Physics Letters B 668 (2008) 312–314

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Anomalous anomalous scaling?

Alessandro Strumia [a,*], Nikolaos Tetradis [b]

[a] Dipartimento di Fisica dell'Università di Pisa and INFN, Pisa, Italy
[b] Department of Physics, University of Athens, Zographou GR-15784, Athens, Greece

ARTICLE INFO

Article history:
Received 15 May 2008
Received in revised form 9 August 2008
Accepted 28 August 2008
Available online 3 September 2008
Editor: G.F. Giudice

ABSTRACT

Motivated by speculations about infrared deviations from the standard behavior of local quantum field theories, we explore the possibility that such effects might show up as an anomalous running of coupling constants. The most sensitive probes are presently given by the anomalous magnetic moments of the electron and the muon, that suggest that $\alpha_{\rm em}$ runs 1.00047 ± 0.00018 times faster than predicted by the Standard Model. The running of $\alpha_{\rm em}$ and α_s up to the weak scale is confirmed with a precision at the % level.

1. Introduction

The range of validity of Quantum Field Theory (QFT) may be limited not only in the UltraViolet (UV), $E \lesssim \Lambda_{\rm UV}$, but also in the InfraRed (IR), $E \gtrsim \Lambda_{\rm IR} \equiv 1/L$, with a non-trivial connection between $\Lambda_{\rm UV}$ and $\Lambda_{\rm IR}$. This possibility has attracted interest due to the following reasons.

On the theoretical side, requiring that the entropy associated with the QFT degrees of freedom $\sim (\Lambda_{\rm UV}/\Lambda_{\rm IR})^3$ saturates the Bekenstein entropy [1] $\sim L^2 M_{\rm Pl}^2$ of a black hole with size L leads to $\Lambda_{\rm IR} \sim \Lambda_{\rm UV}^3/M_{\rm Pl}^2$. Alternatively, it has been suggested that one should require that systems whose size L exceeds their Schwarzschild radius $\sim m/M_{\rm Pl}^2$ do not appear in QFT. For $m \sim \Lambda_{\rm UV}^3 L^3$, this requirement leads to $\Lambda_{\rm IR} \sim \Lambda_{\rm UV}^2/M_{\rm Pl}$ [2].

On the phenomenological side, Refs. [2,3] discussed possible connections of these ideas with the cosmological constant and the supersymmetry breaking puzzles. Indeed, in standard QFT the values of the vacuum energy, scalar masses squared and dimensionless couplings are given by their bare Planck-scale values plus a quantum correction proportional to $\Lambda_{\rm UV}^4$, $\Lambda_{\rm UV}^2$ and $\ln \Lambda_{\rm UV}$, respectively. Such non-local effects could change this power-counting, solving or modifying the hierarchy problems associated with massive parameters [2,3].

We observe here that dimensionless couplings may be similarly affected, leading to an anomalous Renormalization Group (RG) running, and we study how accurately present data test the standard QFT prediction.

2. Speculations

The above ideas about a non-local connection between the IR and UV cutoffs do not have a very precise meaning, and one can debate whether they would lead to any of the effects mentioned above. Rather than arguing in any one direction, we present the uncertain issues.

Firstly, *when do these non-local phenomena appear in particle physics?* The weakest possibility is only when strong gravity effects, such as those arising from black holes, are directly relevant. This would practically mean never, as black hole phenomena (Hawking radiation, etc.) are quantitatively irrelevant in all processes we can realistically observe (possibly unless the true quantum gravity scale is much below $M_{\rm Pl}$). The strongest possibility is that states with energy $E \gtrsim \Lambda_{\rm UV}$ that propagate for more than L do not exist and must be dropped from QFT. However, this possibility seems to contradict experience. We see TeV γ rays from the galactic center, particles with energies up to 10^{20} eV from extragalactic sources, etc.

