation. 21 (2009) 15.
[6] J. Mao, C.T. Wu and M. Sun, Investigation into profile-cutting control of boom-typed excavator. Chinese Journal of Construction Machinery. 5 (2007) 322-328.
[7] D. Guo, Research on profiling cutting theory of longitudinal axis type roadheader. Fuxin: Liaoning Technical University, 2007.
[8] Y. Guo and Y. Zhang, Kinematics analysis and computer simulation on longitudinal cutting head of roadheader. Journal of China Coal Society. 24(2002) 68-72.
[9] X. Li, Research on Key Technologies of Cutting of Roadheaders.Beijing: China Machine Press, 2008.

