Interference and binding effects in decays of possible molecular component of $X(3872)$

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Abstract

It is pointed out that the internal structure of the narrow resonance $X(3872)$ at the $D^0 \bar{D}^{*0}$ threshold can be studied in some detail by measuring the rate and the spectra in the decays $X(3872) \to D^0 \bar{D}^0 \pi^0$ and $X(3872) \to D^0 \bar{D}^0 \gamma$. In particular, if this resonance contains a dominant "molecular" component $D\bar{D}^* \pm \bar{D}D^*$, this component can be revealed and studied by a distinct pattern of interference between the underlying decays of D^{*0} and $\bar{D}^{*0}$ whose coherence is ensured by fixed (but yet unknown) C parity of the $X(3872)$.

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The recently observed [1] by the Belle Collaboration narrow resonance $X(3872)$ at $3872.0 \pm 0.6 \pm 0.5$ MeV (and confirmed by CDF [2] at $3871.4 \pm 0.7 \pm 0.4$ MeV) decaying into $\pi^+ \pi^- J/\psi$ is within 0.2 ± 0.7 MeV from the $D^0 \bar{D}^{*0}$ threshold. The extreme proximity of the resonance to the threshold naturally invites the suggestion [3,4] that its wave function may have a large component with a pair of neutral pseudoscalar and vector D ($\bar{D}$) (anti-)mesons. The spatial separation of the mesons in this component is sufficiently large for the mesons to retain their individual structure. Such configuration would clearly realize the long-standing conjecture [5,6] of existence of "molecular charmonium", i.e., of resonances, which essentially are loosely bound states of charmed hadrons.

Clearly, the approximately 1 MeV or less scale for the energy gap w between the resonance and the threshold is quite likely to result in a completely different weight in the wave function of the $X(3872)$ of the pairs of neutral and charged D mesons, since the threshold for the charged $D^+ \bar{D}^{*-}$ pairs is another 8.1 ± 0.5 MeV higher, which is "far" in the scale of w. Thus the isospin is likely to be strongly violated in the $X(3872)$ resonance, which in the case if $X(3872)$ is even under C parity, would allow the observed decay $X(3872) \to \pi^+ \pi^- J/\psi$ to be in fact occurring due to the decay $X(3872) \to \rho^0 J\psi$ [3, 4], in agreement with the very strong peaking at the maximal value of the spectrum of the invariant mass of the two pions [1]. One can trivially notice that this conjecture can be readily tested by a search for decay involving neutral pions: $X(3872) \to \pi^0 \pi^0 J/\psi$. If indeed the pions emerge from the ρ^0 resonance, the process with neutral pions should be

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absent. The decay with neutral pions is also forbidden, and the observed decay $X(3872) \to \pi^+\pi^- J/\psi$ is manifestly due to an $I = 1$ component of $X(3872)$, in the general case of $C(X) = +1$, even if the ρ^0 dominance is not confirmed. Alternatively, if the discussed resonance is a $C = -1$ state, the decay $X(3872) \to \pi^0\pi^0 J/\psi$ is allowed with the dipion being in the $I = 0$ isospin state, and the relation $\Gamma(X \to \pi^+\pi^- J/\psi) = 2\Gamma(X \to \pi^0\pi^0 J/\psi)$ should hold to a good accuracy. (Any significant presence of an $I = 2$ state of the dipion would obviously be totally exotic.)

Naturally, further study of the properties of $X(3872)$ will likely involve other possible decays of this resonance, including the decays related to the underlying transitions $D^{*0} \to D^0\pi^0$, $D^{*0} \to D^0\gamma$, and the corresponding transitions between the anti-mesons [3,4]. The main purpose of the present Letter is to point out that at the characteristic momenta of the mesons in the wave function of the molecular $D^0\bar{D}^{*0}$ component of the $X(3872)$ resonance the parameters of these decays should likely be measurably different from those of an incoherent sum of decays of free D^{*0} and $\bar{D}^{*0}$ mesons. Rather the rates and the spectra of the decays $X(3872) \to D^0\bar{D}^0\pi^0$ and $X(3872) \to D^0\bar{D}^0\gamma$ should exhibit binding effects and a significant interference between the underlying decays of vector mesons and anti-mesons. Thus an experimental study of these decays may reveal rather fine details of the structure of the $X(3872)$ resonance. In other words, the Dalitz plots of these decays would provide a "CAT scan" of the actual wave function of the mesons inside $X(3872)$.

