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On supersymmetric Chern-Simons-type theories in five dimensions

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Abstract: We present a closed-form expression for the supersymmetric non-Abelian Chern-Simons action in conventional five-dimensional $\mathcal{N} = 1$ superspace. Our construction makes use of the superform formalism to generate supersymmetric invariants. Similar ideas are applied to construct supersymmetric actions for off-shell supermultiplets with an intrinsic central charge. In particular, the large tensor supermultiplet is described in superspace for the first time.

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Contents

1 Introduction

In the 1990s, it was demonstrated [1–3] that in five dimensions (5D) a Chern-Simons term is generated in a supersymmetric Yang-Mills theory by integrating out massive hypermultiplets and keeping only the gauge field of the vector supermultiplet. In a manifestly supersymmetric setting, which takes into account the entire vector supermultiplet, a related one-loop calculation was given in [4], both in the Coulomb and non-Abelian phases. Using the covariant harmonic supergraphs [5, 6] and the heat kernel techniques in harmonic superspace [7, 8], it was shown [4] that the hypermultiplet effective action contains a supersymmetric Chern-Simons (SCS) term.

Within the component approach, the off-shell non-Abelian SCS action (in the presence of conformal supergravity) in five dimensions was first constructed by Kugo and Ohashi [9]. Their approach, however, was not systematic. They started with the Abelian SCS action,[1] which was efficiently derived using the linear supermultiplet action, and then extended it by adding appropriate non-Abelian structures, order by order in the coupling constant, in such

[1] The off-shell Abelian SCS action in five dimensions was constructed for the first time by Zupnik in 5D $\mathcal{N} = 1$ harmonic superspace [10].

a way as to make the action supersymmetric. In the flat space limit, the non-Abelian SCS action is superconformal, which makes this theory very interesting for various applications.

Unlike the component construction of [9], a closed-form expression for the non-Abelian SCS action has never been given in a superspace setting. In the Abelian case, the SCS action was derived in the 5D $\mathcal{N}=1$ harmonic [10] and projective [11] superspaces,[2] and also in terms of 4D $\mathcal{N}=1$ superfields [19].[3] As concerns the non-Abelian case, there exists a unique definition [10] for the variation of the SCS action with respect an infinitesimal deformation of the analytic gauge prepotential, $V^{++} \to V^{++} + \delta V^{++}$, which describes the Yang-Mills supermultiplet within the harmonic-superspace approach.[4] However, it is not yet known how to integrate this variation in a closed form (see the erratum to [10]). In the projective-superspace approach, the variation of the non-Abelian SCS action can be defined similar to [10] using the formalism of [11]. But it is also unclear how to integrate it. For completeness, it is worth mentioning the attempt to construct a non-Abelian SCS action in terms of 4D $\mathcal{N}=1$ superfields [20, 21]. But their action is valid only in a Wess-Zumino gauge, and therefore it is hardly useful.

In this paper we present a closed-form expression for the non-Abelian SCS action in the conventional 5D $\mathcal{N}=1$ superspace setting described in [11]. To achieve this, we do not define the action as an integral over the superspace or its analytic subspace. Instead, we adopt the superform construction of supersymmetric invariants [22–26], also known as the rheonomy approach [22] or the ectoplasm approach [24–26]. More specifically, we will build on the recent papers [27–29] in which $\mathcal{N} \leq 6$ conformal supergravity actions in three dimensions have been constructed efficiently and elegantly via the superform approach by making use of the Chern-Simons form together with a curvature induced form. This method is a generalization of the superform formulation for the linear supermultiplet in four-dimensional $\mathcal{N}=2$ conformal supergravity given in [30].[5] Such an approach can be adapted to five-dimensions and we endeavor to demonstrate this for the non-Abelian SCS theory.

The superform approach can also be used to describe the dynamics of 5D off-shell supermultiplets with an intrinsic central charge. Of course, such theories have been studied in components [9, 35, 36] and in harmonic superspace [11, 37]. In the component setting, however, one has to use rather different ideas in order to describe (i) the non-abelian Chern-Simons theory and (ii) the models for off-shell supermultiplets with an intrinsic

[2]The relationship between the 4D $\mathcal{N}=2$ harmonic [12, 13] and projective [14–16] superspace formulations is spelled out in [17] (see also [18] for a recent review). The same relationship holds in the case of 5D $\mathcal{N}=1$ supersymmetry.

[3]The action given in [19] was derived using an ad hoc procedure; this action is trivially deduced from the systematic projective-superspace construction of [11].

[4]The one-loop calculation in [4] consisted of demonstrating that varying the hypermultiplet effective action produces a SCS action [10] as the leading quantum correction.

[5]This is an example of a known construction where an invariant derived from a closed super d-form can be generated from a closed, gauge-invariant super $(d+1)$-form provided that the latter is Weil trivial, i.e. exact in invariant cohomology (a concept introduced by Bonora, Pasti and Tonin [31] in the context of anomalies in supersymmetric theories). Examples of this include Green-Schwarz actions for various branes [32], as well as some higher-order invariants in other supersymmetric theories which were studied, e.g., in [33, 34].

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central charge. As will be shown below, if the superform approach is employed the two types of theories are formulated uniformly in superspace.

This paper is organized as follows. In section 2, we introduce the superform formulation of the Yang-Mills supermultiplet and use it to construct the Chern-Simons action. To do so we use both the Chern-Simons form and a curvature induced form that we will introduce. In section 3, we turn to supermultiplets with central charge. We provide both the superform formulations for a gauge two-form supermultiplet and the linear supermultiplet with central charge. This immediately leads to the action principle based on the linear supermultiplet. Concluding comments are given in section 4. Finally, in the appendix we analyze the possibility to have a gauge connection that is not annihilated by the central charge.

Throughout the paper, we follow the 5D notation and conventions of [11].

2 Non-abelian Chern-Simons theory

In this section, we describe the non-Abelian SCS theory based on a Yang-Mills supermultiplet and derive the corresponding action via the superform approach.

2.1 Yang-Mills supermultiplet

Conventional 5D $\mathcal{N}=1$ Minkowski superspace $\mathbb{R}^{5|8}$ may be parametrized by the coordinates $z^{\hat{A}}=(x^{\hat{a}},\theta^{\hat{\alpha}}_i)$. One can introduce flat covariant derivatives $D_{\hat{A}}=(\partial_{\hat{a}},D^i_{\hat{\alpha}})$ which obey the algebra

$$[D_{\hat{A}},D_{\hat{B}}\}=T_{\hat{A}\hat{B}}{}^{\hat{C}}D_{\hat{C}}\,, \tag{2.1}$$

with

$$T_{\hat{\alpha}}^{i}{}_{\hat{\beta}}^{j}{}^{\hat{c}}=-2\mathrm{i}\varepsilon^{ij}(\Gamma^{\hat{c}})_{\hat{\alpha}\hat{\beta}} \tag{2.2}$$

the only non-vanishing torsion component.

The non-Abelian vector supermultiplet may be described in superspace by introducing the gauge covariant derivatives[6]

$$\mathcal{D}_{\hat{A}}=(\mathcal{D}_{\hat{a}},\mathcal{D}^i_{\hat{\alpha}})=D_{\hat{A}}+\mathrm{i}V_{\hat{A}}(z)\,,\quad \{\mathcal{D}_{\hat{A}},\mathcal{D}_{\hat{B}}\}=T_{\hat{A}\hat{B}}{}^{\hat{C}}\mathcal{D}_{\hat{C}}+\mathrm{i}F_{\hat{A}\hat{B}}\,, \tag{2.3}$$

where $V_{\hat{A}}$ is a gauge connection taking values in the Lie algebra of the gauge group. The covariant derivatives and field strength may also be written in a coordinate-free way as follows

$$\mathcal{D}=\mathrm{d}+\mathrm{i}V\,,\quad F=\mathrm{d}V-\mathrm{i}V\wedge V\,, \tag{2.4}$$

where

$$\mathcal{D}:=\mathrm{d}z^{\hat{A}}\mathcal{D}_{\hat{A}}\,,\quad V:=\mathrm{d}z^{\hat{A}}V_{\hat{A}}\,,\quad F:=\frac{1}{2}\mathrm{d}z^{\hat{B}}\wedge\mathrm{d}z^{\hat{A}}F_{\hat{A}\hat{B}}\,. \tag{2.5}$$

The covariant derivatives possess the gauge transformation law

$$\mathcal{D}_{\hat{A}}\to \mathrm{e}^{\mathrm{i}\tau}\,\mathcal{D}_{\hat{A}}\,\mathrm{e}^{-\mathrm{i}\tau}\,,\quad \tau^\dagger=\tau\,, \tag{2.6}$$

[6]Keep in mind that the operation of complex conjugation acts as $(D^i_{\hat{\alpha}}F)^*=-(-1)^{\varepsilon(F)}D^{\hat{\alpha}}_iF^*$, where $\varepsilon(F)$ is the Grassmann parity of F, see [11] for details.

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where the Lie-algebra-valued gauge parameter $\tau(z)$ is arbitrary modulo the reality condition imposed. This implies that the gauge connection and field strength transform as follows

$$V \to \mathrm{e}^{\mathrm{i}\tau}\, V\, e^{-\mathrm{i}\tau} - \mathrm{i}\mathrm{e}^{\mathrm{i}\tau}\, \mathrm{d}\mathrm{e}^{-\mathrm{i}\tau}\,, \qquad F \to \mathrm{e}^{\mathrm{i}\tau}\, F\, \mathrm{e}^{-\mathrm{i}\tau}\ . \tag{2.7}$$

The field strength satisfies the Bianchi identity

$$\mathcal{D}F = \mathrm{d}F + \mathrm{i}V \wedge F - \mathrm{i}F \wedge V = 0\,, \quad \mathcal{D}_{[\hat{A}}F_{\hat{B}\hat{C}\}} - T_{[\hat{A}\hat{B}}{}^{\hat{D}}F_{|\hat{D}|\hat{C}\}} = 0\ . \tag{2.8}$$

Upon constraining the lowest mass dimension component of the field strength tensor as [10, 11, 38]

$$F^{i\,j}_{\hat{\alpha}\,\hat{\beta}} = -2\mathrm{i}\varepsilon^{ij}\varepsilon_{\hat{\alpha}\hat{\beta}}W\,, \tag{2.9a}$$

the remaining components are found to be

$$F_{\hat{a}}{}^{j}_{\hat{\beta}} = (\Gamma_{\hat{a}})_{\hat{\beta}}{}^{\hat{\gamma}}\mathcal{D}^j_{\hat{\gamma}}W\,, \quad F_{\hat{a}\hat{b}} = \frac{\mathrm{i}}{4}(\Sigma_{\hat{a}\hat{b}})^{\hat{\alpha}\hat{\beta}}\mathcal{D}^k_{(\hat{\alpha}}\mathcal{D}_{\hat{\beta})k}W\ . \tag{2.9b}$$

