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Theoretical analysis of direct CP violation and differential decay width in $D^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ in phase space around the resonances $\rho^0(770)$ and $f_0(500)$

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Abstract We perform a theoretical study on direct CP violation in $D^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ in phase space around the intermediate states $\rho^0(770)$ and $f_0(500)$. The possible interference between the amplitudes corresponding to the two resonances is taken into account, and the relative strong phase of the two amplitudes is treated as a free parameter. Our analysis shows that by a properly chosen strong phase, both the CP violation strength and the differential decay width accommodate the experimental results.

1 Introduction

Charge-parity (CP) violation has gained extensive attention ever since its first discovery in K^0–$\overline{K^0}$ systems in 1964 [1]. Within the standard model (SM), CP violation originates from a complex phase in Cabibbo–Kobayashi–Maskawa (CKM) matrix, which describes the mixing of weak and mass eigenstates of the quarks [2].

Although it is a small effect in general, CP violation can be relatively large in some decay channels of B and B_s mesons [3–6]. In fact, large CP violation has been confirmed in some two-body decay channel of the B and B_s meson, such as $B^{\pm} \to D_{CP(+1)}\pi^{\pm}$ [7], $B^{\pm} \to \rho^0 K^{\pm}$ [8], $B^{\pm} \to \rho^0 K^*(892)^{\pm}$ [9], $B^{\pm} \to f^0(1370)\pi^{\pm}$[10], $B^0 \to \rho^- K^+$ [11], and $B_s \to \pi^+ K^-$ [12,13]. In recent years, even larger CP violation which localized in three-body decay phase space of B meson was observed by LHCb collaboration in channels such as $B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ and $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$ [14,15]. In view of its anisotropy property for a small invariant mass of $\pi^+\pi^-$ pair (no larger than the mass of $\rho^0(770)$) in phase space, the large localized CP violation was first interpreted as a consequence of the interference of the decay amplitudes corresponding to nearby resonances with different spins [16,17]. Some other explanations such as the re-scattering effects of final states [18,19] were also proposed thereafter.

Since it is believed to be very small within SM, CP violation in the charm sector provides a good place for searching for New Physics. Extensive theoretical analyses on CP violations have been performed in some two-body decay channels of charmed hadrons [20–24], within and without SM. On the experimental side, no CP violation in the charm sector has been established to date. Though there were some hints of CP violation for the channels $D \to \pi\pi$ and $D \to KK$ [25,26], the latest result from the LHCb collaboration showed, however, no evidence of CP violation in these channels [27].

A measurement of CP violation in the three-body decay $D^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ has also been performed by LHCb [28]. With very high statistics, no localized or overall CP asymmetries are found. As has been shown in some aforementioned three-body decay channels of B meson, localized CP asymmetries in phase space can be enhanced by the interference of the decay amplitudes corresponding to two intermediate resonances with different spins, if the relative strong phases between these amplitudes are properly chosen. The same interference effect should also apply in D meson decays, such as the aforementioned channels, $D^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$. In this situation, one can use the CP asymmetry of $D^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ to determine the relative strong phase. Then, by comparing the event distributions determined by the strong phase with that from the experimental data, one can tell whether the interference effect is suitable or not for three-body decays of the D meson.

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2 Formalism of decay amplitudes and CP asymmetries

In the region of the phase space around the resonances $\rho^0(770)$ and $f_0(500)$, the process $D^\pm \to \pi^\pm\pi^+\pi^-$ is dominated by two cascade decays, $D^\pm \to \pi^\pm f_0(500) \to \pi^\pm\pi^+\pi^-$ and $D^\pm \to \pi^\pm\rho^0(770) \to \pi^\pm\pi^+\pi^-$. For the two weak decays $D^\pm \to \pi^\pm f_0(500)$ and $D^\pm \to \pi^\pm\rho^0(770)$, the corresponding effective Hamiltonian can be expressed as [29,30]