We envisage a scenario in which the bulk of the contributions from (real of virtual) modes to quantities such as the energy density or the renormalized couplings is delimited by a scale that depends on the IR cutoff. Individual very energetic modes may still exist, but their effect on the above observables is negligible. For example, one may limit the temperature T of a system of size L by demanding that this system does not collapse into a black hole: $T^2 \lesssim M_{\rm Pl}/L$. States with energy much larger than this limit may exist, but their contribution to the energy density is negligible. Our basic, and possibly contentious, assumption is that gravity enforces a similar constraint to the contributions from virtual fluctuations. This leads to limits on the energy density, the quantum-corrected vacuum energy density (i.e. the minimum of the potential) [2], the Higgs mass (i.e. the curvature of the potential at its minimum) [3], and possibly all couplings (i.e. the derivatives of the potential). The

* Corresponding author.
E-mail address: astrumia@mail.df.unipi.it (A. Strumia).

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2008.08.061

smallness of the cosmological constant is consistent with such an assumption.

Secondly, *what is the precise meaning of $\Lambda_{\rm IR}$?* There are various possibilities, and we mention two: (i) The IR cutoff can be defined by imposing boundary conditions such that QFT lives “in a box” with size $1/\Lambda_{\rm IR}$. (ii) $\Lambda_{\rm IR}$ is the minimal energy scale that appears in loop integrations. In practical cases these choices can lead to different answers. For example, only in the first case the IR cutoff for the cosmological constant would be the Hubble distance $1/H$ (possibly leading to a small $\Lambda_{\rm UV} \sim \sqrt{M_{\rm Pl} H} \sim {\rm eV}$ [2]). On the other hand, H does not appear in the one-loop correction to the vacuum energy, equal to the value of the potential V at its physical local minimum

$$V \simeq V_{\rm bare} + \frac{1}{2}\int_{\Lambda_{\rm IR}}^{\Lambda_{\rm UV}} \frac{d^4k}{(2\pi)^4}\,{\rm Str}\ln\big(k^2 + V''_{\rm bare}\big), \tag{1}$$

unless $\Lambda_{\rm IR} = H$ is imposed.

We finally arrive to the third issue: *what is the precise meaning of $\Lambda_{\rm UV}$ if it is viewed as a function of $\Lambda_{\rm IR}$?* As we explained above, we assume that $\Lambda_{\rm UV}$ sets the upper limit for the bulk of the fluctuations that renormalize a quantum field theory. Modes with characteristic momenta above $\Lambda_{\rm UV}$ may exist, but they do not contribute significantly to the renormalization of the couplings. According to this logic $\Lambda_{\rm UV}$ ($\Lambda_{\rm IR}$) may be used as the upper bound of the momentum integration is expressions such as (1).

If the quantum correction to the minimum of V is naturally small thanks to an anomalous dependence of $\Lambda_{\rm UV}$ on $\Lambda_{\rm IR}$ [2], one expects a similar anomalous running of the whole potential, and in particular of its coupling constants. In standard QFT, the one-loop corrections to any dimensionless coupling (e.g. the gauge boson vertex g) has the form

$$g(p) = g_{\rm bare} - \beta\frac{g^3_{\rm bare}}{8\pi}\left[\ln\frac{\Lambda^2_{\rm UV}}{p^2} + {\rm finite}\right], \tag{2}$$

where p is some combination of the external momenta that sets the IR cutoff in the loop integration. In standard QFT the physical coupling $g(p)$ ‘runs’ with the energy p of the process, and the RG coefficient β is a number that depends on the particle content of the theory above p. (In Eq. (2) we assumed that all the masses are negligibly small.)

If, instead, non-QFT effects produce some physical UV cutoff $\Lambda_{\rm UV}$ that depends on $\Lambda_{\rm IR} \sim p$, one generically obtains an anomalous RG running of $g(p)$. For example, in the one-loop approximation the running is proportional to

$$\beta \to \beta\left(1 - \frac{\partial\ln\Lambda_{\rm UV}}{\partial\ln\Lambda_{\rm IR}}\right) \equiv \beta(1-\delta). \tag{3}$$

The correction appears because fewer UV modes contribute to quantum corrections when the IR cutoff is lowered. The running does not arise only because of the integration of modes around the IR cutoff, but also because the influence of the UV modes is altered. This is the essence of the non-local mechanism we assume in this work.