Here the superfield W is Hermitian, $W^\dagger = W$, and obeys the superfield Bianchi identity

$$\mathcal{D}^{(i}_{\hat{\alpha}}\mathcal{D}^{j)}_{\hat{\beta}}W = \frac{1}{4}\varepsilon_{\hat{\alpha}\hat{\beta}}\mathcal{D}^{\hat{\gamma}(i}\mathcal{D}^{j)}_{\hat{\gamma}}W\ . \tag{2.10}$$

From the above constraint one can derive identities involving products of spinor derivatives acting on W. We list these below:

$$\mathcal{D}^i_{\hat{\alpha}}\mathcal{D}^j_{\hat{\beta}}W = -\frac{1}{2}\varepsilon^{ij}\mathcal{D}^k_{(\hat{\alpha}}\mathcal{D}_{\hat{\beta})k}W + \frac{1}{4}\varepsilon_{\hat{\alpha}\hat{\beta}}\mathcal{D}^{\hat{\gamma}(i}\mathcal{D}^{j)}_{\hat{\gamma}}W - \mathrm{i}\varepsilon^{ij}\mathcal{D}_{\hat{\alpha}\hat{\beta}}W\,, \tag{2.11a}$$

$$\begin{aligned}\mathcal{D}^i_{\hat{\alpha}}\mathcal{D}^{\hat{\gamma}(j}\mathcal{D}^{k)}_{\hat{\gamma}}W &= \frac{1}{3}\varepsilon^{ij}\mathcal{D}_{\hat{\alpha}l}\mathcal{D}^{\hat{\gamma}(k}\mathcal{D}^{l)}_{\hat{\gamma}}W + \frac{1}{3}\varepsilon^{ik}\mathcal{D}_{\hat{\alpha}l}\mathcal{D}^{\hat{\gamma}(j}\mathcal{D}^{l)}_{\hat{\gamma}}W \\ &= 8\mathrm{i}\varepsilon^{i(j}\mathcal{D}_{\hat{\alpha}\hat{\gamma}}\mathcal{D}^{\hat{\gamma}k)}W - 8\varepsilon^{i(j}[W, \mathcal{D}^{k)}_{\hat{\alpha}}W]\,,\end{aligned} \tag{2.11b}$$

$$\mathcal{D}^i_{\hat{\alpha}}\mathcal{D}^k_{(\hat{\beta}}\mathcal{D}_{\hat{\gamma})k}W = 4\mathrm{i}\varepsilon_{\hat{\alpha}(\hat{\beta}}\mathcal{D}_{\hat{\gamma})\hat{\delta}}\mathcal{D}^{\hat{\delta}i}W - 4\mathrm{i}\mathcal{D}_{\hat{\alpha}(\hat{\beta}}\mathcal{D}^i_{\hat{\gamma})}W\ . \tag{2.11c}$$

As a result of the above identities, we may define the independent fields contained in W as

$$\varphi := W|\,, \quad \Psi^i_{\hat{\alpha}} := -\mathrm{i}\mathcal{D}^i_{\hat{\alpha}}W|\,, \quad F_{\hat{\alpha}\hat{\beta}} := \frac{\mathrm{i}}{4}\mathcal{D}^k_{(\hat{\alpha}}\mathcal{D}_{\hat{\beta})k}W|\,, \quad X^{ij} := \frac{\mathrm{i}}{4}\mathcal{D}^{\hat{\alpha}(i}\mathcal{D}^{j)}_{\hat{\alpha}}W|\,, \tag{2.12}$$

where the bar projection of a superfield $U(z) = U(x,\theta)$ is defined by the standard rule $U| := U(x,\theta)|_{\theta=0}$. The component gauge field is identified with $V_{\hat{a}}|$ and we will drop the bar projection when it is clear that we are referring to the component field. The component field strength $F_{\hat{a}\hat{b}}$ can be expressed in terms of the gauge field as follows

$$F_{\hat{a}\hat{b}} = 2\partial_{[\hat{a}}V_{\hat{b}]} + \mathrm{i}\left[V_{\hat{a}}, V_{\hat{b}}\right]\ . \tag{2.13}$$

It is seen that the vector supermultiplet consists of the following component fields: φ, $\Psi^i_{\hat{\alpha}}$, $V_{\hat{a}}$ and X^{ij}.

The supersymmetry transformations of the fields φ, $\Psi^i_{\hat\alpha}$, $V_{\hat a}$ and X^{ij} may be obtained by evaluating the component projection of the identities (2.11). This gives

$$\delta_\xi \varphi = \mathrm{i}\xi^{\hat\gamma}_k \Psi^k_{\hat\gamma} \,, \tag{2.14a}$$

$$\delta_\xi \Psi^i_{\hat\alpha} = -2\xi^{\hat\beta i} F_{\hat\alpha\hat\beta} + \xi_{\hat\alpha j} X^{ij} + \xi^{\hat\beta i}\mathcal{D}_{\hat\beta\hat\alpha}\varphi \,, \tag{2.14b}$$

$$\delta_\xi X^{ij} = -2\mathrm{i}\xi^{\hat\alpha(i}\mathcal{D}_{\hat\alpha}{}^{\hat\beta}\Psi^{j)}_{\hat\beta} - 2\xi^{\hat\alpha(i}[\varphi, \Psi^{j)}_{\hat\alpha}] \,, \tag{2.14c}$$

$$\delta_\xi V_{\hat a} = \xi^{\hat\beta}_j F^j_{\hat\beta\hat a}| = -\mathrm{i}\xi^{\hat\alpha}_j (\Gamma_{\hat a})_{\hat\alpha}{}^{\hat\beta}\Psi^j_{\hat\beta} \,, \tag{2.14d}$$

where we have used $\mathcal{D}_{\hat a}$ to mean its projection, $\mathcal{D}_{\hat a}| = \partial_{\hat a} + \mathrm{i}V_{\hat a}|$, when acting on a component field.

2.2 Superforms and the Chern-Simons action

The SCS action may readily be found in the Abelian case with the use of the action principle based on a linear supermultiplet without central charge. However, a generalization of the action principle to the non-Abelian case is not straightforward. In components, the non-Abelian SCS action was constructed by Kugo and Ohashi [9] by first starting with the Abelian Chern-Simons action. They added non-Abelian structures to the action and checked supersymmetry with the supersymmetry transformations of the non-Abelian theory.

There is a more elegant alternative offered by the superform approach to construct supersymmetric invariants. In conventional 5D superspace $\mathbb{R}^{5|8}$, the formalism makes use of a closed five-form

$$J = \frac{1}{5!}\mathrm{d}z^{\hat E}\wedge \mathrm{d}z^{\hat D}\wedge \mathrm{d}z^{\hat C}\wedge \mathrm{d}z^{\hat B}\wedge \mathrm{d}z^{\hat A} J_{\hat A\hat B\hat C\hat D\hat E} \,, \quad \mathrm{d}J = 0 \,. \tag{2.15}$$

Under an infinitesimal coordinate transformation generated by a vector field $\xi = \xi^A\partial_A$, the five-form varies as

$$\delta_\xi J = \mathcal{L}_\xi J \equiv i_\xi \mathrm{d}J + \mathrm{d}i_\xi J = \mathrm{d}i_\xi J \,. \tag{2.16}$$

If we assume that the components ξ^A vanish at infinity in $\mathbb{R}^{5|8}$ then we have the supersymmetric invariant

$$S = \int_{\mathbb{R}^5} i^* J \,, \tag{2.17}$$

where $i : \mathbb{R}^5 \to \mathbb{R}^{5|8}$ is the inclusion map. This can be represented as

$$S = \int \mathrm{d}^5 x \, {}^*J|_{\theta=0} \,, \qquad {}^*J = \frac{1}{5!}\varepsilon^{\hat a\hat b\hat c\hat d\hat e} J_{\hat a\hat b\hat c\hat d\hat e} \,. \tag{2.18}$$

A suitable action must also be invariant under all *gauge symmetries* of a dynamical system under consideration. If the closed five-form J also transforms by an exact form under the gauge transformations,

$$\delta J = \mathrm{d}\Theta \,, \tag{2.19}$$

then the functional (2.17) is a suitable candidate for an action.

For the Chern-Simons action, following [27, 28], we will construct a gauge invariant closed five-form by first finding two solutions, $\Sigma_{\rm CS}$ and Σ_R, to the superform equation

$$\mathrm{d}\Sigma = \langle F^3\rangle := \mathrm{tr}\Big(F\wedge F\wedge F\Big) \ . \tag{2.20}$$

The first of which is the Chern-Simons form $\Sigma_{\rm CS}$. The existence of the second solution, the curvature induced form Σ_R, is a direct consequence of the constraints we imposed on the geometry, eq. (2.9a). If they transform by an exact form under the gauge group then their difference

$$J = \Sigma_{\rm CS} - \Sigma_R \tag{2.21}$$

will yield an appropriate closed five-form that describes the action.

2.2.1 Chern-Simons five-form

Representing $\langle F^3\rangle = \mathrm{d}\,\Sigma_{\rm CS}$ yields the Chern-Simons form

$$\Sigma_{\rm CS} = \mathrm{tr}\Big(V\wedge F\wedge F + \frac{\mathrm{i}}{2}V\wedge V\wedge V\wedge F - \frac{1}{10}V\wedge V\wedge V\wedge V\wedge V\Big) \ . \tag{2.22}$$

Since $\Sigma_{\rm CS}$ has been constructed by extracting a total derivative from the gauge invariant superform $\langle F^3\rangle$ it must transform by a closed form under the gauge group. In fact, one can show it transforms by an exact form,

$$\Sigma_{\rm CS} \to \Sigma_{\rm CS} - \mathrm{d}\ \mathrm{tr}\Big(\mathrm{d}\tau\wedge\Big(V\wedge F + \frac{\mathrm{i}}{2}V\wedge V\wedge V\Big)\Big) \ . \tag{2.23}$$

2.2.2 Curvature-induced five-form

To construct the curvature-induced five-form we need to find a gauge-invariant solution to

$$\mathrm{d}\Sigma = \mathrm{tr}\Big(F\wedge F\wedge F\Big) \tag{2.24a}$$

or, equivalently,

$$2\mathcal{D}_{[A}\Sigma_{BCDEF\}} \ \ -5T_{[AB}{}^{G}\Sigma_{|G|CDEF\}} = 30\,\mathrm{tr}\Big(F_{[AB}F_{CD}F_{EF\}}\Big)\, , \tag{2.24b}$$

where the gauge covariant derivative $\mathcal{D}_A$ is defined by eq. (2.3). Note that since Σ is a gauge singlet we have

$$\mathcal{D}_A\Sigma = D_A\Sigma \ . \tag{2.25}$$

Keeping this in mind, we will use gauge covariant derivatives everywhere in this section.

On dimensional grounds, it is natural to impose the constraint[7]

$$\Sigma_{\underline{\hat\alpha\hat\beta\hat\gamma\hat\delta\hat\epsilon}} = 0 \ . \tag{2.26}$$

[7]We denote pairs of spinor and isospinor indices, e.g. ${}^{i}_{\hat\alpha}$ by underlined spinor indices, e.g. $\underline{\hat\alpha}$.