$$H_{\Delta C=1} = \frac{G_F}{\sqrt{2}}\left\{\left[\sum_{q=d,s} V_{uq}V_{cq}^*\left(c_1 O_1^q + c_2 O_2^q\right)\right] - V_{ub}V_{cb}^*\sum_{i=3}^{6} c_i O_i\right\} + \text{h.c.}, \qquad (1)$$

where G_F is the Fermi constant, $V_{q_1q_2}$ (q_1 and q_2 represent quarks) is the CKM matrix element, c_i ($i = 1,\ldots,6$) is the Wilson coefficient, and O_i is the four quark operator, which can be written as

$$\begin{aligned}
O_1^q &= \bar{u}_\alpha\gamma_\mu(1-\gamma_5)q_\beta\bar{q}_\beta\gamma^\mu(1-\gamma_5)c_\alpha\,,\\
O_2^q &= \bar{u}\gamma_\mu(1-\gamma_5)q\bar{q}\gamma^\mu(1-\gamma_5)c\,,\\
O_3 &= \bar{u}\gamma_\mu(1-\gamma_5)c\textstyle\sum_{q'}\bar{q}'\gamma^\mu(1-\gamma_5)q'\,,\\
O_4 &= \bar{u}_\alpha\gamma_\mu(1-\gamma_5)c_\beta\textstyle\sum_{q'}\bar{q}'_\beta\gamma^\mu(1-\gamma_5)q'_\alpha\,,\\
O_5 &= \bar{u}\gamma_\mu(1-\gamma_5)c\textstyle\sum_{q'}\bar{q}'\gamma^\mu(1+\gamma_5)q'\,,\\
O_6 &= \bar{u}_\alpha\gamma_\mu(1-\gamma_5)c_\beta\textstyle\sum_{q'}\bar{q}'_\beta\gamma^\mu(1+\gamma_5)q'_\alpha\,,
\end{aligned} \qquad (2)$$

with O_1^q and O_2^q being tree operators, O_3–O_6 being QCD penguin operators, α and β being color indices, and q' running through all the light flavor quarks.

The effective Hamiltonian for the strong decays $\rho^0 \to \pi^+\pi^-$ and $f_0(500) \to \pi^+\pi^-$ can be formally expressed as

$$\mathcal{H}_{\rho^0\pi\pi} = ig_{\rho\pi\pi}\rho^0_\mu(\pi^-\partial^\mu\pi^+ - \pi^+\partial^\mu\pi^-), \qquad (3)$$

$$\mathcal{H}_{f_0\pi\pi} = g_{f_0\pi\pi}f_0(2\pi^+\pi^- + \pi^0\pi^0), \qquad (4)$$

where ρ^0_μ, f_0 and $\pi^\pm$ are the field operators for the ρ^0, $f_0(500)$, and π mesons, respectively, $g_{\rho\pi\pi}$ and $g_{f_0\pi\pi}$ are the effective coupling constants, which can be expressed in terms of the decay widths as

$$g^2_{\rho\pi\pi} = \frac{48\pi}{\left(1-\frac{4m_\pi^2}{m_\rho^2}\right)^{3/2}} \times \frac{\Gamma_{\rho^0\to\pi^+\pi^-}}{m_\rho}, \qquad (5)$$

$$g^2_{f_0\pi\pi} = \frac{4\pi m_{f_0}\Gamma_{f_0\to\pi^+\pi^-}}{\left(1-\frac{4m_\pi^2}{m_{f_0}^2}\right)^{1/2}}. \qquad (6)$$

Both $f_0(500)$ and $\rho^0(770)$ decay into one pion pair dominantly through the strong interaction, and the isospin symmetry of the strong interaction tells us that $\Gamma_{\rho^0} \simeq \Gamma_{\rho^0\to\pi^+\pi^-}$ and $\Gamma_{f_0} \simeq \frac{3}{2}\Gamma_{f_0\to\pi^+\pi^-}$.