It can be noted, that the spatial (momentum) dependence of the main part of the wave function of the mesons can be described, in a way, similar to that used for deuteron [3], and the essential unknown parameter for this part is the overall normalization, which represents the weight of the molecular component in the wave function of the $X(3872)$ resonance. Indeed, at the gap energy $w \lesssim 1$ MeV the dynamics of the $D^0\bar{D}^{*0}$ ($\bar{D}^0 D^{*0}$) meson pair is determined by momenta of order $\kappa = \sqrt{2\mu|\epsilon|} \lesssim 45$ MeV, where $\mu \approx 966$ MeV is the reduced mass of the system made of pseudoscalar and vector neutral D mesons. Thus the characteristic distances κ^{-1} are far beyond the range of the strong interaction, and the wave function at those characteristic distances is in fact given

by the Schrödinger equation for free motion. On the other hand, the value of κ may well be comparable with the momentum p of the pion emitted in the decay $X(3872) \to D^0\bar{D}^0\pi^0$ ($p_0 = 43$ MeV for a decay of a free D^{*0} meson), which would give rise to large binding and interference effects in the decay. In the case of radiative decay $X(3872) \to D^0\bar{D}^0\gamma$, the representative value of the photon momentum k is that in a free D^{*0} decay: $k_0 = 137$ MeV. Although this value looks large as compared to κ, it will be shown that the relative magnitude of the interference effect is determined by the expression $(2\kappa/k_0)\arctan(k_0/2\kappa)$ and is significant for this decay as well.

In the following discussion it is assumed for definiteness that $X(3872)$ is below the $D^0\bar{D}^{*0}$ threshold, so that $w = m + M - M(X)$ is a positive quantity, where $m = M(D^0) \approx 1864.5$ MeV and $M = M(D^{*0}) \approx 2006.7$ MeV. A generalization to the case where $X(3872)$ is just above the threshold can be done by analytical continuation. Also for definiteness it is assumed here that the mesons inside the $X(3872)$ are in the S wave, which quite plausibly is the actual situation. This obviously corresponds to $J^{PC}(X)$ equal to either 1^{++}, or 1^{+-}. If it further turns out that the quantum numbers of $X(3872)$ are different, the orbital motion of the mesons can readily be accounted for by a straightforward modification of the formulas presented below. Thus the wave function of the relative motion of the mesons is considered here as given by the standard S-wave expression

$$\psi(\vec{r}) = \xi\sqrt{\frac{\kappa}{2\pi}}\frac{e^{-\kappa r}}{r}, \tag{1}$$

where ξ^2 is the overall weight of the considered here molecular component in the $X(3872)$ resonance. For a purely molecular system $\xi^2 = 1$, while realistically one would expect $\xi^2 < 1$ thus allowing for some admixture in the wave function of $X(3872)$ of other states (e.g., $c\bar{c}$, D^+D^{*-}, etc.).

The amplitude for the decay $D^{*0} \to D^0\pi^0$ can be written as

$$A_{D^*D\pi} = g(\vec{\epsilon} \cdot \vec{p}), \tag{2}$$

where $\vec{\epsilon}$ is the polarization amplitude of the D^* and $\vec{p}$ is the momentum of the pion. The coupling constant g

is related to the width $\Gamma_\pi \equiv \Gamma(D^{*0} \to D^0\pi^0)$ as[1] $\Gamma_\pi = |g^2| p_0^3/6\pi$. The rate Γ_π can be estimated from the isotopic symmetry and the known [7] total width of D^{*+} (96 ± 22 keV) and the branching ratio $B(D^{*+} \to D^+\pi^0) = (30.7 \pm 0.5)\%$, and also taking into account the slight difference in the kinematics: $\Gamma_\pi = 43 \pm 10$ keV. In terms of the coupling g this leads to a quite reasonable estimate $|g^{-1}| = 315 \pm 36$ MeV.