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Then analyzing the superform equation (2.24) by increasing mass dimension and using the identities (2.11) yields all the remaining components of the curvature induced five-form. One finds the following components:

$$\begin{aligned}
\Sigma_{\hat{a}\hat{\alpha}}{}^{i}{}_{\hat{\beta}}{}^{j}{}_{\hat{\gamma}}{}^{k}{}_{\hat{\delta}}{}^{l} = &- 4\Big(\varepsilon^{ij}\varepsilon^{kl}\big((\Gamma_{\hat{a}})_{\hat{\alpha}\hat{\beta}}\varepsilon_{\hat{\gamma}\hat{\delta}} + (\Gamma_{\hat{a}})_{\hat{\gamma}\hat{\delta}}\varepsilon_{\hat{\alpha}\hat{\beta}}\big) + \varepsilon^{ik}\varepsilon^{jl}\big((\Gamma_{\hat{a}})_{\hat{\alpha}\hat{\gamma}}\varepsilon_{\hat{\beta}\hat{\delta}} + (\Gamma_{\hat{a}})_{\hat{\beta}\hat{\delta}}\varepsilon_{\hat{\alpha}\hat{\gamma}}\big) \\
&+ \varepsilon^{il}\varepsilon^{jk}\big((\Gamma_{\hat{a}})_{\hat{\alpha}\hat{\delta}}\varepsilon_{\hat{\beta}\hat{\gamma}} + (\Gamma_{\hat{a}})_{\hat{\beta}\hat{\gamma}}\varepsilon_{\hat{\alpha}\hat{\delta}}\big)\Big)\mathrm{tr}(W^3)\,, \qquad (2.27a)
\end{aligned}$$

$$\Sigma_{\hat{a}\hat{b}\hat{\alpha}}{}^{i}{}_{\hat{\beta}}{}^{j}{}_{\hat{\gamma}}{}^{k} = - 4\mathrm{i}\Big(\varepsilon^{jk}\varepsilon_{\hat{\beta}\hat{\gamma}}(\Sigma_{\hat{a}\hat{b}})_{\hat{\alpha}}{}^{\hat{\delta}}\mathcal{D}^{i}_{\hat{\delta}} + \varepsilon^{ij}\varepsilon_{\hat{\alpha}\hat{\beta}}(\Sigma_{\hat{a}\hat{b}})_{\hat{\gamma}}{}^{\hat{\delta}}\mathcal{D}^{k}_{\hat{\delta}} + \varepsilon^{ki}\varepsilon_{\hat{\gamma}\hat{\alpha}}(\Sigma_{\hat{a}\hat{b}})_{\hat{\beta}}{}^{\hat{\delta}}\mathcal{D}^{j}_{\hat{\delta}}\Big)\mathrm{tr}(W^3)\,, \qquad (2.27b)$$

$$\begin{aligned}
\Sigma_{\hat{a}\hat{b}\hat{c}\hat{\alpha}}{}^{i}{}_{\hat{\beta}}{}^{j} = &- \frac{3}{4}\varepsilon^{ij}\varepsilon_{\hat{\alpha}\hat{\beta}}\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}(\Sigma^{\hat{d}\hat{e}})^{\hat{\gamma}\hat{\delta}}\mathrm{tr}(W^2\mathcal{D}^{k}_{\hat{\gamma}}\mathcal{D}_{\hat{\delta}k}W + 4W\mathcal{D}^{k}_{\hat{\gamma}}W\mathcal{D}_{\hat{\delta}k}W) \\
&- \frac{3}{2}\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}(\Sigma^{\hat{d}\hat{e}})_{\hat{\alpha}\hat{\beta}}\mathrm{tr}(W^2\mathcal{D}^{\hat{\gamma}(i}\mathcal{D}^{j)}_{\hat{\gamma}}W + 4W\mathcal{D}^{\hat{\gamma}(i}W\mathcal{D}^{j)}_{\hat{\gamma}}W)\,, \qquad (2.27c)
\end{aligned}$$

$$\begin{aligned}
\Sigma_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{\alpha}}{}^{i} = &- \frac{\mathrm{i}}{8}\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}(\Gamma^{\hat{e}})_{\hat{\alpha}}{}^{\hat{\beta}}\mathrm{tr}\Big(6W\{\mathcal{D}^{j}_{(\hat{\beta}}\mathcal{D}_{\hat{\gamma})j}W, \mathcal{D}^{\hat{\gamma}i}W\} + 3W\{\mathcal{D}^{\hat{\gamma}(i}\mathcal{D}^{j)}_{\hat{\gamma}}W, \mathcal{D}_{\hat{\beta}j}W\} \\
&\qquad + 16\mathcal{D}^{\hat{\gamma}(i}W\mathcal{D}^{j)}_{\hat{\gamma}}W\mathcal{D}_{\hat{\beta}j}W\Big) \\
&- \mathrm{i}\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}(\Gamma^{\hat{e}})^{\hat{\beta}\hat{\gamma}}\mathrm{tr}\big(\mathcal{D}^{(i}_{\hat{\beta}}W\mathcal{D}^{j)}_{\hat{\gamma}}W\mathcal{D}_{\hat{\alpha}j}W\big) \\
&+ 3\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}(\Sigma^{\hat{e}\hat{f}})_{\hat{\alpha}}{}^{\hat{\beta}}\mathrm{tr}\Big(W\mathcal{D}_{\hat{f}}\{W, \mathcal{D}^{i}_{\hat{\beta}}W\}\Big) \\
&+ \frac{3}{2}\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}\mathrm{tr}\Big(W\mathcal{D}^{\hat{e}}\{W, \mathcal{D}^{i}_{\hat{\alpha}}W\}\Big)\,. \qquad (2.27d)
\end{aligned}$$

The final component

$$\begin{aligned}
\Sigma_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}} = &- \frac{3}{32}\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}\mathrm{tr}\Big(W\mathcal{D}^{\hat{\gamma}(k}\mathcal{D}^{l)}_{\hat{\gamma}}W\mathcal{D}^{\hat{\delta}}_{(k}\mathcal{D}_{\hat{\delta}l)}W - 2W\mathcal{D}^{(\hat{\gamma}k}\mathcal{D}^{\hat{\delta})}_{k}W\mathcal{D}^{l}_{(\hat{\gamma}}\mathcal{D}_{\hat{\delta})l}W \\
&+ 4\mathcal{D}^{\hat{\gamma}(k}\mathcal{D}^{l)}_{\hat{\gamma}}W\mathcal{D}^{\hat{\delta}}_{k}W\mathcal{D}_{\hat{\delta}l}W - 8\mathcal{D}^{(\hat{\gamma}k}\mathcal{D}^{\hat{\delta})}_{k}W\mathcal{D}^{l}_{\hat{\gamma}}W\mathcal{D}_{\hat{\delta}l}W \\
&- 16W\mathcal{D}^{\hat{f}}\{W, \mathcal{D}_{\hat{f}}W\} + 16\mathrm{i}W[\mathcal{D}_{\hat{\gamma}\hat{\delta}}\mathcal{D}^{\hat{\gamma}k}W, \mathcal{D}^{\hat{\delta}}_{k}W] \\
&- 32W^2\mathcal{D}^{\hat{\gamma}k}W\mathcal{D}_{\hat{\gamma}k}W\Big) \qquad (2.27e)
\end{aligned}$$

is the most important from the point of view of constructing the action. It is obvious that the superform constructed is gauge invariant. The last term in (2.27e), which is quartic in W, disappears in the Abelian case.

Once all components are determined there still remains the final superform component equation

$$5\mathcal{D}_{[\hat{a}}\Sigma_{\hat{b}\hat{c}\hat{d}\hat{e}]\underline{\hat{\alpha}}} - \mathcal{D}_{\underline{\hat{\alpha}}}\Sigma_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}} - 90\mathrm{tr}\Big(F_{[\hat{a}\hat{b}}F_{\hat{c}\hat{d}}F_{\hat{e}]\underline{\alpha}}\Big) = 0\,. \qquad (2.28)$$

However, this only remains as a check as it will always be identically satisfied (see appendix of [39]).

2.2.3 The component non-abelian Chern-Simons action

Making use of the superforms Σ_{CS} and Σ_R one can construct a closed five-form

$$J = \Sigma_{\mathrm{CS}} - \Sigma_R\,, \qquad (2.29)$$

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from which one can derive a supersymmetric action. The gauge invariance of the action, modulo total derivatives, is guaranteed by the fact that $\Sigma_{\rm CS}$ transforms via an exact form while Σ_R is invariant.

In components we have

$$J_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}} = 30{\rm tr}\Big(V_{[\hat{a}}F_{\hat{b}\hat{c}}F_{\hat{d}\hat{e}]} - {\rm i}V_{[\hat{a}}V_{\hat{b}}V_{\hat{c}}F_{\hat{d}\hat{e}]} - \frac{2}{5}V_{[\hat{a}}V_{\hat{b}}V_{\hat{c}}V_{\hat{d}}V_{\hat{e}]}\Big) - \Sigma_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}\,, \tag{2.30a}$$

or, equivalently,

$${}^*J = \frac{1}{4}\varepsilon^{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}{\rm tr}\Big(V_{\hat{a}}F_{\hat{b}\hat{c}}F_{\hat{d}\hat{e}} - {\rm i}V_{\hat{a}}V_{\hat{b}}V_{\hat{c}}F_{\hat{d}\hat{e}} - \frac{2}{5}V_{[\hat{a}}V_{\hat{b}}V_{\hat{c}}V_{\hat{d}}V_{\hat{e}]}\Big) - \frac{1}{5!}\varepsilon^{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}\Sigma_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}\,. \tag{2.30b}$$

Applying eq. (2.17) to the above results and dividing out an irrelevant factor of 3 gives the Chern-Simons action

$$\begin{aligned} S = \int {\rm d}^5x\, {\rm tr}\Big\{&\frac{1}{12}\varepsilon^{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}V_{\hat{a}}F_{\hat{b}\hat{c}}F_{\hat{d}\hat{e}} - \frac{\rm i}{12}\varepsilon^{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}V_{\hat{a}}V_{\hat{b}}V_{\hat{c}}F_{\hat{d}\hat{e}} - \frac{1}{30}\varepsilon^{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}V_{\hat{a}}V_{\hat{b}}V_{\hat{c}}V_{\hat{d}}V_{\hat{e}} \\ &- \frac{1}{2}\varphi F_{\hat{a}\hat{b}}F^{\hat{a}\hat{b}} + \frac{1}{2}\varphi X^{ij}X_{ij} - \frac{\rm i}{2}F_{\hat{a}\hat{b}}(\Psi^k\Sigma^{\hat{a}\hat{b}}\Psi_k) \\ &- \frac{\rm i}{2}X_{ij}(\Psi^i\Psi^j) + \frac{\rm i}{2}\varphi\Psi^k\overleftrightarrow{\not{\mathcal{D}}}\Psi_k - \varphi\mathcal{D}^{\hat{a}}\varphi\mathcal{D}_{\hat{a}}\varphi - \varphi^2\Psi^k\Psi_k\Big\}\,, \end{aligned} \tag{2.31}$$

where we integrated by parts and defined

$$\varphi\Psi^k\overleftrightarrow{\not{\mathcal{D}}}\Psi_k := \varphi\Psi^k\not{\mathcal{D}}\Psi_k - \varphi\not{\mathcal{D}}\Psi^k\Psi_k\,. \tag{2.32}$$

The above action may be compared to the action in [9]. The supersymmetry transformations of the component fields are given by eq. (2.14).