The decay amplitudes for the cascade decays $D^+ \to \rho^0(770)\pi^+ \to \pi^+\pi^+\pi^-$ and $D^+ \to \pi^+ f_0(500) \to \pi^+\pi^+\pi^-$ take the form

$$\mathcal{M}_{\rho^0}(s_\text{low}, s_\text{high}) = \frac{\langle\pi^+\pi^-|\mathcal{H}_{\rho^0\pi\pi}|\rho^0\rangle\langle\rho^0\pi^+|\mathcal{H}_\text{eff}|D^+\rangle}{s_\text{low} - m_\rho^2 + im_\rho\Gamma_\rho}, \qquad (7)$$

$$\mathcal{M}_{f_0}(s_\text{low}, s_\text{high}) = \frac{\langle\pi^+\pi^-|\mathcal{H}_{f_0\pi\pi}|f_0\rangle\langle f_0\pi^+|\mathcal{H}_\text{eff}|D^+\rangle}{s_\text{low} - m_{f_0}^2 + im_{f_0}\Gamma_{f_0}}, \qquad (8)$$

respectively, where s_low and s_high are the invariant mass squared of $\pi^+\pi^-$ pairs with lower and higher invariant masses, respectively, and the summation over the polarization states of ρ^0 in the first equation is understood. We use a naive factorization approach [31,32] to calculate the matrix elements $\langle\rho^0\pi^+|\mathcal{H}_\text{eff}|D^+\rangle$ and $\langle f_0\pi^+|\mathcal{H}_\text{eff}|D^+\rangle$. After some algebra, one has

$$\begin{aligned}
\mathcal{M}_{\rho^0} &= \frac{\sqrt{2}G_F g_{\rho\pi\pi}m_\rho(s_\text{high} - \Sigma_{s_\text{low}})}{s_\text{low} - m_\rho^2 + im_\rho\Gamma_\rho}\\
&\times\left\{V_{ud}V_{cd}^*\left(-\frac{1}{\sqrt{2}}a_1 f_\rho F_1 + a_2 f_\pi A_0\right)\right.\\
&\left.-V_{ub}V_{cb}^*\left[\left(a_4 - \frac{2m_\pi^2 a_6}{(m_c+m_d)(m_u+m_d)}\right)f_\pi A_0\right]\right\},
\end{aligned} \qquad (9)$$

$$\begin{aligned}
\mathcal{M}_{f_0} &= \frac{\sqrt{2}G_F(m_D^2 - m_{f_0}^2)g_{f_0\pi\pi}f_\pi F_0}{s_\text{low} - m_{f_0}^2 + im_{f_0}\Gamma_{f_0}} \times \left\{V_{ud}V_{cd}^* a_2\right.\\
&\left.-V_{ub}V_{cb}^*\left[a_4 - \frac{2m_\pi^2 a_6}{(m_c+m_d)(m_u+m_d)}\right]\right\},
\end{aligned} \qquad (10)$$

where $\Sigma_{s_\text{low}} = (s_\text{high,max} + s_\text{high,min})/2$, with $s_\text{high,max(min)}$ being the maximum (minimum) value of s_high allowed by phase space for each s_low, F_0, F_1, and A_0 are short for the form factors $F_0^{D\to f_0}(m_\pi^2)$, $F_1^{(D\to\pi)}(s_\text{low})$ and $A_0^{(D\to\rho)}(m_\pi^2)$, respectively. All the a_i's are built up from the Wilson coefficients c_i's, and they take the form $a_i = c_i + c_{i+1}/N_c$ for odd i and $a_i = c_i + c_{i-1}/N_c$ for even i.

In the phase space that we are considering, the total decay amplitude for $D^+ \to \pi^+\pi^+\pi^-$ is dominated by $\mathcal{M}_{f_0}$ and $\mathcal{M}_{\rho^0}$. As a result, it can be expressed as

$$\begin{aligned}
\mathcal{M} &= \left[\mathcal{M}_{f_0}(s_\text{low}, s_\text{high}) + \mathcal{M}_{\rho^0}(s_\text{low}, s_\text{high})e^{i\delta}\right]\\
&+[s_\text{low} \leftrightarrow s_\text{high}],
\end{aligned} \qquad (11)$$

where δ is the relative strong phase between the two amplitudes $\mathcal{M}_{f_0}$ and $\mathcal{M}_{\rho^0}$, which, in principle, arises from the long distance effect, $[s_\text{low} \leftrightarrow s_\text{high}]$ represents a term which takes the same form as that in the first square bracket except for an interchange between s_low and s_high. For the calculation of the decay amplitude of the CP conjugate process $D^- \to \pi^+\pi^-\pi^-$, which will be denoted as $\overline{\mathcal{M}}$, all one needs to do is to replace the CKM matrix elements in $\mathcal{M}$ with their complex conjugates.