For a system of a vector and a pseudoscalar mesons, with a definite C parity η the amplitude of decay into $D^0\bar{D}^0\pi^0$ is contributed by both the decay $D^{*0} \to D^0\pi^0$ and its charge-conjugate $\bar{D}^{*0} \to \bar{D}^0\pi^0$. Taking into account that $C(\pi^0) = +1$, and performing the standard transition to the center-of-mass coordinate $\vec{R}$ and the relative coordinate $\vec{r}$ the amplitude of the decay $X(3872) \to D^0\bar{D}^0\pi^0$ can be written as

$$
\begin{aligned}
&\langle D^0(\vec{q}_1)\bar{D}^0(\vec{q}_2)\pi^0(\vec{p})|H_{D^*D\pi}|X(\vec{\epsilon}, \vec{P} = 0)\rangle \\
&= (2\pi)^3\delta^{(3)}(\vec{q}_1 + \vec{q}_2 + \vec{p})g(\vec{\epsilon} \cdot \vec{p}) \\
&\quad \times [\phi(\vec{q}_2) + \eta\phi(\vec{q}_1)],
\end{aligned}
\tag{3}
$$

where $\vec{q}_1$ ($\vec{q}_2$) is the momentum of the final D ($\bar{D}$) meson, $\vec{p}$ is the pion momentum, and $\vec{P}$ is the momentum of the initial $X(3872)$ resonance, which is set to zero, corresponding to consideration in the rest frame of the $X(3872)$. (In Eq. (3) a use is made of the momentum conservation relation in this specific frame: $\vec{q}_1 + \vec{q}_2 + \vec{p} = 0$, which somewhat simplifies the formula and the subsequent discussion.) Finally, $\phi(\vec{q})$ is the wave function of the relative motion in the momentum representation:

$$
\phi(\vec{q}) = \int \psi^*(\vec{r})e^{i\vec{q}\cdot\vec{r}}\, d^3r.
\tag{4}
$$

The "free" wave function in Eq. (1) in the momentum space reads as

$$
\phi(\vec{q}) = \xi\frac{\sqrt{8\pi\kappa}}{q^2 + \kappa^2}.
\tag{5}
$$

It should be noted, that in the expression in Eq. (3) it is assumed that the final D and $\bar{D}$ mesons move as free particles, i.e., that the wave function of each is a plane wave $\exp(i\vec{q} \cdot \vec{r})$. Such assumption looks quite reasonable, since the final D mesons are produced at large distances of order κ^{-1} from each other, i.e.,

beyond the range of strong interaction. This behavior would be invalid if there were a resonance or a bound state of the pseudoscalar D mesons very close to their threshold, similar to the $X(3872)$ state at the threshold of $D^0\bar{D}^{*0}$. Existence of such resonance would certainly be a new phenomenon by itself, and would require a separate consideration. Here it is assumed that no singularity exists in the spectrum of $D\bar{D}$ pairs within at least few MeV near their threshold.

Using Eq. (3) the expression for the decay rate can be written in terms of $\phi(\vec{q})$ in the textbook form:

$$
\begin{aligned}
&d\Gamma(X \to D^0\bar{D}^0\pi^0) \\
&= |g^2|\frac{p^2}{3(2\pi)^5}|\phi(\vec{q}_2) + \eta\phi(\vec{q}_1)|^2 \\
&\quad \times \delta\left(\Delta - w - E_\pi - \frac{q_1^2}{2m} - \frac{q_2^2}{2m}\right) \\
&\quad \times \delta^{(3)}(\vec{q}_1 + \vec{q}_2 + \vec{p})\, d^3q_1\, d^3q_2\, \frac{d^3p}{2E_\pi} \\
&= |g^2|\frac{p^2}{96\pi^3}|\phi(\vec{q}_2) + \eta\phi(\vec{q}_1)|^2\, dq_1^2\, dq_2^2,
\end{aligned}
\tag{6}
$$