In the Abelian case the Chern-Simons action simplifies to[8]

$$\begin{aligned} S = \int {\rm d}^5x\ {\rm tr}\Big(&\frac{1}{12}\varepsilon^{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}V_{\hat{a}}F_{\hat{b}\hat{c}}F_{\hat{d}\hat{e}} - \frac{1}{2}\varphi F_{\hat{a}\hat{b}}F^{\hat{a}\hat{b}} + \frac{1}{2}\varphi X^{ij}X_{ij} - \frac{\rm i}{2}F_{\hat{a}\hat{b}}(\Psi^k\Sigma^{\hat{a}\hat{b}}\Psi_k) \\ &- \frac{\rm i}{2}X_{ij}(\Psi^i\Psi^j) + {\rm i}\varphi\Psi^k\not{\partial}\Psi_k - \varphi\partial^{\hat{a}}\varphi\partial_{\hat{a}}\varphi\Big)\,. \end{aligned} \tag{2.33}$$

In the next section we will derive the above action with the use of the linear supermultiplet.

3 Off-shell supermultiplets with central charge

In this section, we provide a superform description for certain supermultiplets with gauged central charge. Firstly, we discuss how to gauge the central charge in 5D $\mathcal{N}=1$ superspace following [37]. We then give the superform formulation for the linear supermultiplet with central charge and immediately derive the action. Finally, we give the superform formulations for a gauge two-form supermultiplet and for a large tensor supermultiplet.

[8]Due to a typo in [11], the first term in the action differs from the one in [11] by a factor of 4.

3.1 Gauging a central charge in superspace

Let Δ denote a central charge. It can be gauged using an Abelian vector supermultiplet associated with a gauge connection $\mathcal{V}$. The procedure is similar to the one used in subsection 2.1. We simply need to replace the gauge connection V and field strength F in eqs. (2.3) and (2.4) with those associated with the central charge Δ as follows:

$$\mathrm{i}V \to \mathcal{V}\Delta\,, \quad \mathrm{i}F \to \mathcal{F}\Delta\,. \tag{3.1}$$

The central charge commutes with the covariant derivatives and annihilates both $\mathcal{V}$ and $\mathcal{F}$

$$[\Delta, \mathcal{D}_{\hat{A}}] = 0\,, \quad \Delta\mathcal{V} = 0\,, \quad \Delta\mathcal{F} = 0\,. \tag{3.2}$$

Gauge transformations of the covariant derivatives are replaced by

$$\delta\mathcal{D}_{\hat{A}} = [\Lambda\Delta, \mathcal{D}_{\hat{A}}] \implies \delta\mathcal{V}_{\hat{A}} = -D_{\hat{A}}\Lambda\,, \tag{3.3}$$

where the gauge parameter is inert under the central charge, $\Delta\Lambda = 0$. The possibility of allowing the central charge to not annihilate the gauge connection is discussed in the appendix.

The field strength $\mathcal{F}$ is constrained to be formally the same as eq. (2.9) but with W replaced by $\mathcal{W}$. For later reference, we list the components of $\mathcal{F}$ here. They are

$$\mathcal{F}_{\hat{\alpha}}^{i}{}_{\hat{\beta}}^{j} = -2\mathrm{i}\varepsilon^{ij}\varepsilon_{\hat{\alpha}\hat{\beta}}\mathcal{W}\,, \tag{3.4a}$$

$$\mathcal{F}_{\hat{a}}{}_{\hat{\beta}}^{j} = (\Gamma_{\hat{a}})_{\hat{\beta}}{}^{\hat{\gamma}}\mathcal{D}_{\hat{\gamma}}^{j}\mathcal{W}\,, \tag{3.4b}$$

$$\mathcal{F}_{\hat{a}\hat{b}} = \frac{\mathrm{i}}{4}(\Sigma_{\hat{a}\hat{b}})^{\hat{\alpha}\hat{\beta}}\mathcal{D}_{\hat{\alpha}}^{k}\mathcal{D}_{\hat{\beta}k}\mathcal{W}\,, \tag{3.4c}$$

with $\mathcal{W}$ constrained by the Bianchi identity

$$\mathcal{D}_{\hat{\alpha}}^{(i}\mathcal{D}_{\hat{\beta}}^{j)}\mathcal{W} = \frac{1}{4}\varepsilon_{\hat{\alpha}\hat{\beta}}\mathcal{D}^{\hat{\gamma}(i}\mathcal{D}_{\hat{\gamma}}^{j)}\mathcal{W}\,. \tag{3.5}$$

3.2 Linear supermultiplet

Here we construct a superform formulation for the 5D linear supermultiplet[9] with gauged central charge which will naturally lead to the action for the supermultiplet.

3.2.1 Superform formulation for the linear supermultiplet

To construct a superform formulation for a supermultiplet with intrinsic central charge one usually makes some modifications to superspace. In rigid supersymmetry with a central charge, it is well known that one can treat the central charge as a derivative with respect to an additional bosonic coordinate. In fact, this approach was used in 4D to construct

[9] In 4D $\mathcal{N} = 2$ supergravity, the linear supermultiplet was introduced by Breitenlohner and Sohnius [40] (see also [41]) building on the rigid supersymmetric construction due to Sohnius [42]. The 5D $\mathcal{N} = 1$ linear supermultiplet [43, 44] is a natural generalization of its 4D ancestor.

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a superform formulation for the linear vector-tensor supermultiplet [45–47].[10] For certain supermultiplets, e.g. the linear supermultiplet, the approach is equivalent to dimensional reduction of supermultiplets from higher dimensions. However, the situation is more complex in the presence of a gauged central charge. For the linear supermultiplet with gauged central charge in 4D supergravity one finds that it is natural to extend the vielbein to include the central charge gauge one-form [30]. The resulting formulation turns out to be equivalent to a system of superforms. Here we will begin with a generalization of the system of superforms found in [30] and introduce some useful notation that will help us solve certain constraints.

We introduce a five-form $\tilde{\Sigma}$ and a four-form Φ which are coupled by the superform equations

$$\mathcal{D}\tilde{\Sigma} = \mathcal{F} \wedge \Phi\,, \quad \mathcal{D}\Phi = -\Delta\tilde{\Sigma} \tag{3.6}$$

and transform as scalars under the central charge gauge transformations (3.3)

$$\delta\tilde{\Sigma} = \Lambda\Delta\tilde{\Sigma}\,, \quad \delta\Phi = \Lambda\Delta\Phi\ . \tag{3.7}$$

The superforms $\tilde{\Sigma}$ and Φ may be related to the linear supermultiplet with central charge by imposing certain constraints. It will prove useful to first introduce some notation to deal with the superform equations (3.6).

We introduce indices that range over not just $\hat{A}$ but an additional bosonic coordinate, $\hat{\mathcal{A}} = (\hat{A}, 6)$. Then we may rewrite eq. (3.6) in components as

$$\mathcal{D}_{[\hat{\mathcal{A}}}\Sigma_{\hat{\mathcal{B}}\hat{\mathcal{C}}\hat{\mathcal{D}}\hat{\mathcal{E}}\hat{\mathcal{F}}\}} - \frac{5}{2}T_{[\hat{\mathcal{A}}\hat{\mathcal{B}}}{}^{\hat{\mathcal{G}}}\Sigma_{|\hat{\mathcal{G}}|\hat{\mathcal{C}}\hat{\mathcal{D}}\hat{\mathcal{E}}\hat{\mathcal{F}}\}} = 0\,, \tag{3.8}$$

where we have made the identifications

$$T_{\hat{A}\hat{B}}{}^{6} = \mathcal{F}_{\hat{A}\hat{B}}\,, \quad T_{6\hat{B}}{}^{\hat{\mathcal{A}}} = T_{\hat{B}6}{}^{\hat{\mathcal{A}}} = 0\,, \quad \mathcal{D}_6 = \Delta \tag{3.9}$$

and

$$\begin{aligned}\tilde{\Sigma} &= \frac{1}{5!}\mathrm{d}z^{\hat{E}} \wedge \mathrm{d}z^{\hat{D}} \wedge \mathrm{d}z^{\hat{C}} \wedge \mathrm{d}z^{\hat{B}} \wedge \mathrm{d}z^{\hat{A}}\Sigma_{\hat{A}\hat{B}\hat{C}\hat{D}\hat{E}}\,,\\ \Phi &= \frac{1}{4!}\mathrm{d}z^{\hat{D}} \wedge \mathrm{d}z^{\hat{C}} \wedge \mathrm{d}z^{\hat{B}} \wedge \mathrm{d}z^{\hat{A}}\Sigma_{6\hat{A}\hat{B}\hat{C}\hat{D}}\ .\end{aligned} \tag{3.10}$$

We now impose simple constraints on the lowest mass dimension components

$$\begin{aligned}&\Sigma_{\underline{\hat{\alpha}}\underline{\hat{\beta}}\underline{\hat{\gamma}}\underline{\hat{\delta}}\underline{\hat{\epsilon}}} = \Sigma_{\hat{a}\underline{\hat{\alpha}}\underline{\hat{\beta}}\underline{\hat{\gamma}}\underline{\hat{\delta}}} = \Sigma_{\hat{a}\hat{b}\underline{\hat{\alpha}}\underline{\hat{\beta}}\underline{\hat{\gamma}}} = \Sigma_{6\underline{\hat{\alpha}}\underline{\hat{\beta}}\underline{\hat{\gamma}}\underline{\hat{\delta}}} = \Sigma_{6\hat{a}\underline{\hat{\beta}}\underline{\hat{\gamma}}\underline{\hat{\delta}}} = 0\,,\\ &\Sigma_{6\hat{a}\hat{b}\underline{\hat{\alpha}}\underline{\hat{\beta}}} = 4\mathrm{i}(\Sigma_{\hat{a}\hat{b}})_{\hat{\alpha}\hat{\beta}}L^{ij}\,,\end{aligned} \tag{3.11}$$

[10] The superform formulation and action for the linear supermultiplet with rigid central charge in 5D was given in [48]. However, the case of a gauged central charge was not studied.