The differential CP asymmetry for $D^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ is defined as

$$A_{\rm CP} = \frac{|\mathcal{M}|^2 - |\overline{\mathcal{M}}|^2}{|\mathcal{M}|^2 + |\overline{\mathcal{M}}|^2}, \tag{12}$$

while the localized CP asymmetry can be expressed as

$$A_{\rm CP}^{R} = \frac{\int_R ds_{\rm high} ds_{\rm low} (|\mathcal{M}|^2 - |\overline{\mathcal{M}}|^2)}{\int_R ds_{\rm high} ds_{\rm low} (|\mathcal{M}|^2 + |\overline{\mathcal{M}}|^2)}, \tag{13}$$

where R represents a certain region of the phase space we are considering.

3 Numerical analysis

Table 1 lists the input parameters and the corresponding references we used in this paper. In the following, we give some comments on these input parameters. We use the Wolfenstein parameterization for the CKM matrix elements, which, up to the order of λ^8, can be expressed as [33,34]

$$\begin{aligned}
V_{ud} &= 1 - \tfrac{\lambda^2}{2} - \tfrac{\lambda^4}{8} - \tfrac{\lambda^6}{16}[1 + 8A^2(\rho^2 + \eta^2)] \\
&\quad - \tfrac{\lambda^8}{128}[5 - 32A^2(\rho^2 + \eta^2)], \\
V_{cd} &= -\lambda + \tfrac{\lambda^5}{2}A^2[1 - 2(\rho + i\eta)] + \tfrac{\lambda^7}{2}A^2(\rho + i\eta), \\
V_{ub} &= \lambda^3 A(\rho - i\eta), \\
V_{cb} &= A\lambda^2 - \tfrac{\lambda^8}{2}A^3(\rho^2 + \eta^2),
\end{aligned} \tag{14}$$

with A, ρ, η, and λ being the Wolfenstein parameters. To all orders in λ, the relation between ρ, η and $\overline{\rho}$, $\overline{\eta}$ can be expressed as [34]

$$\rho + i\eta = \frac{\sqrt{1 - A^2\lambda^4}(\overline{\rho} + i\overline{\eta})}{\sqrt{1 - \lambda^2}[1 - A^2\lambda^4(\overline{\rho} + i\overline{\eta})]}. \tag{15}$$

For the invariant mass dependence of the form factors $F_1^{D\to\pi}$ and $A_0^{D\to\rho}$, we use a model from Ref. [35], which takes the form

$$F(s) = \frac{F(0)}{1 - a^X \times \frac{s}{m_D^2} + b^X \times \left(\frac{s}{m_D^2}\right)^2}, \tag{16}$$

where F and X can be $F_1^{D\to\pi}$ and π, or $A_0^{D\to\rho}$ and ρ, respectively. The form factor $F^{D\to f_0}(m_\pi^2)$ which we use here is a rough estimation, and is consistent with the branching ratio of $D^+ \to f_0(500)\pi^+$ extracted from a Dalitz analysis of the data [36].