Even within this framework, it is still possible for the anomalous running not to arise. For example, the Standard Model (SM) at energies below $M_{\rm Pl}$ and above a scale Λ_f may be replaced by some other model where couplings do not run (e.g. an UV-finite theory, or some fixed point of the RG flow). If the values of $\Lambda_{\rm UV}$ that correspond to the range of $\Lambda_{\rm IR}$ of interest are above Λ_f, no anomalous running appears. Similar behaviour is expected if the SM couplings come from a scalar field (e.g. the dilaton in string theory) whose vacuum expectation value is determined by low energy dynamics.

By using the β function of Eq. (3) within the dimensional regularization formalism, we get for the one-loop RG running of a gauge coupling α

$$\frac{1}{\alpha(\mu')} - \frac{1}{\alpha(\mu)} \simeq \beta\ln\frac{\mu^{1-\delta}}{\mu'^{1-\delta}} + \cdots. \tag{4}$$

This is equivalent to the standard expression, with the Minimal Subtraction mass scale μ replaced by $\mu^{1-\delta}$. In theories with several particle masses and sizable RG corrections, the factor δ generically can become some unknown function of the energy. In order to compute it, we would need to know the physics around $\Lambda_{\rm UV}$.

In the next sections, we explore the most sensitive experimental probes of anomalous RG running, assuming for definiteness that all Standard Model formula get modified as in Eq. (2), with a constant δ to be extracted from data.

3. Running of $\alpha_{\rm em}$ from m_e to m_μ

The measurements of the anomalous magnetic moments of the electron [4]

$$g_e/2 = 1.00115965218085(76) \tag{5}$$

and of the muon [5]

$$g_\mu/2 = 1.00116592080(63), \tag{6}$$

together with the assumption of the validity of the Standard Model, allow us to infer the electromagnetic coupling $\alpha_{\rm em}(\mu)$ at the scales $\mu = m_e$ and m_μ, in view of the theoretical prediction

$$g_i = 2 + \alpha_{\rm em}(m_i)/\pi + \cdots, \tag{7}$$

where $\cdots$ denotes higher-order effects. We recall that g_e gives the most precise determination of $\alpha_{\rm em}$, that is consistent with lower-energy probes from atomic physics [4]. Assuming the anomalous running

$$\frac{1}{\alpha_{\rm em}(m_e)} - \frac{1}{\alpha_{\rm em}(m_\mu)} = \frac{1-\delta}{3\pi}\ln\frac{m_\mu}{m_e} + \cdots \tag{8}$$

one gets the presently most precise determination of δ:

$$\delta = -(0.047 \pm 0.018)\%. \tag{9}$$

The central value of δ is about 3σ below zero, because, for $\delta = 0$, g_μ is about 3σ above the SM prediction, $(g_\mu - g_\mu^{\rm SM})/2 = (23 \pm 9) \times 10^{-10}$, with the precise number depending on how one deals with the theoretical uncertainties on higher-order QCD corrections to g_μ: relying on e^-e^+ data and/or on τ-decay data [5].

The usual new-physics interpretation of the $g_\mu - 2$ anomaly is that new particles with heavy mass M, like supersymmetric particles, affect g_μ giving an extra contribution $\Delta g_\mu \sim \alpha_2 m^2_\mu/M^2$. They also affect precision data at higher energies, but have a negligible influence on g_e in view of $m_\mu \gg m_e$.

We point out that the relative incompatibility between g_μ and g_e could instead be due to a ‘too fast’ RG running of $\alpha_{\rm em}$. We show that g_e and g_μ presently give the most sensitive probes to δ: this kind of new physics is best seen with higher precision than with higher energy.

