Numerical solution of an inverse problem of gravimetry for a contact surface

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Abstract

An inverse problem of gravimetry for a contact surface is considered. Using the critical component method, shapes of a surface are determined numerically at a known depth.

Keywords: Gravitational prospecting; Inverse problem

1. Introduction

Gravitational prospecting is used to study a deep Earth’s crust structure, tectonic and petrophysical zoning of large regions, geological mapping of closed territories, search for and prediction of oil and gas deposits, search for and prospecting of solid minerals: coal, ores, and nonmetallics. Gravitational prospecting is applied also to solve some problems of engineering geology, geodesic problems, which study the Earth’s figure.

An important problem of interpretation of gravitational anomalies is the problem of determining the parameters of ore bodies using the gravitational values at the Earth’s surface. If the density of ore
bodies is known from other sources, the problem consists in determining the surface geometry of an ore body [1]. In this paper, the inverse problem is solved with the use of a critical components method [2,3].

2. Problem statement

Let us introduce the Cartesian system of coordinates, in which the axis $z$ is directed vertically upwards. A linearized statement of the inverse problem for a contact surface $z(x)$ (the form of the contact surface) infinitely long along the axis $y$ is reduced to the solution of the integral Fredholm equation of the first kind [4,5]:

$$
\int_a^b \frac{z(\eta)d\eta}{(x-\eta)^2 + H^2} = f(x).
$$

(1)

Here $H$ is the deposit depth.

It is known [6,7] that the solution of Eq. (1) is an ill-posed problem. To solve Eq. (1) in [8,9], regularization algorithms are proposed and justified. In [1], an iterative method is proposed.

Using the trapezium method, Eq. (1) is reduced to a system of the linear algebraic equations:

$$
Au = F.
$$

(2)

Here $A = (A_{ij})$ is the $m \times n$ dimensionality matrix, $u$ is an unknown vector of dimensionality $n$, and $F$ is the known vector of dimensionality $m$.

3. Numerical experiments

In accordance with [1], we choose an unknown function of the following form:

$$
z(\eta) = \begin{cases}
0, & \text{at } \eta < -1; \\
1 + \eta, & \text{at } -1 \leq \eta \leq 0; \\
1 - \eta, & \text{at } 0 < \eta < 1; \\
0, & \text{at } \eta > 1.
\end{cases}
$$

Substituting this function into (1), we calculate the function $f(x)$ at three different points $x = -0.5, 0, 0.5$ for $H = 0.5, a = -1$, and $b = 1$, i.e. we assume in (2) that $m = 3$, and $F = (f(-0.5), f(0), f(0.5))^T$, where $T$ is the transposition sign.

Let us set $n = 21$. The exact solution to system of Eq. (2) is a vector of the form

$$
\tilde{u} = (0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, \\
1.0, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, 0.0)^T.
$$

To solve the system of linear algebraic equations (2), we use the critical components method [2]. In this method, the numerical solution of systems $BX = Y$ with ill-posed matrices is reduced to the problem of stable solution of the reduced systems. The ill-posed components of the solutions $x_{lk}$ are considered separately, and do not participate in the recurrent processes of deriving any components of the solution. Therefore, these components are called critical. The critical component method allows us to find numerically an approximate normal minimum solution, which provides a minimum of the discrepancy.
norm:

\[(X^+ = B^+Y) : \|BX^+ - Y\| = \inf_{X \in X_B} \|BX - Y\|, \|X^+\| = \inf_{X \in X_B} \|X\|,\]

where \(X_B\) is the totality of all solutions of the system \(BX = Y\), and the unique matrix \(B^+\) satisfies the conditions

\[\|B^+B - E\| = \inf_{\tilde{B}^{-1} \in \Omega_B} \|\tilde{B}^{-1}B - E\|, \|B^+\| = \inf_{\tilde{B}^{-1} \in \Omega_B} \|\tilde{B}^{-1}\|, B^+B = BB^+.\]

Here \(E\) is a unit matrix and \(\Omega_B\) is the totality of all \(\tilde{B}^{-1}\), which are “pseudoinverse” to \(B\).

In Table 1, approximate solutions to the system of equations (2) are represented. The relative solution errors \(\delta = \frac{\|u - \tilde{u}\|}{\|\tilde{u}\|}\) at the corresponding relative perturbations of the right-hand sides of \(\varepsilon = \frac{\|F - \tilde{F}\|}{\|\tilde{F}\|}\) are also given.

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