As is observed by LHCb, the CP asymmetries in the vicinities of $f_0(500)$ and $\rho^0(770)$ have opposite signs for small and large values of $s_{\rm high}$ in the case of the B meson decay channel $B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ [15]. In view of the above, for the case of $D^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$, we will focus on the CP asymmetries of two regions, denoted Ω^+ and Ω^-, where Ω^+ (Ω^-) represents the phase space satisfying $s_{\rm high} > (<) \Sigma_{s_{\rm low}}$, and $s_{\rm high} > m_\rho^2$. The CP asymmetry difference of the aforementioned two regions is

Table 1 Input parameters used in this paper

Parameters	Input data	References
Fermi constant (in GeV^{-2})	$G_F = 1.16638 \times 10^{-5}$	[37]
Wilson coefficients	$c_1 = -0.6941,\ c_2 = 1.3777,$	[30]
	$c_3 = 0.0652,\ c_4 = -0.0627,$	
	$c_5 = 0.0206,\ c_6 = -0.1355,$	
Masses and decay widths (in GeV)	$m_{D^\pm} = 1.86961,\ \tau_{D^\pm} = 1.040 \times 10^{-12}s$	[37]
	$\mathrm{BR}(D^+ \to \pi^+\pi^-\pi^+) = 3.18 \times 10^{-3}$	
	$m_{\rho^0(770)} = 0.775,\ \Gamma_{\rho^0(770)} = 0.150,$	
	$m_{f_0(500)} = 0.5,\ \Gamma_{f_0(500)} = 0.5,$	
	$m_\pi = 0.13957,$	
	$m_u = 0.0023,\ m_d = 0.0048,$	
	$m_s = 0.095,\ m_c = 1.275,$	
Form factors	$F_1^{D\to\pi}(0) = 0.67,\ A_0^{D\to\rho}(0) = 0.64,$	[35]
	$a^\pi = 1.19,\ b^\pi = 0.36$	
	$a^\rho = 1.07,\ b^\rho = 0.54$	
	$F_0^{F\to f_0}(m_\pi^2) = 0.33,$	–
Decay constants (in GeV)	$f_\pi = 0.13041,\ f_K = 0.1562,$	[37]
	$f_\rho = 0.216,$	[38]
Wolfenstein parameters of CKM matrix	$\lambda = 0.22548^{+0.00068}_{-0.00034},\ A = 0.810^{+0.018}_{-0.024},$	[39]
	$\overline{\rho} = 0.145^{+0.013}_{-0.007},\ \overline{\eta} = 0.343^{+0.011}_{-0.012},$	

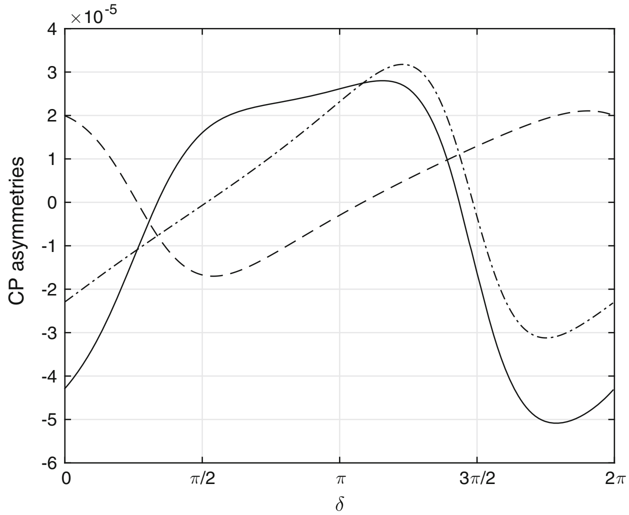

Fig. 1 The CP asymmetry difference ΔA_{CP} (*solid line*), the CP asymmetry of region Ω^- (*dashed line*) and Ω^+ (*dash-dotted line*) as a function of the strong phase δ

$$\Delta A_{\mathrm{CP}} = A_{\mathrm{CP}}^{\Omega^+} - A_{\mathrm{CP}}^{\Omega^-}. \tag{17}$$

In Fig. 1, the CP asymmetries $A_{\mathrm{CP}}^{\Omega^+}$, $A_{\mathrm{CP}}^{\Omega^-}$ and their difference ΔA_{CP} are shown as a function of the strong phase δ, where the strong phase δ is assumed to be a constant with respect to s_{low} and s_{high}.