where $\Delta = M - m = 142.12 \pm 0.07$ MeV is the difference between the masses of the vector and pseudoscalar neutral D mesons, and $E_\pi = \sqrt{p^2 + m_\pi^2}$ is the energy of the pion. The intermediate expression with unintegrated delta-functions is convenient for discussing the limiting case of loose binding, $\kappa \to 0$, while the final one is the standard Dalitz type and thus is convenient for discussing the decay parameters in terms of the Dalitz plot. (Clearly, in the latter expression the value of p^2 is uniquely determined through the conservation laws by the values of q_1 and q_2.)

In the limit of no binding ($\kappa \to 0$) the momentum space wave function can be replaced as $|\phi(\vec{q})|^2 \to \xi^2(2\pi)^3\delta^{(3)}(\vec{q})$, and the intermediate expression splits into two noninterfering terms, corresponding to independent "free" decays $D^{*0} \to D^0\pi^0$ ($\vec{q}_2 = 0$), and $\bar{D}^{*0} \to \bar{D}^0\pi^0$ ($\vec{q}_1 = 0$), thus recovering the naive expression for the total width $\Gamma = 2\xi^2\Gamma_\pi$ (and the trivial kinematics). However, at a value of κ comparable with p_0 the spread of the momentum space wave function and the interference effects are essential.

In order to present the results of calculation of the rate from Eq. (6), we write the total rate of the

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[1] The nonrelativistic normalization is used here for the wave functions of the D mesons, but not for the pion.
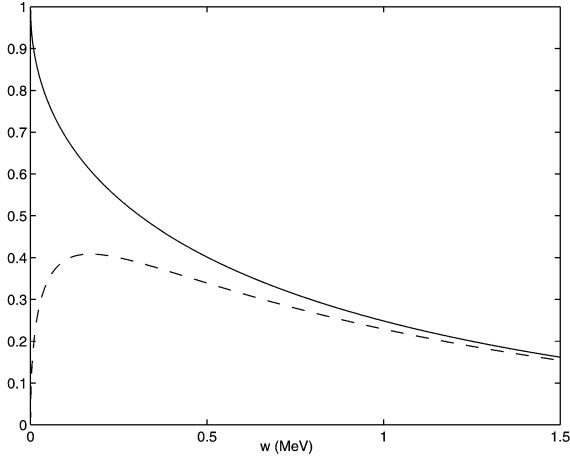
Fig. 1. The noncoherent contribution $A(w)$ (solid line) and the interference term $B(w)$ (dashed line) as defined in Eq. (7), calculated by a numerical integration in Eq. (6).

discussed decay in the form

$$\Gamma\big(X \to D^0\bar{D}^0\pi^0\big) = 2\xi^2\Gamma_\pi\big[A(w) + \eta B(w)\big], \qquad (7)$$

where $A(w)$ describes the incoherent contribution of the decays of individual D^{*0} and $\bar{D}^{*0}$, and $B(w)$ describes the effect of the interference between these two processes. The result of a numerical calculation of the terms A and B with the wave function from Eq. (5) is shown in Fig. 1. It is seen from the plot, that the discussed effects reach quite sizeable magnitude starting already from small values of the binding energy $w \sim 0.1$ MeV. In particular, the interference between the two wave functions in Eq. (6) significantly enhances the discussed decay if the C parity of $X(3872)$ is positive ($\eta = +1$) and suppresses the rate in the case of negative C parity ($\eta = -1$).