and analyze eq. (3.8). The remaining components are fixed as follows:

$$
\begin{aligned}
\Sigma_{\hat{a}\hat{b}\hat{c}\underline{\hat{\alpha}\hat{\beta}}} &= 2\mathrm{i}\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}(\Sigma^{\hat{d}\hat{e}})_{\hat{\alpha}\hat{\beta}}\mathcal{W}L^{ij}\,,\\
\Sigma_{6\hat{a}\hat{b}\hat{c}\underline{\hat{\alpha}}} &= -\frac{1}{3}\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}(\Sigma^{\hat{d}\hat{e}})_{\hat{\alpha}}{}^{\hat{\beta}}\mathcal{D}_{\hat{\beta}j}L^{ji}\,,\\
\Sigma_{\hat{a}\hat{b}\hat{c}\hat{d}\underline{\hat{\alpha}}} &= -\frac{1}{3}\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}(\Gamma^{\hat{e}})_{\hat{\alpha}}{}^{\hat{\beta}}(\mathcal{W}\mathcal{D}_{\hat{\beta}j}L^{ji}+3\mathcal{D}_{\hat{\beta}j}\mathcal{W}L^{ji})\,,\\
\Sigma_{6\hat{a}\hat{b}\hat{c}\hat{d}} &= \frac{\mathrm{i}}{24}\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}(\Gamma^{\hat{e}})^{\hat{\alpha}\hat{\beta}}\mathcal{D}^i_{\hat{\alpha}}\mathcal{D}^j_{\hat{\beta}}L_{ij}\,,\\
\Sigma_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}} &= \frac{\mathrm{i}}{24}\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}(\mathcal{W}\mathcal{D}^{\hat{\gamma}i}\mathcal{D}^j_{\hat{\gamma}}L_{ij}+3\mathcal{D}^{\hat{\gamma}i}\mathcal{D}^j_{\hat{\gamma}}\mathcal{W}L_{ij}+8\mathcal{D}^{\hat{\gamma}i}\mathcal{W}\mathcal{D}^j_{\hat{\gamma}}L_{ij})\,,
\end{aligned}
\tag{3.12}
$$

where L^{ij} satisfies the constraint for the linear supermultiplet

$$\mathcal{D}^{(i}_{\hat{\alpha}}L^{jk)} = 0\ . \tag{3.13}$$

In the above we did not assume anywhere that L^{ij} is annihilated by the central charge. However, if L^{ij} is inert under the central charge, $\Delta L^{ij} = 0$, we have

$$\mathrm{d}\Phi = 0 \tag{3.14}$$

and L^{ij} becomes a gauge three-form supermultiplet, also known as the $\mathcal{O}(2)$ supermultiplet.

3.2.2 Action principle

Making use of the components $\Sigma_{\hat{\mathcal{A}}\hat{\mathcal{B}}\hat{\mathcal{C}}\hat{\mathcal{D}}\hat{\mathcal{E}}}$ one can construct a closed five-form. The appropriate closed form is simply given by

$$J = \tilde{\Sigma} - \mathcal{V}\wedge\Phi\ . \tag{3.15}$$

All that one must check is closure,

$$\mathrm{d}J = \mathrm{d}\tilde{\Sigma} - \mathcal{V}\wedge\mathrm{d}\Phi - \mathrm{d}\mathcal{V}\wedge\Phi = \mathcal{D}\tilde{\Sigma} - \mathcal{V}\wedge\Delta\tilde{\Sigma} - \mathcal{V}\wedge\mathcal{D}\Phi - \mathcal{F}\wedge\Phi = 0\,, \tag{3.16}$$

and the transformation law under central charge transformations,

$$
\begin{aligned}
\delta_\Lambda J &= \delta_\Lambda\Sigma + \delta_\Lambda\mathcal{V}\wedge\Phi + \mathcal{V}\wedge\delta_\Lambda\Phi\\
&= \Lambda\Delta\Sigma - \mathrm{d}\Lambda\wedge\Phi + \mathcal{V}\wedge(\Lambda\Delta\Phi) = \mathrm{d}(\Lambda\Delta\Phi)\ .
\end{aligned}
\tag{3.17}
$$

In components we have

$$J_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}} = \tilde{\Sigma}_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}} - 5\mathcal{V}_{[\hat{a}}\Phi_{\hat{b}\hat{c}\hat{d}\hat{e}]}\,, \tag{3.18a}$$

which gives

$${}^*J = \frac{1}{5!}\varepsilon^{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}\tilde{\Sigma}_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}} - \frac{1}{4!}\varepsilon^{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}\mathcal{V}_{\hat{a}}\Phi_{\hat{b}\hat{c}\hat{d}\hat{e}}\ . \tag{3.18b}$$

The action is then

$$
\begin{aligned}
S &= -\frac{\mathrm{i}}{24}\int\mathrm{d}^5x\Big(\mathcal{W}\mathcal{D}^{\hat{\gamma}i}\mathcal{D}^j_{\hat{\gamma}}L_{ij}+3\mathcal{D}^{\hat{\gamma}i}\mathcal{D}^j_{\hat{\gamma}}\mathcal{W}L_{ij}+8\mathcal{D}^{\hat{\gamma}i}\mathcal{W}\mathcal{D}^j_{\hat{\gamma}}L_{ij}+\mathcal{V}_{\hat{a}}(\mathcal{D}^i\Gamma^{\hat{a}}\mathcal{D}^j)L_{ij}\Big)\Big|\\
&= -\frac{1}{2}\int\mathrm{d}^5x\Big(\varphi G + X^{ij}\ell_{ij} + 2\Psi^{\hat{\gamma}k}\chi_{\hat{\gamma}k} - 2\mathcal{V}_{\hat{a}}\phi^{\hat{a}}\Big)\ ,
\end{aligned}
\tag{3.19}
$$

where the component fields of $\mathcal{W}$ are defined as in eq. (2.12) and we have defined the component fields of the linear supermultiplet as follows:

$$\ell^{ij} := L^{ij}| \,, \qquad \chi^i_{\hat\alpha} := \frac{1}{3}\mathcal{D}_{\hat\alpha j}L^{ij}| \,, \qquad G := \frac{\mathrm{i}}{12}\mathcal{D}^{\hat\gamma i}\mathcal{D}^j_{\hat\gamma}L_{ij}| \,, \tag{3.20a}$$

$$\phi^{\hat a} := \frac{\mathrm{i}}{24}(\Gamma^{\hat a})^{\hat\alpha\hat\beta}\mathcal{D}^i_{\hat\alpha}\mathcal{D}^j_{\hat\beta}L_{ij}| = \Phi^{\hat a}| \,, \qquad \Phi_{\hat a\hat b\hat c\hat d} = \varepsilon_{\hat a\hat b\hat c\hat d\hat e}\Phi^{\hat e} \,. \tag{3.20b}$$

The supersymmetry transformations for the linear supermultiplet follow from the constraint (3.13) and are found to be

$$\delta_\xi \ell^{ij} = -2\xi^{\hat\alpha(i}\chi^{j)}_{\hat\alpha} \,, \tag{3.21a}$$

$$\delta_\xi \chi^i_{\hat\alpha} = -\frac{\mathrm{i}}{2}\xi^i_{\hat\alpha}G + \mathrm{i}\xi^{\hat\beta i}\phi_{\hat\alpha\hat\beta} + \mathrm{i}\xi^{\hat\beta}_j\mathcal{D}_{\hat\beta\hat\alpha}\ell^{ij} \,, \tag{3.21b}$$

$$\delta_\xi G = -2\xi^{\hat\alpha}_i\mathcal{D}_{\hat\alpha}{}^{\hat\beta}\chi^i_{\hat\beta} - 2\mathrm{i}\xi^{\hat\alpha}_i\Psi_{\hat\alpha j}\Delta\ell^{ij} \,, \tag{3.21c}$$

$$\delta_\xi \phi_{\hat a} = 2\xi^{\hat\alpha}_i(\Sigma_{\hat a\hat b})_{\hat\alpha}{}^{\hat\beta}\mathcal{D}^{\hat b}\chi^i_{\hat\beta} - \mathrm{i}\xi^{\hat\alpha}_i(\Gamma_{\hat a})_{\hat\alpha}{}^{\hat\beta}\Psi_{\hat\beta j}\Delta\ell^{ij} - \xi^{\hat\alpha}_i(\Gamma_{\hat a})_{\hat\alpha}{}^{\hat\beta}\varphi\Delta\chi^i_{\hat\beta} \,. \tag{3.21d}$$

The action (3.19) and the supersymmetry transformations (3.21) agree with those given in [35]. These results hold for the linear multiplet both with or without central charge.

It is worth noting that checking invariance of the component action (3.19) under the central charge is nontrivial and requires having to derive some nontrivial identities. However, within the superform approach invariance follows much more easily. Furthermore, the superform formulation for the linear supermultiplet tells us more than just the action. For instance, taking the component projection of the Bianchi identity

$$5\mathcal{D}_{[\hat a}\Phi_{\hat b\hat c\hat d\hat e\}} = \Delta\tilde\Sigma_{\hat a\hat b\hat c\hat d\hat e} \,, \tag{3.22}$$

gives the differential constraint on the component field $\phi_{\hat a}$

$$2\mathcal{D}^{\hat a}\phi_{\hat a} = \Delta(\varphi G + X^{ij}\ell_{ij} + 2\psi^{\hat\gamma k}\chi_{\hat\gamma k}) \,. \tag{3.23}$$

The supersymmetry transformations are also encoded in the Bianchi identities (3.8). This provides an efficient means of computing some of the supersymmetry transformations. In particular, the supersymmetry transformation of $\phi_{\hat a}$, eq. (3.21d), follows directly from the component projection of the Bianchi identity

$$\mathcal{D}^i_{\hat\alpha}\Phi_{\hat a\hat b\hat c\hat d} = -4\mathcal{D}_{[\hat a}\Phi_{\hat b\hat c\hat d]}{}^i_{\hat\alpha} + \Delta\tilde\Sigma_{\hat a\hat b\hat c\hat d}{}^i_{\hat\alpha} \,. \tag{3.24}$$

Using the action for the linear supermultiplet, one can derive the Abelian Chern-Simons action by taking [11]

$$L^{ij} = \mathrm{i}\mathcal{D}^{\hat\gamma(i}W\mathcal{D}^{j)}_{\hat\gamma}W + \frac{\mathrm{i}}{2}W\mathcal{D}^{\hat\gamma(i}\mathcal{D}^{j)}_{\hat\gamma}W \,. \tag{3.25}$$

Using the above choice of L^{ij} and the action principle for the linear supermultiplet one derives (after removing a total derivative from the Lagrangian and dividing out an irrelevant factor of 6) the Abelian action (2.33).

3.3 Gauge two-form supermultiplet

We have seen how to derive the Abelian Chern-Simons action both by constructing a curvature induced form and by making use of the linear supermultiplet. The vector supermultiplet turns out to be dual to a gauge two-form supermultiplet, which possesses an intrinsic central charge and may be coupled to additional vector supermultiplets via Chern-Simons terms. The supermultiplet is also called the gauge tensor multiplet or small tensor multiplet in [36].[11] In superspace, it is described, similar to the 4D $\mathcal{N}=2$ vector-tensor supermultiplet [53], by a constrained real superfield L coupled to the vector supermultiplet gauging the central charge [37]. In this subsection, we will turn to deriving a superform formulation for this supermultiplet.