It can be seen from Fig. 1 that ΔA_{CP} is negative (positive) when δ is around 0 (π). The magnitude of ΔA_{CP} can reach a value as large as 0.5×10^{-4} for some values of δ. Especially, Fig. 1 shows that our mechanism indicates possibilities for ΔA_{CP} being zero. This is interesting because the experimental result from the LHCb collaboration shows no CP violation in this channel. One can read off two zero points for ΔA_{CP} from Fig. 1, which are $\delta_1 = 4.50$ and $\delta_2 = 1.06$.

Figures 2 and 3 present in the phase space for $\delta = 4.50$ and $\delta = 1.06$, respectively, the relative differential decay width γ of $D^+ \to \pi^+\pi^-\pi^+$, which is defined as

$$\gamma(s_{\mathrm{low}}, s_{\mathrm{high}}) \equiv \frac{1}{\Gamma_{D^+\to\pi^+\pi^-\pi^+}} \times \frac{\mathrm{d}^2\Gamma_{D^+\to\pi^+\pi^-\pi^+}}{\mathrm{d}s_{\mathrm{low}}\mathrm{d}s_{\mathrm{high}}} = \frac{1}{256\pi^3 m_D^3 \Gamma_{D^+\to\pi^+\pi^-\pi^+}} |\mathcal{M}|^2. \tag{18}$$

For comparison, we also present the relative differential decay width in Fig. 4 for $\delta = 0$, and in Fig. 5 for the situation that only the resonance $\rho^0(770)$ is taken into account. Experimental data from LHCb shows that symmetries of event distribution around the $\rho^0(770)$ resonance are badly destroyed. The number of events around the $\rho^0(770)$ resonance for $s_{\mathrm{high}} < \Sigma_{s_{\mathrm{low}}}$ are much larger than that for $s_{\mathrm{high}} > \Sigma_{s_{\mathrm{low}}}$, as is shown in Ref. [28]. Besides, LHCb results also shows an enhancement of event distributions in the region of phase space where $\sqrt{s_{\mathrm{low}}}$ and $\sqrt{s_{\mathrm{high}}}$ are around the masses of $f_0(500)$ and $\rho^0(770)$, respectively. These behaviors are roughly the same as those shown in Fig. 2, which indicates that our mechanism is consistent with the experimental data when $\delta = 4.50$.

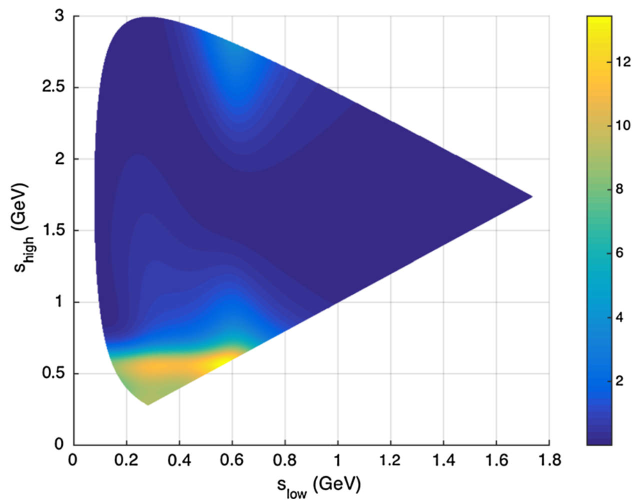

Fig. 2 Differential branching ratio (in GeV^{-4}) of $D^+ \to \pi^+\pi^-\pi^+$ when $\delta = 4.50$