The sign of the interference term is reversed in the radiative decay $X(3872) \to D^0\bar{D}^0\gamma$ due to the negative C parity of the photon. The general expression for the decay rate, analogously to Eq. (6), has the form

$$
\begin{aligned}
&d\Gamma\big(X \to D^0\bar{D}^0\gamma\big) \\
&= \frac{\Gamma_\gamma}{k_0^3}\frac{k^2}{(2\pi)^4}\big|\phi(\vec{q}_2) - \eta\phi(\vec{q}_1)\big|^2 \\
&\quad \times \delta\left(\Delta - w - k - \frac{q_1^2}{2m} - \frac{q_2^2}{2m}\right) \\
&\quad \times \delta^{(3)}(\vec{q}_1 + \vec{q}_2 + \vec{k})\, d^3q_1\, d^3q_2\, \frac{d^3k}{2k}, \qquad (8)
\end{aligned}
$$

where $\vec{k}$ is the momentum of the photon, Γ_γ is the width of the "free" decay $D^{*0} \to D^0\gamma$ (from the available data one can estimate $\Gamma_\gamma \approx 26 \pm 7$ keV), and $k_0 \approx 137$ MeV is the photon energy in the "free" decay. Since the binding energy w is in any case very small in comparison with Δ, one can neglect the small shift in the energy k of the photon in the decay of $X(3872)$ in comparison with k_0. Furthermore, the effect of the recoil of the heavy mesons, when their momentum changes on the scale of both κ and k_0, contributes very little to the energy balance, and one can perform the integration over one of the heavy meson momenta by neglecting the kinematical constraint on it. Also making use of the normalization condition: $\int |\phi(\vec{q})|^2\, d^3q/(2\pi)^3 = \xi^2$, one readily arrives at the following expression for the total rate

$$
\begin{aligned}
&\Gamma\big(X \to D^0\bar{D}^0\gamma\big) \\
&= 2\xi^2\Gamma_\gamma\left[1 - \frac{\eta}{\xi^2}\int \phi(\vec{k}+\vec{q})\phi(\vec{q})\,\frac{d^3q}{(2\pi)^3}\right]. \qquad (9)
\end{aligned}
$$

Using the expression (5) for the momentum space wave function, one finally finds

$$\Gamma\big(X \to D^0\bar{D}^0\gamma\big) = 2\xi^2\Gamma_\gamma\left(1 - \eta\frac{2\kappa}{k_0}\arctan\frac{k_0}{2\kappa}\right). \qquad (10)$$

Clearly, the interference term described by this formula is quite substantial even at very moderate values of κ/k_0: e.g., it amounts to 0.32 already at $w = 0.1$ MeV, and to 0.71 at $w = 0.5$ MeV.

One might argue that the integral for the interference term in Eq. (9) is mainly contributed by the behavior of the wave function at momenta of order $q \sim k_0/2 \approx 70$ MeV, i.e., larger than κ. However, the corresponding distances $r \sim 2/k_0$ are still beyond the range of strong forces, and the "free" approximation (1) for the wave function should still be applicable.

As is already discussed, the free motion wave function (1) is justified only at distances beyond the range of strong interactions, and thus it fails to properly describe the dynamics at shorter distances, $r \lesssim m_\pi^{-1}$. At those distances, i.e., in the "core" of the system, the mesons strongly overlap, and the whole "molecular" picture of individual heavy mesons is likely to be inapplicable. It is not known at present, how significant the nonmolecular core part of the wave function is, and in particular what is its contribution

to the amplitudes of the discussed decays. It is clear however that the possible core contribution to these decays should lead to significantly larger than κ values of the momentum transfer to the final heavy mesons in the decays. Thus this contribution can be revealed by studying the momentum distribution of the D and $\bar{D}$ mesons produced in the decays. The core contribution should rather uniformly populate the Dalitz plot, including the events, where both heavy mesons recoil with a momentum significantly larger than κ, up to the kinematical limits of the Dalitz plot. On the contrary, the discussed here "molecular" contribution mainly populates the regions, where one of the heavy mesons (the spectator) has a recoil momentum of order κ. In other words, a study of the Dalitz plot of the discussed decays would allow to literally scan the internal structure of the $X(3872)$ resonance and, possibly, to see both the molecular and core components of its internal dynamics.

Note added

When this work was finished, there appeared the paper [8], where possible properties of the $X(3872)$ resonance are discussed in connection with a further study of the decay $X(3872) \to \pi^+\pi^- J/\psi$, including the possibility of this resonance being dominantly a molecular type state.

Acknowledgements

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