We start with the superspace setting of subsection 3.1 in which the central charge is gauged by a vector supermultiplet $\mathcal{W}$. However, we will also include coupling to an additional Yang-Mills supermultiplet W (see subsection 2.1). Therefore in this subsection we will make use of covariant derivatives which include both gauge connections[12]

$$\mathcal{D} = \mathrm{d} + \mathcal{V}\Delta + \mathrm{i}V\,, \quad \mathcal{D}_{\hat{A}} = D_{\hat{A}} + \mathcal{V}_{\hat{A}}\Delta + \mathrm{i}V_{\hat{A}}\,. \tag{3.26}$$

We introduce a gauge two-form, $B = \frac{1}{2}E^B E^A B_{AB}$ and its three-form field strength H defined by[13]

$$H := \mathcal{D}B - \mathrm{tr}\Big(V\wedge F + \frac{\mathrm{i}}{3}V\wedge V\wedge V\Big)\,, \tag{3.27}$$

where V and F is the Yang-Mills connection and field strength corresponding to the superfield W.[14] Here we do not assume B to be annihilated by the central charge. The (infinitesimal) transformation law for the system of superforms is

$$\begin{aligned} \delta\mathcal{V} &= -\mathrm{d}\Lambda\,, & \Delta\Lambda &= 0\,,\\ \delta V &= -\mathrm{d}\tau\,, & \Delta\tau &= 0\,,\\ \delta B &= \Lambda\Delta B - \mathrm{tr}(\tau\wedge \mathrm{d}V) + \mathrm{d}\Gamma\,, & \Delta\Gamma &= 0\,, \end{aligned} \tag{3.28}$$

where Λ, τ and Γ generate the gauge transformations of $\mathcal{V}$, V and B respectively. The field strength H transforms covariantly

$$\delta H = \Lambda\Delta H \tag{3.29}$$

and satisfies the Bianchi identity

$$\mathcal{D}H = \mathcal{F}\wedge\Delta B - \mathrm{tr}(F\wedge F)\,. \tag{3.30}$$

[11]On-shell tensor multiplets in 5D gauged supergravity were introduced by Günaydin and Zagermann [49] (see also [50]) as a generalization of the earlier work by Günaydin, Sierra and Townsend [51, 52] on 5D supergravity-matter systems with vector supermultiplets.

[12]The central charge commutes with the Yang-Mills gauge group.

[13]Both B and H are Yang-Mills singlets.

[14]The special case of n Abelian vector supermultiplets may be obtained by taking $\mathrm{tr}(V\wedge F)\to \eta_{IJ}V^I F^J$, where η is a symmetric, $\eta_{IJ}=\eta_{JI}$, coupling constant and V^I and F^I are the gauge connections and field strengths of the Abelian vector supermultiplets.

Using the notation that was introduced in subsection 3.2.1, it is possible to extend the Bianchi identity by introducing an additional bosonic index, $\hat{\mathcal{A}} = (\hat{A}, 6)$. To do this we first note that we also have the additional superform equation

$$\Delta H = \mathcal{D}(\Delta B) \; . \tag{3.31}$$

We then extend the Bianchi identity (3.30) and the additional equation (3.31) to

$$\mathcal{D}_{[\hat{\mathcal{A}}} H_{\hat{\mathcal{B}}\hat{\mathcal{C}}\hat{\mathcal{D}}\}} - \frac{3}{2} T_{[\hat{\mathcal{A}}\hat{\mathcal{B}}}{}^{\hat{\mathcal{E}}} H_{|\hat{\mathcal{E}}|\hat{\mathcal{C}}\hat{\mathcal{D}}\}} + \frac{3}{2} \mathrm{tr}(F_{[\hat{\mathcal{A}}\hat{\mathcal{B}}} F_{\hat{\mathcal{C}}\hat{\mathcal{D}}\}}) = 0 \, , \tag{3.32}$$

where we have defined

$$H_{6\hat{A}\hat{B}} := \Delta B_{\hat{A}\hat{B}} \, , \qquad F_{6\hat{\mathcal{A}}} = F_{\hat{\mathcal{A}}6} = 0 \, , \tag{3.33a}$$

$$T_{\hat{A}\hat{B}}{}^{6} := \mathcal{F}_{\hat{A}\hat{B}} \, , \qquad T_{\hat{\mathcal{A}}6}{}^{\hat{\mathcal{B}}} = T_{6\hat{\mathcal{A}}}{}^{\hat{\mathcal{B}}} = 0 \, , \qquad \mathcal{D}_6 := \Delta \; . \tag{3.33b}$$

We now impose simple constraints on the lowest components of $H_{\hat{\mathcal{A}}\hat{\mathcal{B}}\hat{\mathcal{C}}}$

$$H_{\underline{\hat{\alpha}}\underline{\hat{\beta}}\underline{\hat{\gamma}}} = 0 \, , \quad H_{6\underline{\hat{\alpha}}\underline{\hat{\beta}}} = -2\mathrm{i}\varepsilon^{ij}\varepsilon_{\hat{\alpha}\hat{\beta}} L \; . \tag{3.34}$$

The remaining components of $H_{\hat{\mathcal{A}}\hat{\mathcal{B}}\hat{\mathcal{C}}}$ can be found by analyzing eq. (3.32) subject to the constraints (3.34) and the identifications (3.33). They are found to be:

$$H_{\hat{a}\underline{\hat{\beta}}\underline{\hat{\gamma}}} = -2\mathrm{i}\varepsilon^{jk}(\Gamma_{\hat{a}})_{\hat{\beta}\hat{\gamma}}(\mathcal{W}L - \mathrm{tr}(W^2)) \, , \tag{3.35a}$$

$$H_{6\hat{a}}{}^{j}_{\hat{\beta}} = (\Gamma_{\hat{a}})_{\hat{\beta}}{}^{\hat{\gamma}} \mathcal{D}^{j}_{\hat{\gamma}} L \, , \tag{3.35b}$$

$$H_{\hat{a}\hat{b}}{}^{k}_{\hat{\gamma}} = 2(\Sigma_{\hat{a}\hat{b}})_{\hat{\gamma}}{}^{\hat{\delta}} \mathcal{D}^{k}_{\hat{\delta}}(\mathcal{W}L - \mathrm{tr}(W^2)) \, , \tag{3.35c}$$

$$\begin{aligned} H_{6\hat{a}\hat{b}} &= \frac{\mathrm{i}}{4}(\Sigma_{\hat{a}\hat{b}})^{\hat{\alpha}\hat{\beta}} \mathcal{D}^{k}_{\hat{\alpha}} \mathcal{D}_{\hat{\beta}k} L \, , \\ H_{\hat{a}\hat{b}\hat{c}} &= -\frac{\mathrm{i}}{8}\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}(\Sigma^{\hat{d}\hat{e}})^{\hat{\alpha}\hat{\beta}} \Big(\mathcal{D}^{k}_{\hat{\alpha}} \mathcal{D}_{\hat{\beta}k}(\mathcal{W}L - \mathrm{tr}(W^2)) \\ &\qquad + 2\mathcal{D}^{k}_{\hat{\alpha}} \mathcal{W} \mathcal{D}_{\hat{\beta}k} L - 2\mathrm{tr}(\mathcal{D}^{k}_{\hat{\alpha}} W \mathcal{D}_{\hat{\beta}k} W) \Big) \, , \end{aligned} \tag{3.35d}$$

where L satisfies the constraints

$$\mathcal{D}^{(i}_{\hat{\alpha}} \mathcal{D}^{j)}_{\hat{\beta}} L = \frac{1}{4}\varepsilon_{\hat{\alpha}\hat{\beta}} \mathcal{D}^{\hat{\gamma}(i} \mathcal{D}^{j)}_{\hat{\gamma}} L \, , \tag{3.36a}$$

$$\mathcal{D}^{\hat{\gamma}(i} \mathcal{D}^{j)}_{\hat{\gamma}} (\mathcal{W}L - \mathrm{tr}(W^2)) = -2\mathcal{D}^{\hat{\gamma}(i} \mathcal{W} \mathcal{D}^{j)}_{\hat{\gamma}} L + 2\mathrm{tr}(\mathcal{D}^{\hat{\gamma}(i} W \mathcal{D}^{j)}_{\hat{\gamma}} W) \; . \tag{3.36b}$$

The constraints derived from the geometry precisely agree with those in [37]. The remarkable feature of this analysis is that it highlights how the constraints (3.36) follow from requiring the presence of a two-form and simple constraints on its field-strength.

The corresponding superfield Lagrangian may be taken as [37] (formally the same as that of a vector supermultiplet)

$$L^{ij} = \frac{\mathrm{i}}{2}\big(2\mathcal{D}^{\hat{\alpha}(i} L \mathcal{D}^{j)}_{\hat{\alpha}} L + L\mathcal{D}^{\hat{\alpha}(i} \mathcal{D}^{j)}_{\hat{\alpha}} L\big) \; . \tag{3.37}$$

The equation of motion for this model proves to be $\Delta L = 0$.

The off-shell component action for the gauge two-form supermultiplet (in supergravity) together with its Chern-Simons couplings was constructed in [36]. The formulation of the Chern-Simons couplings was inspired by the general form of vector-tensor supermultiplet couplings in the superconformal framework [54].

3.4 Large tensor supermultiplet

In [36] it was discovered that there also exists the large tensor supermultiplet, which consists of 16 (boson) + 16 (fermion) component fields. The large tensor supermultiplet can also be seen to naturally originate in superspace. It may be viewed as a generalization of the gauge two-form supermultiplet in which the constraints (3.36) are weakened. To show this let $\mathcal{L}$ be a superfield constrained in the same way as eq. (3.36a),

$$\mathcal{D}^{(i}_{\hat\alpha}\mathcal{D}^{j)}_{\hat\beta}\mathcal{L} = \frac{1}{4}\varepsilon_{\hat\alpha\hat\beta}\mathcal{D}^{\hat\gamma(i}\mathcal{D}^{j)}_{\hat\gamma}\mathcal{L}\ . \tag{3.38}$$

Requiring only the above constraint, it is possible to show that consistency requires us to have [37]

$$\begin{aligned} 0 &= \Delta\Big\{\mathcal{D}^{\hat\gamma(i}\mathcal{D}^{j)}_{\hat\gamma}(\mathcal{W}\mathcal{L}) + 2\mathcal{D}^{\hat\gamma(i}\mathcal{W}\mathcal{D}^{j)}_{\hat\gamma}\mathcal{L}\Big\} \\ &= \mathcal{D}^{\hat\gamma(i}\mathcal{D}^{j)}_{\hat\gamma}(\mathcal{W}\Delta\mathcal{L}) + 2\mathcal{D}^{\hat\gamma(i}\mathcal{W}\mathcal{D}^{j)}_{\hat\gamma}\Delta\mathcal{L}\,, \end{aligned} \tag{3.39}$$

which is automatically satisfied for the gauge two-form supermultiplet. Here we will take eq. (3.39) as a second constraint on $\mathcal{L}$. The constraints (3.38) and (3.39) allow us to construct a superform framework describing the large tensor supermultiplet.