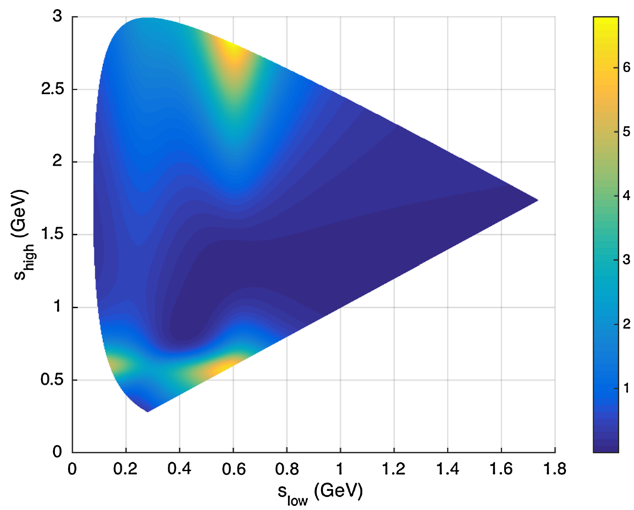

Fig. 3 Differential branching ratio (in GeV^{-4}) of $D^+ \to \pi^+\pi^-\pi^+$ when $\delta = 1.06$

4 Discussion

Rather than perturbative QCD factorization [40], QCD factorization [41], and soft collinear effective theory [42], we used a naive factorization approach in the calculation of the decay amplitudes. The reason is simply that the large part of the region in phase space that we focused on is off the mass-shell of $\rho^0(770)$ and $f_0(500)$, making the advantages of the aforementioned three kinds of factorization approaches smeared out by the off-shell effect. On the other hand, since the phase space that we are focus on is still very close to

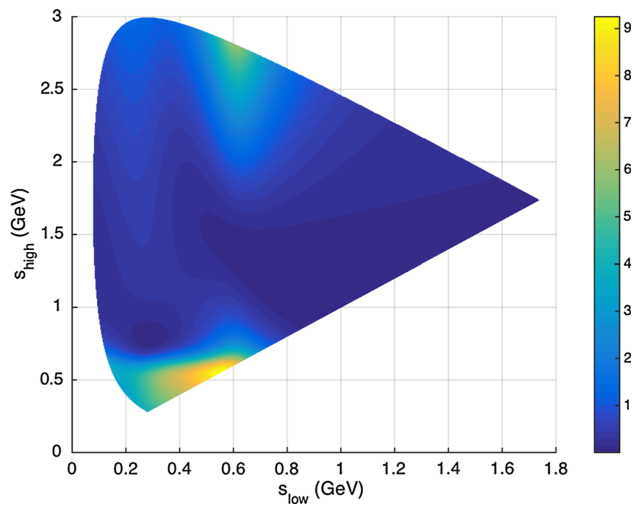

Fig. 4 Differential branching ratio (in GeV^{-4}) of $D^+ \to \pi^+\pi^-\pi^+$ when $\delta = 0$

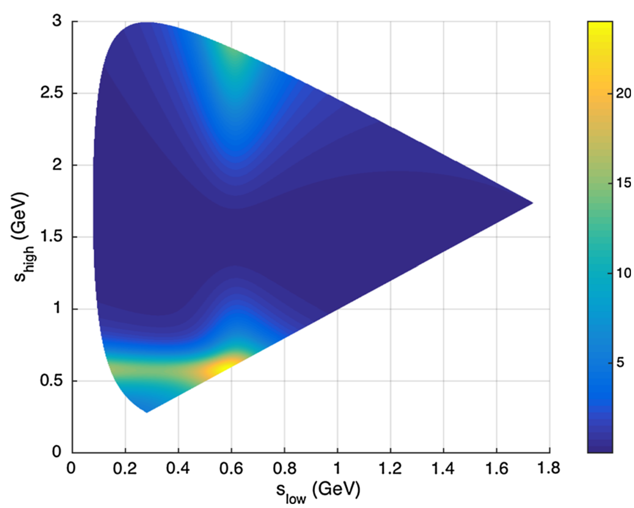

Fig. 5 Differential branching ratio (in GeV^{-4}) of $D^+ \to \pi^+\pi^-\pi^+$, where only the amplitudes corresponding to $\rho^0(770)$ are included

the mass-shell of both $\rho^0(770)$ and $f_0(500)$, a factorization approach is still reliable.