4. Running of $\alpha_{\rm em}$ from m_μ to M_Z

Precision tests at the Z pole offer another precision determination of the electromagnetic coupling. By performing a global fit within the SM with Higgs mass m_h [6] we find

$$\frac{1}{\alpha_{\rm em}(M_Z)} = 128.92 + 0.23\ln\frac{m_h}{M_Z} \pm 0.06. \tag{10}$$

This value can be compared with the RG extrapolation from m_e, m_μ up to M_Z [7]

$$\frac{1}{\alpha_{\rm em}(M_Z)} = 128.937 + 8.1\delta \pm 0.028, \tag{11}$$

where the uncertainty comes from QCD thresholds. So

$$\delta = \left(-0.2 + 2.9 \ln \frac{m_h}{M_Z} \pm 0.9\right)\%. \tag{12}$$

The precise measurement of the muon lifetime does not give another probe of δ, as the anomalous dimension of the associated Fermi operator

$$[\bar\mu \gamma_\mu P_L \nu_\mu][\bar\nu_e \gamma_\mu P_L e]$$

is zero: indeed, the electromagnetic current is not renormalized, and this operator can be related to it, times a neutrino current not affected by electromagnetic interactions.

5. Running of α_s from m_τ to M_Z

Another sensitive probe to δ comes from the running of the strong coupling α_s: in view of its large value, α_s runs fast. The strong coupling constant has been measured at various scales, and the two most precise determinations are at m_τ and M_Z. By performing a global fit of electroweak precision data within the SM with Higgs mass m_h [6] we find

$$\alpha_s(M_Z) = 0.121 + 0.0008 \ln \frac{m_h}{M_Z} \pm 0.0023. \tag{13}$$

On the other hand, the measurement of the strong coupling from τ decays, $\alpha_s(m_\tau) = 0.334 \pm 0.009$, extrapolated up to M_Z gives [8]

$$\alpha_s(M_Z) = 0.1212 + 0.08\,\delta \pm 0.0011. \tag{14}$$

So

$$\delta = \left(-0.4 + 1.1 \ln \frac{m_h}{M_Z} \pm 3.3\right)\%. \tag{15}$$

Finally, flavor-physics observations allow us to test the QCD running of various operators from the weak scale down to the bottom or charm mass. However, the uncertainty on δ is at the level of several tens of percent.

6. Conclusions

Motivated by possible deviations from the standard QFT predictions for the RG running of couplings, we rescaled β functions by $1-\delta$ and studied how data probe the new-physics parameter δ that parameterizes an anomalous running. Unlike in ordinary new physics, the most sensitive probe to δ is given by precision experiments at low energies $E \gtrsim m_e$: the measurements of the magnetic moments of the electron and the muon determine δ with a 0.018% uncertainty, excluding order-one effects. However, the anomaly in the anomalous magnetic moment of the muon indicates a best fit value for δ which is 3σ below zero. Running of $\alpha_{\rm em}$ and α_s up to M_Z is confirmed with a 1% and 3% precision, respectively.

Acknowledgements

We thank Gino Isidori for comments.

References

[1] J. Bekenstein, Lett. Nuovo Cimento 4 (1972) 737;
G. 't Hooft, gr-qc/9310026;
R. Bousso, J. High Energy Phys. 9907 (1999) 004, hep-th/9905177.
[2] A.G. Cohen, D.B. Kaplan, A.E. Nelson, Phys. Rev. Lett. 82 (1999) 4971, hep-th/9803132.
[3] T. Banks, hep-th/0007146.
[4] B. Odom, D. Hanneke, B. D'Urso, G. Gabrielse, Phys. Rev. Lett. 97 (2006) 030801;
G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, B.C. Odom, Phys. Rev. Lett. 97 (2006) 030802, and Erratum.
[5] G.W. Bennett, et al., Phys. Rev. D 73 (2006) 072003, hep-ex/0602035;
For a recent review of theoretical computations see: Z. Zhang, arXiv: 0801.4905.
[6] The data included in our fit are summarized in Tables 2 and 3 of R. Barbieri, A. Pomarol, R. Rattazzi, A. Strumia, Nucl. Phys. B 703 (2004) 127, hep-ph/0405040, updated at the light of the most recent data, as summarized in http://lepewwg.web.cern.ch/LEPEWWG;
In particular, we use the top mass from Tevatron Electroweak Working Group, arXiv: 0803.1683.
[7] K. Hagiwara, A.D. Martin, D. Nomura, T. Teubner, Phys. Lett. B 649 (2007) 173.
[8] M. Davier, et al., arXiv: 0803.0979.