We begin by introducing a two-form $\mathcal{B}$, transforming homogeneously under the local central charge transformations

$$\delta\mathcal{B} = \Lambda\Delta\mathcal{B}\,, \tag{3.40}$$

and an associated three form $\mathcal{H}$

$$\mathcal{H} = \mathcal{D}\mathcal{B}\ . \tag{3.41}$$

Imposing the constraints

$$\mathcal{H}_{\underline{\hat\alpha}\underline{\hat\beta}\underline{\hat\gamma}} = 0\,, \quad \mathcal{H}_{6\underline{\hat\alpha}\underline{\hat\beta}} = -2\mathrm{i}\varepsilon^{ij}\varepsilon_{\hat\alpha\hat\beta}\Delta\mathcal{L} \tag{3.42}$$

and solving the Bianchi identities yields the components of $\mathcal{H}$:

$$\mathcal{H}_{\hat a\underline{\hat\beta}\underline{\hat\gamma}} = -2\mathrm{i}\varepsilon^{jk}(\Gamma_{\hat a})_{\hat\beta\hat\gamma}\mathcal{W}\Delta\mathcal{L}\,, \tag{3.43a}$$

$$\mathcal{H}_{6\hat a}{}^{j}_{\hat\beta} = (\Gamma_{\hat a})_{\hat\beta}{}^{\hat\gamma}\mathcal{D}^j_{\hat\gamma}\Delta\mathcal{L}\,, \tag{3.43b}$$

$$\mathcal{H}_{\hat a\hat b}{}^{k}_{\hat\gamma} = 2(\Sigma_{\hat a\hat b})_{\hat\gamma}{}^{\hat\delta}\mathcal{D}^k_{\hat\delta}(\mathcal{W}\Delta\mathcal{L})\,, \tag{3.43c}$$

$$\begin{aligned} \mathcal{H}_{6\hat a\hat b} &= \frac{\mathrm{i}}{4}(\Sigma_{\hat a\hat b})^{\hat\alpha\hat\beta}\mathcal{D}^k_{\hat\alpha}\mathcal{D}_{\hat\beta k}\Delta\mathcal{L}\,, \\ \mathcal{H}_{\hat a\hat b\hat c} &= -\frac{\mathrm{i}}{8}\varepsilon_{\hat a\hat b\hat c\hat d\hat e}(\Sigma^{\hat d\hat e})^{\hat\alpha\hat\beta}\Big(\mathcal{D}^k_{\hat\alpha}\mathcal{D}_{\hat\beta k}(\mathcal{W}\Delta\mathcal{L}) + 2\mathcal{D}^k_{\hat\alpha}\mathcal{W}\mathcal{D}_{\hat\beta k}\Delta\mathcal{L}\Big)\,, \end{aligned} \tag{3.43d}$$

where $\mathcal{L}$ is constrained by eqs. (3.38) and (3.39) and $\mathcal{H}_{6\hat{A}\hat{B}} = \Delta\mathcal{B}_{\hat{A}\hat{B}}$. There are still too many component fields and to eliminate them we impose the constraint on $\mathcal{B}$

$$\mathcal{B}_{\hat{\alpha}}^{i}{}_{\hat{\beta}}^{j} = -2\mathrm{i}\varepsilon^{ij}\varepsilon_{\hat{\alpha}\hat{\beta}}\mathcal{L}\,, \tag{3.44}$$

which fixes the remaining components via eq. (3.41) as

$$\mathcal{B}_{\hat{a}}{}_{\hat{\beta}}^{j} = (\Gamma_{\hat{a}})_{\hat{\beta}}{}^{\hat{\gamma}}\mathcal{D}_{\hat{\gamma}}^{j}\mathcal{L}\,, \quad \mathcal{B}_{\hat{a}\hat{b}} = \frac{\mathrm{i}}{4}(\Sigma_{\hat{a}\hat{b}})^{\hat{\alpha}\hat{\beta}}\mathcal{D}_{\hat{\alpha}}^{k}\mathcal{D}_{\hat{\beta}k}\mathcal{L}\,. \tag{3.45}$$

At the highest dimension eq. (3.41) gives

$$3\mathcal{D}_{[\hat{a}}\mathcal{B}_{\hat{b}\hat{c}]} = -\frac{\mathrm{i}}{8}\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}(\Sigma^{\hat{d}\hat{e}})^{\hat{\alpha}\hat{\beta}}\Delta\Big(\mathcal{D}_{\hat{\alpha}}^{k}\mathcal{D}_{\hat{\beta}k}(\mathcal{W}\mathcal{L}) + 2\mathcal{D}_{\hat{\alpha}}^{k}\mathcal{W}\mathcal{D}_{\hat{\beta}k}\mathcal{L}\Big)\,. \tag{3.46}$$

The conditions (3.39) and (3.46) correspond to the ones imposed in [36] from requiring closure of the supersymmetry transformations. In contrast with the gauge two-form supermultiplet, which was based on the stronger constraints (3.36), the component fields of the large tensor supermultiplet

$$\Delta\mathcal{D}_{\alpha}^{i}\mathcal{L}|\,, \quad \Delta^2\mathcal{L}| \tag{3.47}$$

are no longer composite. We should remark that the above constraints can naturally be generalized to include couplings to the Yang-Mills supermultiplet.

We can construct an action for an even number of large tensor supermultiplets $\mathcal{L}^I$. To do so we make use of the superfield Lagrangian

$$\mathcal{L}^{ij} = \mathcal{L}_{\mathrm{kin}}^{ij} + \mathcal{L}_{\mathrm{mass}}^{ij}\,, \tag{3.48}$$

where

$$\mathcal{L}_{\mathrm{mass}}^{ij} = \frac{\mathrm{i}}{2}m_{IJ}\big(2\mathcal{D}^{\hat{\alpha}(i}\mathcal{L}^I\mathcal{D}_{\hat{\alpha}}^{j)}\mathcal{L}^J + \mathcal{L}^I\mathcal{D}^{\hat{\alpha}(i}\mathcal{D}_{\hat{\alpha}}^{j)}\mathcal{L}^J\big)\,, \qquad m_{IJ} = m_{JI}\,, \tag{3.49a}$$

$$\mathcal{L}_{\mathrm{kin}}^{ij} = \frac{\mathrm{i}}{4}k_{IJ}\big(2\mathcal{D}^{\hat{\alpha}(i}\mathcal{L}^I\overleftrightarrow{\Delta}\mathcal{D}_{\hat{\alpha}}^{j)}\mathcal{L}^J + \mathcal{L}^I\overleftrightarrow{\Delta}\mathcal{D}^{\hat{\alpha}(i}\mathcal{D}_{\hat{\alpha}}^{j)}\mathcal{L}^J\big)\,, \qquad k_{IJ} = -k_{JI}\,. \tag{3.49b}$$

The constant matrices m_{IJ} and k_{IJ} are assumed to be nonsingular. The Lagrangian $\mathcal{L}^{ij}$ may be seen to be a linear supermultiplet. The component action in supergravity is given in [36].

On-shell each large tensor supermultiplet describes 4 + 4 degrees of freedom [9]. The equations of motion for the large-tensor supermultiplets are given by the superfield constraint

$$k_{IJ}\Delta\mathcal{L}^J + m_{IJ}\mathcal{L}^J = 0\,. \tag{3.50}$$

Under the above constraint (3.46) becomes a duality condition on $\mathcal{B}$,

$$\frac{1}{2}k_{IJ}\varepsilon^{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}\mathcal{D}_{\hat{c}}\mathcal{B}_{\hat{d}\hat{e}}^J = -m_{IJ}\Big(\mathcal{W}\mathcal{B}^{J\hat{a}\hat{b}} + F^{\hat{a}\hat{b}}\mathcal{L}^J + \mathrm{i}(\Sigma^{\hat{a}\hat{b}})^{\hat{\alpha}\hat{\beta}}\mathcal{D}_{\hat{\alpha}}^{k}\mathcal{W}\mathcal{D}_{\hat{\beta}k}\mathcal{L}^J\Big)\,. \tag{3.51}$$

Furthermore, the 16 + 16 independent component fields

$$\mathcal{L}|\,, \quad \mathcal{D}_{\hat{\alpha}}^{i}\mathcal{L}|\,, \quad \Delta\mathcal{L}|\,, \quad \mathcal{B}_{\hat{a}\hat{b}}|\,, \quad \mathcal{D}^{\hat{\gamma}(i}\mathcal{D}_{\hat{\gamma}}^{j)}\mathcal{L}|\,, \quad \Delta\mathcal{D}_{\hat{\alpha}}^{i}\mathcal{L}|\,, \quad \Delta^2\mathcal{L}|\,, \tag{3.52}$$

reduce to[15]

$$\mathcal{L}| , \quad \mathcal{D}^i_{\hat\alpha}\mathcal{L}| , \quad \mathcal{B}_{\hat a\hat b}| \ . \tag{3.53}$$

These components correspond to only $4+4$ degrees of freedom. To see this, we note that the self-duality condition (3.51) implies that $\mathcal{B}_{\hat a\hat b}|$ now possesses only 3 degrees of freedom. Therefore we have $3+1=4$ bosonic degrees of freedom. The remaining component field $\mathcal{D}^i_{\hat\alpha}\mathcal{L}|$ contributes to the remaining 4 fermionic degrees of freedom.

4 Discussion

The closed-form expression for the non-Abelian SCS action in 5D $\mathcal{N}=1$ superspace is one of the main results of this paper. The component action was constructed by Kugo and Ohashi more than ten years ago [9]. However, our work has provided the first systematic, unambiguous and purely geometric derivation of this action. Our construction can readily be generalized to the locally supersymmetric case by making use of the superspace formulation for 5D $\mathcal{N}=1$ conformal supergravity [55]. Moreover, we believe our construction makes it it possible to address another long-standing problem — to formulate the 5D $\mathcal{N}=1$ non-Abelian SCS action in terms of 4D $\mathcal{N}=1$ superfields. For this one has to use the relations (2.27e) and (2.30b) in conjunction with the formalism of reduced superspace introduced in [11]. We hope to elaborate on this issue elsewhere.

The idea of generalizing the gauge two-form supermultiplet in the way described in subsection (3.4) may have an immediate application for the vector-tensor supermultiplet in four-dimensions. To see this, we first recall that in superspace the vector-tensor supermultiplet with gauged central charge $\mathbb{L}$ satisfies the constraint[16]

$$\mathcal{D}^{(i}_\alpha\bar{\mathcal{D}}^{j)}_{\dot\alpha}\mathbb{L}=0 \ . \tag{4.1}$$

The above constraint can only be consistent if the following additional constraint is imposed [53]

$$\begin{aligned}0 &= \Delta\Big(\mathcal{D}^{\alpha(i}\mathcal{D}^{j)}_\alpha(\mathcal{W}\mathbb{L})+\bar{\mathcal{D}}^{(i}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha j)}(\bar{\mathcal{W}}\mathbb{L})-\mathbb{L}\mathcal{D}^{\alpha(i}\mathcal{D}^{j)}_\alpha\mathcal{W}\Big)\\ &= \mathcal{D}^{\alpha(i}\mathcal{D}^{j)}_\alpha(\mathcal{W}\Delta\mathbb{L})+\bar{\mathcal{D}}^{(i}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha j)}(\bar{\mathcal{W}}\Delta\mathbb{L})-\mathcal{D}^{\alpha(i}\mathcal{D}^{j)}_\alpha\mathcal{W}\Delta\mathbb{L}\,,\end{aligned} \tag{4.2}$$

where $\mathcal{W}$ is the chiral field strength of the 4D $\mathcal{N}=2$ central charge vector supermultiplet,

$$\mathcal{D}^{\alpha(i}\mathcal{D}^{j)}_\alpha\mathcal{W}=\bar{\mathcal{D}}^{(i}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha j)}\bar{\mathcal{W}} \ . \tag{4.3}$$

Although stronger constraints are usually chosen for $\mathbb{L}$, our analysis of the large tensor supermultiplet suggests that we could instead choose eq. (4.2) as a second constraint and look for a consistent superform formulation. Furthermore, a similar possibility exists for the variant vector-tensor supermultiplet [39, 56, 57].[17] Whether the more general constraints will lead to consistent supermultiplets is still an open problem.