In determining the strong phase δ, we used the CP asymmetry difference of two regions of phase space instead of the differential CP asymmetry. The reason is that the use of the differential CP asymmetry as a tool to determine the strong phase δ is not an appropriate approach at all. On one hand, if we use the differential CP asymmetry, one would find that no strong phase can accommodate the data. On the other hand, if one checks the nonzero differential CP asymmetries distributed in phase space for $\delta = 4.50$, one would find that the large differential CP asymmetries always come with a very small differential decay amplitude $\mathcal{M}$, indicating a cancelation between $\mathcal{M}_\rho e^{i\delta}$ and $\mathcal{M}_{f_0}$. In this situation, the dominance of these two amplitudes is no longer valid. Consequently, in order to deduce the differential CP asymmetries in this kind of regions, we should, in principle, consider other contributions to the decay amplitude $\mathcal{M}$ in these phase space regions, which is out of the scope of this paper.

We choose the right boundary of the two regions Ω^+ and Ω^- in phase space to be m_ρ^2. Although it is not an unique choice, the boundary should not be far away from the vicinity of $\rho^0(770)$, in which case the allowed strong phase δ is not sensitive to the choice. On the other hand, either it is too large or too small than m_ρ^2; the dominance of two resonance ρ^0 and $f_0(500)$ of the total decay amplitude is no longer valid.

Other resonances can also contribute to the decay amplitude. For a resonance such as $f_0(980)$, only a small part of the total resonance lies in the region. As a result, this contribution is small compared with $\rho^0(770)$. The resonance ω can enter in the amplitude through an isospin breaking effect, which is called the ρ^0–ω mixing mechanism. This mechanism can generate large differential CP asymmetries in the vicinity of Ω. However, the width of ω is small, its contribution to regional CP violation is small. More importantly, the contribution of ω to CP asymmetry is independent of s_{high}, and hence it has no contribution to ΔA_{CP}.[1]

From Fig. 1, one can see that the CP asymmetries of the two regions Ω^+ and Ω^- are nonzero for $\delta = 4.50$, which seems to be a disadvantage of our work. However, these two CP asymmetries are small; both are 1.1×10^{-5}. In principle, these small CP asymmetries are understandable by, for example, an inclusion of an s_{low} or s_{high} dependence on δ. What is important is our division of the phase space; $f_0(500)$ and $\rho^0(770)$ enlarge the effect of CP violations caused by the interference and, consequently, can be used to determine the strong phase.

5 Conclusion

In this paper, we study the localized CP violation and differential decay width of the decay channel $D^\pm \to \pi^+\pi^-\pi^+$. We focus our attention on the phase space where the invariant masses of $\pi^+\pi^-$ are in the vicinities of $\rho^0(770)$ and $f_0(500)$. We consider a mechanism which can generate large CP asymmetries on three-body decays of B meson, that is, localized CP asymmetries caused by the interference of amplitudes corresponding to resonances with different spins. We found that by properly choosing a relative strong phase δ, the interference of amplitude corresponding to resonances $f_0(500)$ and $\rho^0(770)$ gives predictions that are consistent with the

[1] Strictly speaking, since there are two identical particles in the final state for $D^\pm \to \pi^\pm\pi^+\pi^-$, the terms of the amplitudes are doubled by $[s_{low} \leftrightarrow s_{high}]$, as is shown in Eq. 11. As a result, the contribution of the resonance ω in ΔA_{CP} cannot be canceled exactly. Besides, the interference between the amplitudes corresponding to $f_0(500)$ and ω is also small due to the smallness of ω's width.

experimental data both on CP asymmetries and differential decay widths. Our results generate no CP asymmetry differences ($\Delta A_{\mathrm{CP}} = 0$) when the strong phase $\delta = 4.50$. At the same time, the behavior of event distribution in the vicinity of $\rho^0(770)$ and $f_0(500)$ is also understandable.

From the numerical results one can see that the interference effect works well for the decay process $D^{\pm} \to \pi^{\pm}\pi^{-}\pi^{+}$. In order to give a more accurate description, more effects such as Final State Interactions (FSI) should be taken into account. In fact, a recent study shows that FSI are important in the decay channel $D^+ \to K^+\pi^-\pi^+$ [43]. However, the inclusion of FSI in our work would introduce more parameters, which will reduce the predictability.

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