[15]The component field $\mathcal{D}^{\hat\gamma(i}\mathcal{D}^{j)}_{\hat\gamma}\mathcal{L}|$ is composite as a result of eqs. (3.39) and (3.50).
[16]There are also additional constraints which are not important here.
[17]The analogue of (4.2) for the variant vector-tensor supermultiplet may be found in [39].

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A Alternative gauging of the central charge

In subsection (3.1) we made use of a vector supermultiplet to gauge the central charge. This requires the gauge potential $\mathcal{V}$ to be inert under the action of the central charge, $\Delta\mathcal{V} = 0$. However, it was shown in [56, 57] that in 4D it is possible to gauge the central charge with a gauge connection that is not annihilated by the central charge. To the best of our knowledge, the possibility of gauging the central charge with a gauge connection that is no longer inert under the central charge has never been properly analyzed in 5D. In this appendix, we follow an approach similar to that given in [39, 58]. We do not assume that the gauge one-form is annihilated by the central charge and analyze the possibilities under reasonable constraints.

We begin as in subsection (3.1) by introducing gauge covariant derivatives

$$\mathcal{D}_{\hat{A}} = (\mathcal{D}_{\hat{a}}, \mathcal{D}^i_{\hat{\alpha}}) = D_{\hat{A}} + \mathcal{V}_{\hat{A}}\Delta\,, \quad [\Delta, D_{\hat{A}}] = 0\,, \tag{A.1}$$

where $\mathcal{V}_{\hat{A}}$ is a one-form gauge connection associated with the central charge Δ and $\Delta\mathcal{V}_{\hat{A}} \neq 0$. Here the gauge transformation of the gauge connection $\mathcal{V}_{\hat{A}}$ becomes $\mathcal{V}_{\hat{A}}$ to be

$$\delta\mathcal{V}_{\hat{A}} = -D_{\hat{A}}\Lambda + \Lambda\Delta\mathcal{V}_{\hat{A}} \implies \delta\mathcal{D}_{\hat{A}} = [\Lambda\Delta, \mathcal{D}_{\hat{A}}]\,, \tag{A.2}$$

where the gauge parameter is annihilated by the central charge, $\Delta\Lambda = 0$.

The commutation relations for the gauged covariant derivatives are

$$[\mathcal{D}_{\hat{A}}, \mathcal{D}_{\hat{B}}\} = T_{\hat{A}\hat{B}}{}^{\hat{C}}\mathcal{D}_{\hat{C}} + \mathcal{F}_{\hat{A}\hat{B}}\Delta\,, \tag{A.3a}$$

$$[\Delta, \mathcal{D}_{\hat{A}}] = \mathcal{F}_{6\hat{A}}\Delta\,, \tag{A.3b}$$

where we define the field strengths

$$\mathcal{F}_{\hat{A}\hat{B}} := 2\mathcal{D}_{[\hat{A}}\mathcal{V}_{\hat{B}\}} - T_{\hat{A}\hat{B}}{}^{\hat{C}}\mathcal{V}_{\hat{C}}\,, \tag{A.4a}$$

$$\mathcal{F}_{6\hat{A}} := \Delta\mathcal{V}_{\hat{A}}\ . \tag{A.4b}$$

Here the field strengths $\mathcal{F}_{\hat{A}\hat{B}}$ and $\mathcal{F}_{6\hat{A}}$ are covariant with respect to gauge transformations of $\mathcal{V}_{\hat{A}}$

$$\delta\mathcal{F}_{\hat{A}\hat{B}} = \Lambda\Delta\mathcal{F}_{\hat{A}\hat{B}}\,, \quad \delta\mathcal{F}_{6\hat{A}} = \Lambda\Delta\mathcal{F}_{6\hat{A}}\ . \tag{A.5}$$

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The Bianchi identities satisfied by $\mathcal{F}_{\hat{A}\hat{B}}$ and $\mathcal{F}_{6\hat{A}}$ can be combined into one equation by extending the indices to include an additional bosonic coordinate, $\hat{\mathcal{A}} = (\hat{A}, 6)$. The extended object $\mathcal{F}_{\hat{\mathcal{A}}\hat{\mathcal{B}}} = (\mathcal{F}_{\hat{A}\hat{B}}, \mathcal{F}_{6\hat{A}})$ satisfies the Bianchi identity

$$\mathcal{D}_{[\hat{\mathcal{A}}}\mathcal{F}_{\hat{\mathcal{B}}\hat{\mathcal{C}}\}} - T_{[\hat{\mathcal{A}}\hat{\mathcal{B}}}{}^{\hat{\mathcal{D}}}\mathcal{F}_{\hat{\mathcal{D}}\hat{\mathcal{C}}\}} = 0\,, \tag{A.6}$$

where we have made the identifications

$$T_{\hat{A}\hat{B}}{}^{6} = \mathcal{F}_{\hat{A}\hat{B}}\,, \quad T_{6\hat{A}}{}^{6} = -T_{\hat{A}6}{}^{6} = \mathcal{F}_{6\hat{A}}\,, \quad T_{6\hat{A}}{}^{\hat{B}} = -T_{\hat{A}6}{}^{\hat{B}} = 0\,. \tag{A.7}$$

We may now impose constraints on the field strength and analyze the consequences of the Bianchi identities (A.6). We choose the simple constraint

$$\mathcal{F}_{\underline{\hat{\alpha}\hat{\beta}}} = -2\mathrm{i}\varepsilon^{ij}\varepsilon_{\hat{\alpha}\hat{\beta}}M\,, \tag{A.8}$$

where M is initially assumed to be an unconstrained superfield. Analyzing the Bianchi identities yields the components

$$\mathcal{F}_{\hat{a}}{}^{j}_{\hat{\beta}} = (\Gamma_{\hat{a}})_{\hat{\beta}}{}^{\hat{\gamma}}(\mathcal{D}^{j}_{\hat{\gamma}}M - M\mathcal{F}_{6}{}^{j}_{\hat{\gamma}})\,, \tag{A.9a}$$

$$\mathcal{F}_{6\hat{a}} = \frac{\mathrm{i}}{8}(\Gamma_{\hat{a}})_{\hat{\alpha}\hat{\beta}}\mathcal{D}^{\hat{\alpha}k}\mathcal{F}_{6}{}^{\hat{\beta}}_{k}\,, \tag{A.9b}$$

$$\mathcal{F}_{\hat{a}\hat{b}} = \frac{\mathrm{i}}{4}(\Sigma_{\hat{a}\hat{b}})^{\hat{\alpha}\hat{\beta}}(\mathcal{D}^{k}_{\hat{\alpha}}\mathcal{D}_{\hat{\beta}k}M + M\mathcal{D}_{\hat{\alpha}k}\mathcal{F}_{6}{}^{k}_{\hat{\beta}} + 2\mathcal{D}_{\hat{\alpha}k}M\mathcal{F}_{6}{}^{k}_{\hat{\alpha}}) \tag{A.9c}$$

and the constraints

$$\mathcal{D}^{k}_{\hat{\gamma}}\mathcal{F}_{6}{}^{\hat{\gamma}}_{k} = -8\mathrm{i}\Delta M\,, \tag{A.10a}$$

$$\mathcal{D}^{(i}_{(\hat{\alpha}}\mathcal{F}_{6}{}^{j)}_{\hat{\beta})} = 0\,, \tag{A.10b}$$

$$\begin{aligned}\mathcal{D}^{(i}_{\hat{\alpha}}\mathcal{D}^{j)}_{\hat{\beta}}M &= \frac{1}{4}\varepsilon_{\hat{\alpha}\hat{\beta}}\mathcal{D}^{\hat{\gamma}(i}\mathcal{D}^{j)}_{\hat{\gamma}}M - \frac{1}{2}\varepsilon_{\hat{\alpha}\hat{\beta}}\mathcal{D}^{\hat{\gamma}(i}M\mathcal{F}_{6}{}^{j)}_{\hat{\gamma}} - \frac{1}{4}\varepsilon_{\hat{\alpha}\hat{\beta}}M\mathcal{D}^{\hat{\gamma}(i}\mathcal{F}_{6}{}^{j)}_{\hat{\gamma}}\\ &\quad + 2\mathcal{D}^{(i}_{[\hat{\alpha}}M\mathcal{F}_{6}{}^{j)}_{\hat{\beta}]} + M\mathcal{D}^{(i}_{[\hat{\alpha}}\mathcal{F}_{6}{}^{j)}_{\hat{\beta}]}\,.\end{aligned} \tag{A.10c}$$

If we first assume that $\Delta M \neq 0$ and all components of $\mathcal{F}_{\hat{A}\hat{B}}$ are expressible in terms of M and its covariant derivatives then the constraints (A.10a) and (A.10b) are solved by

$$\mathcal{F}_{6}{}^{j}_{\hat{\beta}} = \mathcal{D}^{j}_{\hat{\beta}}\ln M\,. \tag{A.11}$$

Putting this expression into the last constraint gives the condition

$$\mathcal{D}^{(i}_{\hat{\alpha}}M\mathcal{D}^{j)}_{\hat{\beta}}M = \frac{1}{4}\varepsilon_{\hat{\alpha}\hat{\beta}}\mathcal{D}^{\hat{\gamma}(i}M\mathcal{D}^{j)}_{\hat{\gamma}}M\,, \tag{A.12}$$

which implies

$$\mathcal{D}^{(i}_{\hat{\alpha}}M\mathcal{D}^{j}_{\hat{\beta}}M\mathcal{D}^{k)}_{\hat{\gamma}}M = 0\,. \tag{A.13}$$

The only sensible solution to the above constraint is

$$\mathcal{D}^{i}_{\hat{\alpha}}M = 0\,. \tag{A.14}$$

However, this constraint implies that $\Delta M = 0$, which is a contradiction.

Choosing $\Delta M = 0$ reduces M to that of a vector supermultiplet

$$M = W\,, \quad \mathcal{F}_6{}_{\hat\beta}^{j} = 0 \tag{A.15}$$

with components given by eqs. (3.4) and (3.5).

The result of our analysis is in stark contrast to the situation in 4D. In 4D it was pointed out by Theis [56, 57] that it is possible to gauge the central charge with the use of a different supermultiplet whose novel feature is that its gauge one-form is not annihilated by the central charge. The supermultiplet was later generalized to supergravity in [39, 58] and called the variant vector-tensor supermultiplet. The component structure of the supermultiplet is similar to that of the vector supermultiplet, possessing both a one-form and a two-form gauge field.[18] However, our analogous analysis in 5D shows that (under the reasonable assumptions made) the only supermultiplet suitable to gauge the central charge is the vector supermultiplet.

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